

The global ocean circulation: an elegant dynamical system

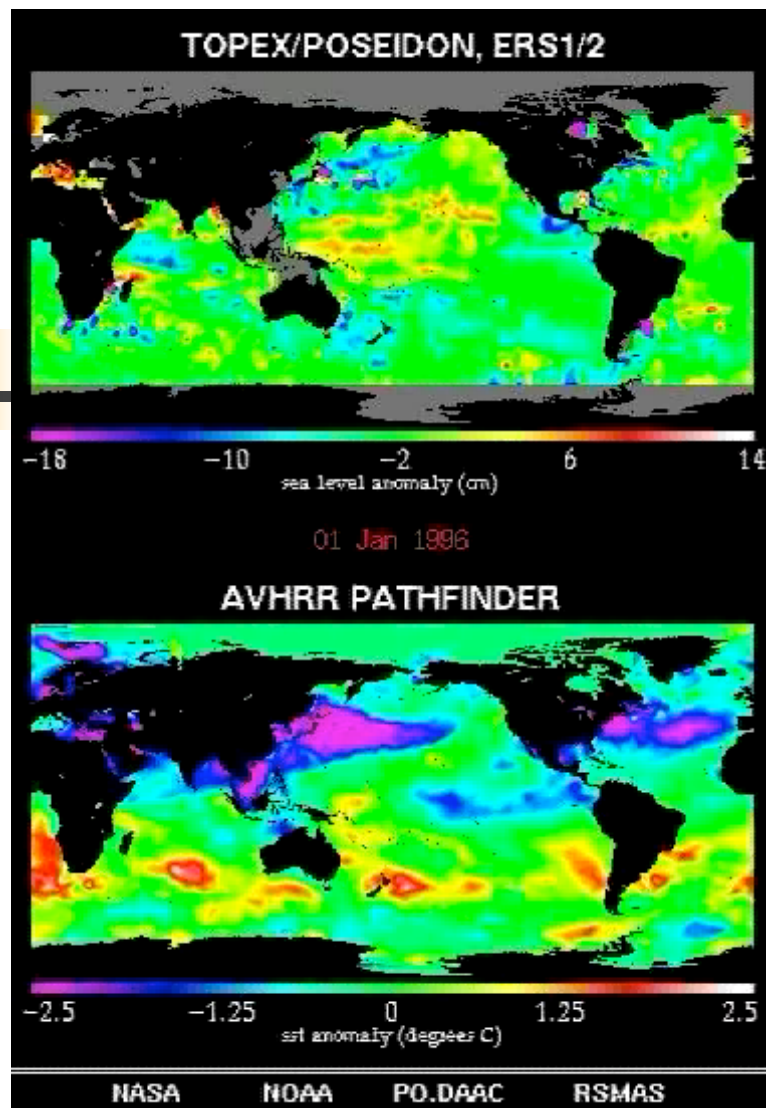


Henk Dijkstra

Institute for Marine and Atmospheric research Utrecht (IMAU),
Department of Physics and Astronomy, Utrecht University, The
Netherlands

Work with: [Lianke te Raa \(IMAU\)](#) & [Jeroen Gerrits \(OSU\)](#)

Variability at the ocean surface

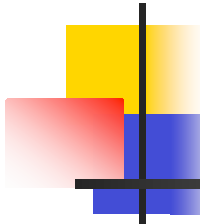


Sea surface height anomaly
Jan 1996 - Nov 1999
Scale -18 cm to +14 cm

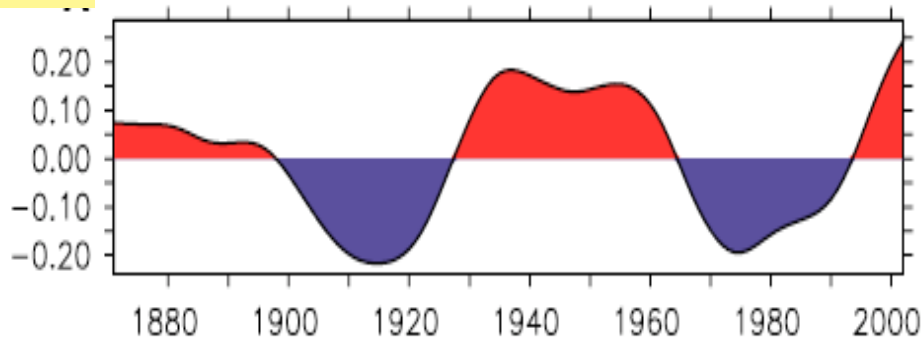
Sea surface temperature
anomaly (SSTA)
Jan 1996 - Nov 1999
Scale -2.5 C to +2.5 C

Courtesy NASA/JPL-Caltech

The Atlantic Multidecadal Oscillation (AMO)

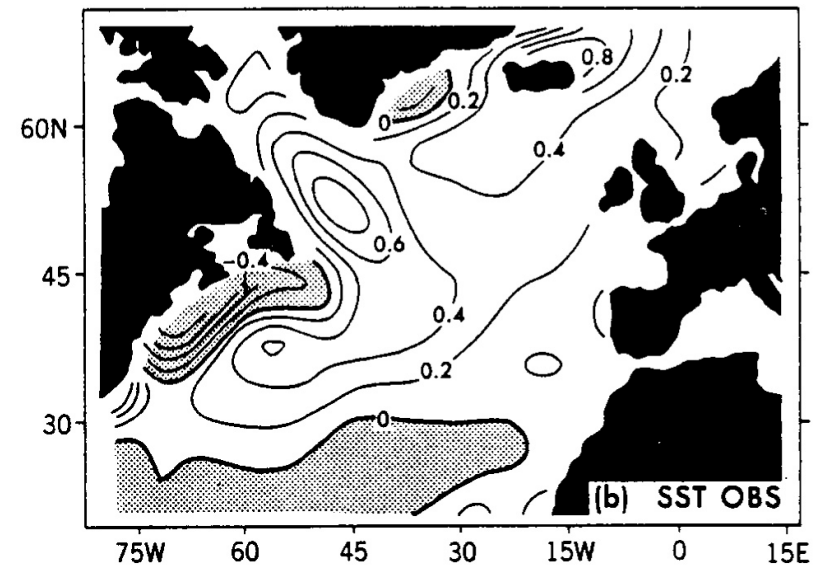


SSTA



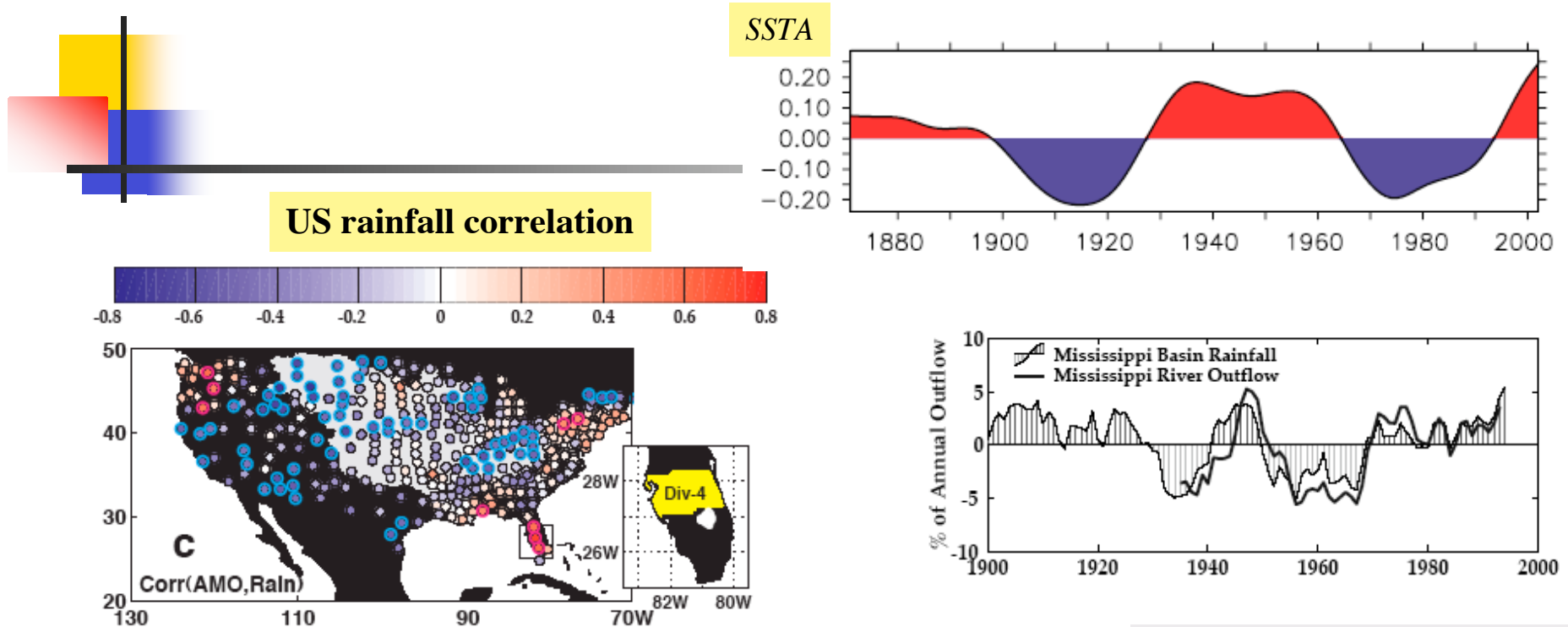
Average SSTA over the entire North Atlantic basin

SST (1950-1964) - SST (1970-1984)



Kushnir, J. Clim, 7, 141, 1994

Importance of the AMO

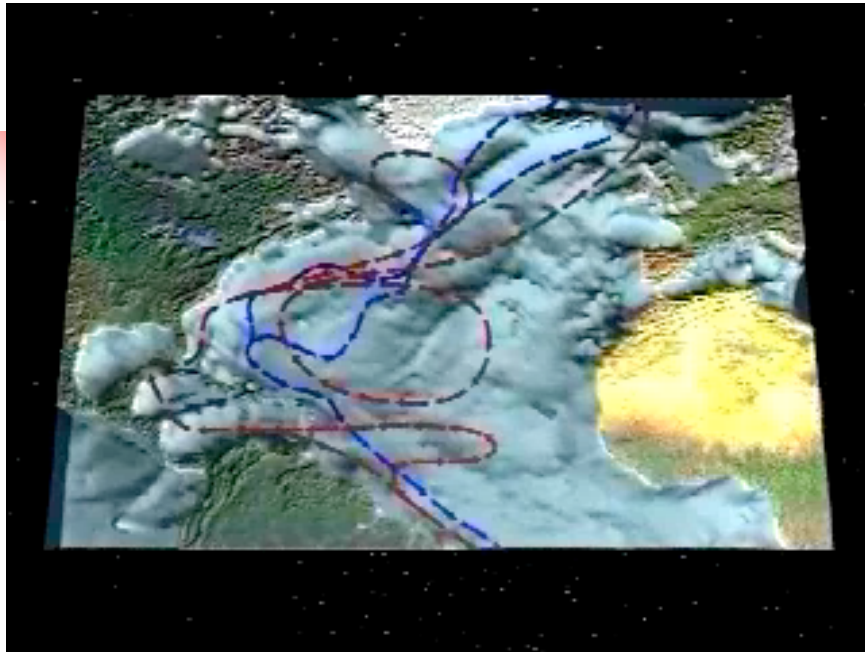


Enfield et al., GRL, 28, 2077, 2001

Question: What is the physics of the multidecadal variability in the North Atlantic?

Wanted: explanation for (i) AMO time scale, (ii) AMO pattern

The North Atlantic Ocean Circulation

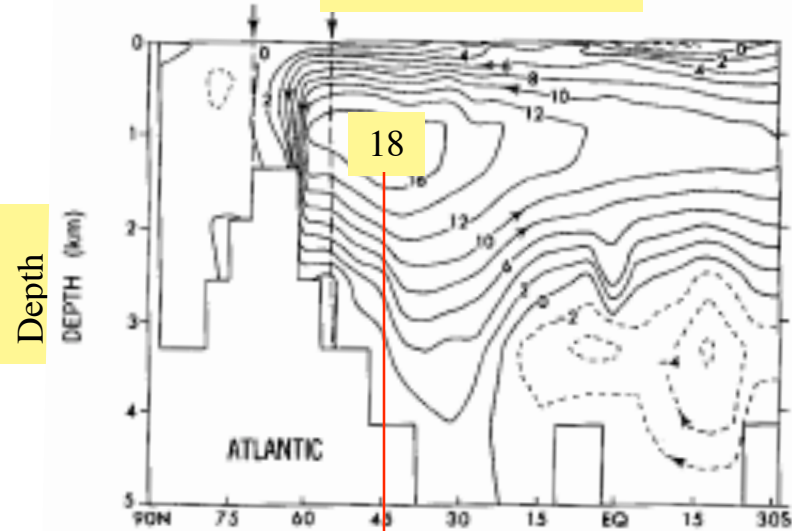


Zonally integrated transport

Meridional overturning streamfunction

Ψ

GFDL-R15 model



Depth

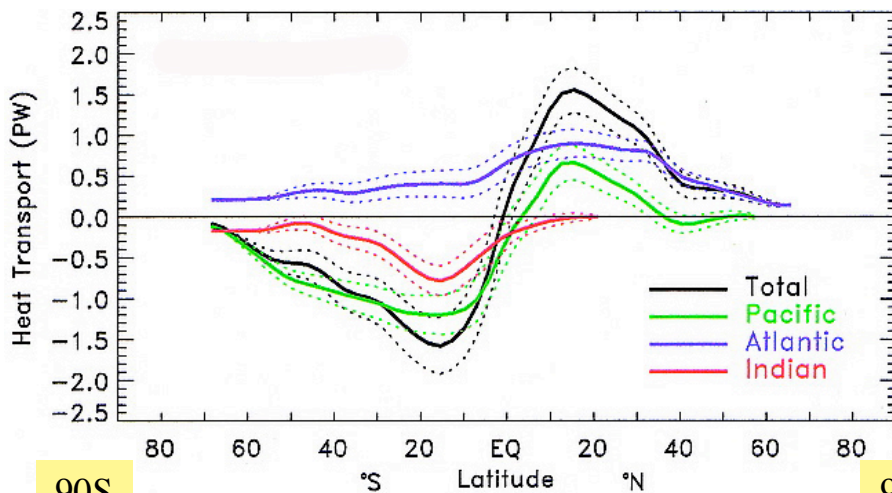
90N

30S

THC-index

1 Sv = 1,000,000 m³/s

Heat transport (PW)

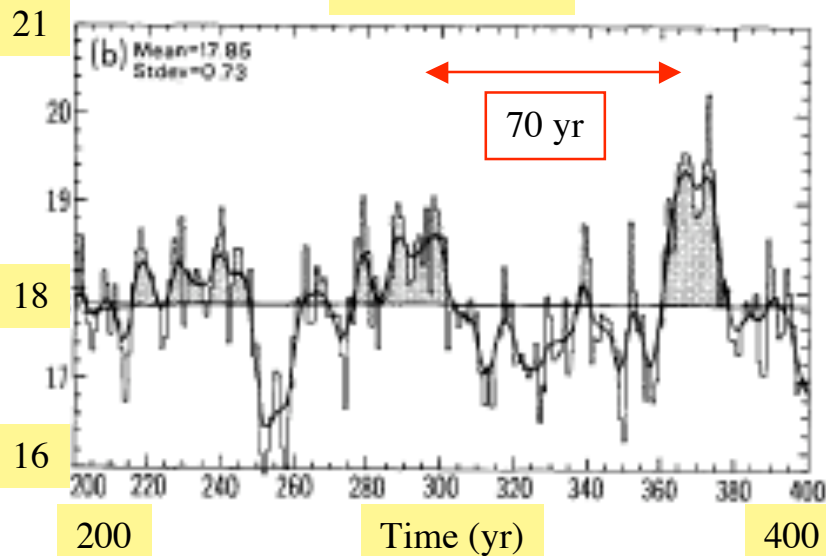


90S

90N

GFDL R 15 climate model results

THC-index

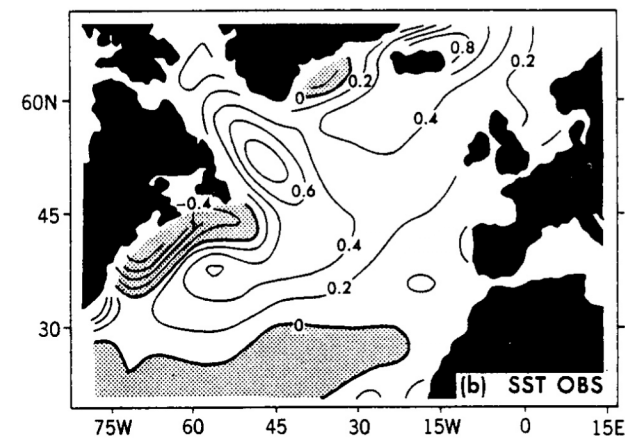
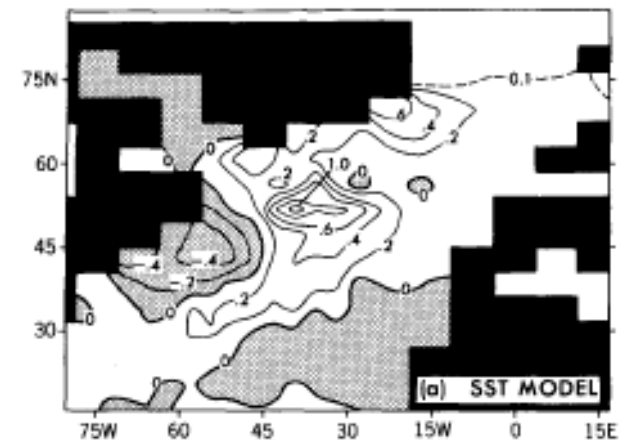


Oceanic grid: 96 x 40 x 12

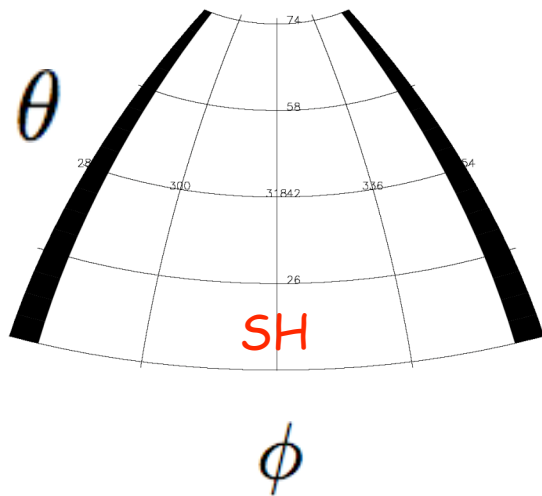
Atmospheric grid ~ 48 x 40 x 11

Simulation: 1000 yr

SST High THC index - Low THC index



Single-sector basin ocean model



Sector [286,350] x [10,74] (4 degree)
Depth 4000 m (16 levels)

$$\rho_0 \left(\frac{d\mathbf{u}}{dt} + 2\boldsymbol{\Omega} \wedge \mathbf{u} \right) = -\nabla p - g\rho\hat{z} + \mathcal{F}$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{dT}{dt} = K_H \nabla_H^2 T + K_V \frac{\partial^2 T}{\partial z^2}$$

$$\rho = \rho_0 (1 - \alpha(T - T_0))$$

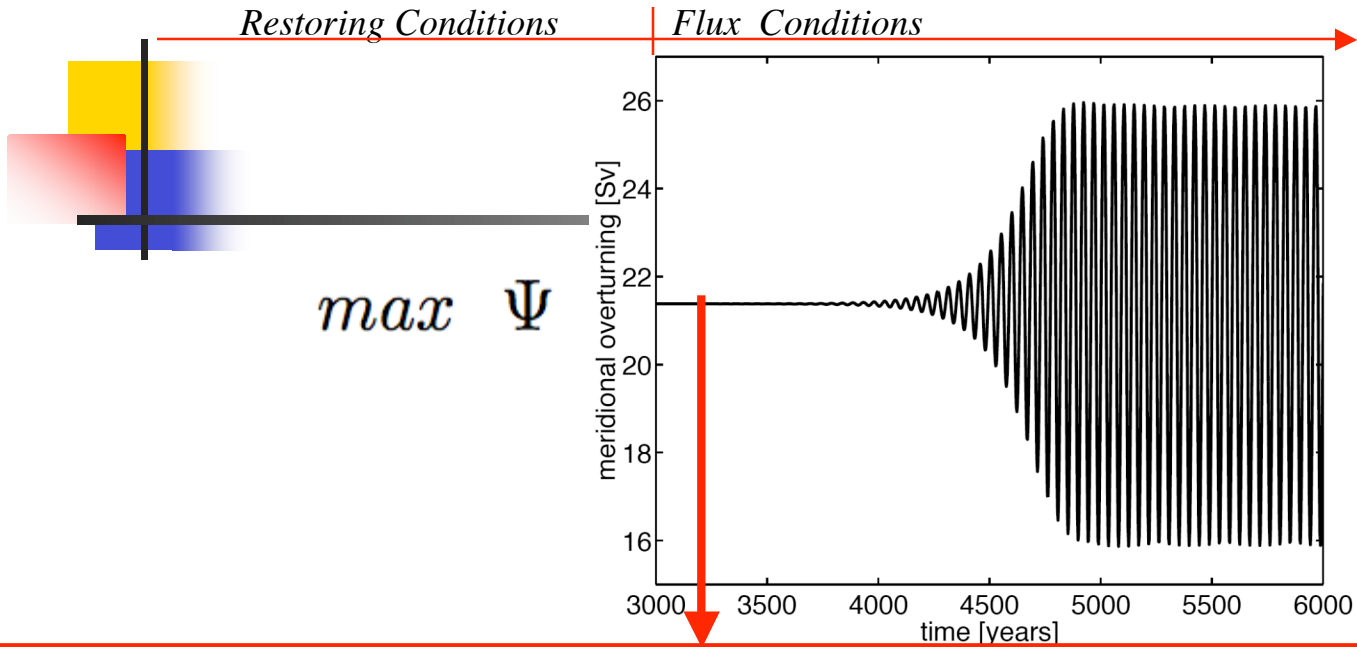
Restoring: $z = 0 : T = \Delta T T_S(\phi, \theta)$

or

Flux: $z = 0 : K_V \frac{\partial T}{\partial z} = \frac{Q_H}{\rho_0 C_p}$

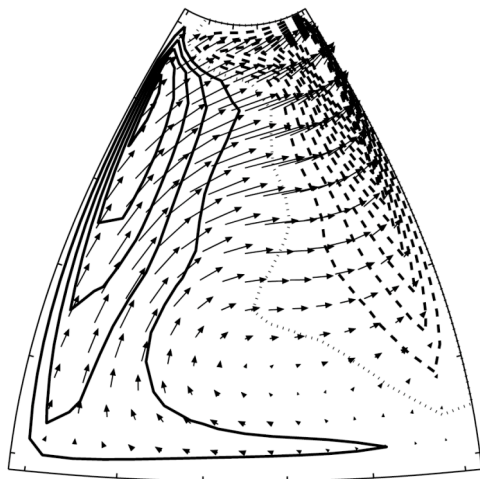
Control parameters: $K_H, \Delta T$

Ocean-only model results

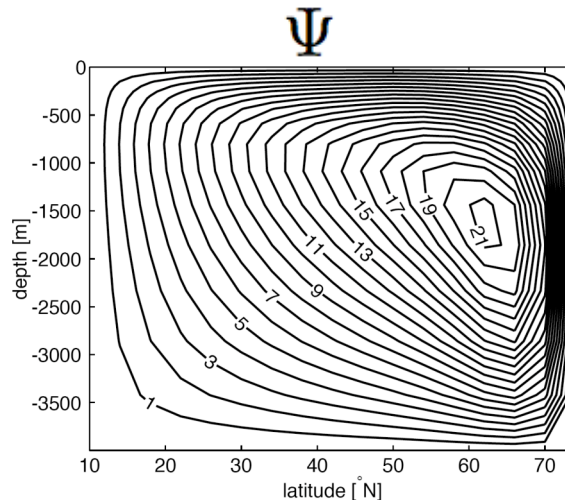


Spontaneous Oscillation:

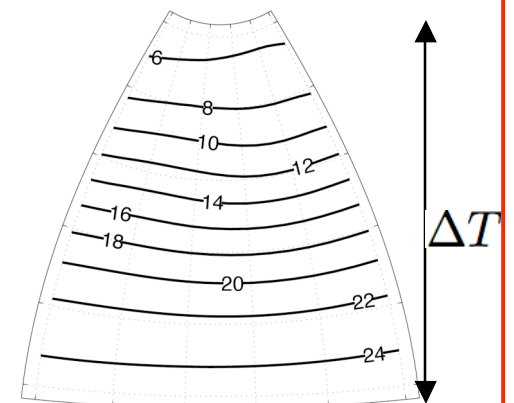
Period ~ 54 year



Surface velocities



Meridional Overturning



Surface temperature

Specific questions & approach

- Does the oscillation appear through an instability of the steady flow to a multidecadal mode (MM)?
 - Physical mechanism of such a MM.
- Do the spatial pattern and time scale of the AMO arise through the presence of such a MM?
 - Consider the MM in more complex models.

Continuation methods

$$M \frac{dx}{dt} + G(x, \lambda) = 0, x \in \mathbb{R}^d$$

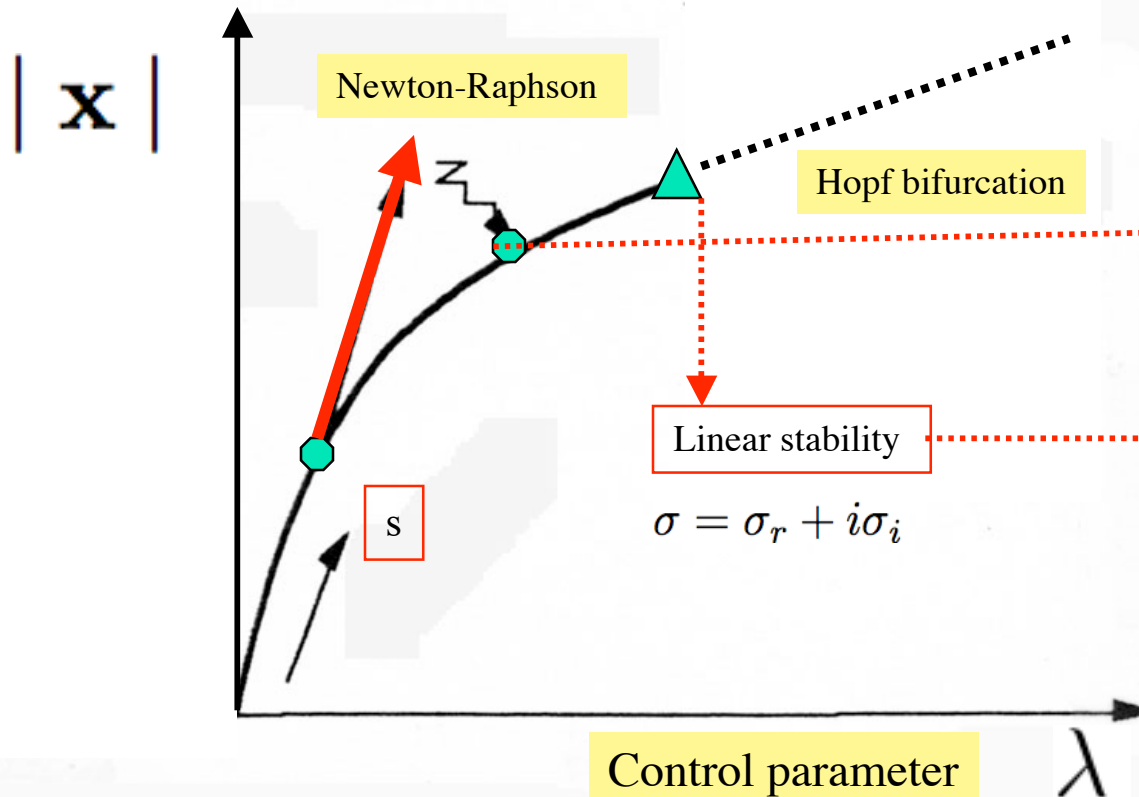
Numerics

Steady states

$$G(x, \lambda) = 0 \Rightarrow$$

$$J(x^k, \lambda^k) \Delta x^{k+1} = -G(x^k, \lambda^k)$$

$Ax = b$
 $A : d \times d$
 MRILU
 GMRES



Linear stability

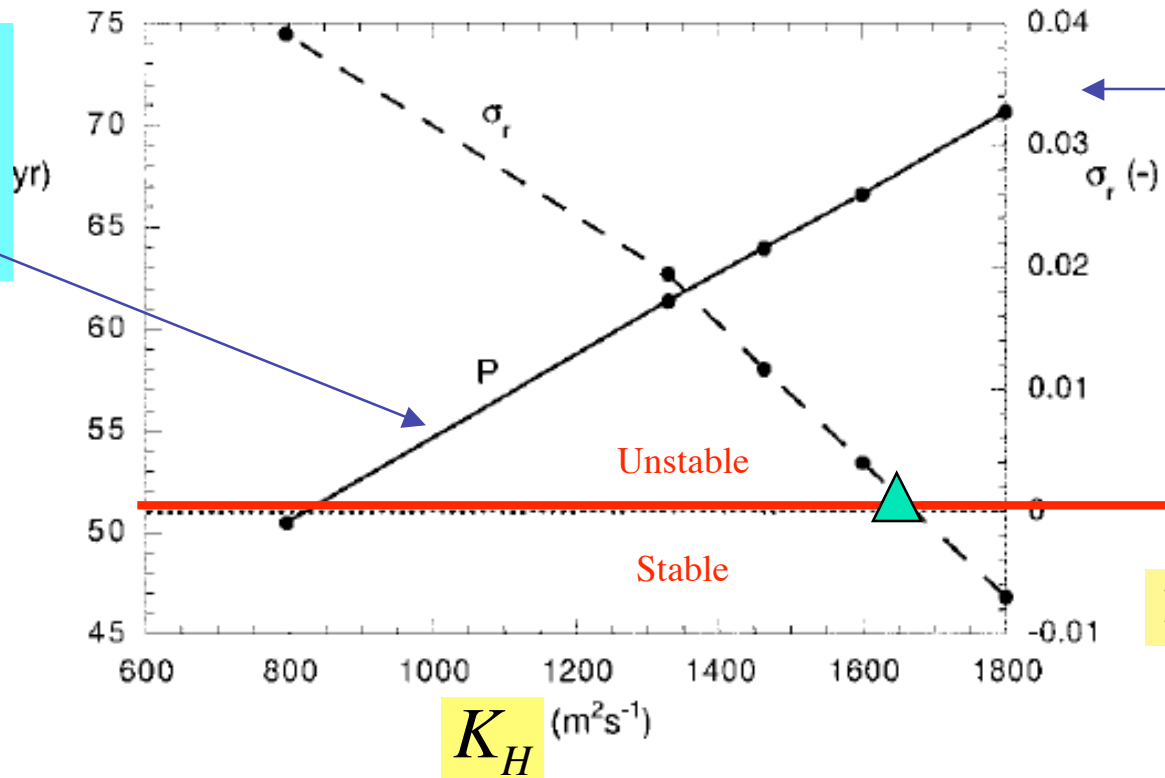
$$\sigma = \sigma_r + i\sigma_i$$

$Ax = \sigma Bx$
 $A, B : d \times d$
 JDQZ

$d = 50,000$

Linear stability of the steady states

Period
(years)

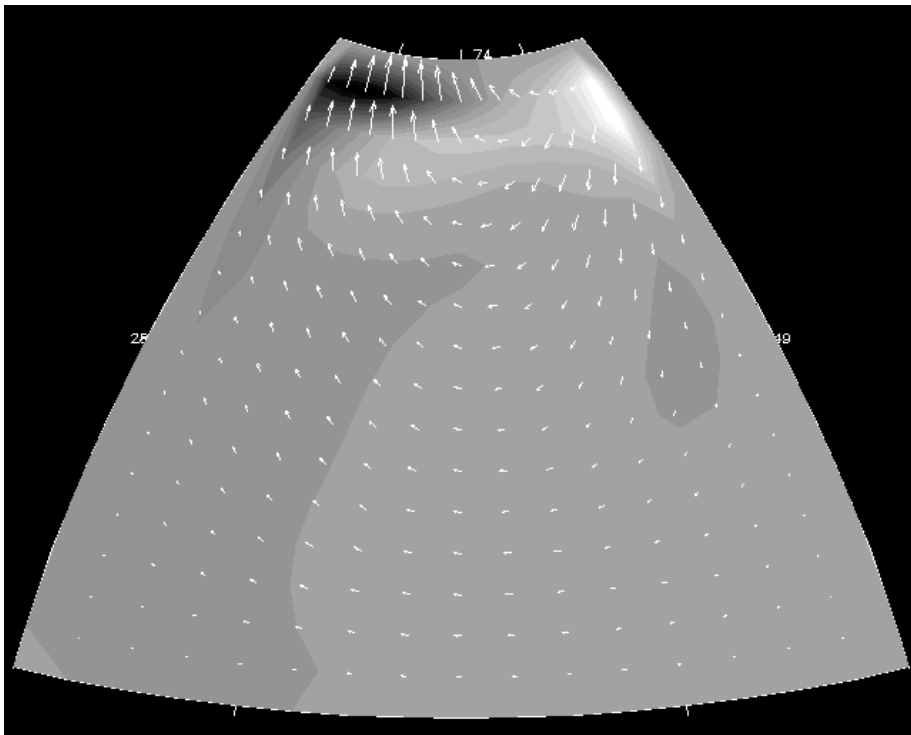


Growth
Rate of
MM

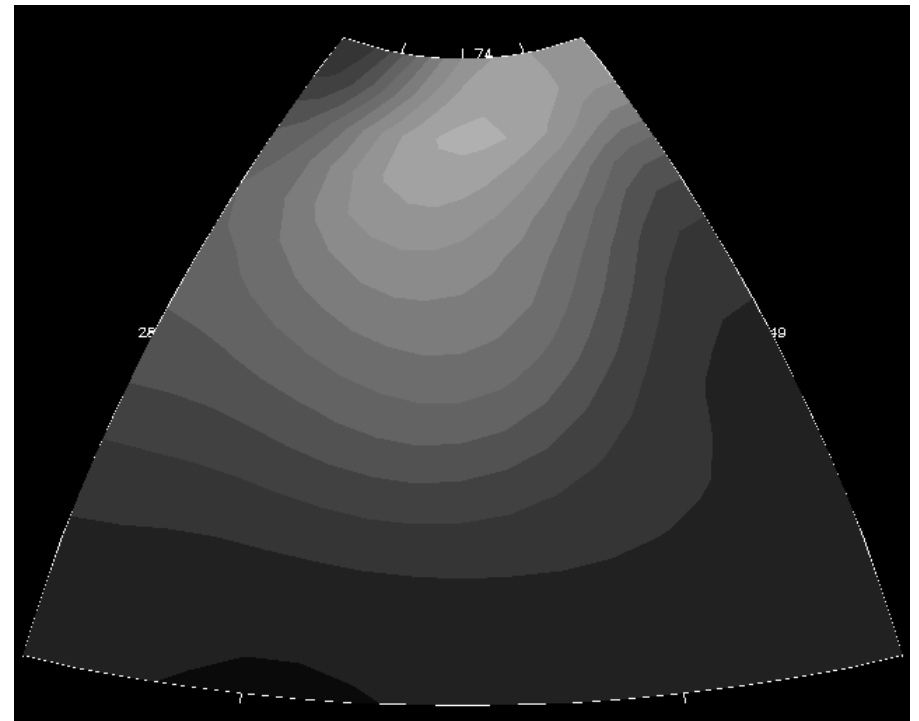
Hopf bifurcation

Lateral mixing of heat

Patterns of the MM near Hopf bifurcation



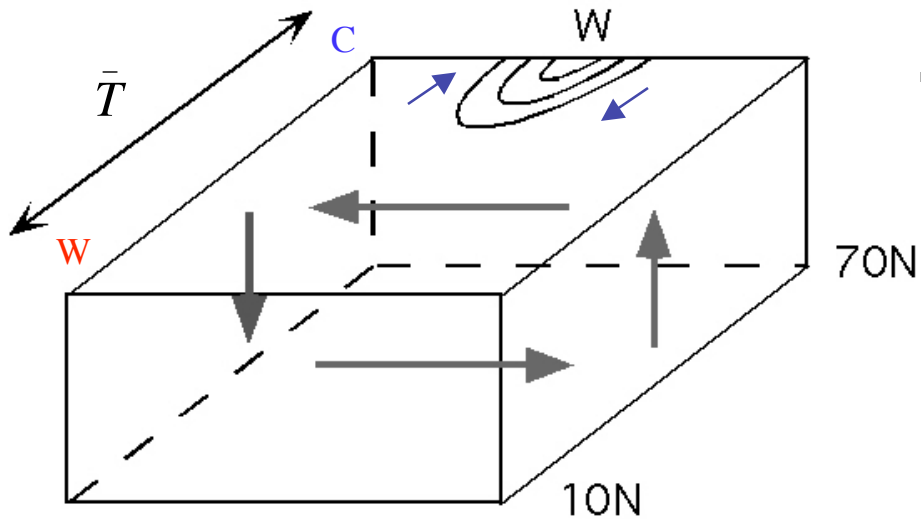
Surface velocity



Surface temperature

Control parameter: Horizontal mixing coefficient of heat, $K_H = 1600 \text{ m}^2 \text{ s}^{-1}$

Physics of the oscillation

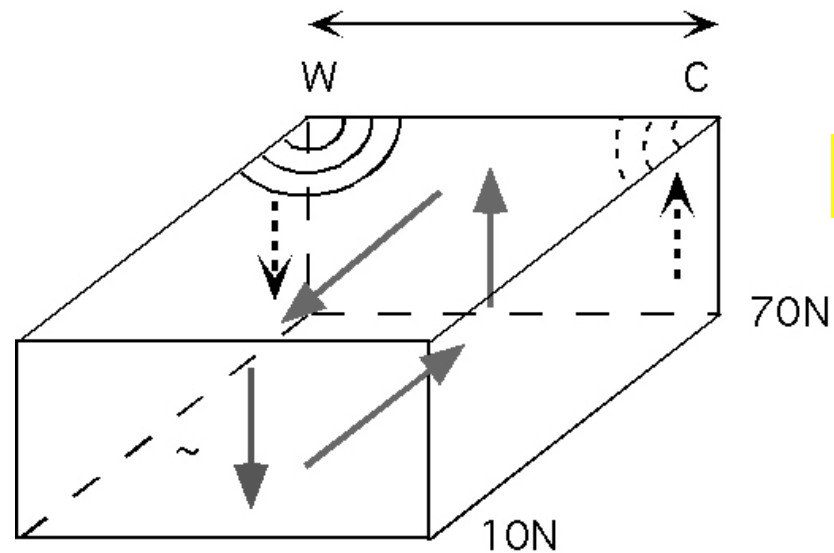


Thermal Wind:

$$\sin \theta \frac{\partial u}{\partial z} \sim -\frac{\partial T}{\partial \theta}$$

$$\sin \theta \frac{\partial v}{\partial z} \sim \frac{1}{\cos \theta} \frac{\partial T}{\partial \phi}$$

Phase $t = 0$

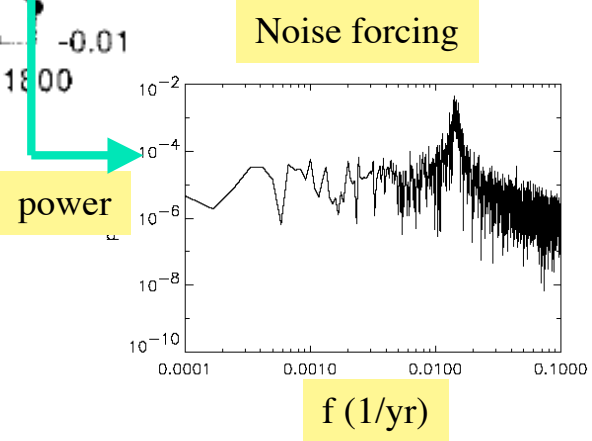
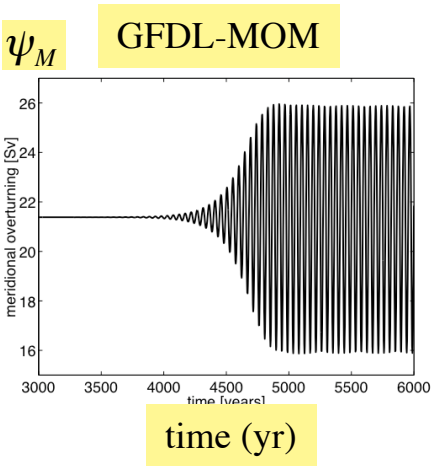
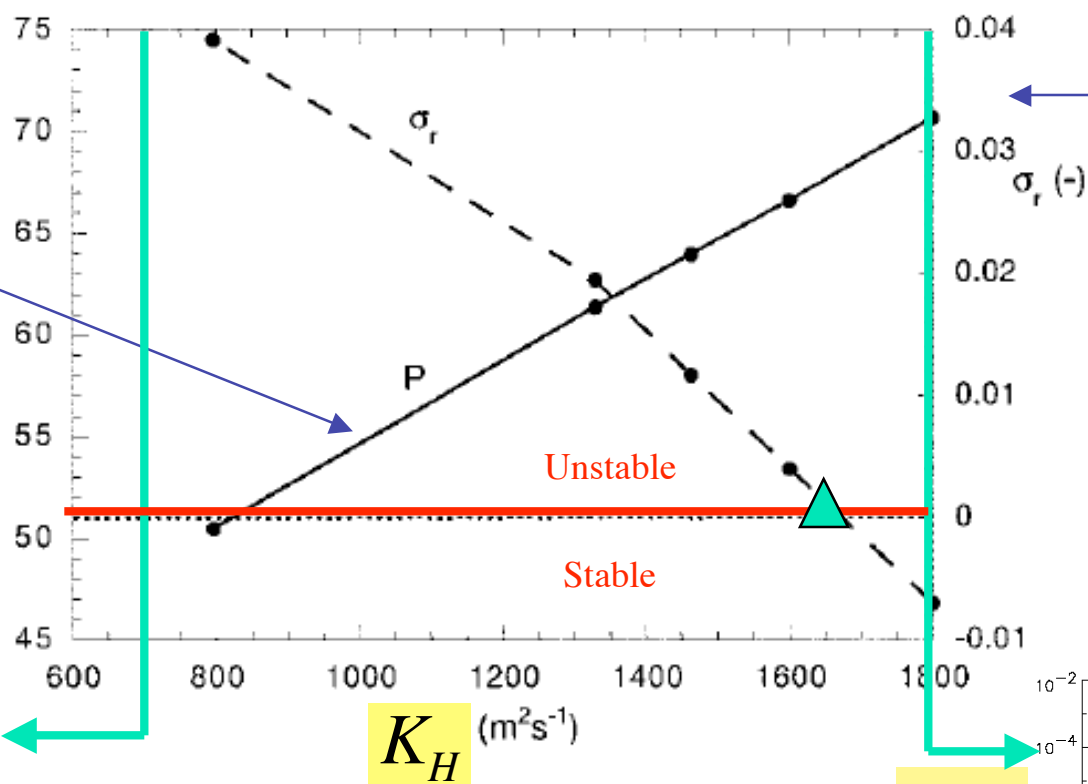


Phase $t = P/4$

Transient behavior

Period
(years)

Growth
Rate of
MM

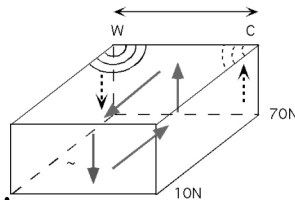
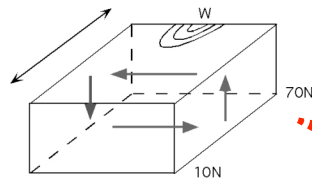


K_H ($m^2 s^{-1}$)

Lateral mixing of heat

Mechanistic indicators

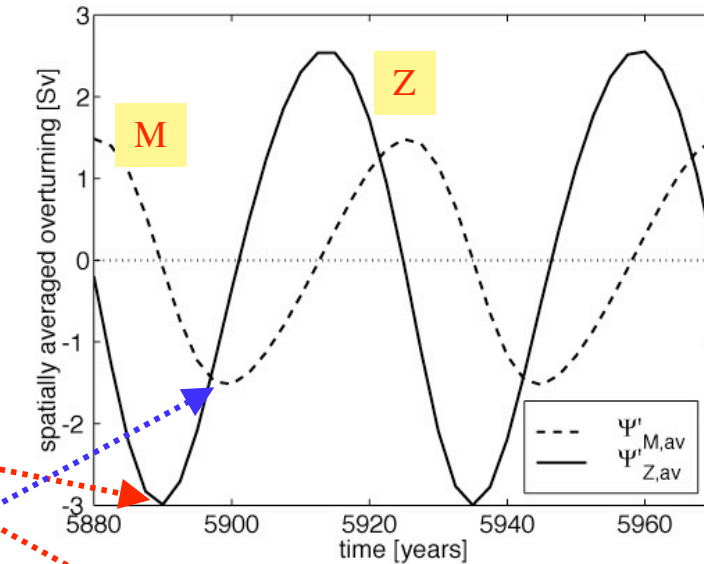
- **M1**: Phase difference between zonal (Z) and meridional (M) overturning anomalies



