

Pictorial Rules for Quantum Circuits

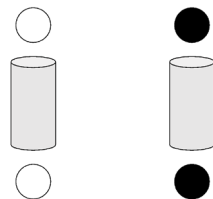
Pictorial Rules

Bits are represented by geometrical shapes like circles, triangles, and squares. Bits are stored on different physical objects of a circuit, such as conductors inside a regular computer. The color of these shapes, white or black, indicate whether each bit is 0 (white) or 1 (black).

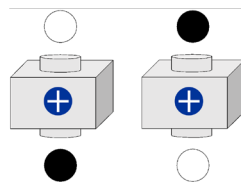


Gates are elements of the circuits that allow a signal to pass or not (like an on and off switch). There are different types of gates, each has a specific function.

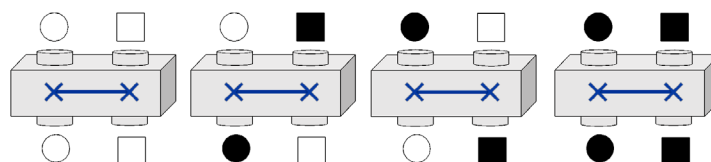
Identity gate is labeled by two parallel lines. The identity gate does not change the bit.



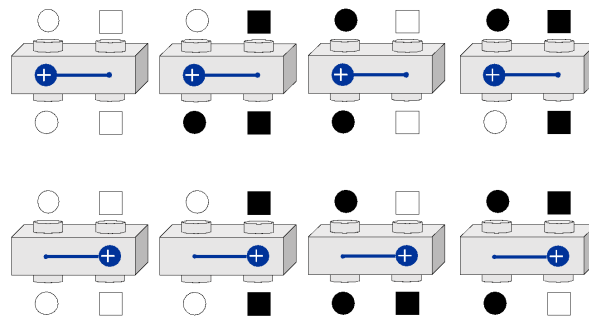
NOT gate is a square box labeled with a target symbol. The NOT gate inverts the color of the bit



SWAP gate is a rectangle label with two Xs connected by a line. It changes the color of all the bits. The SWAP gate can act on two (or more) bits at the same time.



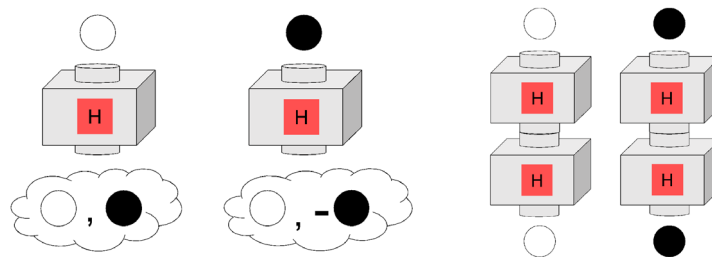
Controlled-NOT or CNOT gate is a rectangle labeled with a target symbol on either end of the box and a dot on the opposite side. The CNOT gate changes the color of the bit on the target symbol side only when the color of the other bit is black (1).



Quantum bits or Qubits are equivalent to bits but with special quantum properties: be black and white simultaneously, which is called superposition. A qubit in super position is represented by a mist.

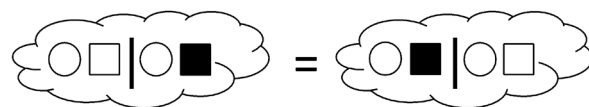


Hadamard gate is a quantum gate, which can create superposition states. It is represented by a square labeled with an H. In addition to producing mists from non-mists, the Hadamard box can also do the opposite: It can turn mists into non-mists. This also means that if we stack two H boxes, we find that the circle returns to its original color



Mist rules:

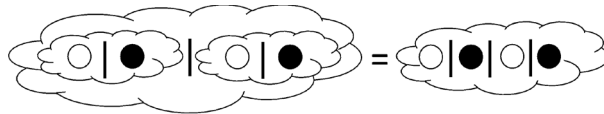
Entry order doesn't matter: Order of the different entries within a mist does not matter



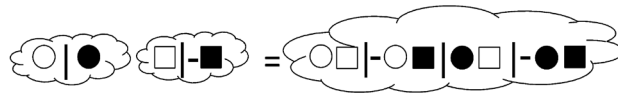
Cancelling entries: If two equal shapes are the same color but with opposite signs, then these bits cancel and can be removed from the mist.



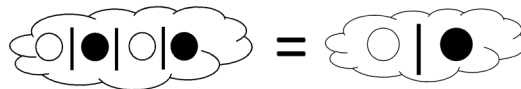
Removing inner mist: Mists within mists can be eliminated.



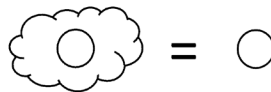
Combining mist: Two or more mists can be combined into a single mist in a manner that is identical to the FOIL ("firsts, outers, inners, lasts") mnemonic in algebra, e.g., $(a + b)(c + d) = ac + ad + bc + bd$.



Removing redundant entries: If a mist contains several identical entries, with the same color, shape, and sign, then redundant entries can be deleted so long as the ratios of distinct entries remain the same.

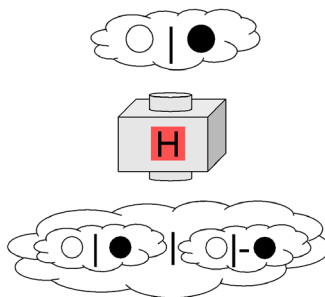


Dropping mist for single entry: If there is only one entry in the mist, then the mist can be dropped. For example:

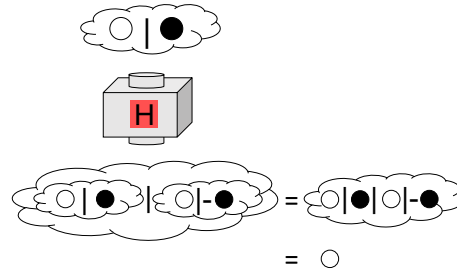


Passing mist through gates:

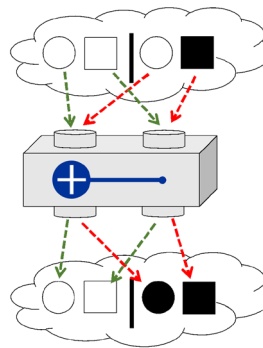
Each qubit that enters the mist is replaced by the qubit pair that results from passing through the H gate. For example, the white qubit when passing through H gate transforms into a white, positive black mist combination and the black qubit transforms into the white, negative black mist combination, as seen in the example above.



If we then follow the mist rules, then we remove the inner mist and then we *cancel entries* and *remove redundant entries*. Finishing with the output of one bit in 0 state



If a two-qubit mist is input into a two-qubit gate, the output can be obtained by separately passing each entry in the mist through the two-qubit gate. The shapes in each entry should be inserted into the input ports in the same order that they appear in the entry: the left shape (circle) is inserted into the left port, the right shape (square) into the right port. In this example involving the CNOT gate, quantum entanglement is created, because while the input mist can be factorized according to the mist rules, the same is not true of the output. Quantum entanglement refers precisely to this inability to factorize states.



Additional mist rules (less frequently used)

Factorizing mist: If the first few shapes in every entry have the same sequence of colors, then these shapes can be factored out of the mist on the left. Similarly, if the last few shapes in every entry have the same color sequence, these shapes can be factored out to the right. This is similar to factoring out a in an expression such as $ax + ay = a(x + y)$. For example:

