Sedimentation of particles

How can such a simple problem be so difficult?

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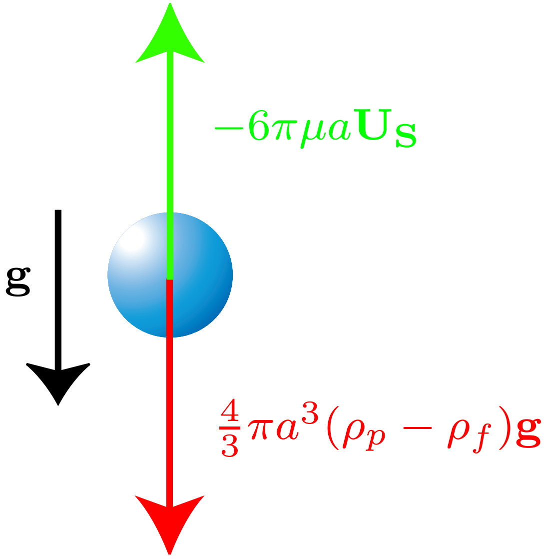
Sedimentation of particles

- $Re = aU/\nu \ll 1 \quad \Rightarrow$ Stokes' flow
- $Pe = aU/D \gg 1 \quad \Rightarrow$ Only hydrodynamics
- Solid and monodisperse particles for simplicity!

Sedimentation of particles

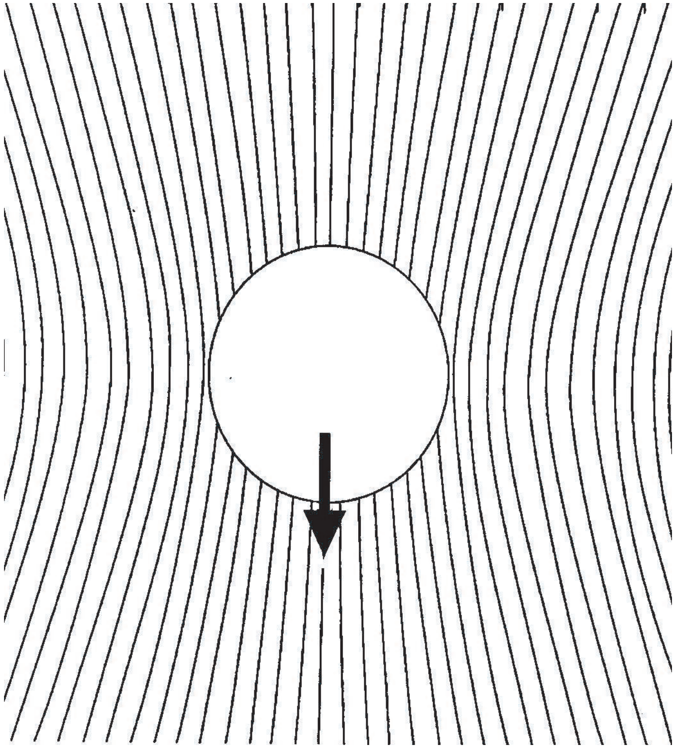
- Sedimentation of a suspension of spheres
- Sedimentation of a suspension of fibers
- Sedimentation of a cloud of particles

Sedimentation of a single sphere



Stokes’ velocity

$$\mathbf{U_S} = 2(\rho_p - \rho_f)a^2\mathbf{g}/9\mu$$



Pozrikidis 1997

Long-range interactions

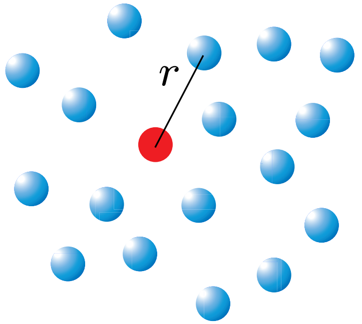
$$u \sim O(\frac{aU_S}{r})$$

Stokes 1851

Uniformly dispersed spheres

- Velocity of a pair of spheres at a separation r:

$$U_S + \Delta U(r)$$



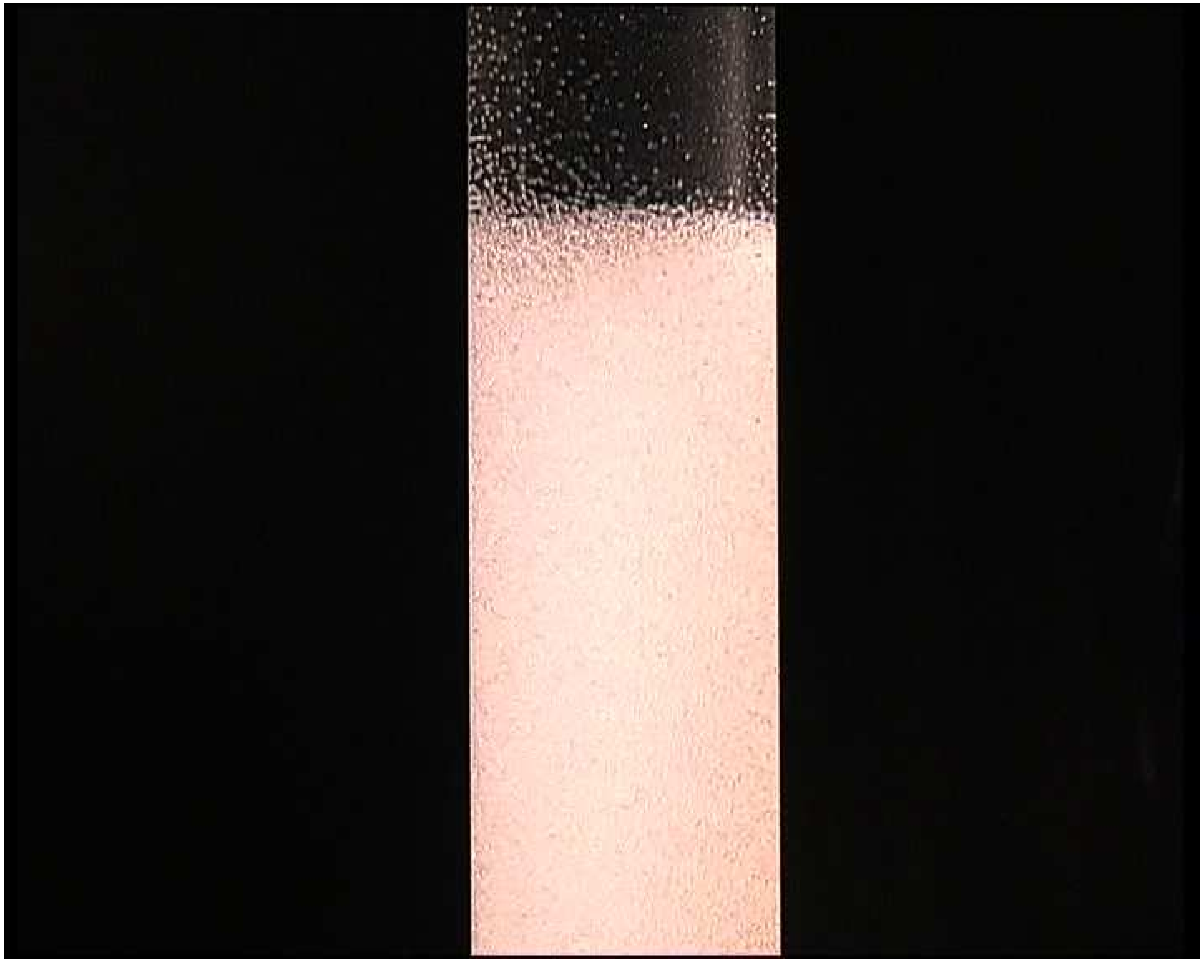
- Averaging over all possible separations occurring with probability $p(r)$:

$$\langle U \rangle = U_S + \int \quad \Delta U(r)\, p(r)\, dr^3 \qquad \text{diverges!}$$

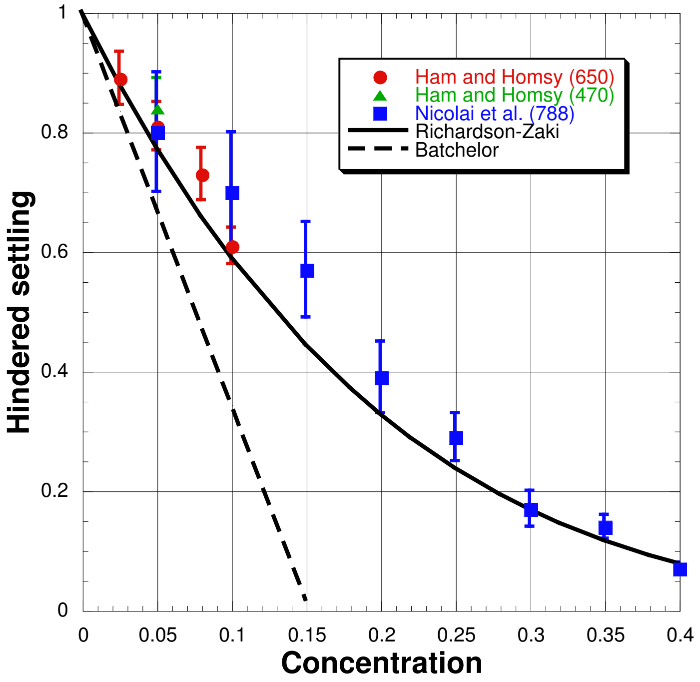
$$O\left(\frac{aU}{r}\right) O(1) O(r^2) \qquad \text{as} \quad r \to \infty$$