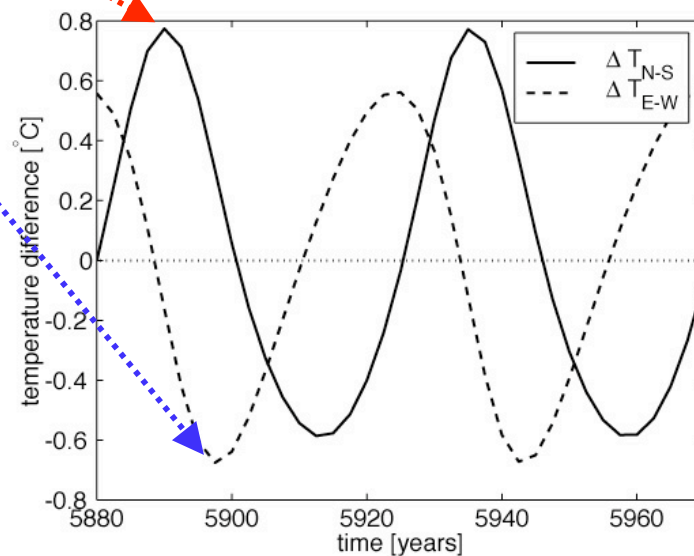
- **M2**: Phase difference between E-W and N-S temperature differences

Others: terms in energy balances ...

$$K_H = 700 \text{ m}^2 \text{ s}^{-1}$$



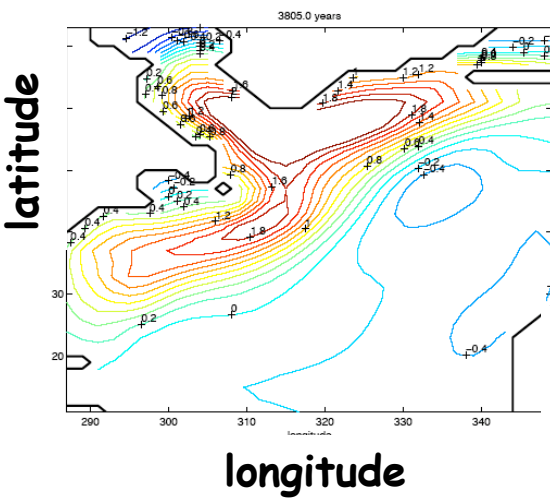
M1



M2

Effects of continents

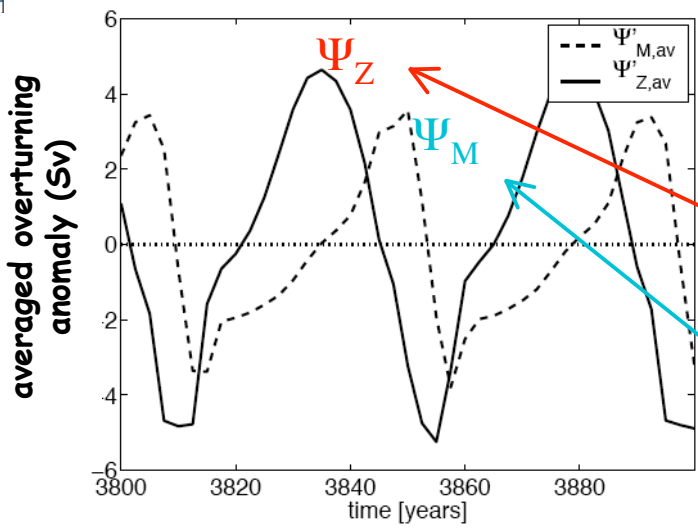
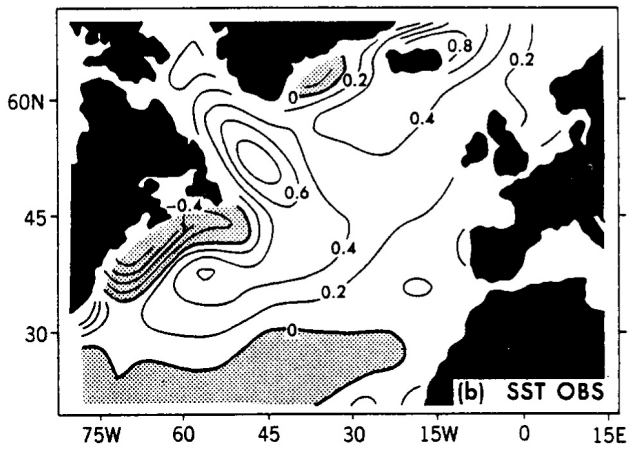
GFDL-MOM
 SST (Ψ_M max.) -
 SST (Ψ_M min.)



$P=45$ yr

Observations

SST (1950-1964) -
 SST (1970-1984)

