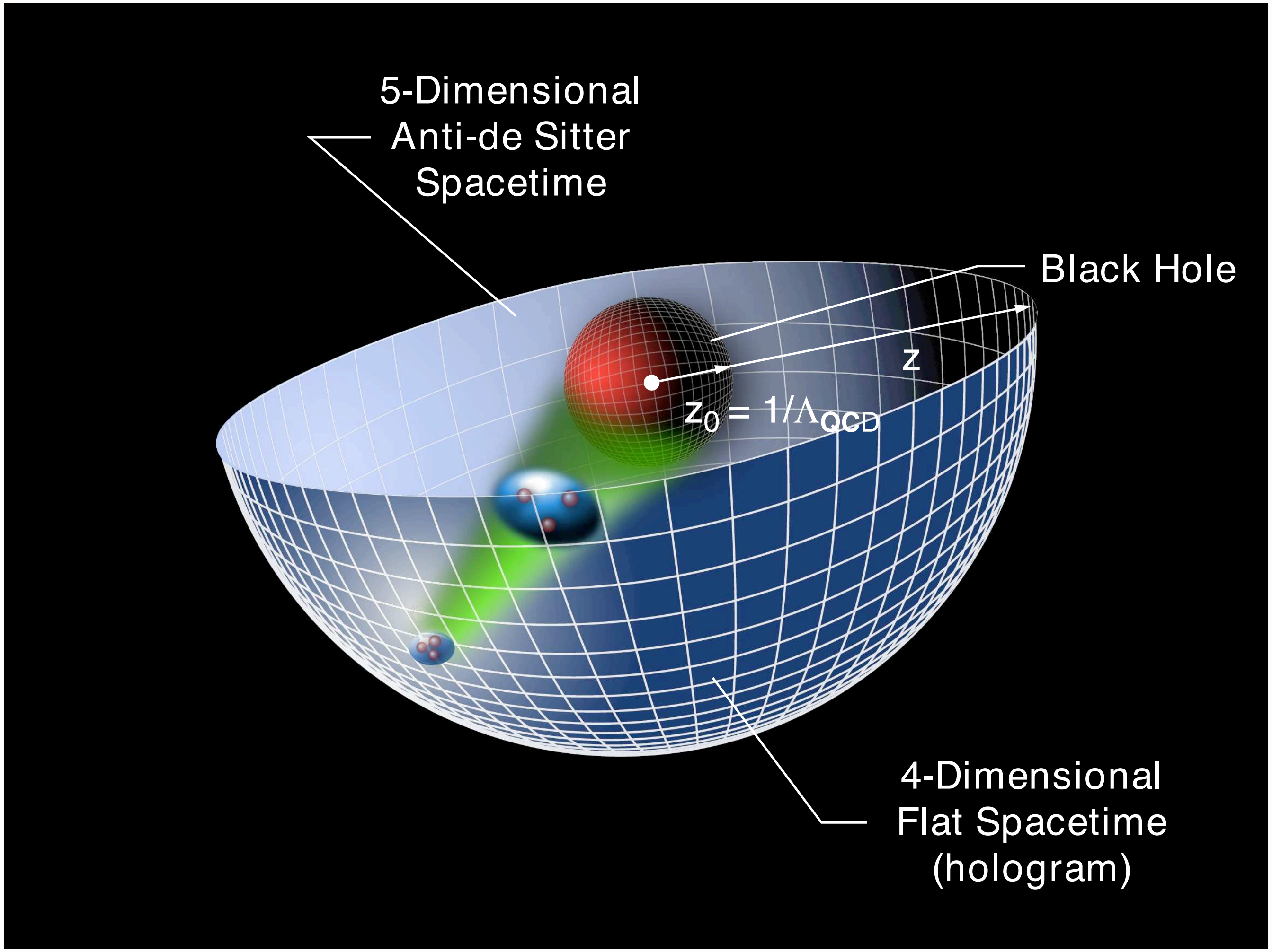
Applications of AdS/CFT to QCD



Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

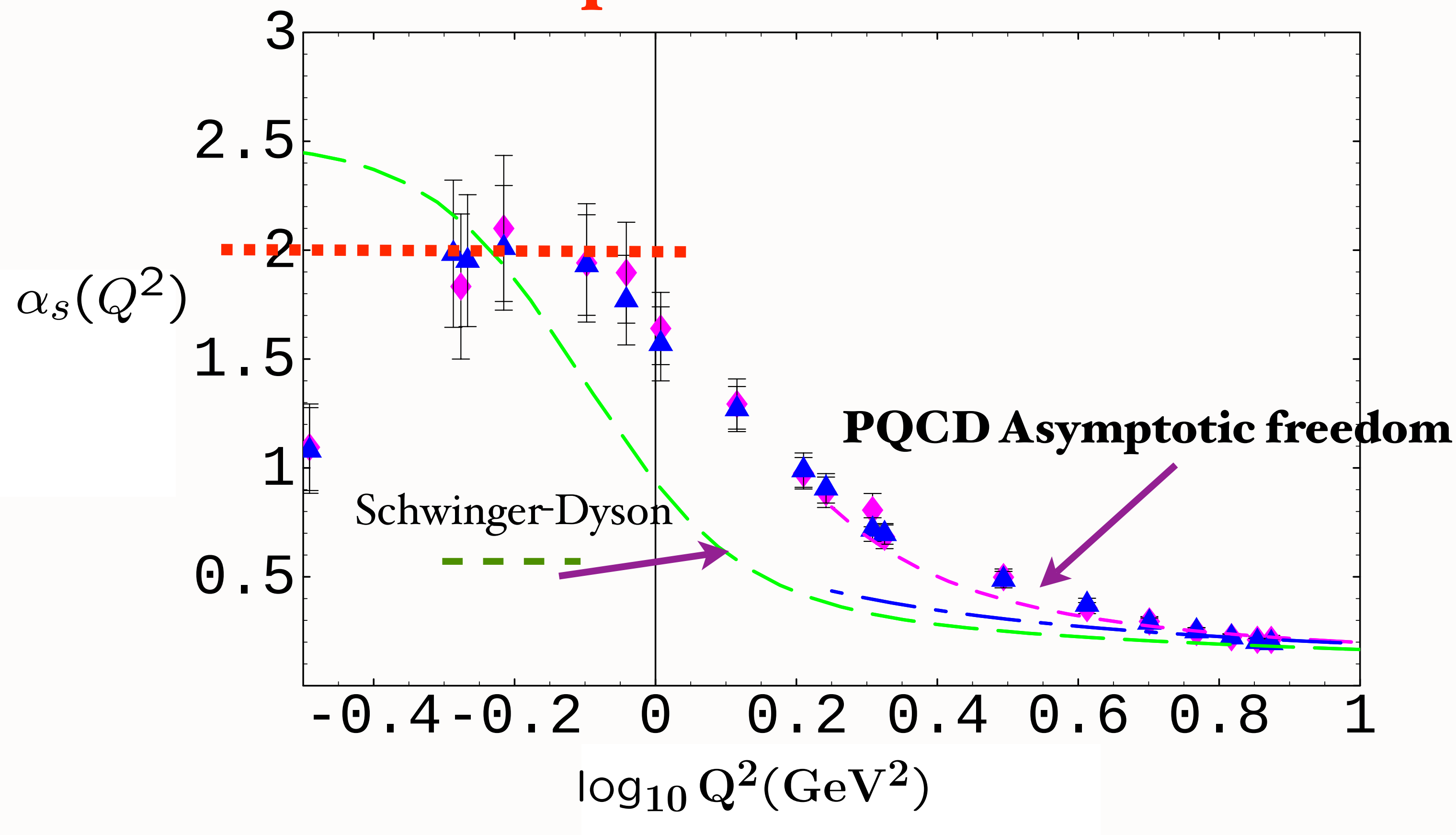
Maldacena:

Map AdS_5 X S_5 to conformal N=4 SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const at small } Q^2$
- **Use mathematical mapping of the conformal group SO(4,2) to AdS5 space**

Conformal window
Infrared fixed-point

$$\beta(Q^2) = \frac{d\alpha_s(Q^2)}{d\log Q^2} \to 0$$



Shirkov
Gribov
Dokshitser
Siminov
Maxwell
Cornwall

lattice: Furui, Nakajima (MILC)

DSE: Alkofer, Fischer, von Smekal et al.

IR Fixed-Point for QCD?

- *Dyson-Schwinger Analysis:* **QCD Coupling has IR Fixed Point**
  Alkofer, Fischer, von Smekal et al.

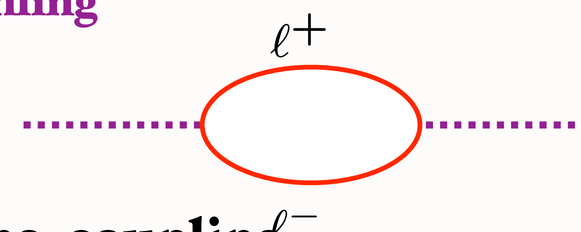
- *Evidence from Lattice Gauge Theory* Furui, Nakajima

- Define coupling from observable: **indications of IR fixed point for QCD effective charges**

- Confined or massive gluons: **Decoupling of QCD vacuum polarization at small Q^2**

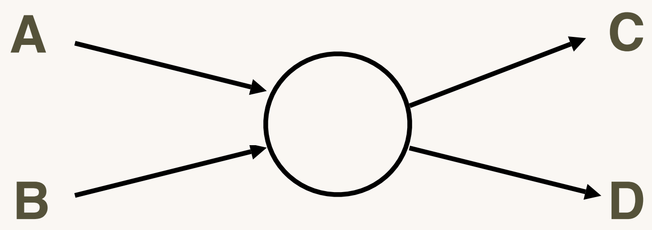
Serber-Uehling

$$\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi}\frac{Q^2}{m^2} \qquad Q^2 << 4m^2$$



- **Justifies application of AdS/CFT in strong-coupling conformal window**

Constituent Counting Rules



$$n_{tot} = n_A + n_B + n_C + n_D$$

Fixed t/s or $\cos\theta_{cm}$

$$\frac{d\sigma}{dt}(s,t) = \frac{F(\theta_{\rm cm})}{s^{[n_{\rm tot}-2]}} \qquad s = E^2_{\rm cm}$$

$$F_H(Q^2) \sim [\frac{1}{Q^2}]^{n_H - 1}$$

Farrar & sjb; Matveev, Muradyan, Tavkhelidze

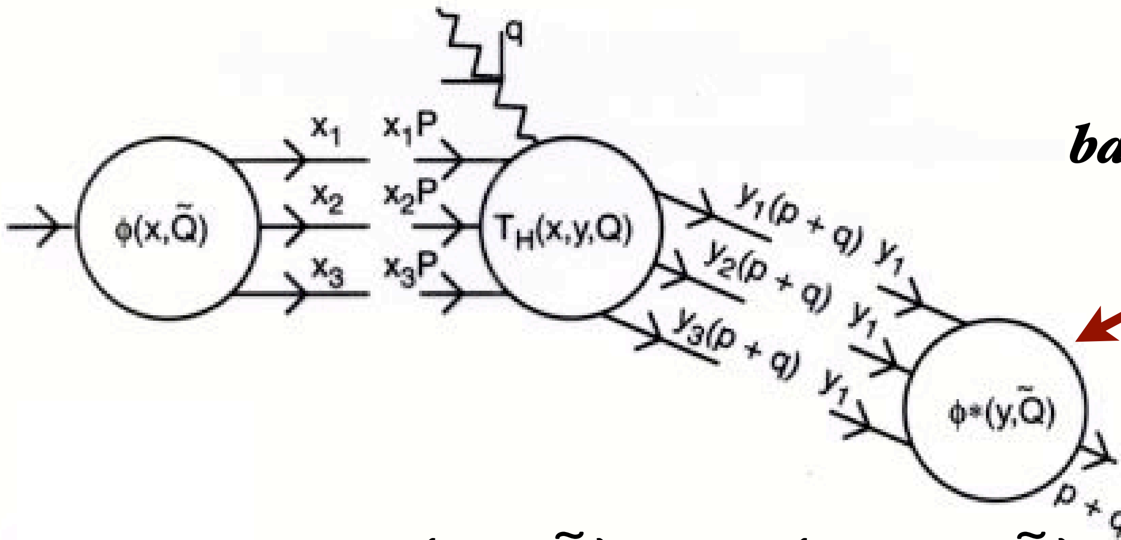
Conformal symmetry and PQCD predict leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

Characteristic scale of QCD: 300 MeV

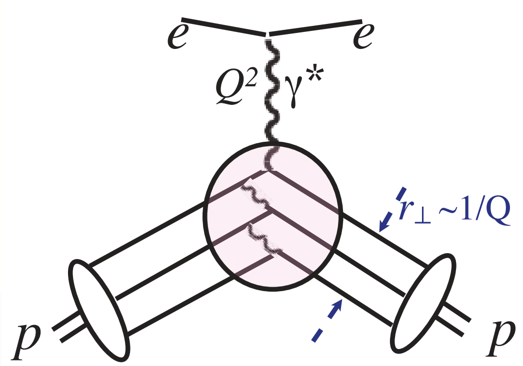
Many new J-PARC, GSI, J-Lab, Belle, Babar tests

Leading-Twist PQCD Factorization for form factors, exclusive amplitudes

Lepage, sjb



$$M = \int \prod dx_i dy_i \phi_F(x_i, \tilde{Q}) \times T_H(x_i, y_i, \tilde{Q}) \times \phi_I(y_i, \tilde{Q})$$



If $\alpha_s(\tilde{Q}^2) \simeq$ constant

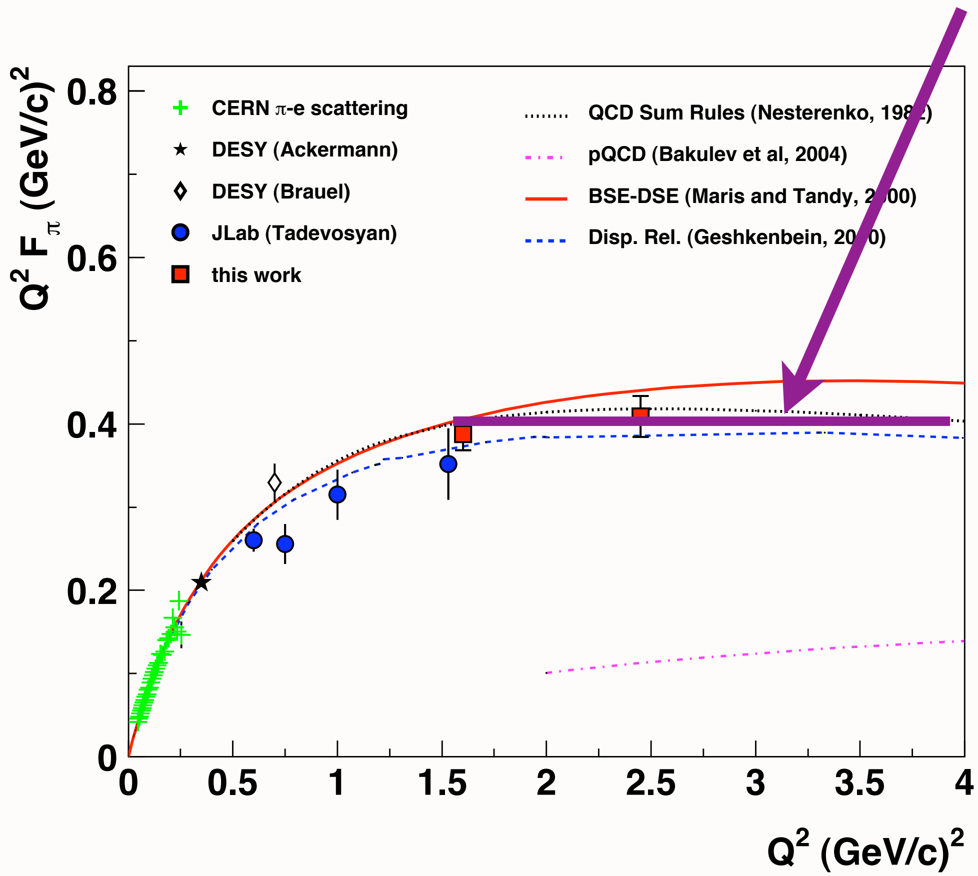
$Q^4 F_1(Q^2) \simeq$ constant

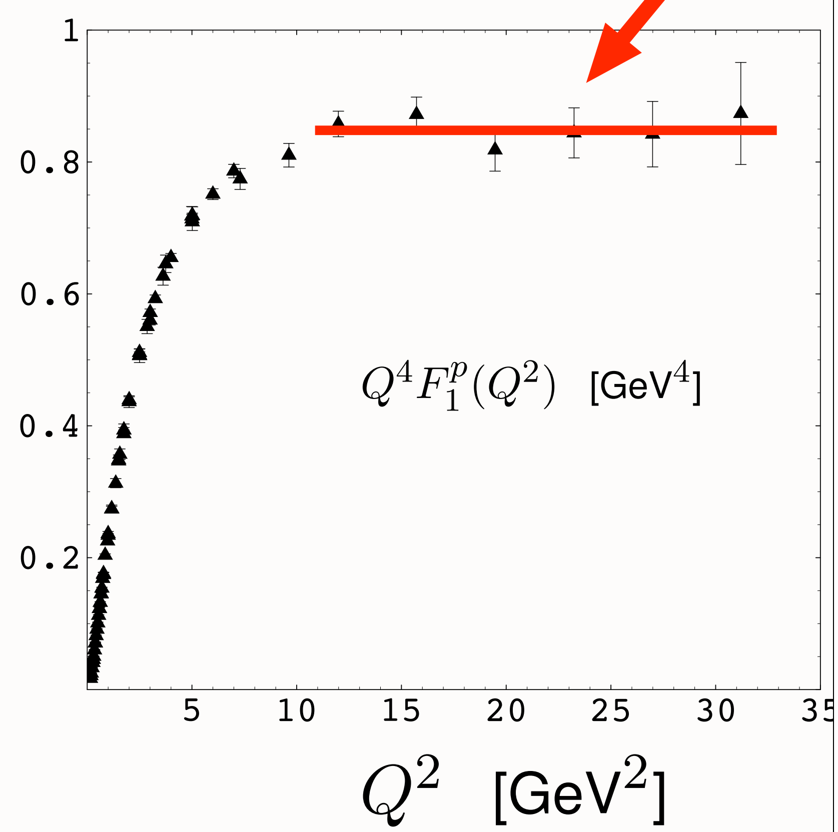
Features of Hard Exclusive Processes in PQCD

- Factorization of perturbative hard scattering subprocess amplitude and nonperturbative distribution amplitudes $M = \int T_H \times \Pi \phi_i$
- Dimensional counting rules reflect conformal invariance: $M \sim \frac{f(\theta_{CM})}{Q^{N_{tot}-4}}$
- Hadron helicity conservation: $\sum_{initial} \lambda_i^H = \sum_{final} \lambda_j^H$
- Color transparency Mueller, sjb;
- Hidden color Ji, Lepage, sjb;
- Evolution of Distribution Amplitudes Lepage, sjb; Efremov, Radyushkin

Conformal behavior: $Q^2 F_\pi(Q^2) \to$ const

$Q^4 F_1(Q^2) \to$ const





Determination of the Charged Pion Form Factor at Q2=1.60 and 2.45 (GeV/c)2.
By Fpi2 Collaboration (T. Horn *et al.*). Jul 2006. 4pp.
e-Print Archive: **nucl-ex/0607005**

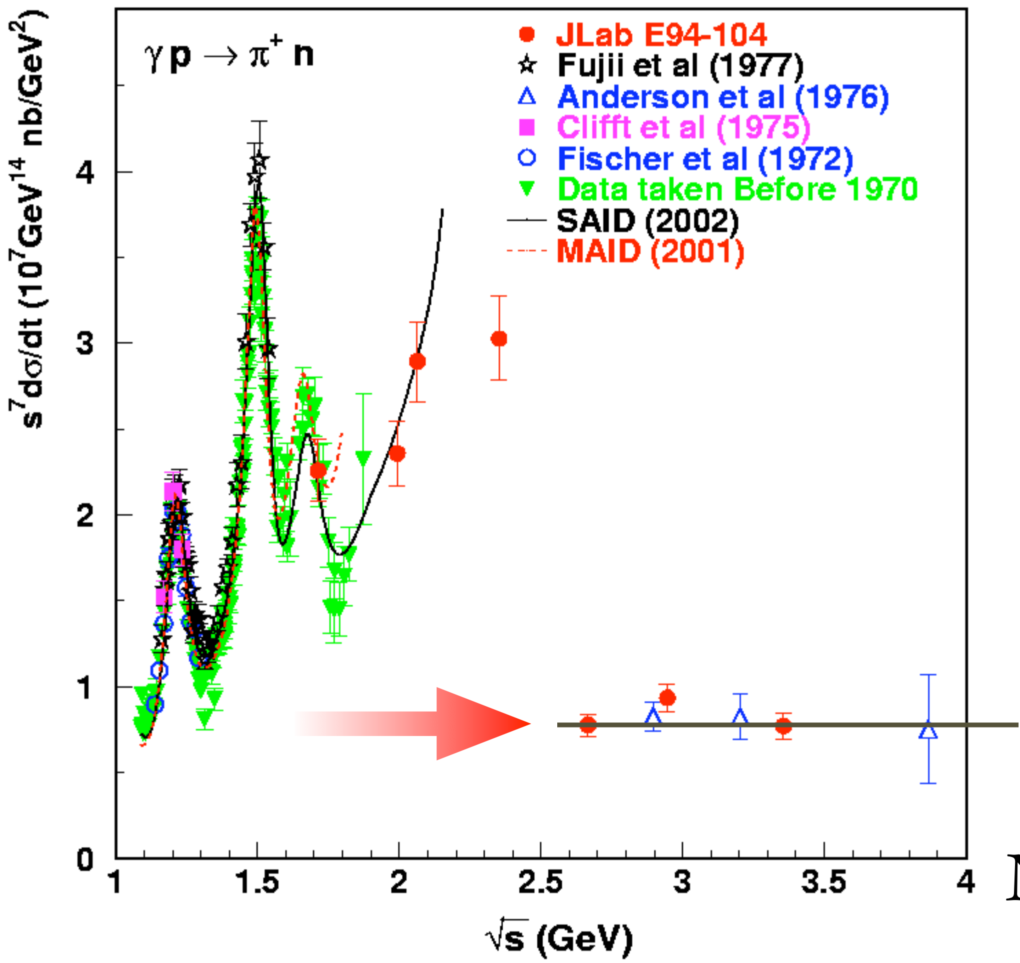
G. Huber

Generalized parton distributions from nucleon form-factor data.
M. Diehl (**DESY**) , **Th. Feldmann** (**CERN**) ,
R. Jakob, **P. Kroll** (**Wuppertal U.**) .
DESY-04-146, CERN-PH-04-154, WUB-04-08, Aug 2004. 68pp.
Published in Eur.Phys.J.C39:1-39,2005
e-Print Archive: **hep-ph/0408173**

Test of PQCD Scaling

Constituent counting rules

Farrar, sjb; Muradyan, Matveev, Taveklidze



$s^7 d\sigma/dt(\gamma p \to \pi^+ n) \sim const$

fixed θ_{CM} scaling

PQCD and AdS/CFT:

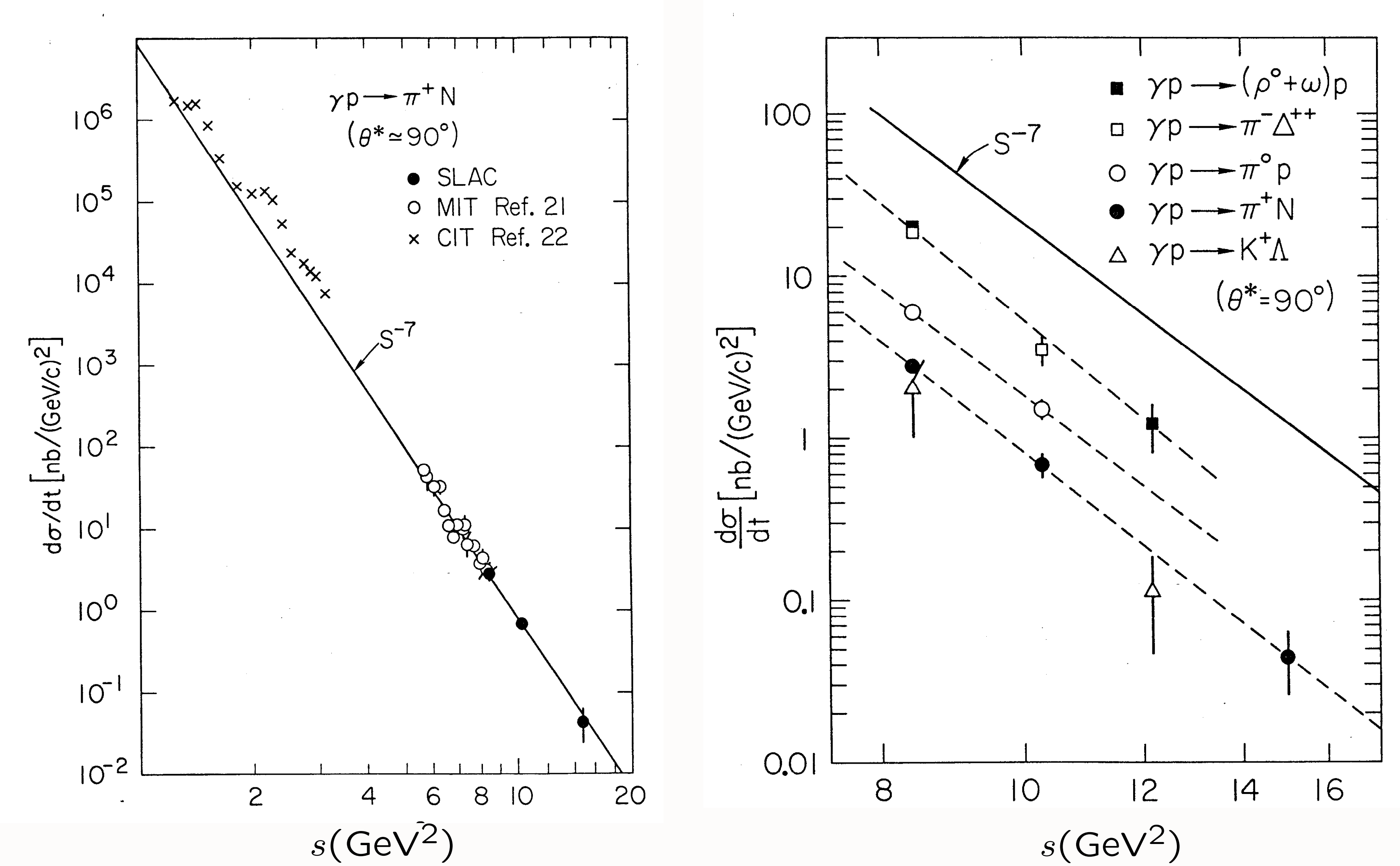
$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B \to C+D) = F_{A+B\to C+D}(\theta_{CM})$$

$$s^7\frac{d\sigma}{dt}(\gamma p \to \pi^+ n) = F(\theta_{CM})$$

$$n_{tot} = 1+3+2+3 = 9$$

No sign of running coupling

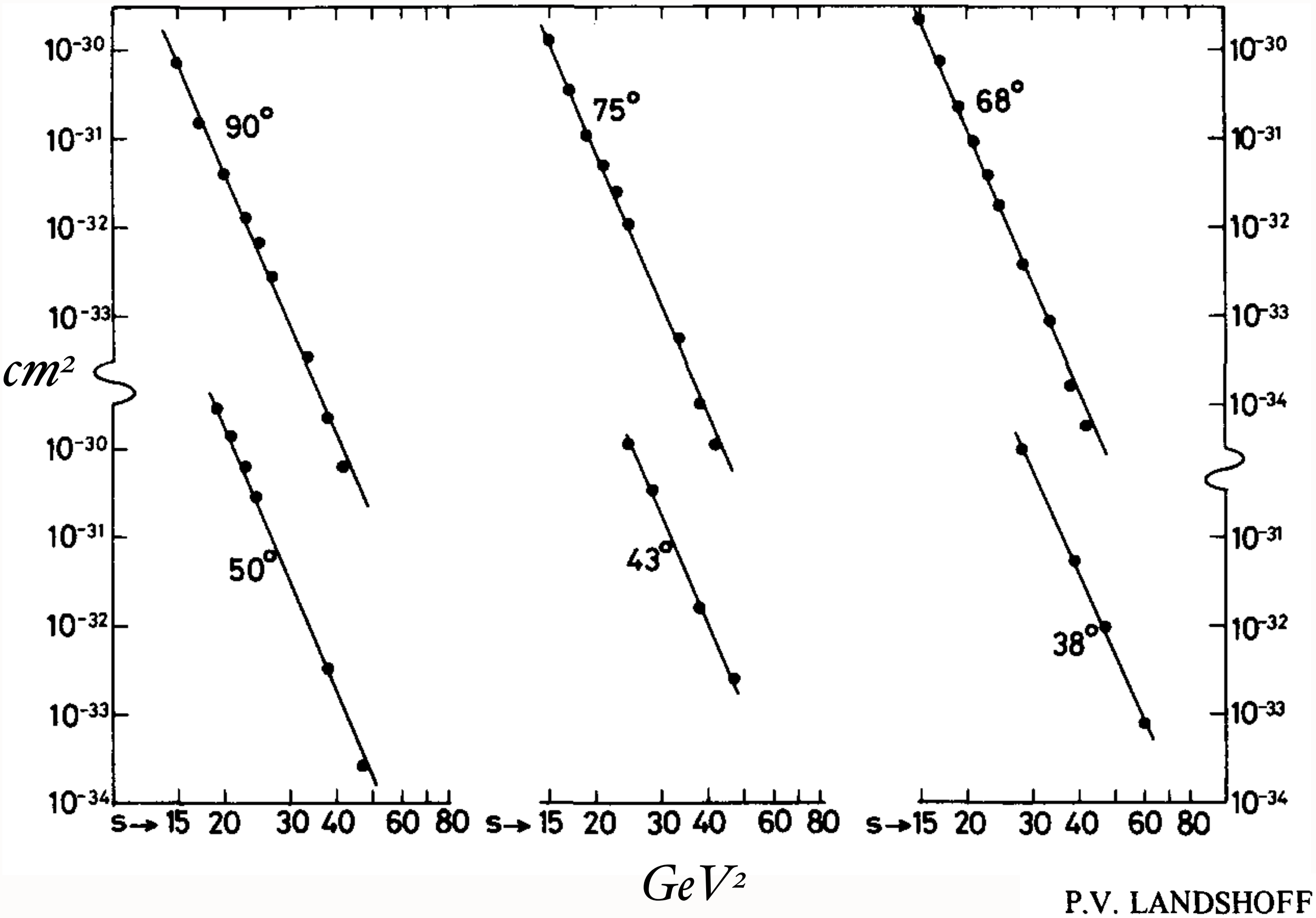
Conformal invariance



Conformal Invariance:

$$\frac{d\sigma}{dt}(\gamma p \to MB) = \frac{F(\theta_{cm})}{s^7}$$

Quark-Counting : $\frac{d\sigma}{dt}(pp \to pp) = \frac{F(\theta_{CM})}{s^{10}}$ $n = 4 \times 3 - 2 = 10$



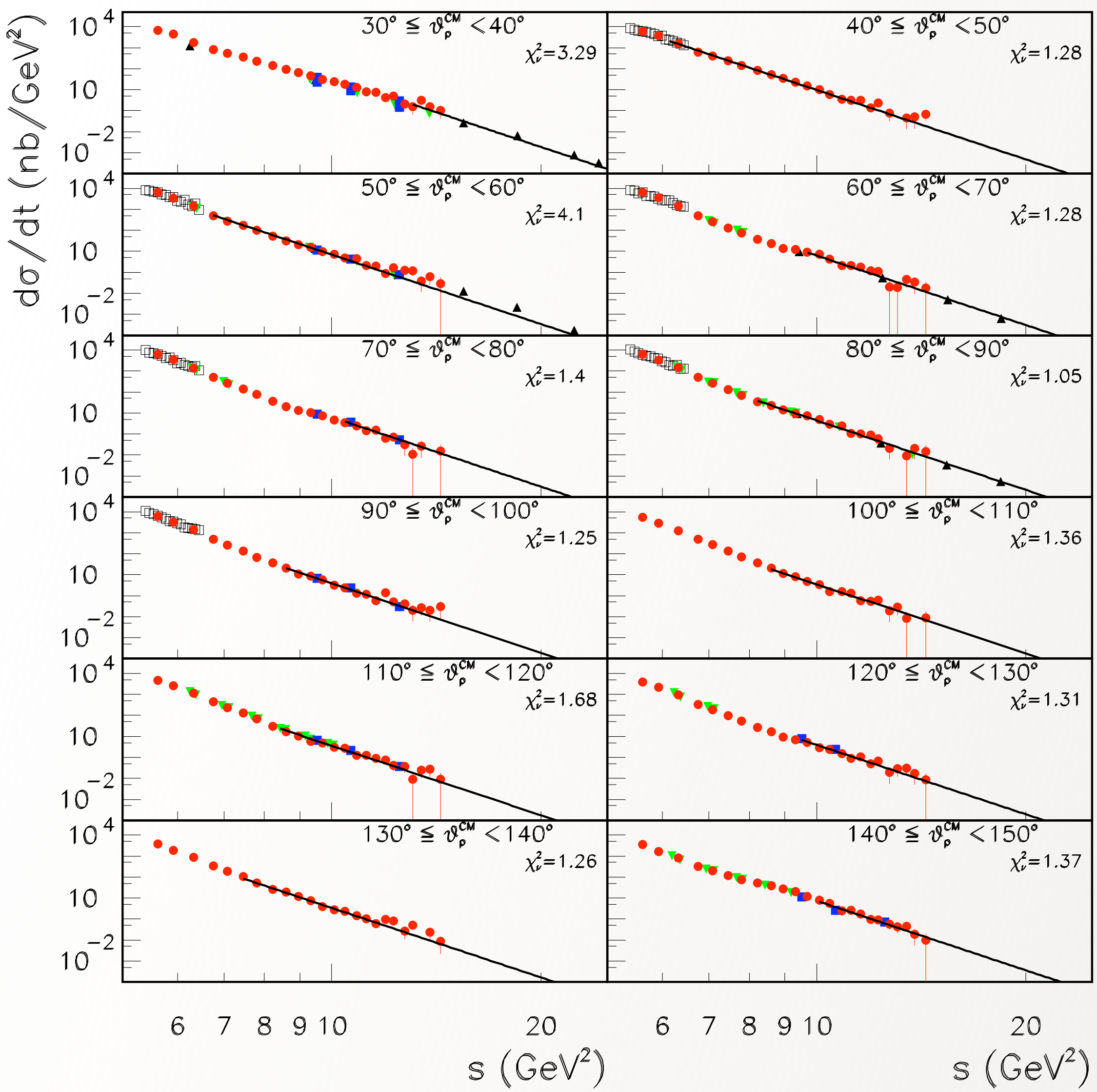
P.V. LANDSHOFF and J.C. POLKINGHORNE

Best Fit

$n = 9.7 \pm 0.5$

Reflects underlying conformal scale-free interactions

Deuteron Photodisintegration



J-Lab

PQCD and AdS/CFT:

$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B\to C+D) = F_{A+B\to C+D}(\theta_{CM})$$

$$s^{11}\frac{d\sigma}{dt}(\gamma d\to np) = F(\theta_{CM})$$

$$n_{tot}-2 = (1+6+3+3) - 2 = 11$$

Reflects conformal invariance

Why do dimensional counting rules work so well?

- **PQCD predicts log corrections from powers of α_s, logs, pinch contributions** Lepage, sjb; Efremov, Radyushkin; Landshoff; Mueller, Duncan

- **DSE: QCD coupling (mom scheme) has IR Fixed point** Alkofer, Fischer, von Smekal et al.

- **Lattice results show similar flat behavior** Furui, Nakajima

- **PQCD exclusive amplitudes dominated by integration regime where α_s is large and flat**

Goal:

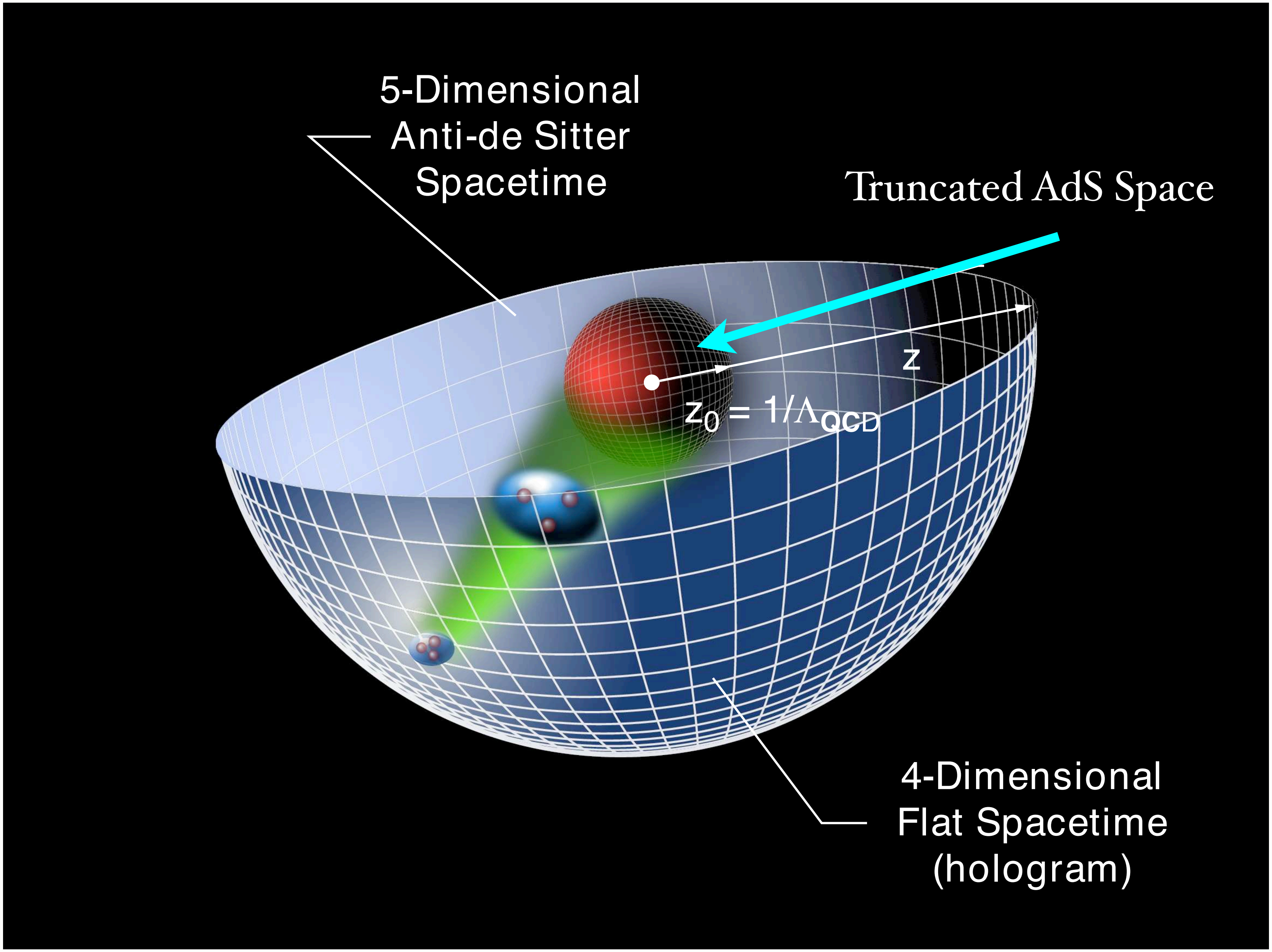
- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrodinger Equation for Atomic Physics**
- *AdS/QCD Holographic Model*

Conformal Theories are invariant under the Poincare and conformal transformations with

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5



5-Dimensional
Anti-de Sitter
Spacetime
Truncated AdS Space
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)

- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances; counting rules for form factors and hard exclusive processes; non-perturbative derivation

- Goal: Use AdS/CFT to provide an approximate model of hadron structure with confinement at large distances, conformal behavior at short distances

- de Teramond, sjb: AdS/QCD Holographic Model: Initial "semi-classical" approximation to QCD. Predict light-quark hadron spectroscopy, form factors.