Multibody long-range hydrodynamic interactions

Sedimentation of spheres in a vessel

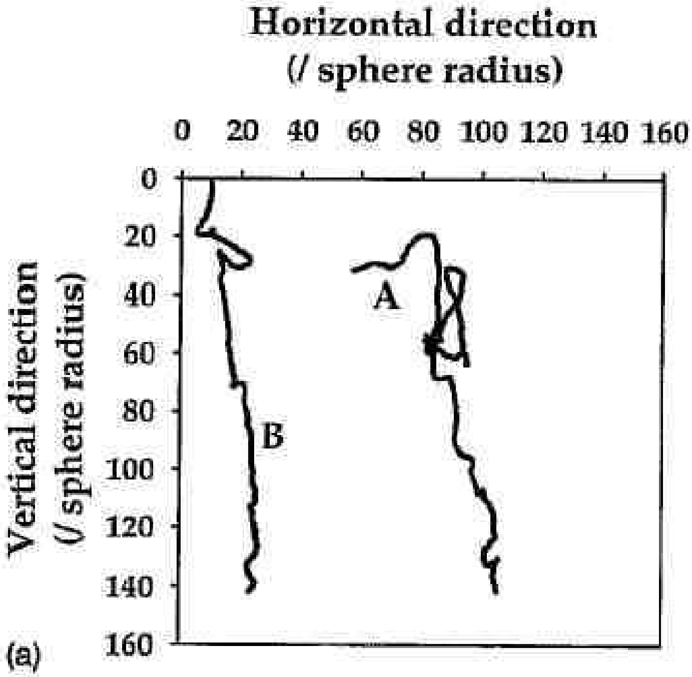


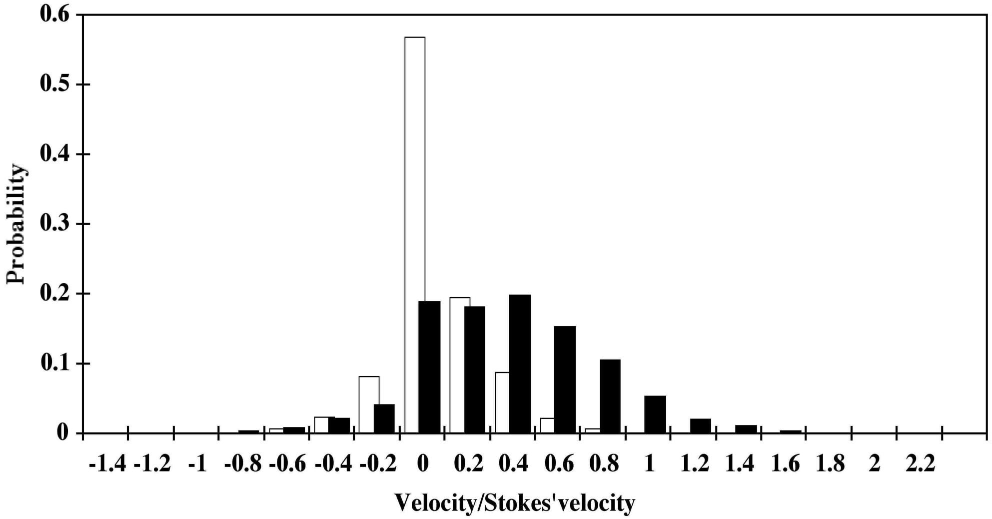
Mean velocity



- Hindered settling: $\langle U \rangle = U_S f(\phi)$ Richardson-Zaki 1954: $f(\phi) = (1-\phi)^5$
- Main effect = Back-flow
- Batchelor 1972: $f(\phi) = 1 - 6.55\phi + O(\phi^2)$ assuming uniformly dispersed spheres
- Results depend on microstructure in turn determined by hydrodynamics

Velocity fluctuations

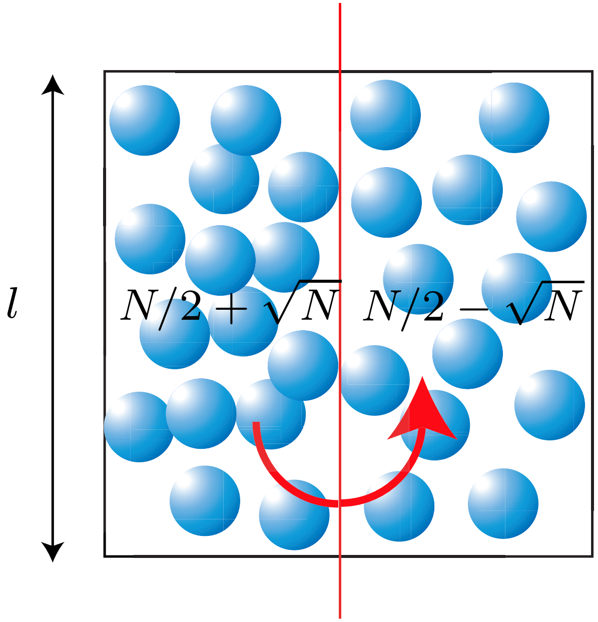




Large anisotropic fluctuations

Sphere-tracking in an index-matched suspension
Ham & Homsy 1988, Nicolai *et al.* 1995

Divergence of velocity fluctuations?