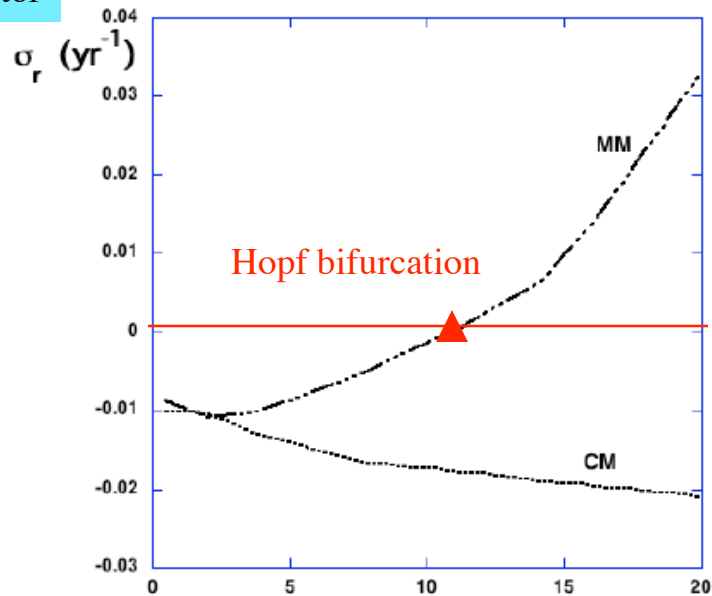


Zonal overturning

Meridional overturning

Spectral origin of the MM

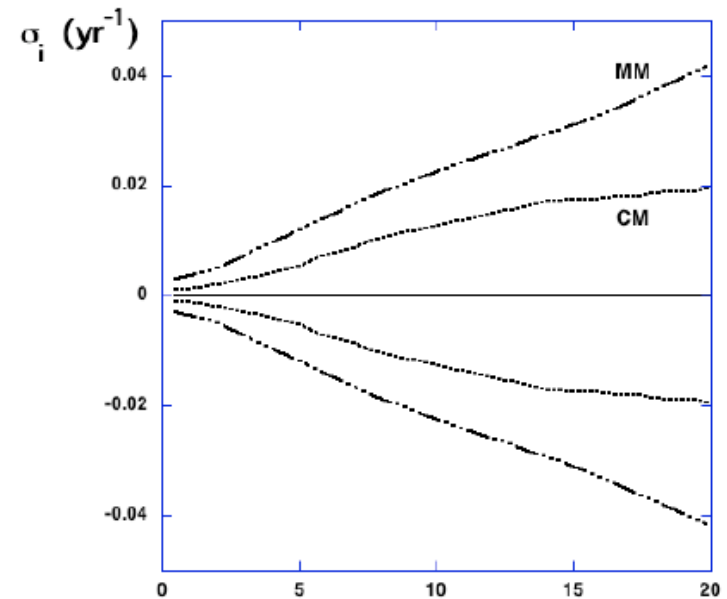
Growth factor



Temperature difference

ΔT

Angular frequency

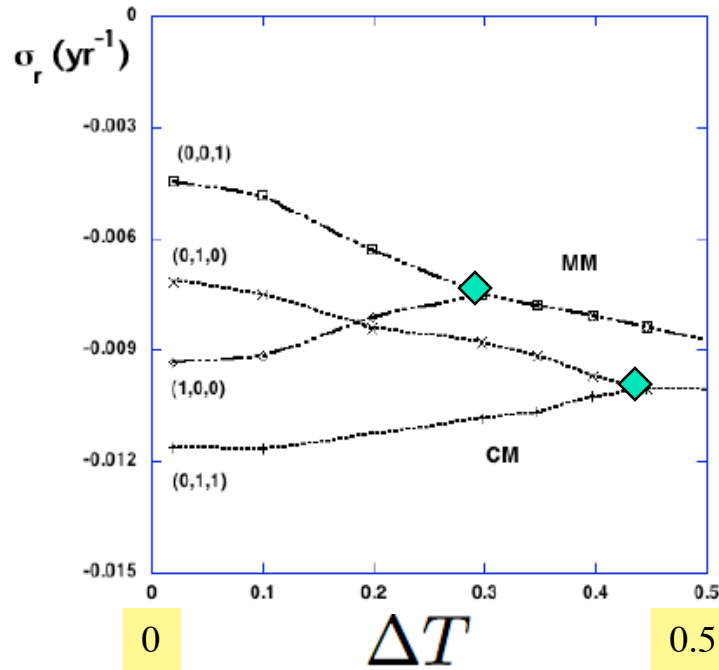


Temperature difference

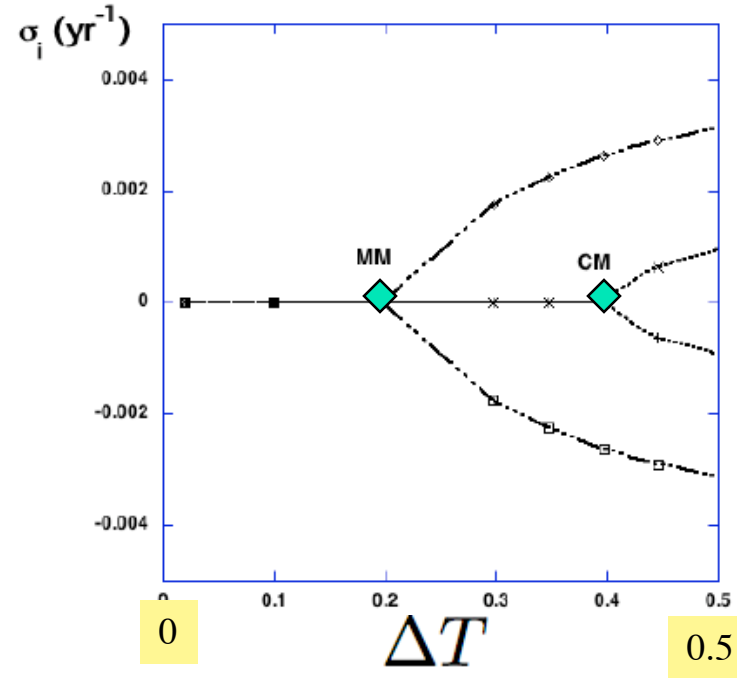
ΔT

SST-mode merger

Growth rate



Frequency

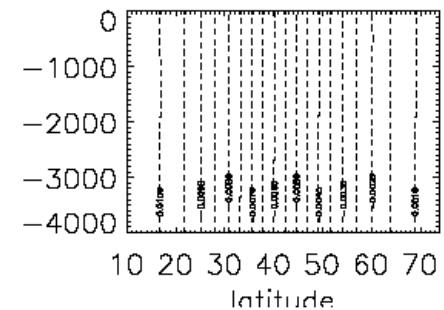
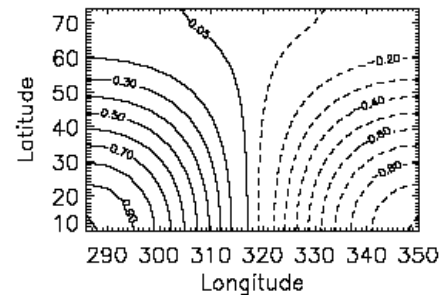


Ex: (1,0,0)

$\Delta T \rightarrow 0$: SST - modes

$$\frac{\partial T}{\partial t} = K_H \nabla_H^2 T + K_V \frac{\partial^2 T}{\partial z^2}$$

+ BC's



Mode mergers provide a framework to understand low-frequency variability of the global ocean circulation

Momentum equations:

$$\mathcal{M}_\lambda \frac{\partial \mathbf{v}}{\partial t} = \mathcal{F}_\lambda(\mathbf{v}, \bar{T}) \quad \blacksquare \quad \mathbf{v} = \hat{\mathbf{v}} e^{\sigma t} \Rightarrow \mathcal{J}_\lambda \hat{\mathbf{v}} = \sigma \mathcal{M}_\lambda \hat{\mathbf{v}} \quad \longrightarrow$$

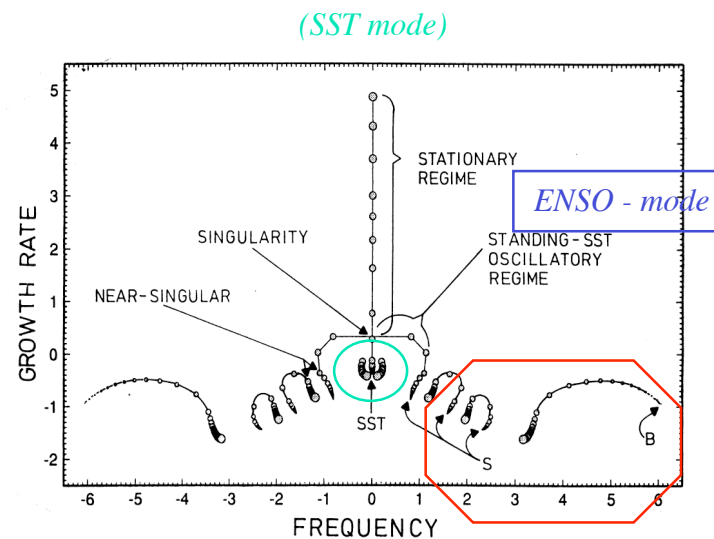
Ocean dynamics modes:
Ex. Rossby Basin modes, Gyre modes

Energy equation:

$$\frac{\partial T}{\partial t} = \mathcal{G}_\lambda(\bar{\mathbf{v}}, T) \quad \blacksquare \quad T = \hat{T} e^{\sigma t} \Rightarrow \mathcal{I}_\lambda \hat{T} = \sigma \hat{T} \quad \longrightarrow$$

Sea surface temperature modes:
(SST-modes)

Prominent example:
El Nino mode



(equatorial dynamics modes)

Summary of main points

A multidecadal mode (MM) exists in models of the North Atlantic climate system.

Its spectral origin is an SST-mode merger

Its propagation mechanism is a lagged response of the zonal and meridional flow perturbations due to propagating temperature (density) perturbations.

The MM may be an important factor in the AMO:

Time scale: basin crossing time

Pattern: deformation of the pattern of the MM due to the continental boundaries

Atmospheric noise may have a substantial effect on the amplitude of the AMO

Further reading ...

Nonlinear Physical Oceanography

**A Dynamical Systems Approach to the Large
Scale Ocean Circulation and El Niño
2nd Revised and Enlarged Edition**

by

Henk A. Dijkstra



Atmospheric and
Oceanographic
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April 15, 2005

Price: \$ 89