- Karch, Katz, Son, Stephanov: Linear Confinement

- Mapping of AdS amplitudes to 3+ 1 Light-Front equations, wavefunctions

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing H^{LF}_{QCD} ; variational methods

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \rightarrow \lambda x^\mu, \; z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.

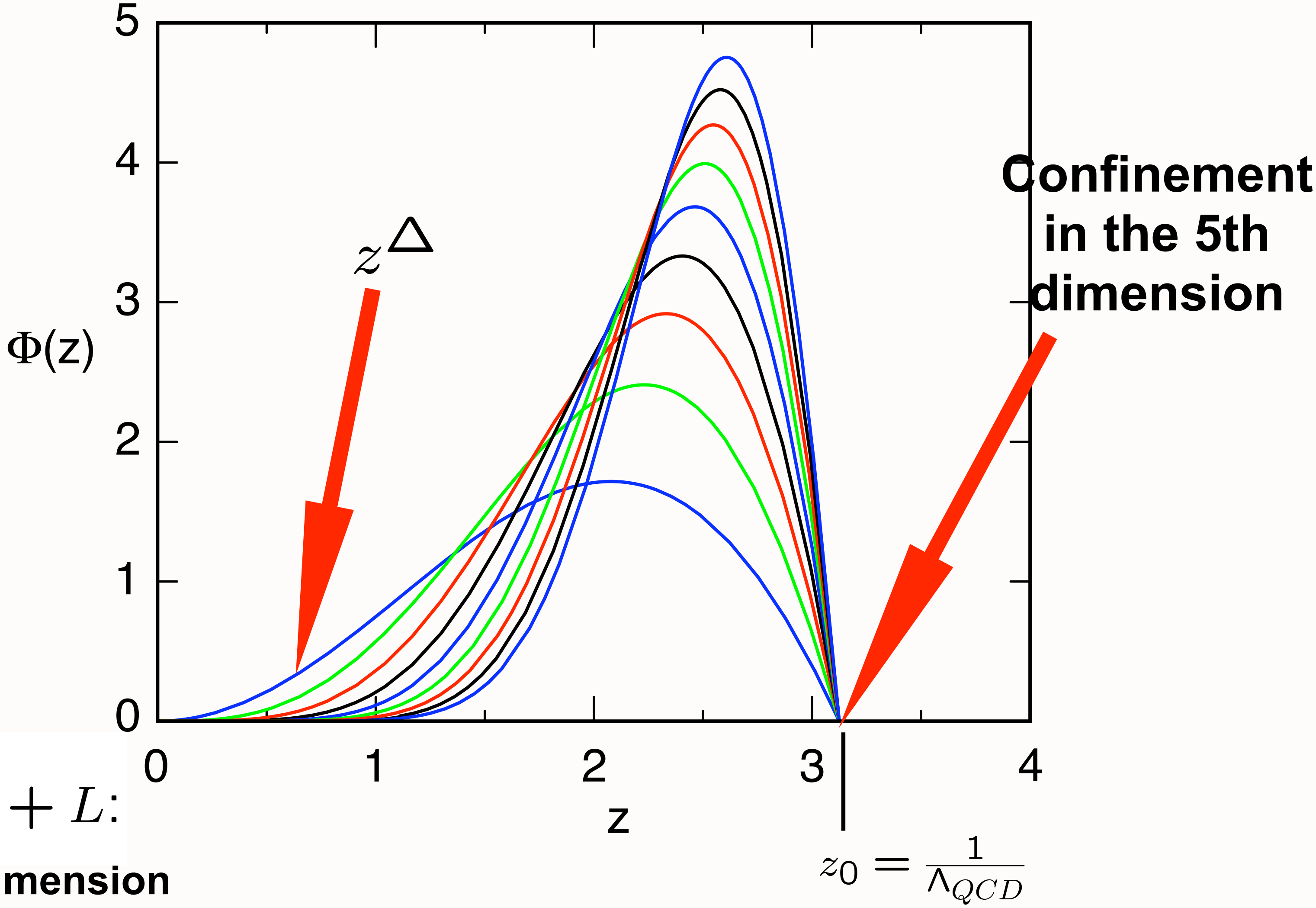
Guy de Teramond
SJB

AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5
- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2 \qquad z \to \lambda z$
- Holographic model: Confinement at large distances and conformal symmetry in interior $0 < z < z_0$
- Match solutions at small z to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \to 0$
- Truncated space simulates "bag" boundary conditions

$$\psi(z_0) = 0 \qquad z_0 = \frac{1}{\Lambda_{QCD}}$$

Identify hadron by its interpolating operator at z -- > 0



$\Delta = 3 + L$: Twist dimension of baryon

de Teramond, sjb

$$\Phi(z) = z^{3/2}\phi(z)$$

AdS Schrodinger Equation for bound state of two scalar constituents

$$[-\frac{d^2}{dz^2} + V(z)]\phi(z) = M^2\phi(z)$$

Truncated space

$$V(z) = -\frac{1-4L^2}{4z^2}$$

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$

Alternative: Harmonic oscillator confinement

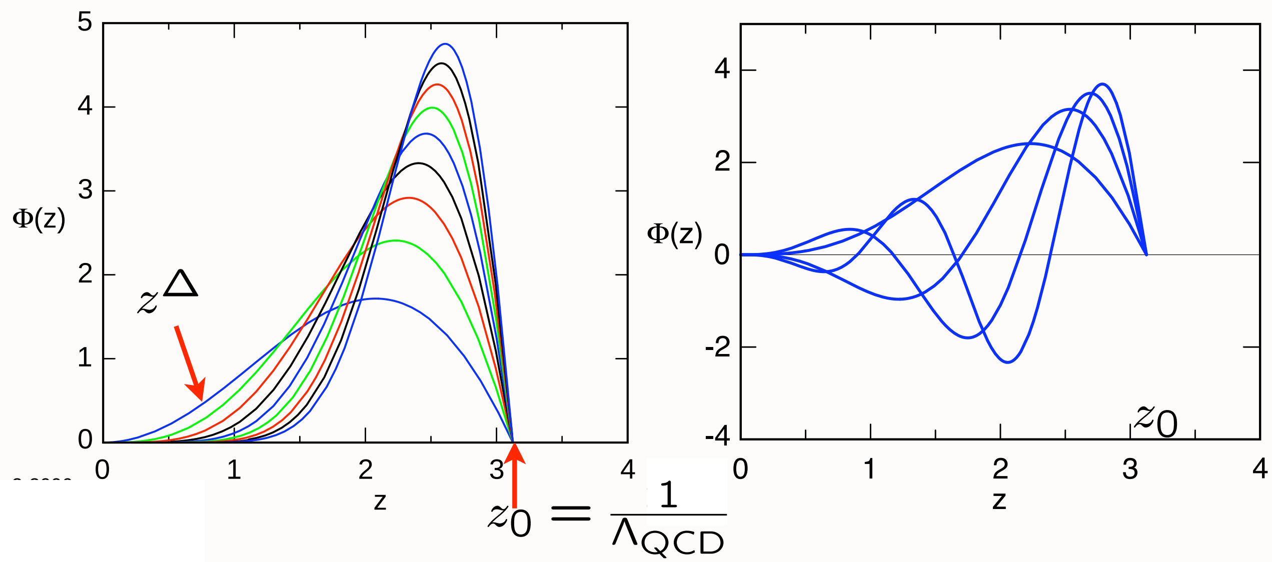
$$V(z) = -\frac{1-4L^2}{4z^2} + \kappa^4 z^2$$

Karch, et al.

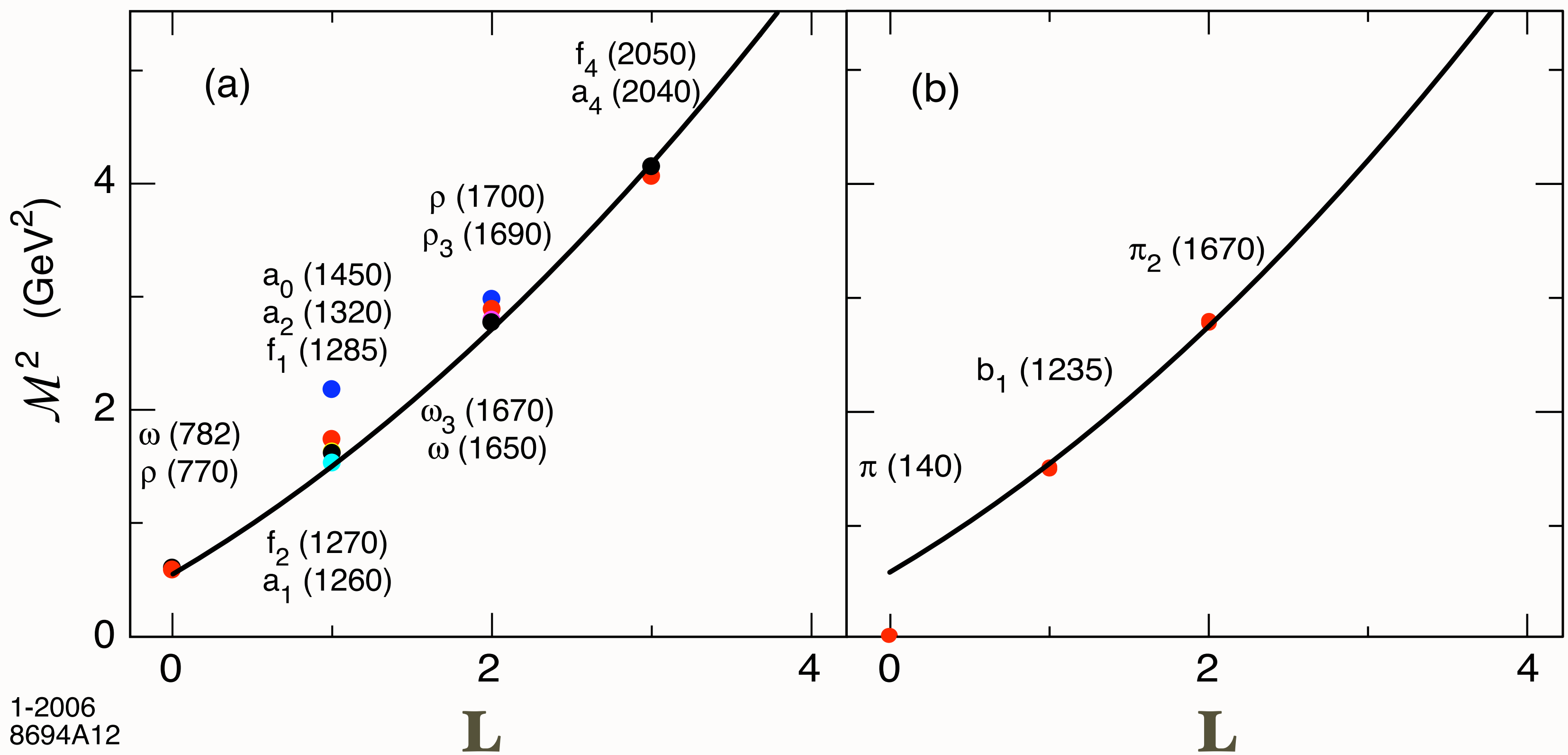
Derived from variation of Action in AdS5

Match fall-off at small z to conformal twist dimension at short distances

- Pseudoscalar mesons: $\mathcal{O}_{3+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge).
- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$



Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.



Light meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Guy de Teramond
SJB

Baryon Spectrum

- Baryon: twist-three, dimension $\frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

Wave Equation: $\boxed{\left[z^2\, \partial_z^2 - 3z\, \partial_z + z^2 \mathcal{M}^2 - \mathcal{L}_\pm^2 + 4\right] f_\pm(z) = 0}$

with $\mathcal{L}_+ = L + 1$, $\mathcal{L}_- = L + 2$, and solution

$$\Psi(x, z) = C e^{-iP\cdot x} z^2 \Big[J_{1+L}(z\mathcal{M})\, u_+(P) + J_{2+L}(z\mathcal{M})\, u_-(P) \Big].$$

- 4-d mass spectrum $\Psi(x, z_o)^\pm = 0 \implies$ parallel Regge trajectories for baryons !

$$\mathcal{M}^+_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}, \qquad \mathcal{M}^-_{\alpha,k} = \beta_{\alpha+1,k} \Lambda_{QCD}.$$

- Ratio of eigenvalues determined by the ratio of zeros of Bessel functions !

Prediction from AdS/QCD

Only one parameter!

Entire light quark baryon spectrum

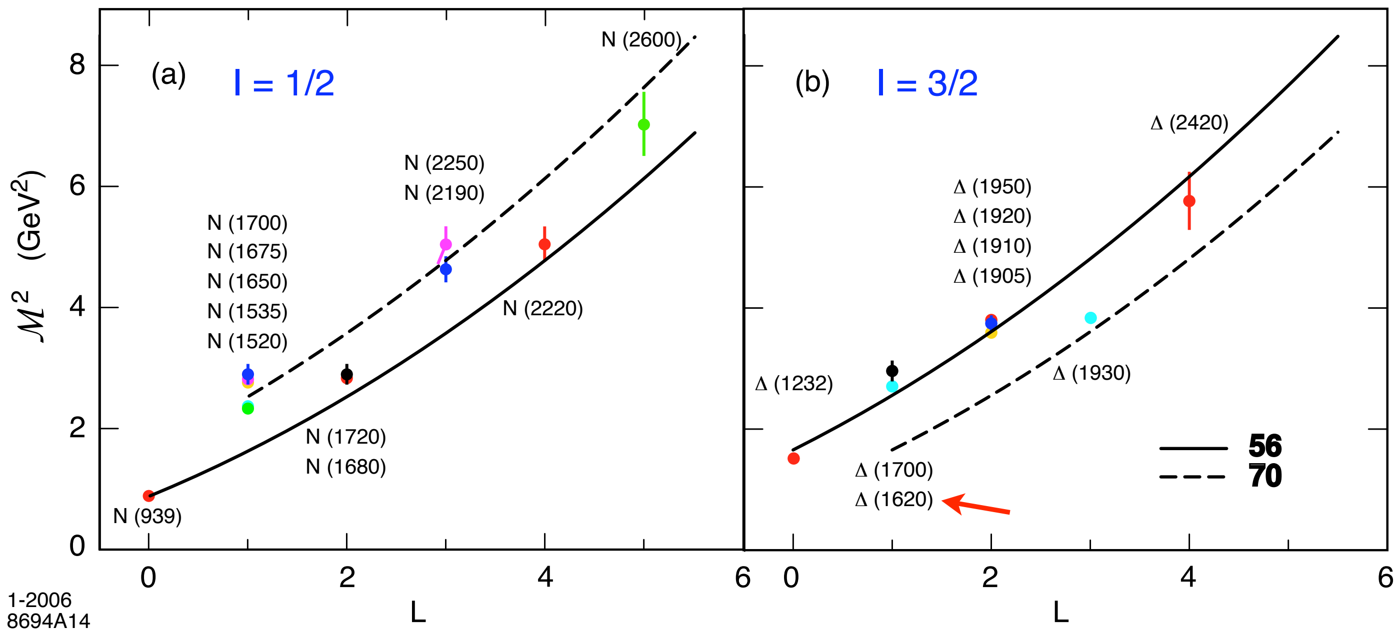


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

Guy de Teramond
SJB

- $SU(6)$ multiplet structure for N and Δ orbital states, including internal spin S and L.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

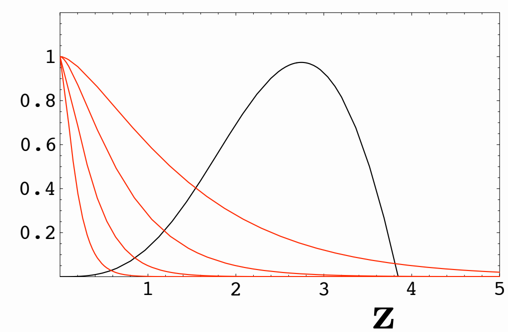
Hadron Form Factors from AdS/CFT

- Propagation of external perturbation suppressed inside AdS. $J(Q,z) = zQK_1(zQ)$

- At large Q^2 the important integration region is $z \sim 1/Q$.