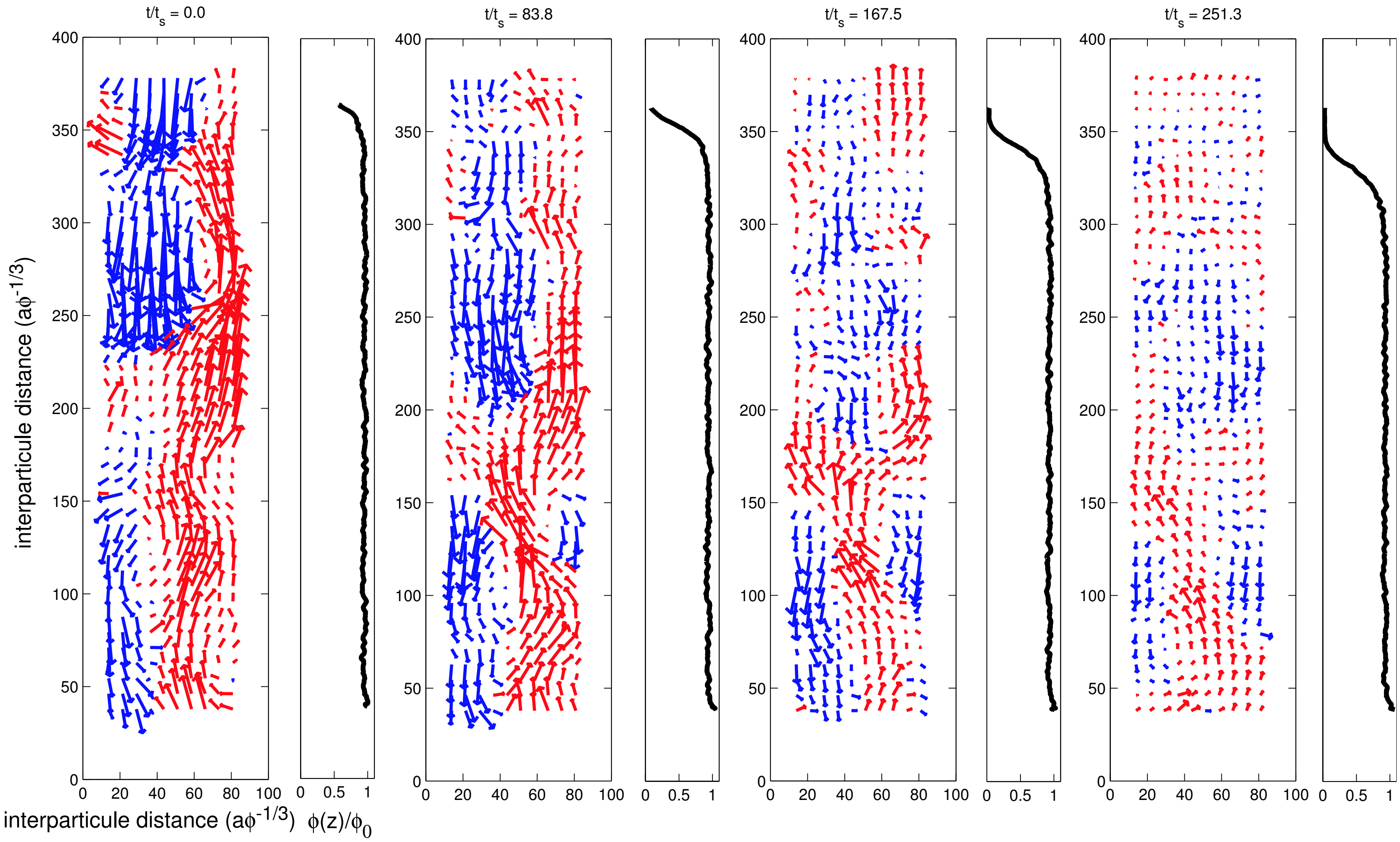
- Randomly distributed particles
- Box of size $a\phi^{-1/3} < l < L$
- Statistical fluctuations $\sqrt{N} \rightarrow$
  $$\Delta U_{\parallel} \sim \frac{\sqrt{N}\frac{4}{3}\pi a^3(\rho_s-\rho)g}{6\pi\mu l} \sim U_S\sqrt{\phi\frac{l}{a}}$$
- Large-scale fluctuations are dominant
  $\Delta U_{\parallel} \sim U_S\sqrt{\phi\frac{L}{a}}$ diverges!
  Caflisch & Luke 1985, Hinch 1988

BUT no such divergence seen in experiments
Nicolai & Guazzelli 1995, Segrè *et al.* 1997, Guazzelli 2001

More theories . . .