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

$\mathbf{J}(\mathbf{Q},\mathbf{z}), \ \mathbf{\Phi}(\mathbf{z})$



Polchinski, Strassler
de Teramond, sjb

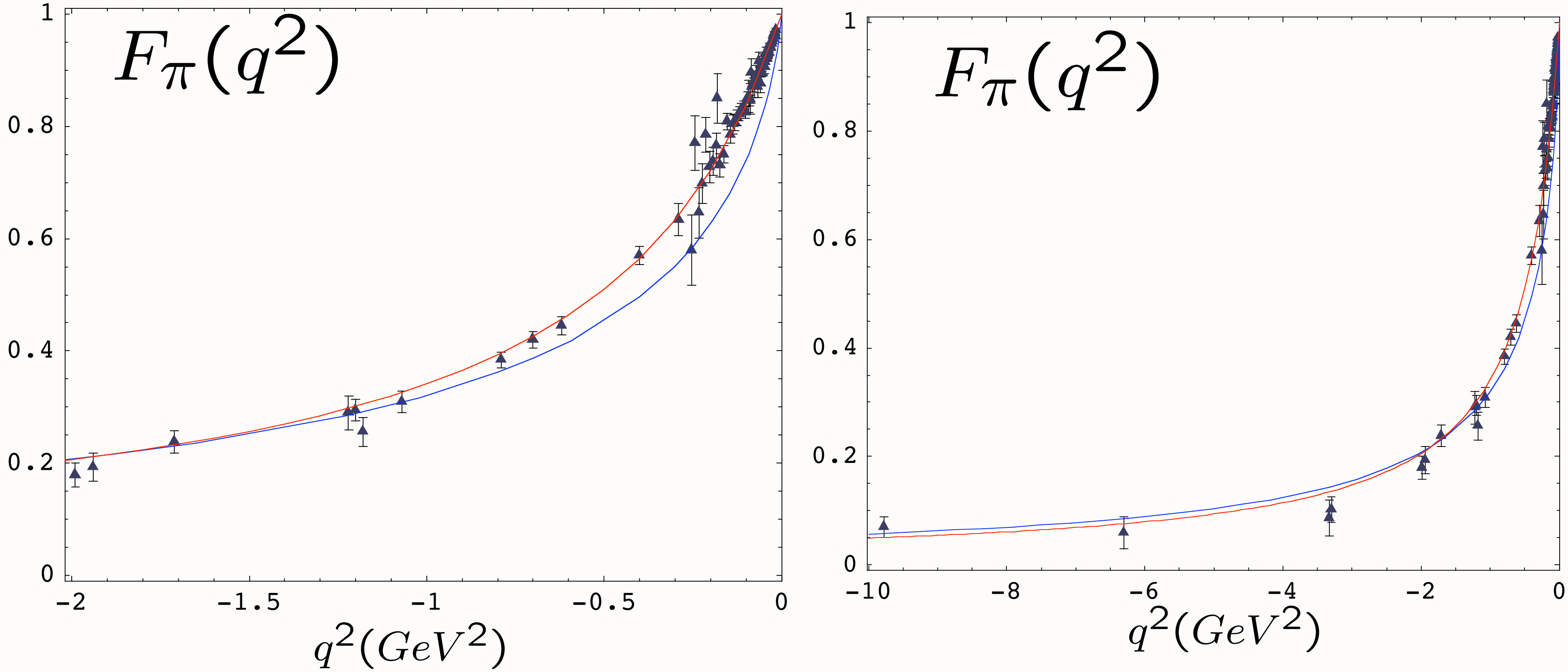
- Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules: General result from AdS/CFT

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^n \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Spacelike pion form factor from AdS/CFT



Data Compilation from Baldini, Kloe and Volmer

Harmonic Oscillator Confinement

Truncated Space Confinement

One parameter - set by pion decay constant

G. de Teramond, sjb

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} \Phi_{P'}(z) J(Q,z) \Phi_P(z).$$

$$\Phi(z) = \frac{\sqrt{2}\kappa}{R^{3/2}} z^2 e^{-\kappa^2 z^2/2}.$$

$$J(Q,z) = zQK_1(zQ).$$

$$F(Q^2) = 1 + \frac{Q^2}{4\kappa^2} \exp\left(\frac{Q^2}{4\kappa^2}\right) Ei\left(-\frac{Q^2}{4\kappa^2}\right)$$

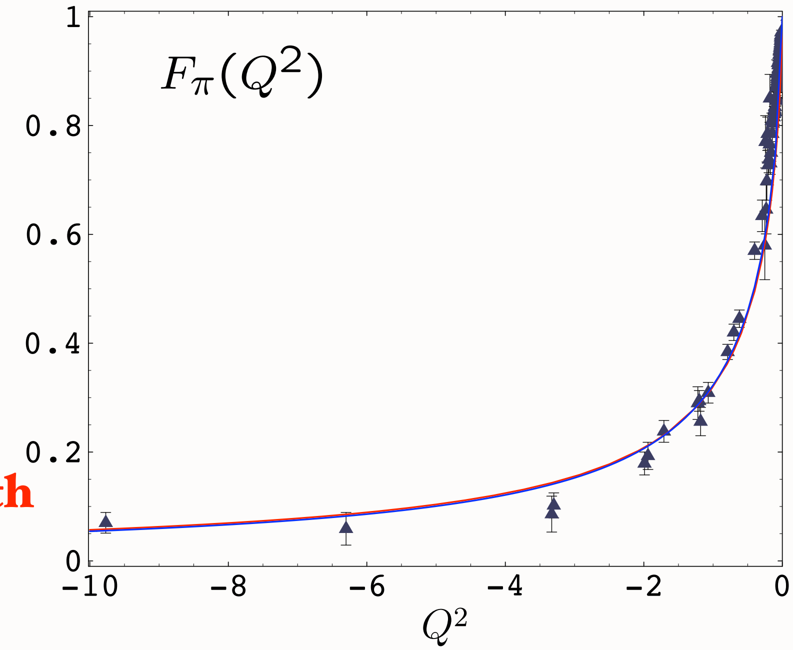
$$Ei(-x) = \int_\infty^x e^{-t} \frac{dt}{t}.$$

Space-like Pion Form Factor

$\kappa = 0.4$ GeV

$\Lambda_{QCD} = 0.2$ GeV.

Identical Results for both confinement models

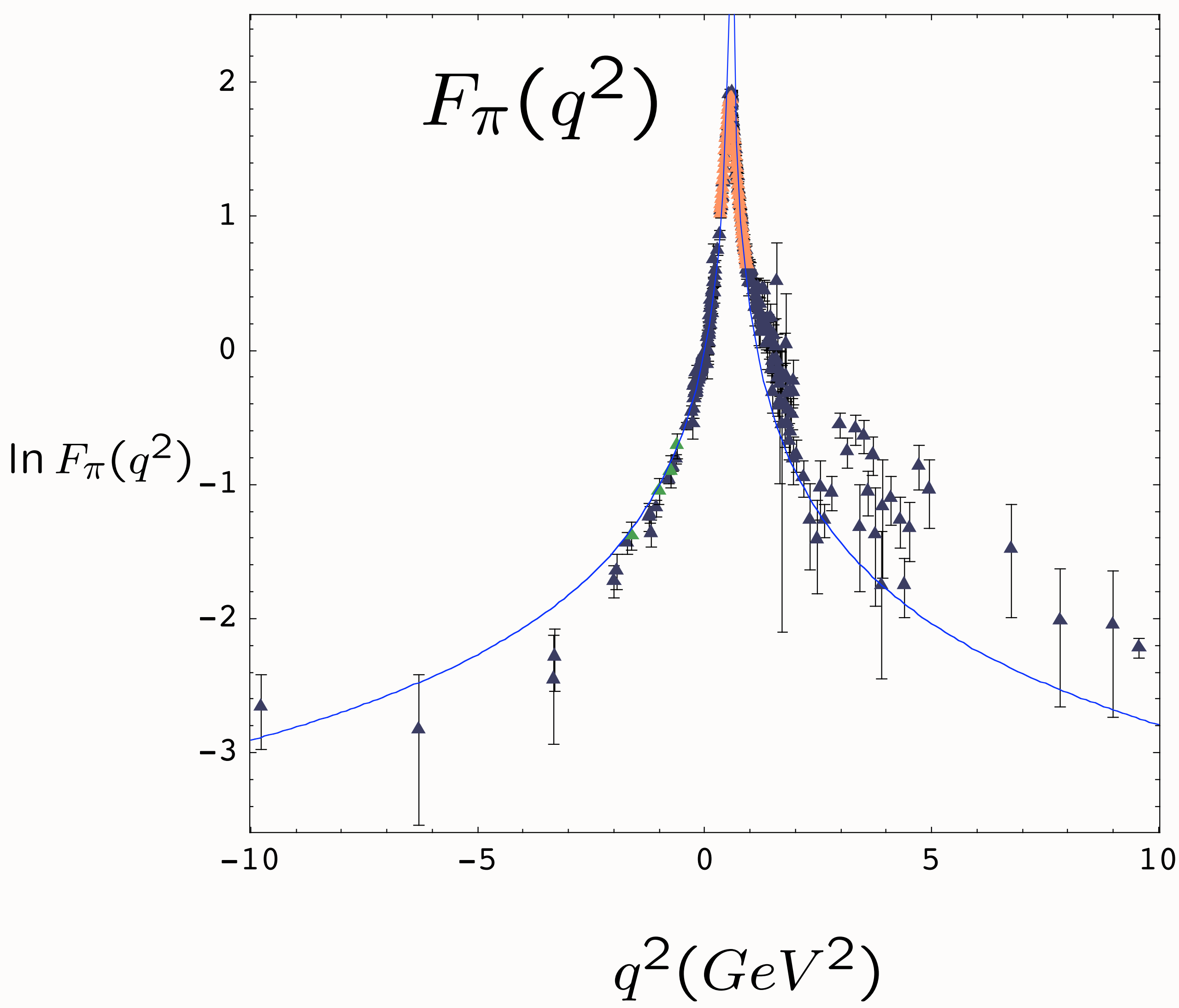


$$F(Q^2) \to \frac{4\kappa^2}{Q^2}$$

$$\kappa = 2\Lambda_{QCD}$$

Spacelike and Timelike Pion form factor from AdS/CFT

G. de Teramond, sjb



Harmonic Oscillator Confinement scale set by pion decay constant

$\kappa = 0.38$ GeV

Baryon Form Factors

- Coupling of the extended AdS mode with an external gauge field $A^\mu(x,z)$

$$ig_5 \int d^4x\, dz\, \sqrt{g}\, A_\mu(x,z)\, \overline{\Psi}(x,z)\gamma^\mu\Psi(x,z),$$

where

$$\Psi(x,z) = e^{-iP\cdot x}\left[\psi_+(z)u_+(P) + \psi_-(z)u_-(P)\right],$$

$$\psi_+(z) = Cz^2 J_1(zM), \qquad \psi_-(z) = Cz^2 J_2(zM),$$

and

$$u(P)_\pm = \frac{1 \pm \gamma_5}{2}\, u(P).$$

$$\psi_+(z) \equiv \psi^\uparrow(z), \quad \psi_-(z) \equiv \psi^\downarrow(z),$$

the LC $\pm$ spin projection along $\hat{z}$.

- Constant C determined by charge normalization:

$$C = \frac{\sqrt{2}\Lambda_{\rm QCD}}{R^{3/2}\left[-J_0(\beta_{1,1})J_2(\beta_{1,1})\right]^{1/2}}.$$

Nucleon Form Factors

- Consider the spin non-flip form factors in the infinite wall approximation

$$F_+(Q^2) = g_+ R^3 \int \frac{dz}{z^3}\, J(Q,z)\, |\psi_+(z)|^2,$$
$$F_-(Q^2) = g_- R^3 \int \frac{dz}{z^3}\, J(Q,z)\, |\psi_-(z)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(z)$ and $\psi_-(z)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = R^3 \int \frac{dz}{z^3}\, J(Q,z) |\psi_+(z)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} R^3 \int \frac{dz}{z^3}\, J(Q,z) \left[|\psi_+(z)|^2 - |\psi_-(z)|^2\right],$$

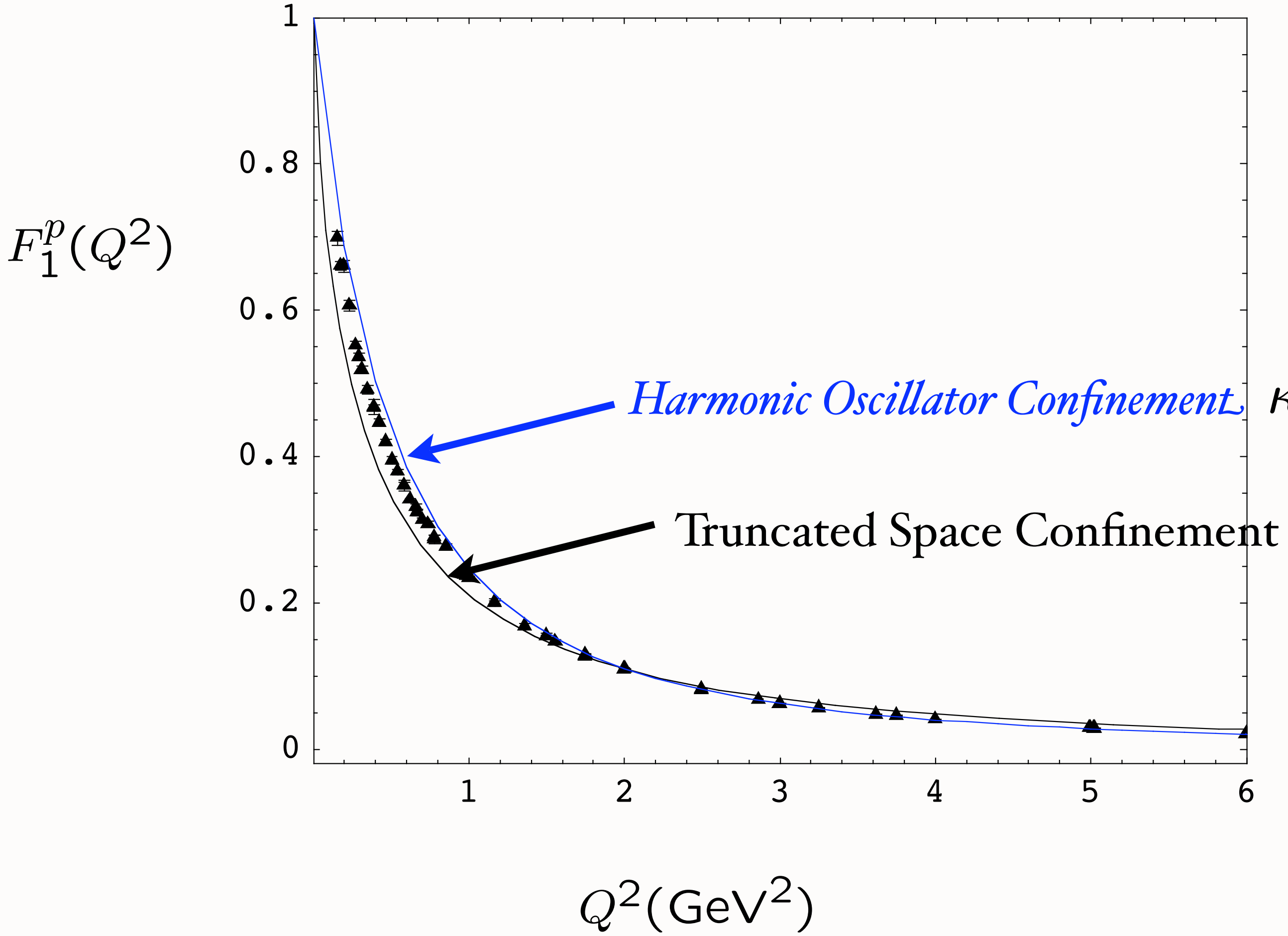
where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Large Q power scaling: $F_1(Q^2) \to \left[1/Q^2\right]^2$.

G. de Teramond, sjb

G. de Teramond, sjb

Preliminary



$\kappa = 0.424$ GeV

$\Lambda = 0.2$ GeV

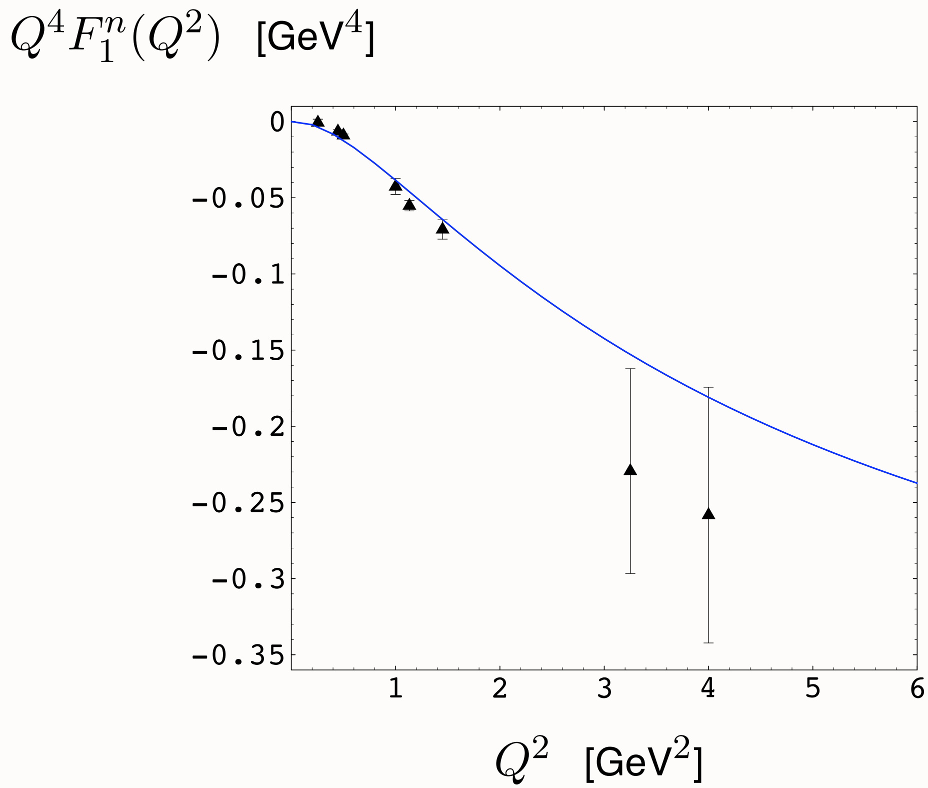
Current modified by metric

$$F_1(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi_F^{\uparrow}(z) J(Q,z) \Phi_I^{\uparrow}(z)$$

Dirac Neutron Form Factor

(Valence Approximation)

Truncated Space Confinement

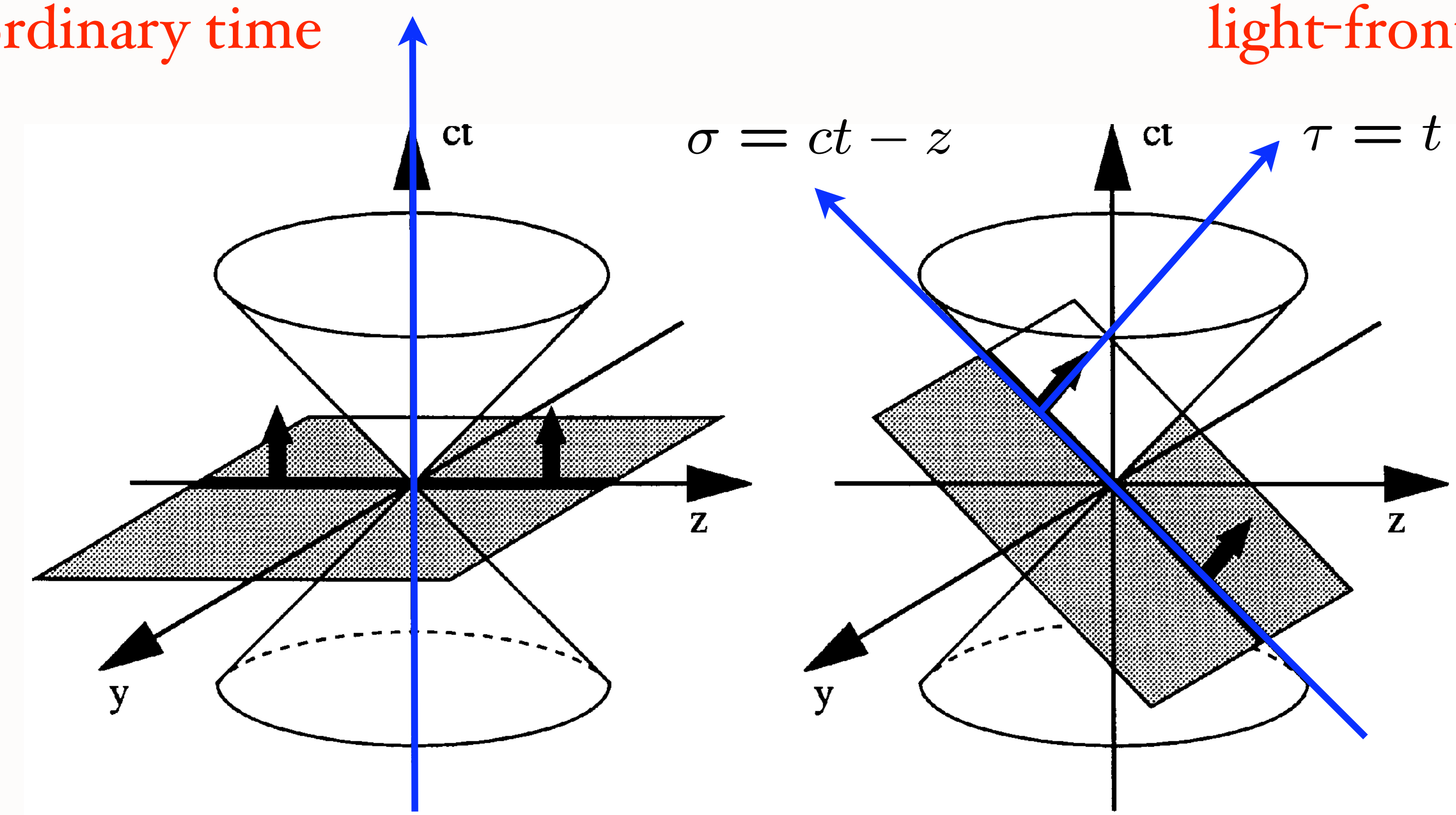


Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

Dirac's Amazing Idea:
The "Front Form"

Evolve in ordinary time

Evolve in light-front time!



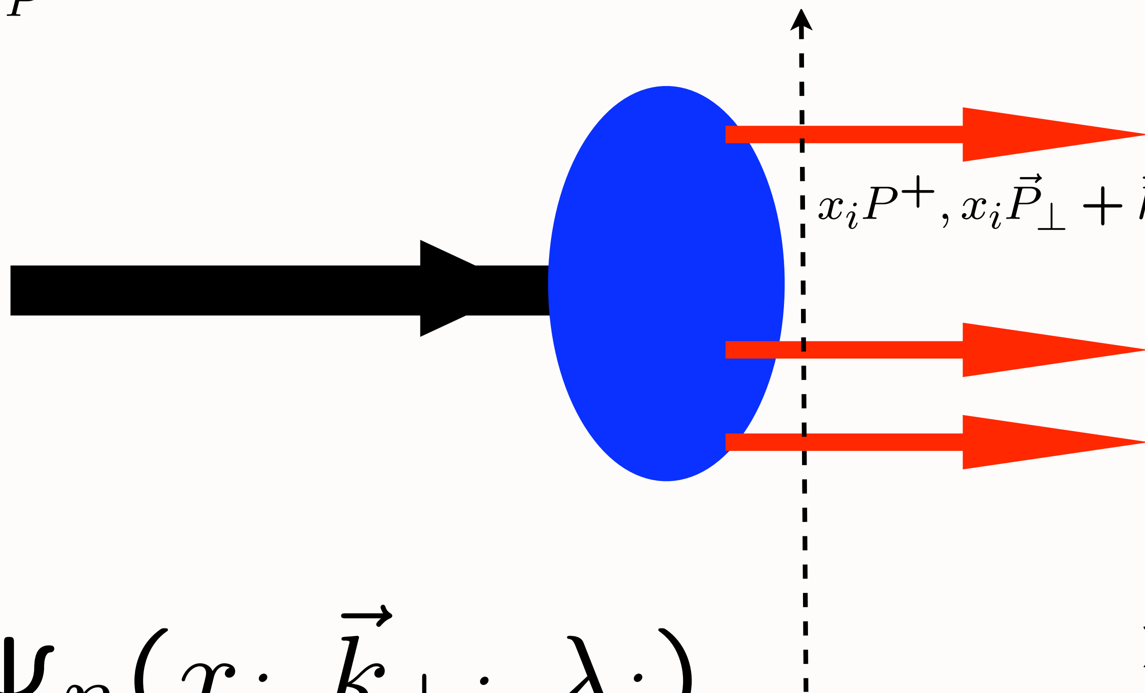
Instant Form

Front Form

Light-Front Wavefunctions

Fixed $\tau = t + z/c$

$P^+ = P^0 + P^z$



$P^+, \vec{P}_\perp$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

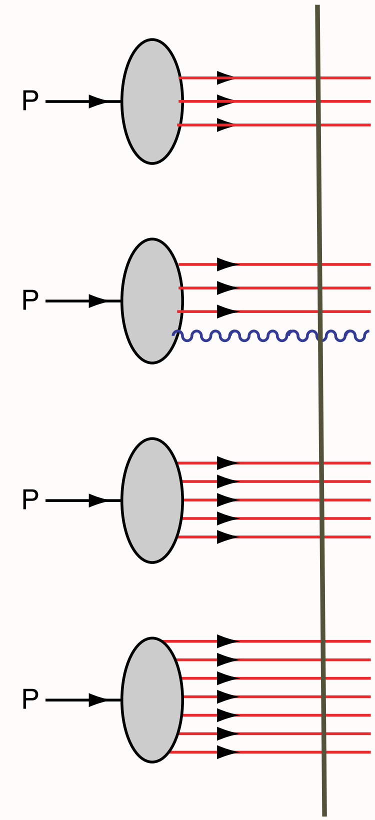
Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi_n(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$



$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Intrinsic gluons, sea quarks, asymmetries

Angular Momentum on the Light-Front

$A^+=0$ gauge: No unphysical degrees of freedom

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

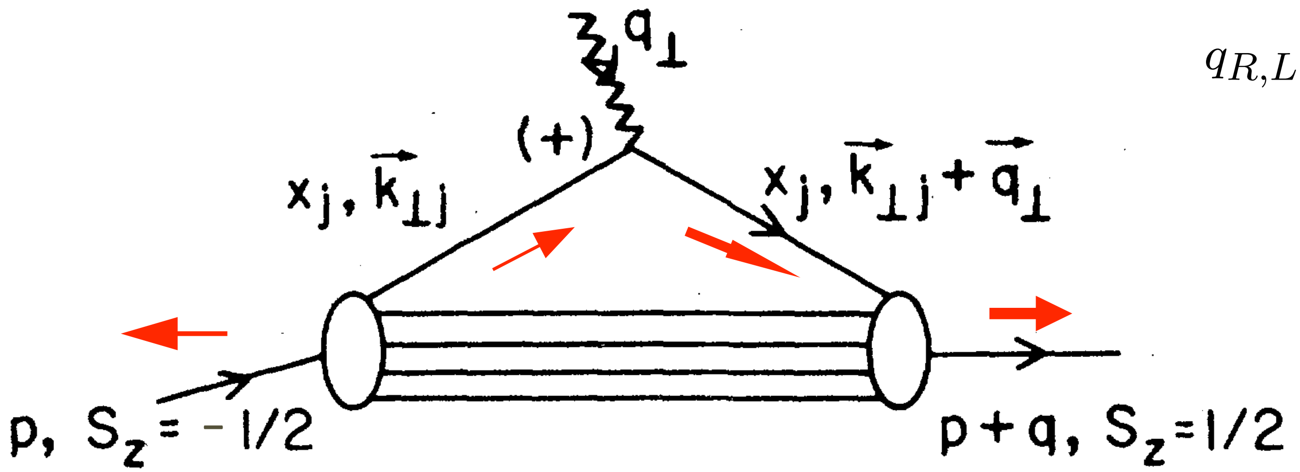
Nonzero Anomalous Moment requires Nonzero orbital angular momentum

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \, \times$$

Drell, sjb

$$\Big[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$



$$q_{R,L} = q^x \pm i q^y$$

Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Light-Front Representation of Meson Form Factor

- Drell-Yan-West form factor

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find ($b = |\vec{b}_\perp|$) :

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0(bqx)\, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

Soper

Identical DYW and AdS5 Formulae: Two parton case

- Change the integration variable $\zeta = |\vec{b}_\perp|\sqrt{x(1-x)}$

$$F(Q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int_0^{\zeta_{max}=\Lambda^{-1}_{\rm QCD}} \zeta \, d\zeta \, J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) \left|\widetilde{\psi}(x,\zeta)\right|^2,$$

- Compare with AdS form factor for arbitrary Q. Find:

Same result for LF and AdS5

$\zeta \leftrightarrow \mathbf{z}$

$$J(Q,\zeta) = \int_0^1 dx J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for the electromagnetic potential in AdS space, and

$$\boxed{\widetilde{\psi}(x,\vec{b}_\perp) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{0,1}\Lambda_{QCD}\right) \theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right)}$$

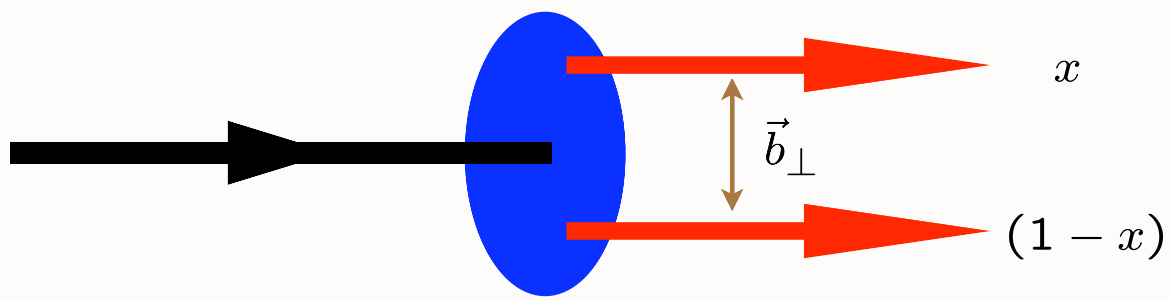
the holographic LFWF for the valence Fock state of the pion $\psi_{\overline{q}q/\pi}$.

- The variable ζ, $0 \le \zeta \le \Lambda^{-1}_{QCD}$, represents the scale of the invariant separation between quarks and is also the holographic coordinate $\zeta = z$!

LF(3+1) AdS$_5$

$\psi(x, \vec{b}_\perp)$ ↔ $\phi(z)$

$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$ ↔ z



$$\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)}\ \phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

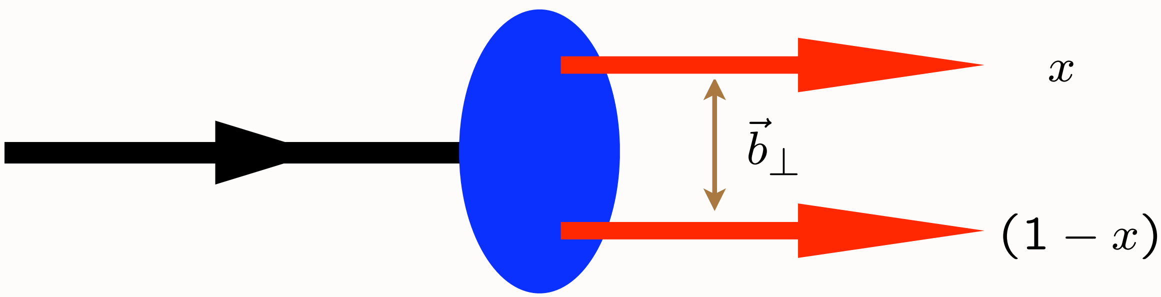
Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic radial equation: *Frame Independent*

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

G. de Teramond and sjb

Map AdS/CFT to 3+1 LF Theory

Effective radial equation:

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Effective conformal potential:

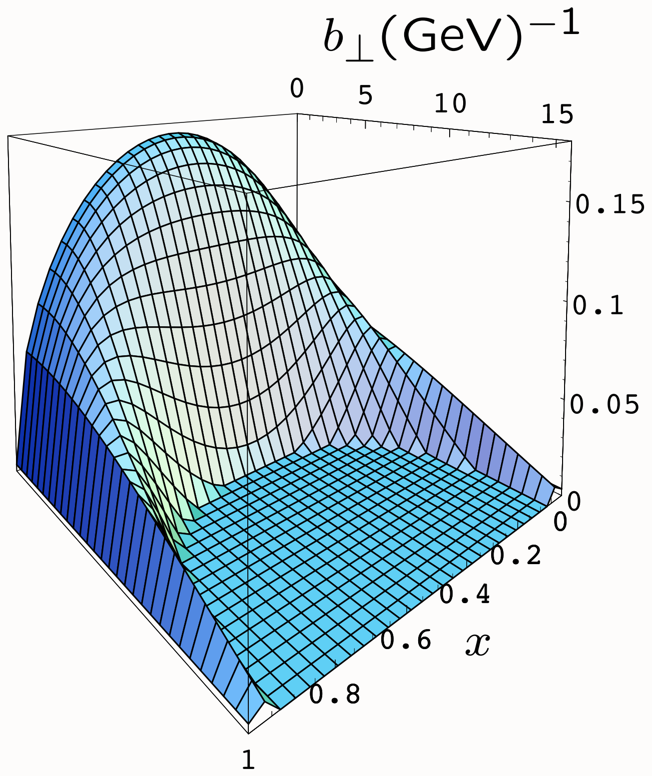
$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

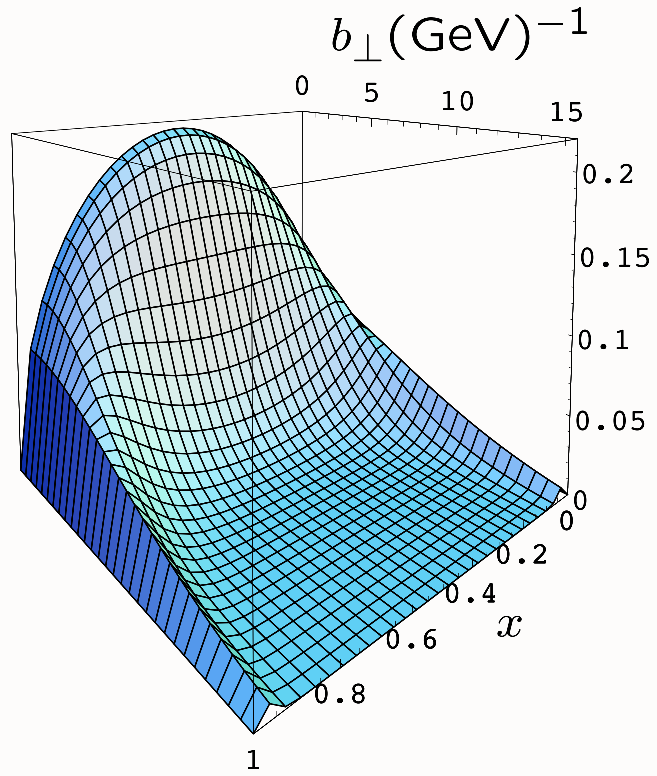
General solution:

$$\widetilde{\psi}_{L,k}(x,\vec{b}_\perp) = B_{L,k}\sqrt{x(1-x)}$$

$$J_L\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{L,k}\Lambda_{\text{QCD}}\right)\theta\left(\vec{b}_\perp^{\,2} \leq \frac{\Lambda_{\text{QCD}}^{-2}}{x(1-x)}\right),$$

AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$



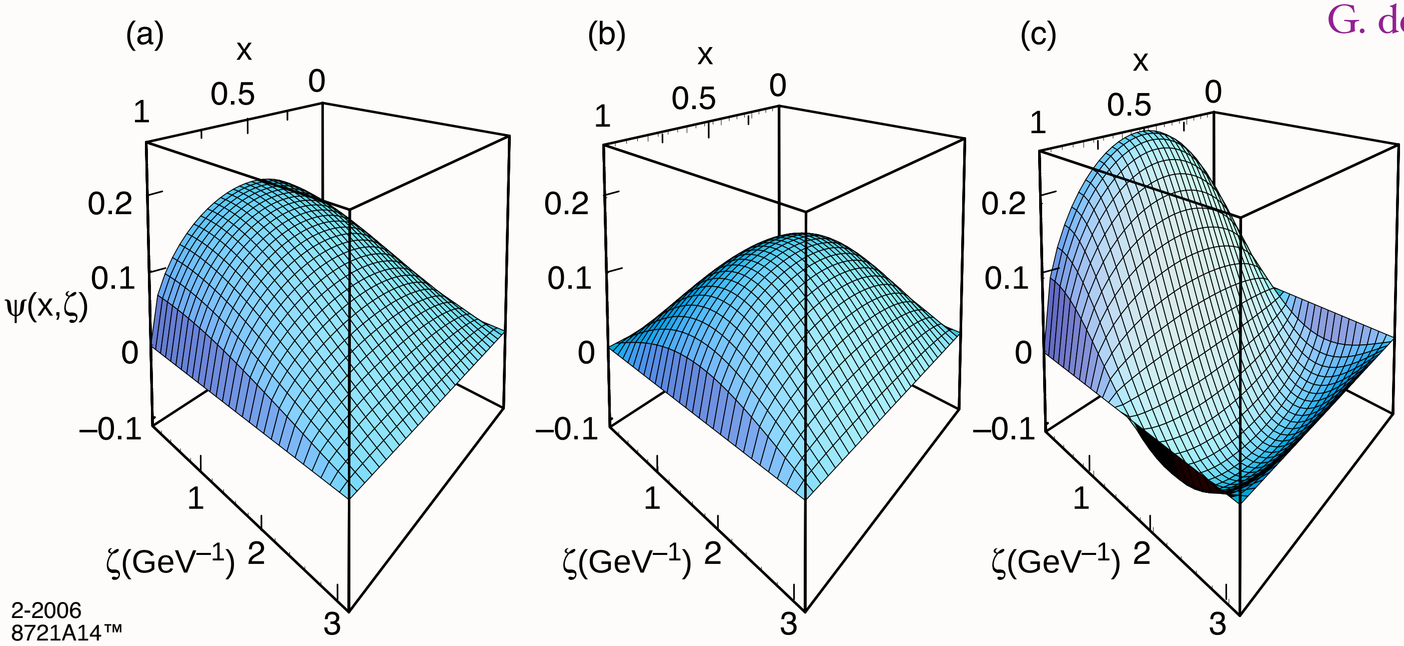


$\Lambda_{QCD} = 0.32 \text{ GeV}$ $\kappa = 0.76 \text{ GeV}$

Truncated Space ***Harmonic Oscillator***

AdS/CFT Prediction for Meson LFWF

G. de Teramond
SJB



Two-parton holographic LFWF in impact space $\widetilde{\psi}(x,\zeta)$ for $\Lambda_{QCD} = 0.32$ GeV: (a) ground state $L=0,\ k=1$; (b) first orbital exited state $L=1,\ k=1$; (c) first radial exited state $L=0,\ k=2$. The variable ζ is the holographic variable $z = \zeta = |b_\perp|\sqrt{x(1-x)}$.

$$\widetilde{\psi}(x,\zeta) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0\left(\zeta \beta_{0,1} \Lambda_{QCD}\right) \theta\left(z \le \Lambda^{-1}_{\rm QCD}\right)$$

- Define effective single particle transverse density by (Soper, Phys. Rev. D **15**, 1141 (1977))

$$F(q^2) = \int_0^1 dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp)$$

- From DYW expression for the FF in transverse position space:

$$\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j \, d^2\vec{b}_{\perp j} \, \delta(1 - x - \sum_{j=1}^{n-1} x_j) \, \delta^{(2)}(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp) |\psi_n(x_j, \vec{b}_{\perp j})|^2$$