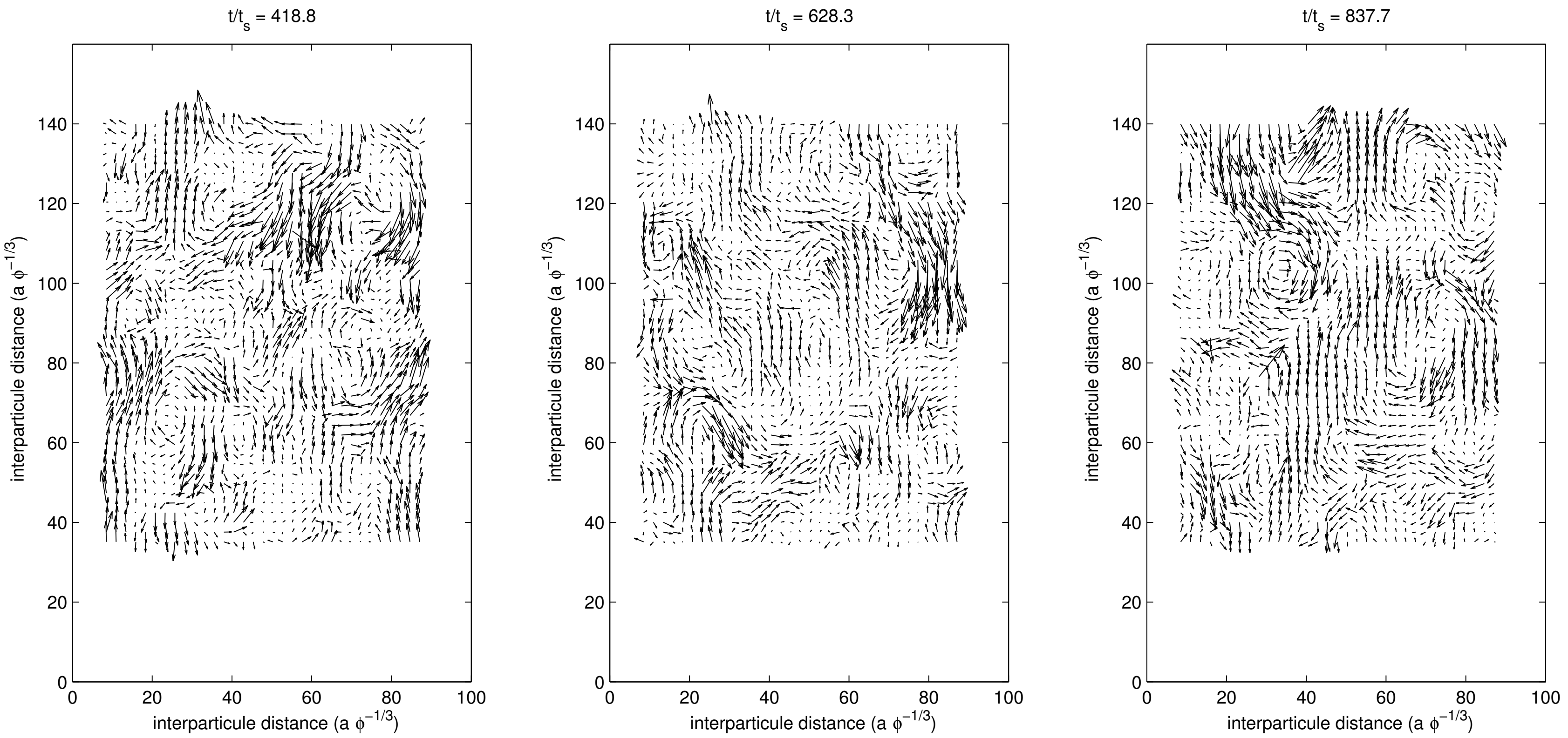
- Koch & Shaqfeh 1991: a non-random microstructure
- Tong & Ackerson 1998: turbulent convection analogy
- Levine *et al.* 1998: stochastic model
- da Cunha 1995, Ladd 2002: impenetrable bottom
- Brenner 1999: wall effect
- Luke 2000: stratification → fluctuation decay
- Tee *et al.* 2002, Mucha *et al.* 2003-04: diffusive spreading of the front → stratification → fluctuation decay
- Nguyen & Ladd 2005: polydispersity → stratification
- Hinch 1985, Asmolov 2004, Luke 2005: bottom and top = sink of large-scale disturbances

Relaxation of large-scale fluctuations



Initially, the large-scale fluctuations dominate the dynamics. But, they are transient as the heavy parts settle to the bottom and light parts raise to the top.

Left with smaller-scale fluctuations



Then, smaller-scale fluctuations are dominant until the arrival of the upper sedimentation front.

Why fluctuation length-scale $\sim 20a\phi^{-1/3}$?