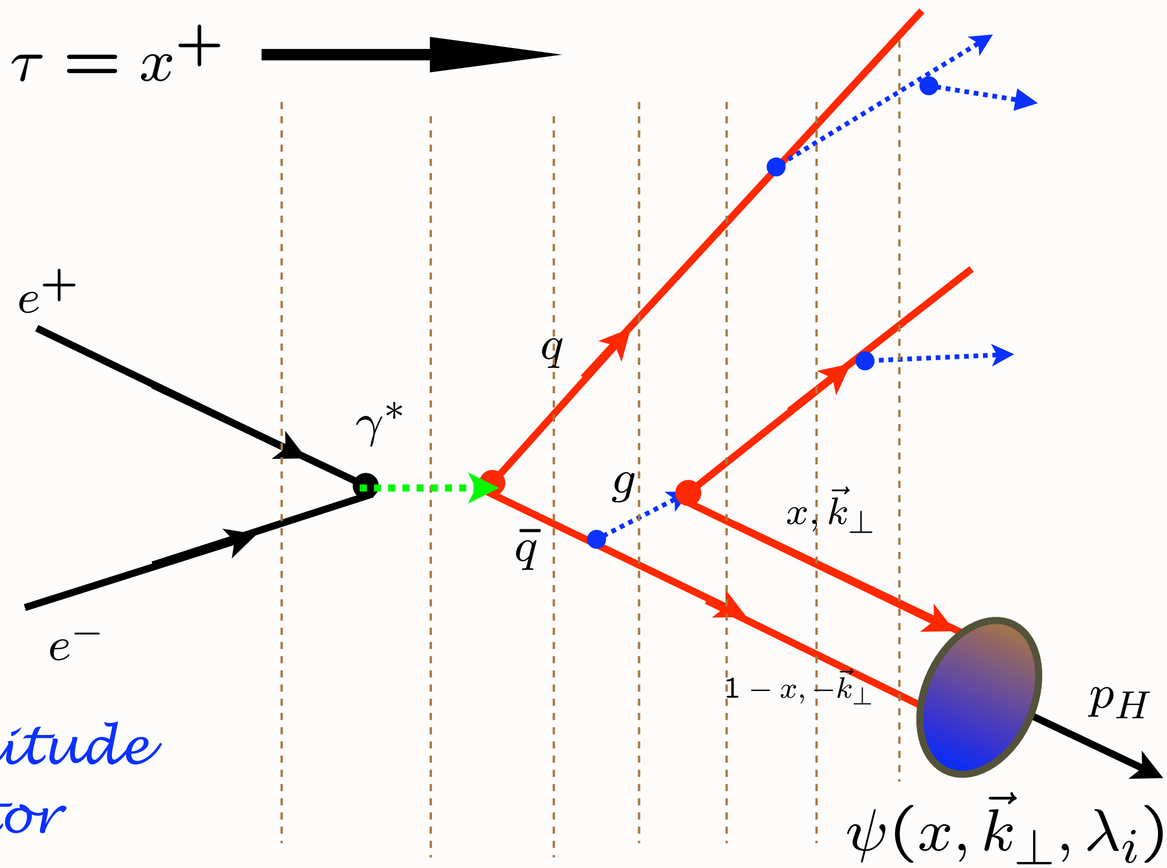
- Compare with the the form factor in AdS space for arbitrary Q:

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) \, J(Q, z) \, \Phi_P(z)$$

- Holographic variable z is expressed in terms of the average transverse separation distance of the spectator constituents $\vec{\eta} = \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j}$

$$z = \sqrt{\frac{x}{1-x}} \, \Big| \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j} \Big|$$

Hadronization at the Amplitude Level

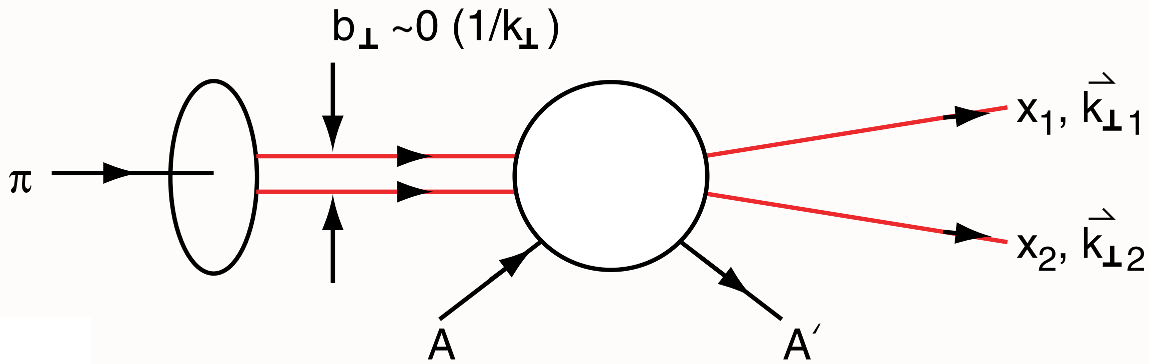


Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.



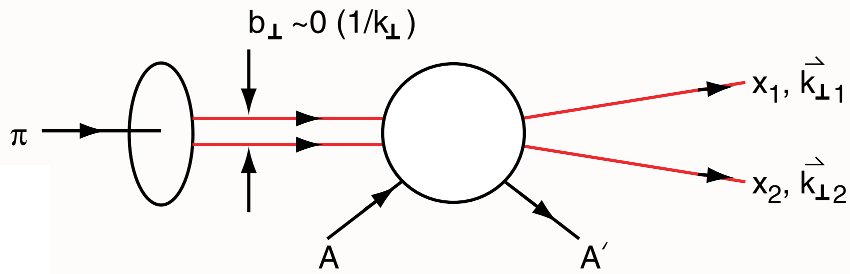
$$M \propto \frac{\partial^2}{\partial^2 k_\perp}\psi_\pi(x, k_\perp)$$

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus

Nucleus left Intact!

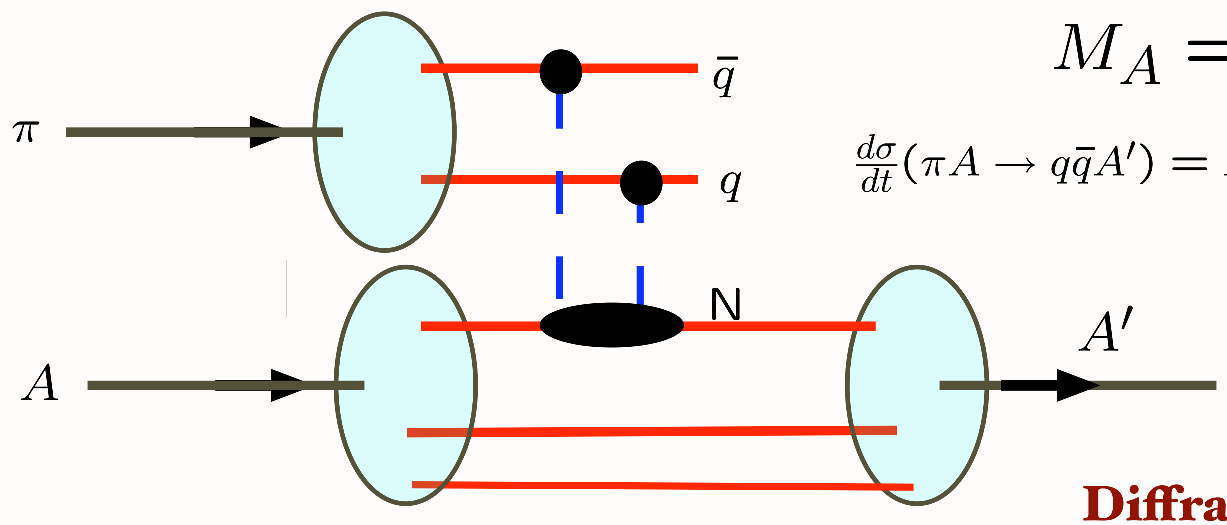
Key Ingredients in E791 Experiment



Brodsky Mueller
Frankfurt Miller Strikman

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

<u>*QCD COLOR Transparency*</u>



$$M_A = A\ M_N$$

$$\frac{d\sigma}{dt}(\pi A \rightarrow q\bar{q}A') = A^2\ \frac{d\sigma}{dt}(\pi N \rightarrow q\bar{q}N')\ F^2_A(t)$$

Target left intact

Diffraction, Rapidity gap

Color Transparency

Bertsch, Gunion, Goldhaber, sjb
A. H. Mueller, sjb

- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets

Mueller, sjb; Bertsch et al;
Frankfurt, Miller, Strikman

Measure pion LFWF in diffractive dijet production
Confirmation of color transparency

A-Dependence results: $\sigma \propto A^{\alpha}$

k_t range (GeV/c)	α	α (CT)
$1.25 < k_t < 1.5$	1.64 +0.06 -0.12	1.25
$1.5 < k_t < 2.0$	1.52 ± 0.12	1.45
$2.0 < k_t < 2.5$	1.55 ± 0.16	1.60

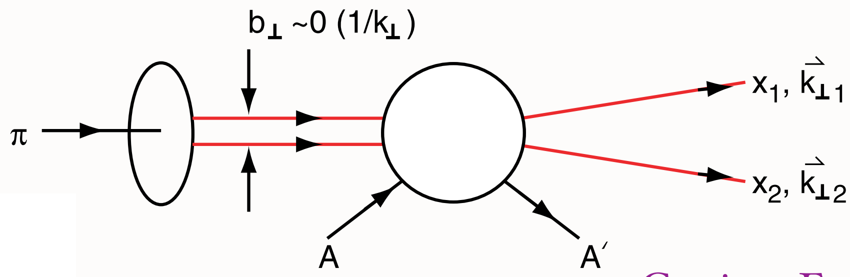
Ashery E791

α (Incoh.) = 0.70 ± 0.1

Conventional Glauber Theory Ruled Out !

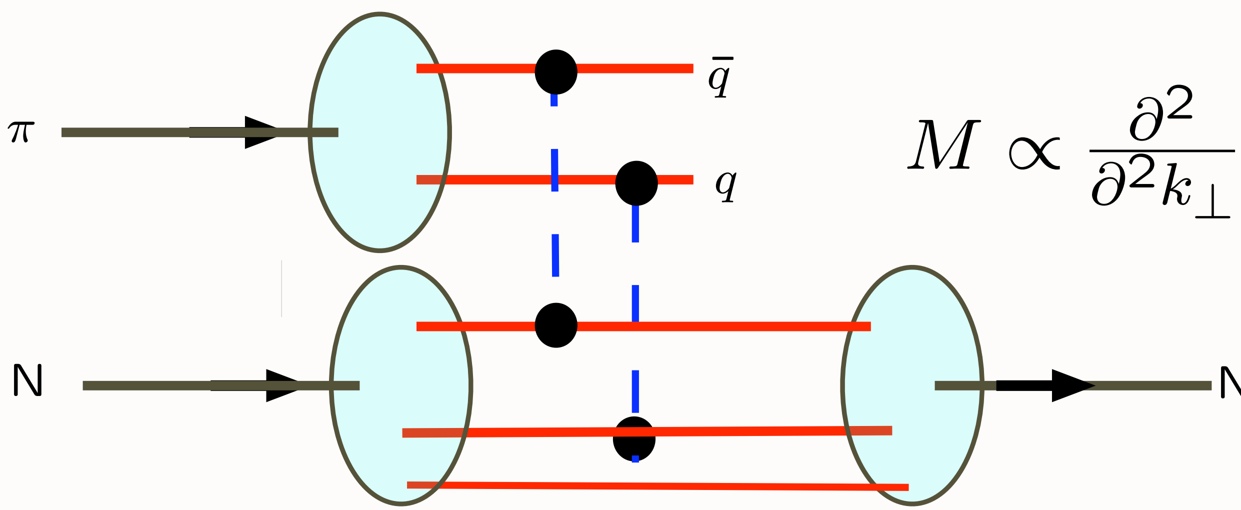
Factor of 7

Key Ingredients in Ashery Experiment



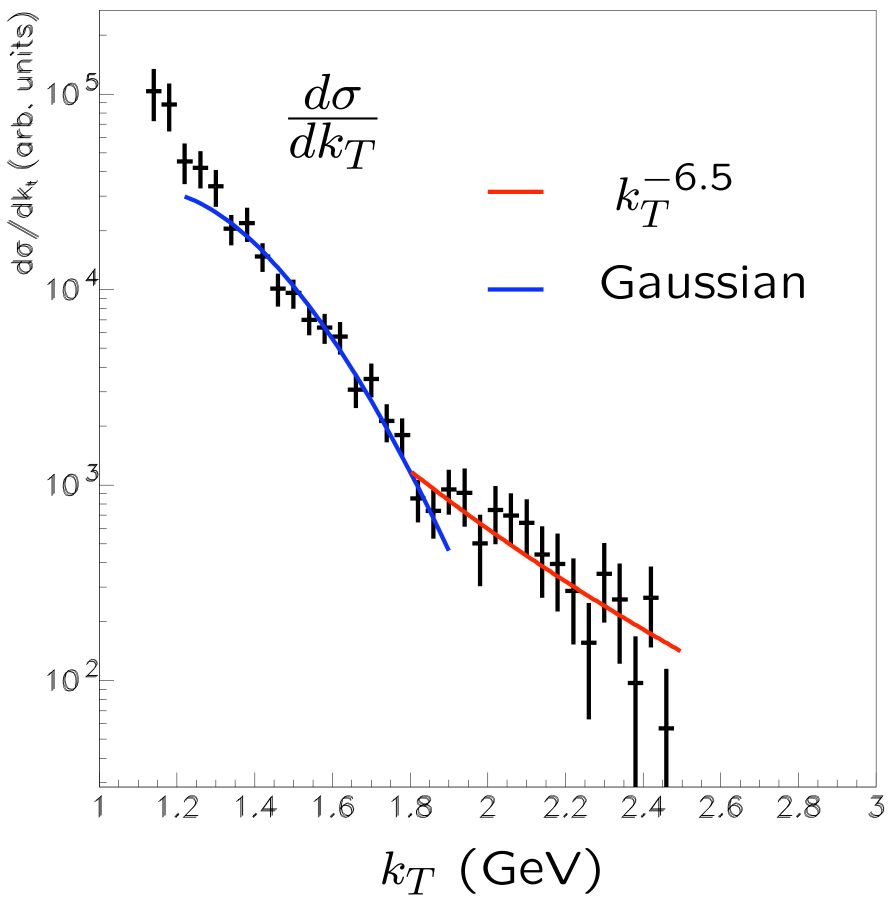
Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman

Two-gluon exchange measures the second derivative of the pion light-front wavefunction



$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

E791 Diffractive Di-Jet transverse momentum distribution

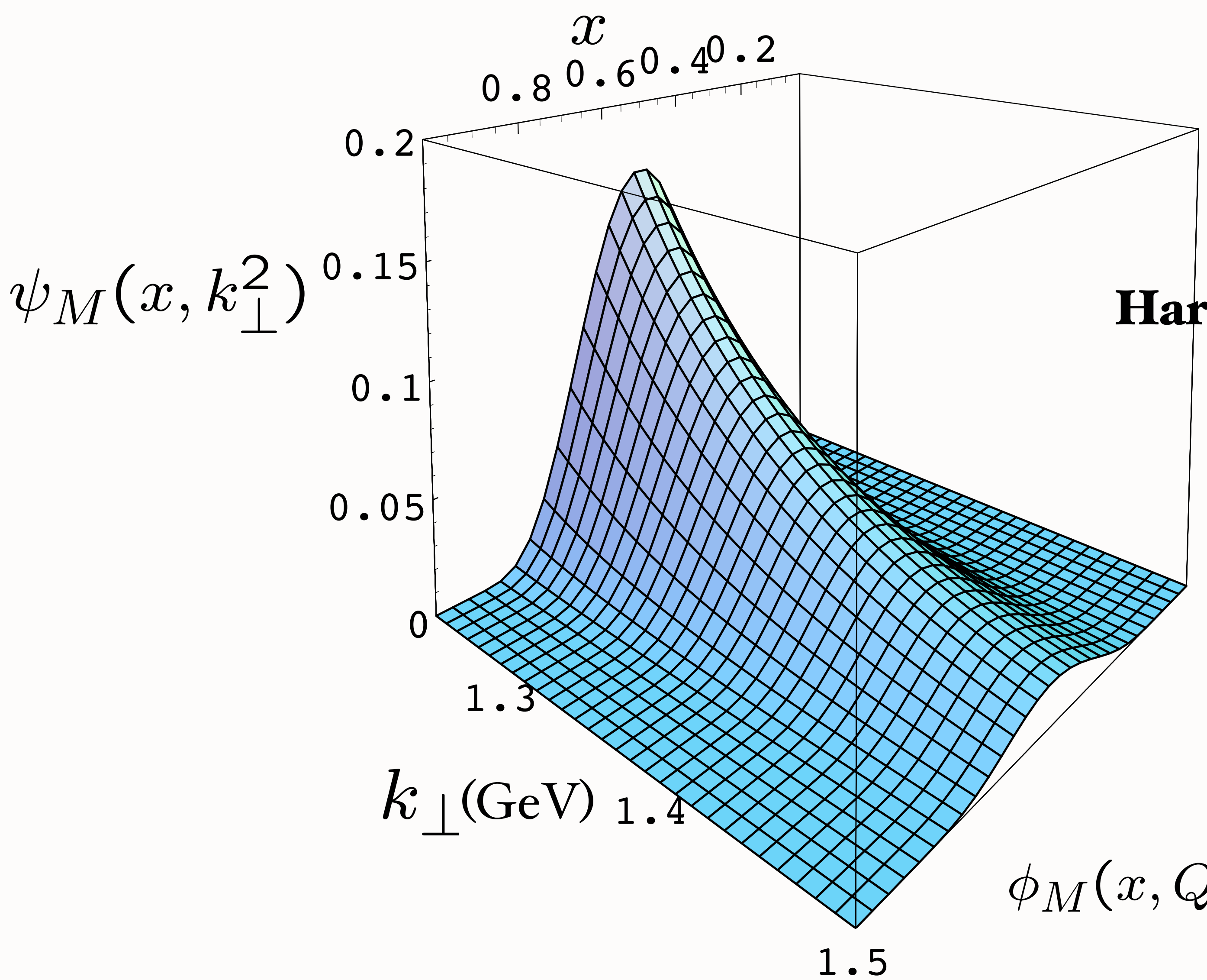


Two Components

High Transverse momentum dependence $k_T^{-6.5}$ *consistent with PQCD, ERBL Evolution*