Chehata, Bergougnoux, Guazzelli, & Hinch 2005

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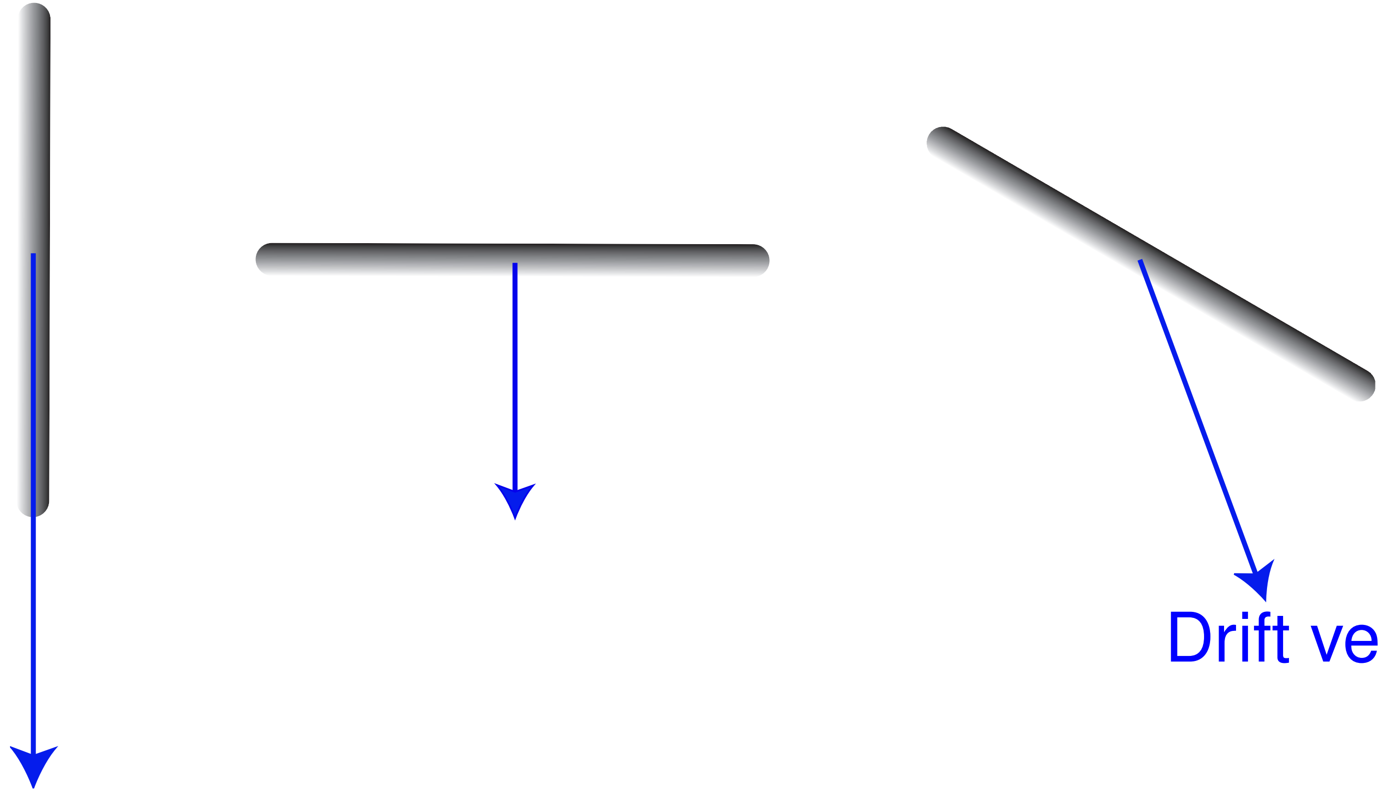
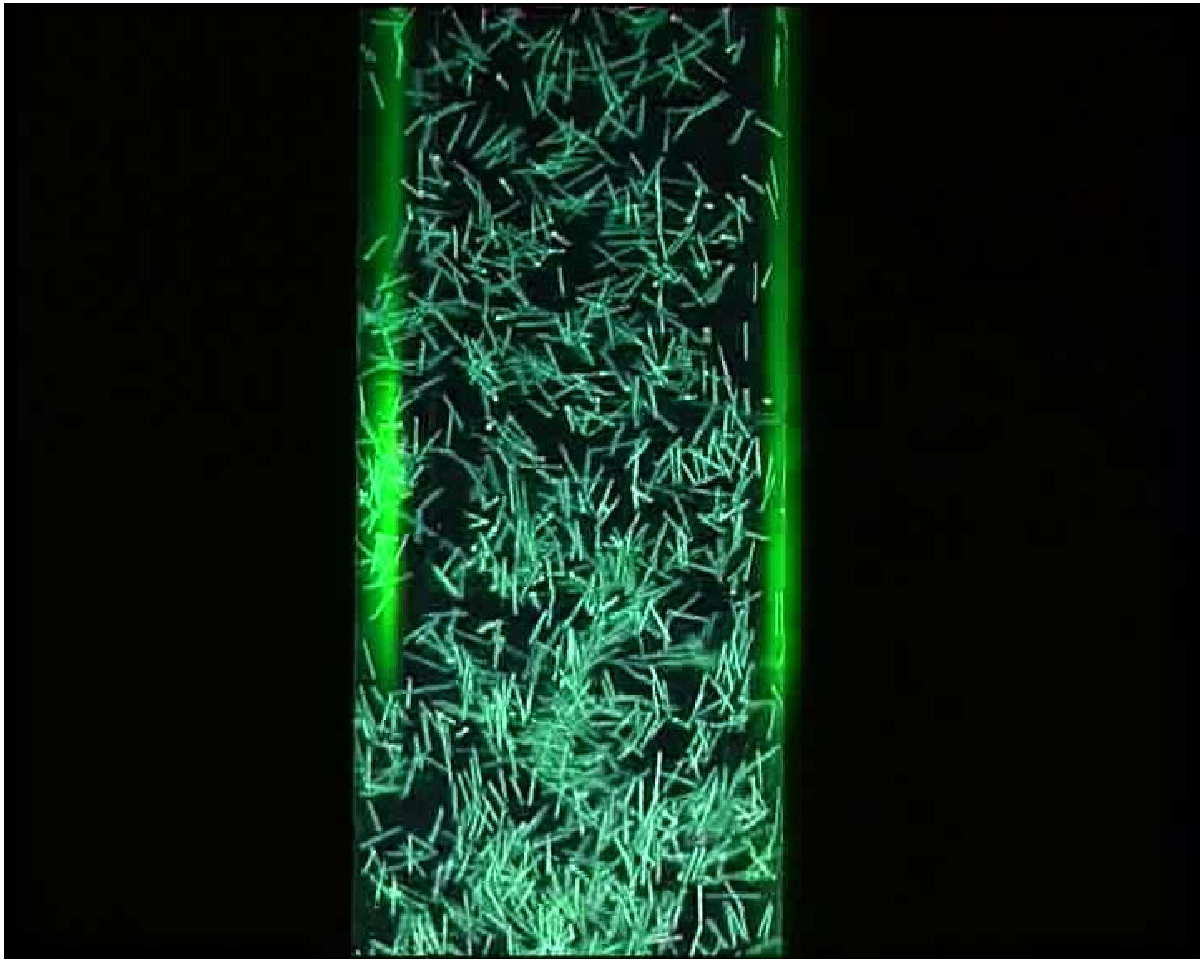
Sedimentation of a single fiber