Gaussian component similar to AdS/CFT HO LFWF

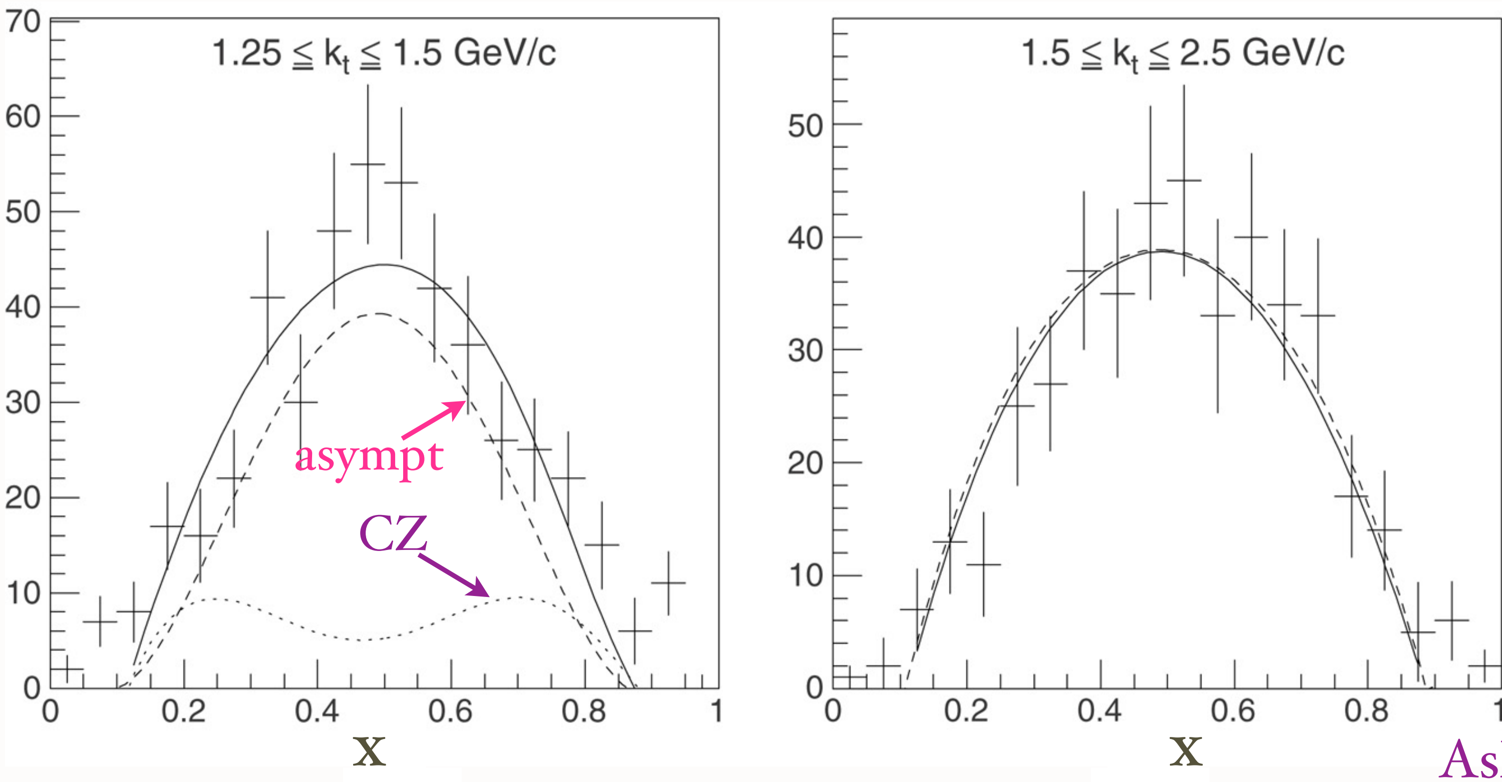
Prediction from AdS/CFT: Meson LFWF



Harmonic oscillator model

de Teramond, sjb

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$



Narrowing of x distribution at higher jet transverse momentum

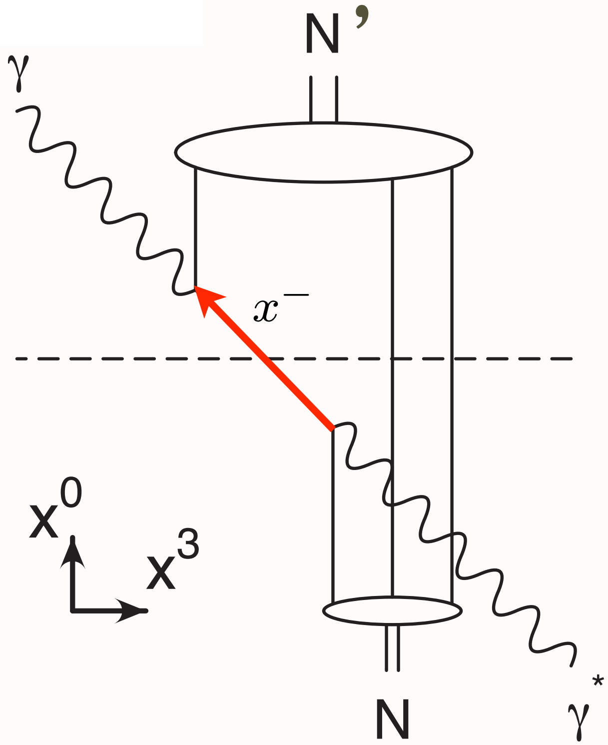
x: distribution of diffractive dijets from the platinum target for $1.25 \le k_t \le 1.5$ GeV/c (left) and for $1.5 \le k_t \le 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components:
Nonperturbative (AdS/CFT) and
Perturbative (ERBL)
Evolution to asymptotic distribution

Space-time picture of DVCS

P. Hoyer

$\sigma = \frac{1}{2} x^- P^+$



$x^+ = \mathbf{x}_\perp = 0$

The position of the struck quark differs by x^- in the two wave functions

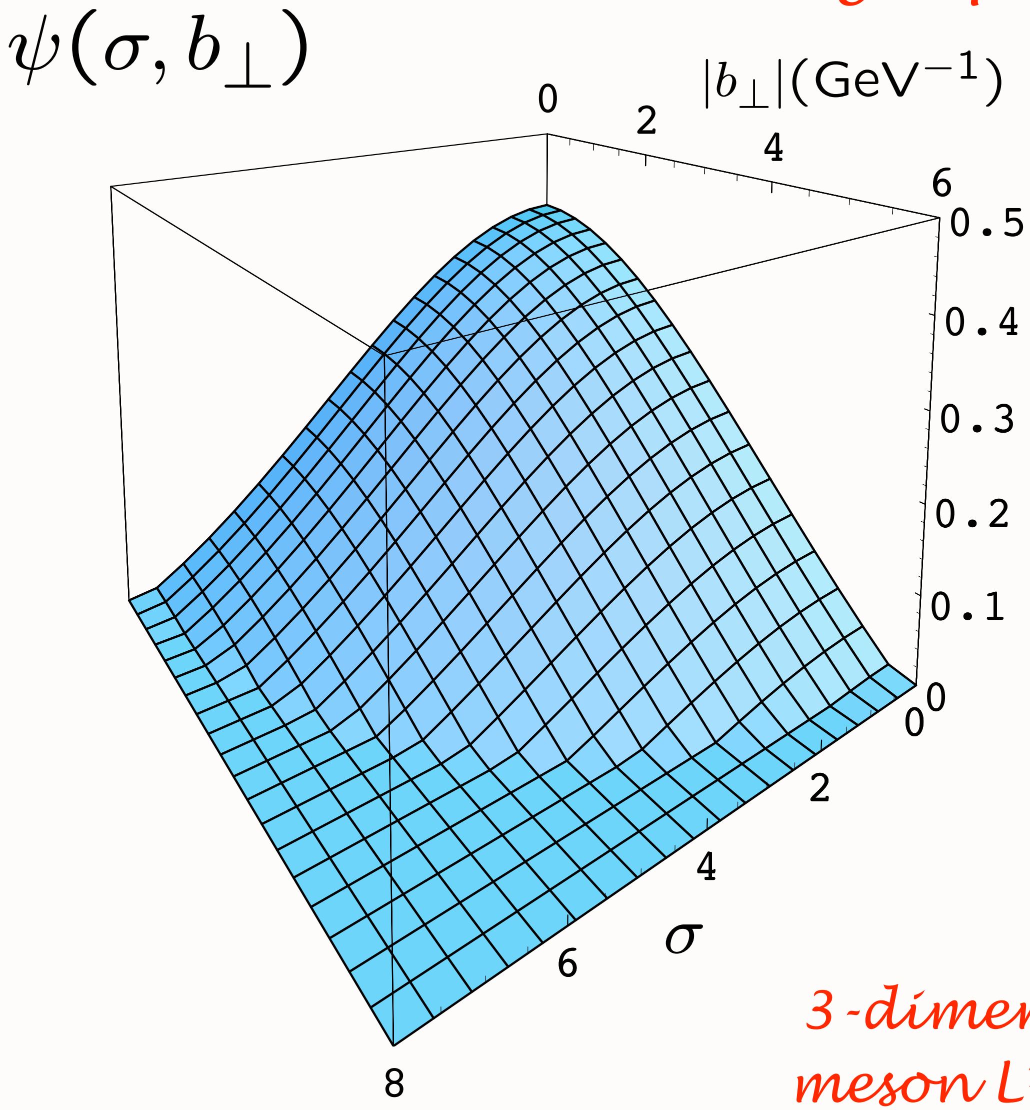
Measure x- distribution from DVCS:
Take Fourier transform of skewness,
the longitudinal momentum transfer

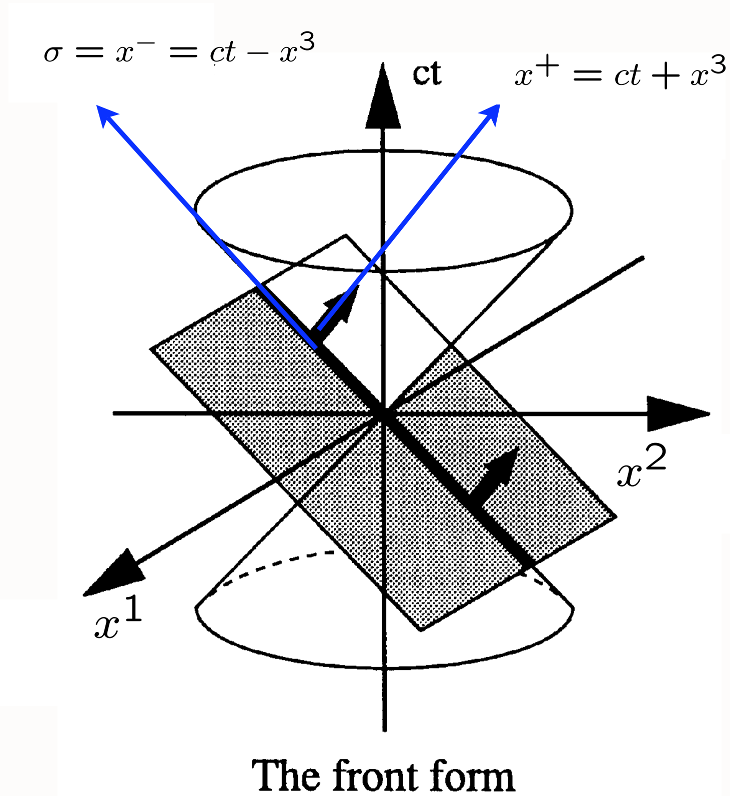
$$\zeta = \frac{Q^2}{2p \cdot q}$$

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

AdS/CFT Holographic Model

G. de Teramond
SJB





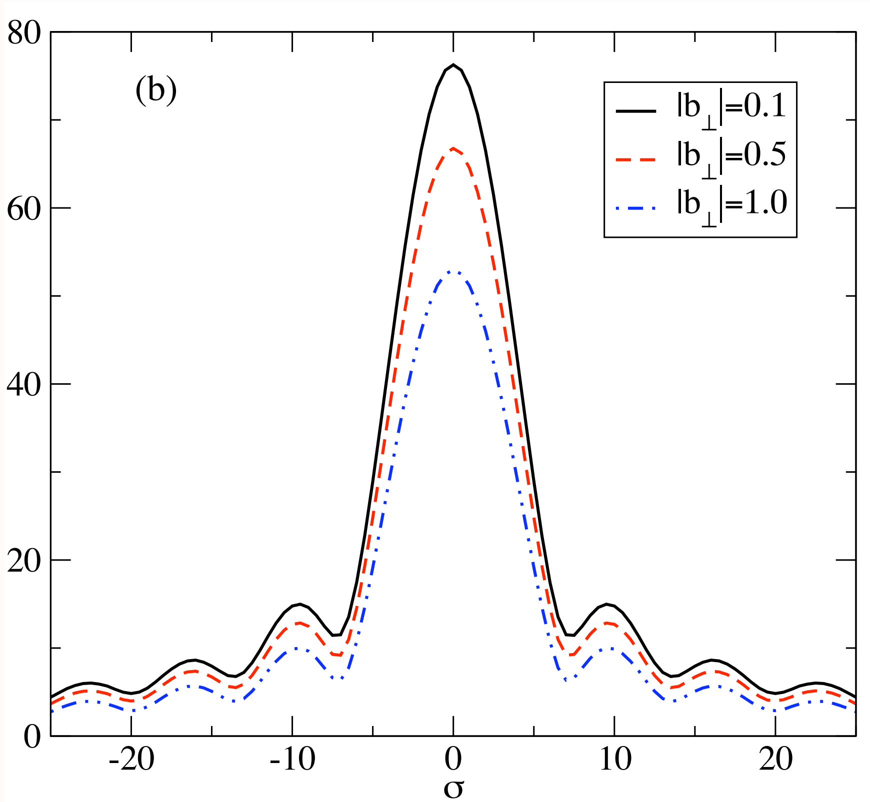
3-dimensional photograph:
meson LFWF at fixed LF Time

S. J. Brodsky[a], D. Chakrabarti[b], A. Harindranath[c], A. Mukherjee[d], J. P. Vary[e,a,f]

Hadron Optics

$$A(\sigma, b_\perp) = \frac{1}{2\pi} \int d\zeta e^{i\sigma\zeta} \tilde{A}(b_\perp, \zeta)$$

$$\sigma = \tfrac{1}{2} x^- P^+ \qquad \zeta = \frac{Q^2}{2p\cdot q}$$

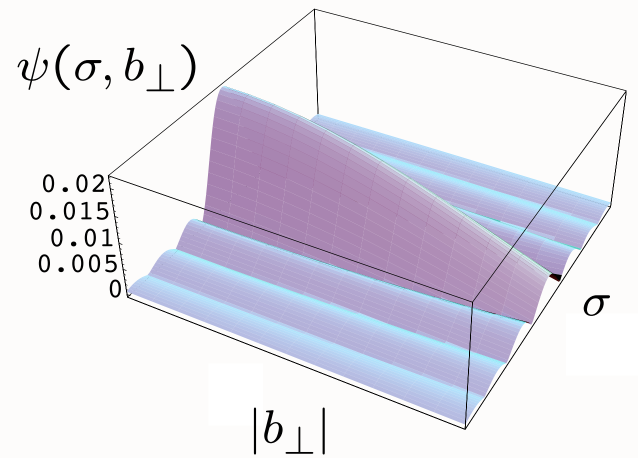


The Fourier Spectrum of the DVCS amplitude in σ space for different fixed values of $|b_\perp|$.

GeV units

DVCS Amplitude using holographic QCD meson LFWF

$$\Lambda_{QCD} = 0.32$$

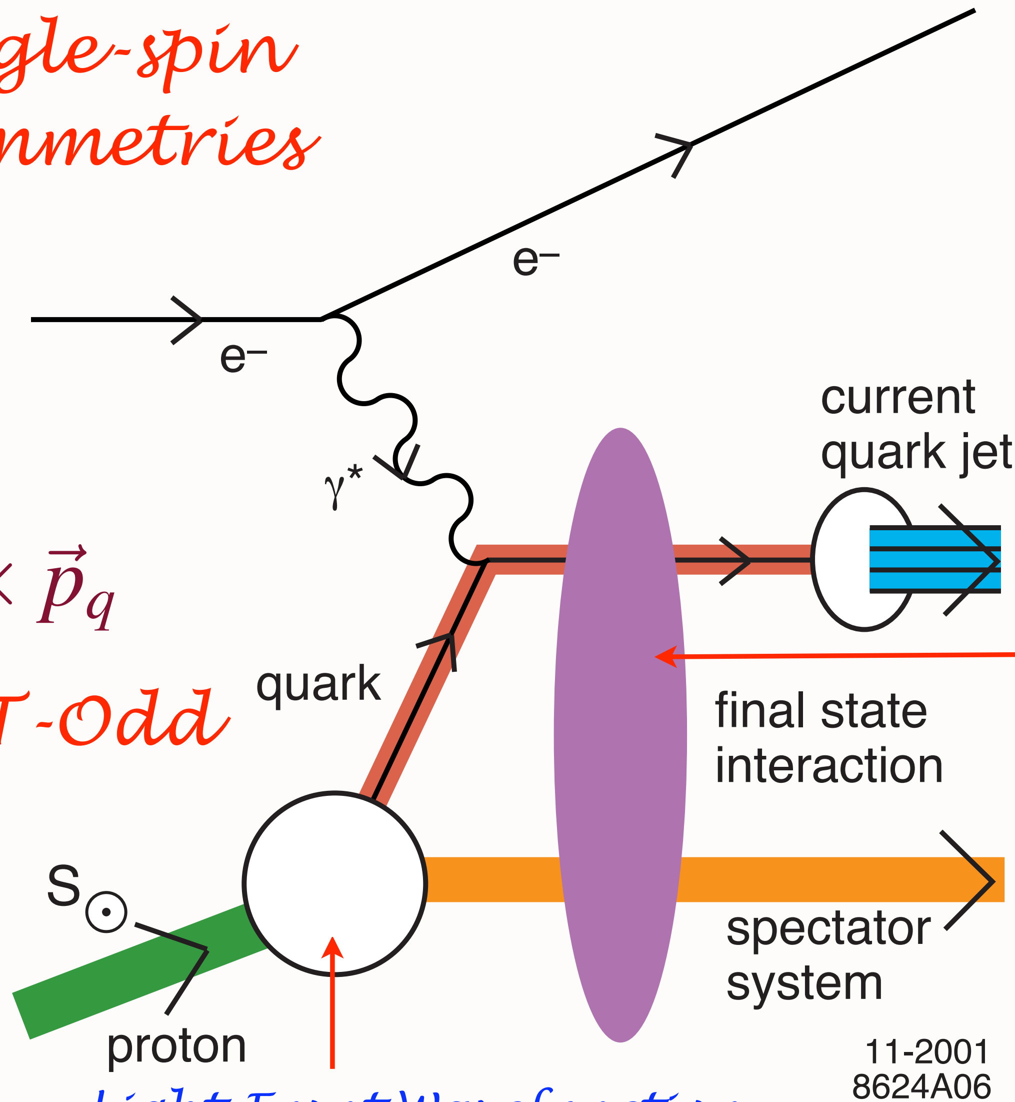


Hadron Dynamics at the Amplitude Level

- LFWFS are the universal hadronic amplitudes which underlie structure functions, GPDs, exclusive processes.
- Relation of spin, momentum, and other distributions to physics of the hadron itself.
- Connections between observables, orbital angular momentum
- Role of FSI and ISIs--Sivers effect

Single-spin asymmetries

Leading Twist Sivers Effect



$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo-T-Odd

QCD S- and P-Coulomb Phases

Light-Front Wavefunction S and P- Waves

D. S. Hwang, I. A. Schmidt, sjb

Features of Light-Front Formalism

- *Hidden Color* Nuclear Wavefunction
- *Color Transparency, Opaqueness*
- *Intrinsic glue, sea quarks, intrinsic charm*
- Simple proof of Factorization theorems for hard processes (Lepage, sjb)
- *Direct mapping to AdS/CFT* (de Teramond, sjb)
- New Effective LF Equations (de Teramond, sjb)
- Light-Front Amplitude Generator

AdS/CFT and Integrability

- Conformal Symmetry plus Confinement: Reduce AdS/QCD Equations to Linear Form
- Generate eigenvalues and eigenfunctions using Ladder Operators
- Apply to Covariant Light-Front Radial Dirac and Schrodinger Equations
- L. Infeld, "On a new treatment of some eigenvalue problems", Phys. Rev. 59, 737 (1941).

AdS/CFT LF Equation for Mesons with HO Confinement

$$\nu = L$$

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\kappa^2(\nu+1) + \mathcal{M}^2\right)\phi_\nu(\zeta) = 0$$

LF Hamiltonian

$$H^\nu_{LF}\phi_\nu = \mathcal{M}^2_\nu\phi_\nu$$

Bilinear $H^\nu_{LF} = \Pi^\dagger_\nu\Pi_\nu,$

where

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta} - \kappa^2\zeta\right),$$

and its adjoint

de Teramond, sjb

$$\Pi^\dagger_\nu(\zeta) = -i\left(\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right),$$

with commutation relations

$$\left[\Pi_\nu(\zeta), \Pi^\dagger_\nu(\zeta)\right] = \frac{2\nu+1}{\zeta^2} - 2\kappa^2.$$

AdS/CFT LF Equation for Mesons with HO Confinement

$$\left(\frac{d^2}{d\zeta^2} + \frac{1-4\nu^2}{4\zeta^2} - \kappa^4\zeta^2 - 2\kappa^2(\nu+1) + \mathcal{M}^2\right)\phi_\nu(\zeta) = 0$$

Define $$b^\dagger_\nu = -i\Pi_\nu = \frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta$$

$$b_\nu = \frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta \qquad b^\dagger_\nu b_\nu = b_{\nu+1} b^\dagger_{\nu+1}$$

Ladder Operator $$b^\dagger_\nu|\nu\rangle = c_\nu|\nu+1\rangle$$

$$\left(-\frac{d}{d\zeta} + \frac{\nu+\frac{1}{2}}{\zeta} + \kappa^2\zeta\right)\phi_\nu(\zeta) = c_\nu\phi_{\nu+1}(\zeta)$$

$$\phi_\nu(z) = C z^{1/2+\nu} e^{-\kappa^2\zeta^2/2} G_\nu(\zeta),$$

$$2xG_\nu(x) - G'(x) = xG_{\nu+1}(x)$$

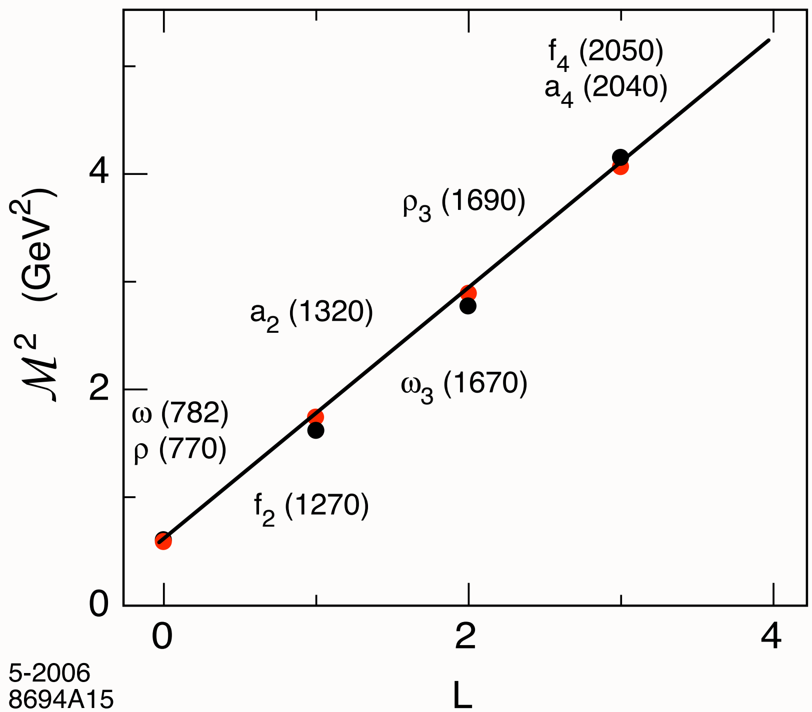
defines the associated Laguerre function $L_n^{\nu+1}(x^2)$

$$\phi_\nu(z) = C_\nu z^{1/2+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2).$$

Subtract Vacuum Energy

$$\mathcal{M}^2 \to \mathcal{M}^2 - 2\kappa^2,$$

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + \frac{1}{2}).$$

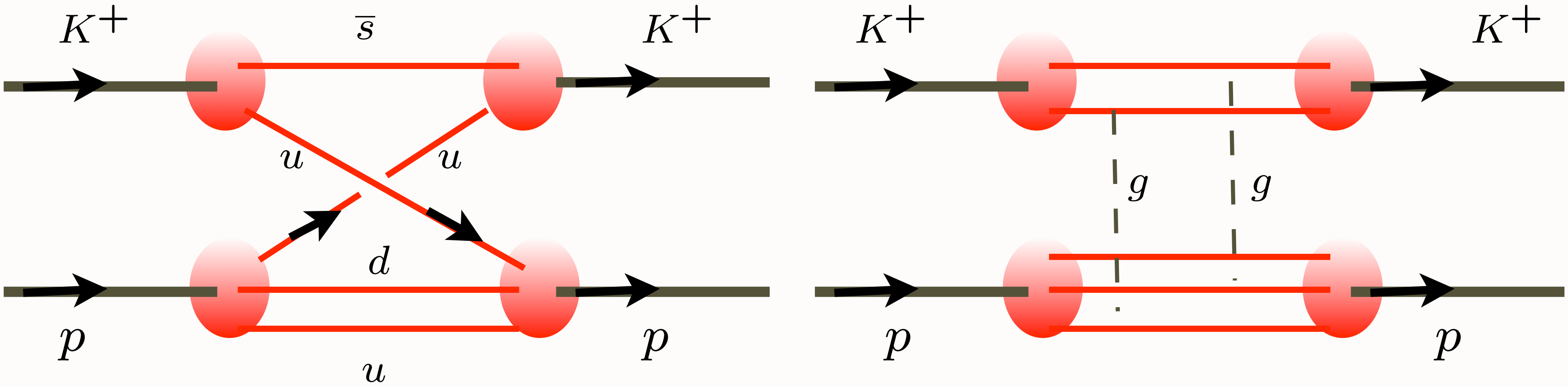


$J = L + 1$ vector meson Regge trajectory for $\kappa \simeq 0.54$ GeV

New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

CIM: Blankenbecler, Gunion, sjb



Quark Interchange
(Spin exchange in atom-atom scattering)

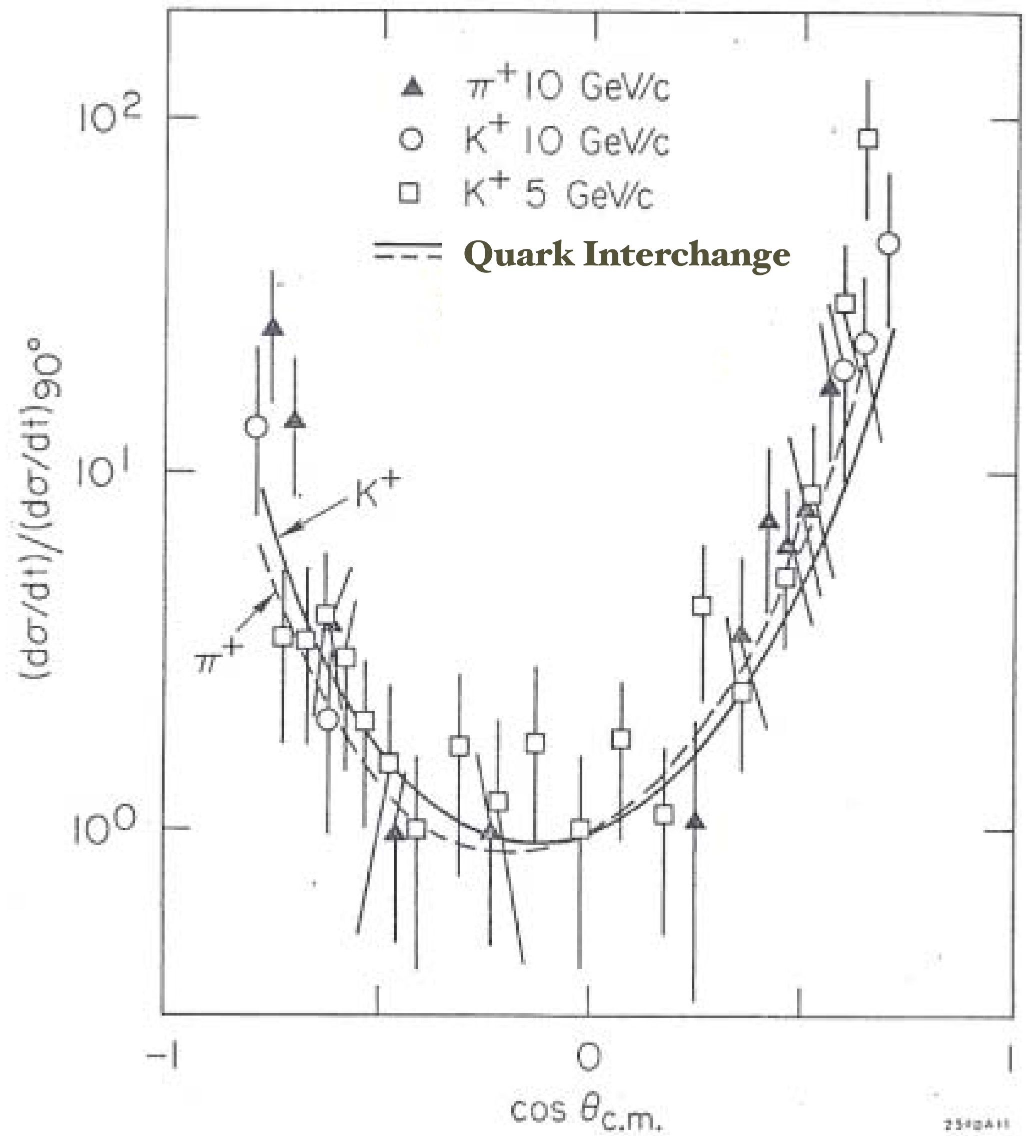
Gluon Exchange
(Van der Waal -- Landshoff)

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$

$M(s,t)_{\text{gluonexchange}} \propto sF(t)$

MIT Bag Model (de Tar), large N_C, ('t Hooft), AdS/CFT all predict dominance of quark interchange:



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

Non-linear Regge behavior:

$$\alpha_R(t) \to -1$$

Why is quark-interchange dominant over gluon exchange?

Example: $M(K^+ p \to K^+ p) \propto \frac{1}{ut^2}$

Exchange of common u quark

$M_{QIM} = \int d^2k_\perp dx \; \psi^\dagger_C \psi^\dagger_D \Delta \psi_A \psi_B$

Holographic model (Classical level):

Hadrons enter 5th dimension of AdS_5

Quarks travel freely within cavity as long as separation $z < z_0 = \frac{1}{\Lambda_{QCD}}$

LFWFs obey conformal symmetry producing quark counting rules.

Comparison of Exclusive Reactions at Large t

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Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^{\pm}p \to p\pi^{\pm}, p\rho^{\pm}, \pi^{+}\Delta^{\pm}, K^{+}\Sigma^{\pm}, (\Lambda^0/\Sigma^0)K^0$; $K^{\pm}p \to pK^{\pm}$; $p^{\pm}p \to pp^{\pm}$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.

$\pi^{\pm}p \to p\pi^{\pm}$,

$K^{\pm}p \to pK^{\pm}$,

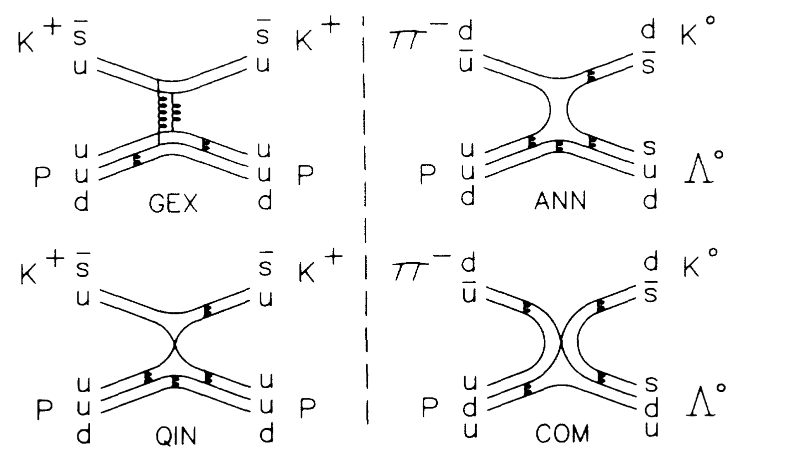
$\pi^{\pm}p \to p\rho^{\pm}$,

$\pi^{\pm}p \to \pi^{+}\Delta^{\pm}$,

$\pi^{\pm}p \to K^{+}\Sigma^{\pm}$,

$\pi^{-}p \to \Lambda^0K^0, \Sigma^0K^0$,

$p^{\pm}p \to pp^{\pm}$.



Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis

Pauli, Hornbostel, Hiller, McCartor, sjb

- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion

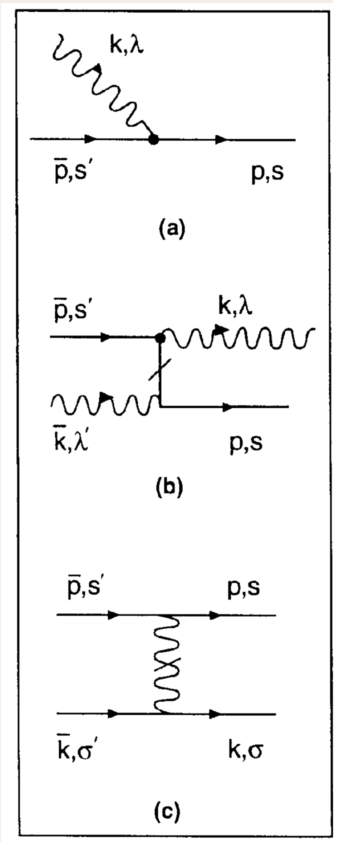
Vary, Harinandrath, sjb

- Similar to Shell Model calculations

Light-Front QCD Heisenberg Equation

$$H^{QCD}_{LC} |\Psi_h\rangle = \mathcal{M}^2_h \, |\Psi_h\rangle$$

DLCQ



n	Sector	1 $q\bar q$	2 gg	3 $q\bar q\, g$	4 $q\bar q\, q\bar q$	5 $gg\, g$	6 $q\bar q\, gg$	7 $q\bar q\, q\bar q\, g$	8 $q\bar q\, q\bar q\, q\bar q$	9 $gg\, gg$	10 $q\bar q\, gg\, g$	11 $q\bar q\, q\bar q\, gg$	12 $q\bar q\, q\bar q\, q\bar q\, g$	13 $q\bar q\, q\bar q\, q\bar q\, q\bar q$
1	$q\bar q$					•		•	•	•	•	•	•	•
2	gg				•			•	•		•	•	•	•
3	$q\bar q\, g$								•	•		•	•	•
4	$q\bar q\, q\bar q$		•			•				•	•		•	•
5	$gg\, g$	•			•			•	•			•	•	•
6	$q\bar q\, gg$								•				•	•
7	$q\bar q\, q\bar q\, g$	•	•			•				•				•
8	$q\bar q\, q\bar q\, q\bar q$	•	•	•		•	•			•	•			
9	$gg\, gg$	•		•	•			•	•			•	•	•
10	$q\bar q\, gg\, g$	•	•		•				•				•	•
11	$q\bar q\, q\bar q\, gg$	•	•	•		•				•				•
12	$q\bar q\, q\bar q\, q\bar q\, g$	•	•	•	•	•	•			•	•			
13	$q\bar q\, q\bar q\, q\bar q\, q\bar q$	•	•	•	•	•	•	•		•	•	•		

Use AdS/QCD basis functions

Pauli, Pinsky, sjb

String Theory

↓

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

↓

Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

AdS/QCD

Conformal behavior at short distances
+ Confinement at large distance

↓

Semi-Classical QCD / Wave Equations

↓ Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

↓ Integrable!

Hadron Spectra, Wavefunctions, Dynamics

AdS/QCD

- New initial approximation to QCD based on conformal invariance, and confinement
- Underlying principle: Conformal Window
- AdS5: Mathematical representation of conformal gauge theory
- Systematically improve using DLCQ
- Successes: Hadron spectra, LFWFs, dynamics
- QCD at the Amplitude Level

AdS/CFT and QCD

Bottom-Up Approach

- Nonperturbative derivation of dimensional counting rules of hard exclusive glueball scattering for gauge theories with mass gap dual to string theories in warped space:

  Polchinski and Strassler, hep-th/0109174.

- Deep inelastic structure functions at small x:

  Polchinski and Strassler, hep-th/0209211.

- Derivation of power falloff of hadronic light-front Fock wave functions, including orbital angular momentum, matching short distance behavior with string modes at AdS boundary:

  Brodsky and de Téramond, hep-th/0310227. E. van Beveren et al.

- Low lying hadron spectra, chiral symmetry breaking and hadron couplings in AdS/QCD:

  Boschi-Filho and Braga, hep-th/0212207; de Téramond and Brodsky, hep-th/0501022; Erlich, Katz, Son and Stephanov, hep-ph/0501128; Hong, Yong and Strassler, hep-th/0501197; Da Rold and Pomarol, hep-ph/0501218; Hirn and Sanz, hep-ph/0507049; Boschi-Filho, Braga and Carrion, arXiv:hep-th/0507063; Katz, Lewandowski and Schwartz, arXiv:hep-ph/0510388.

- Gluonium spectrum (top-bottom):

  Csaki, Ooguri, Oz and Terning, hep-th/9806021; de Mello Kock, Jevicki, Mihailescu and Nuñez, hep-th/9806125; Csaki, Oz, Russo and Terning, hep-th/9810186; Minahan, hep-th/9811156; Brower, Mathur and Tan, hep-th/0003115, Caceres and Nuñez, hep-th/0506051.

- D3/D7 branes (top-bottom):

  Karch and Katz, hep-th/0205236; Karch, Katz and Weiner, hep-th/0211107; Kruczenski, Mateos, Myers and Winters, hep-th/0311270; Sakai and Sonnenschein, hep-th/0305049; Babington, Erdmenger, Evans, Guralnik and Kirsch, hep-th/0312263; Nuñez, Paredes and Ramallo, hep-th/0311201; Hong, Yoon and Strassler, hep-th/0312071; hep-th/0409118; Kruczenski, Pando Zayas, Sonnenschein and Vaman, hep-th/0410035; Sakai and Sugimoto, hep-th/0412141; Paredes and Talavera, hep-th/0412260; Kirsh and Vaman, hep-th/0505164; Apreda, Erdmenger and Evans, hep-th/0509219; Casero, Paredes and Sonnenschein, hep-th/0510110.

- Other aspects of high energy scattering in warped spaces:

  Giddings, hep-th/0203004; Andreev and Siegel, hep-th/0410131; Siopsis, hep-th/0503245.

- Strongly coupled quark-gluon plasma ($\eta/s = 1/4\pi$):

  Policastro, Son and Starinets, hep-th/0104066; Kang and Nastase, hep-th/0410173 . . .

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A Theory of Everything Takes Place

String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists' best hope for unifying gravity and quantum theory -- into a single coherent theory.

Frank and Ernest