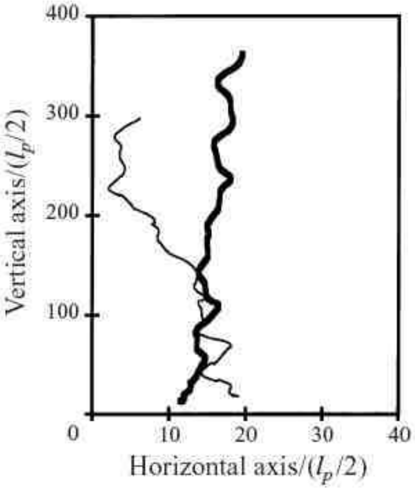
Drift velocity

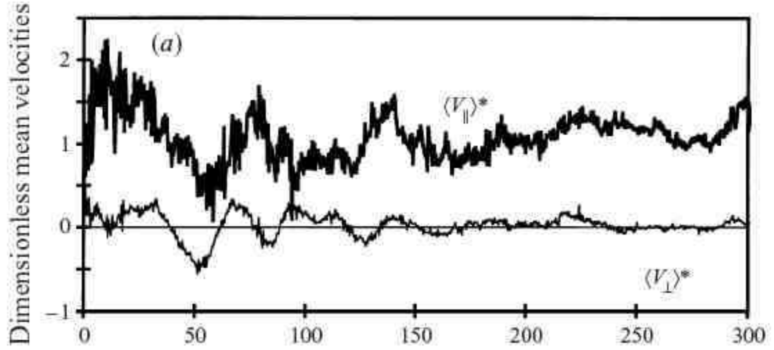
Coupling between orientation and velocity

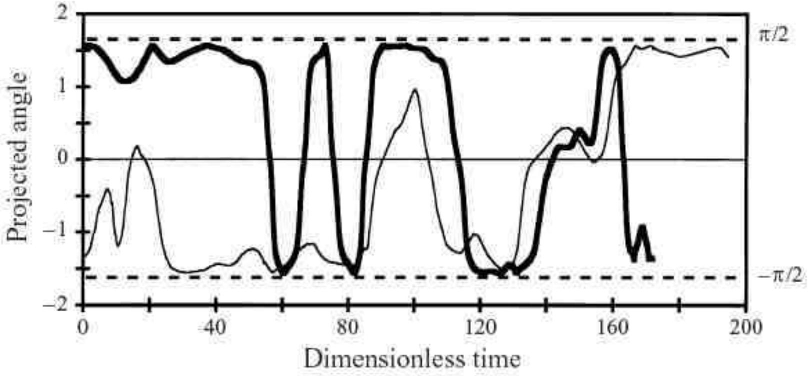
Sedimentation of fibers in a vessel

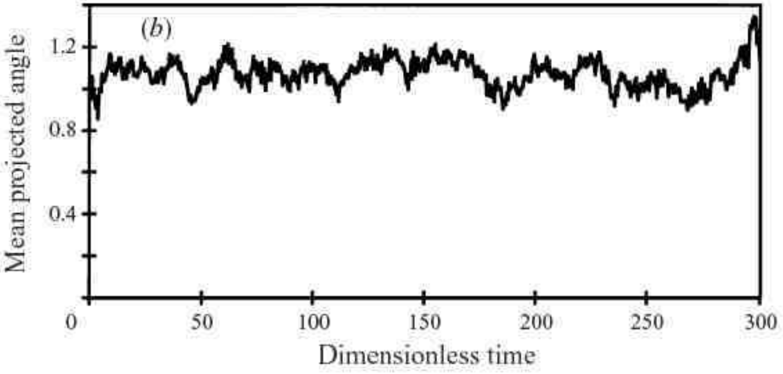


Mean Velocity and orientation





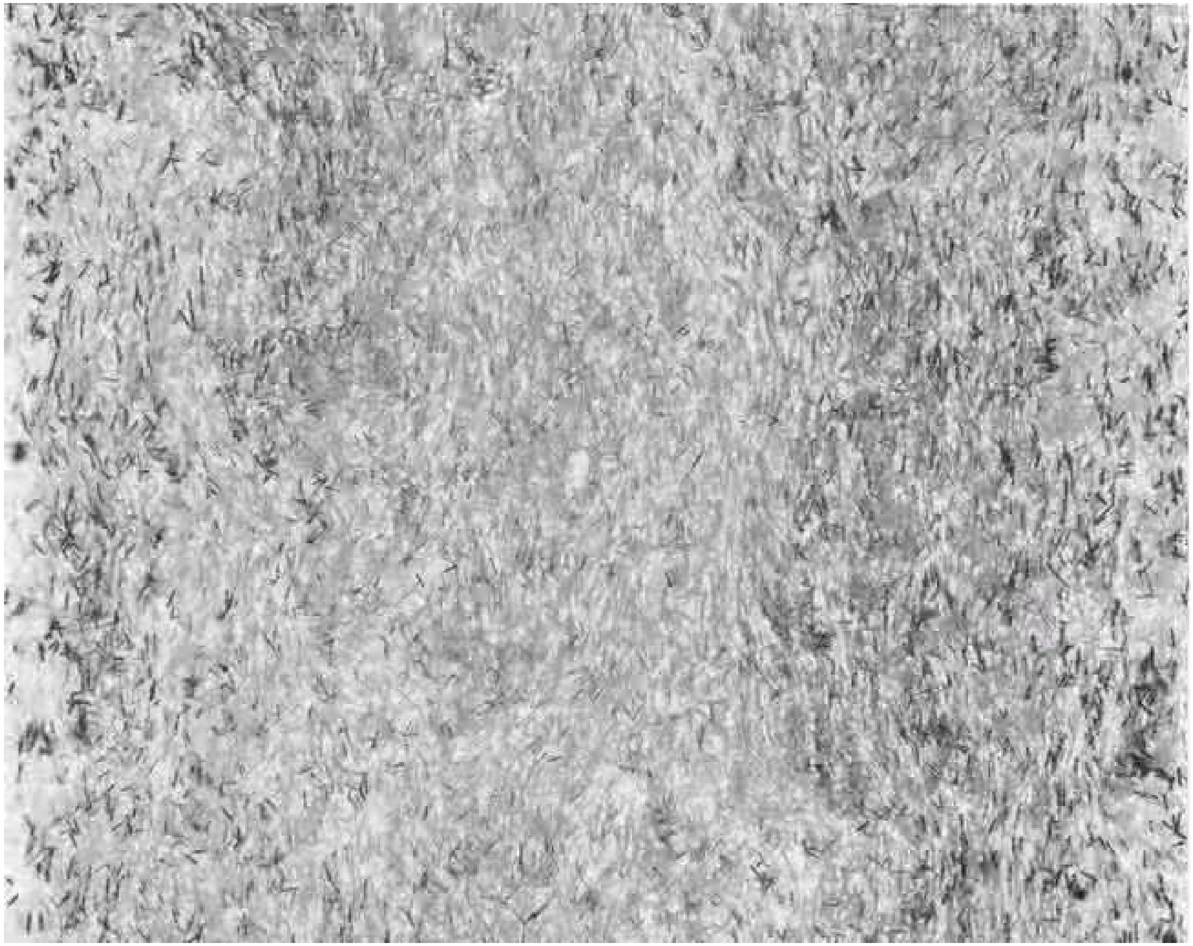




Enhanced sedimentation and vertical orientation

Fiber-tracking in an index-matched suspension
Herzhaft & Guazzelli 1999

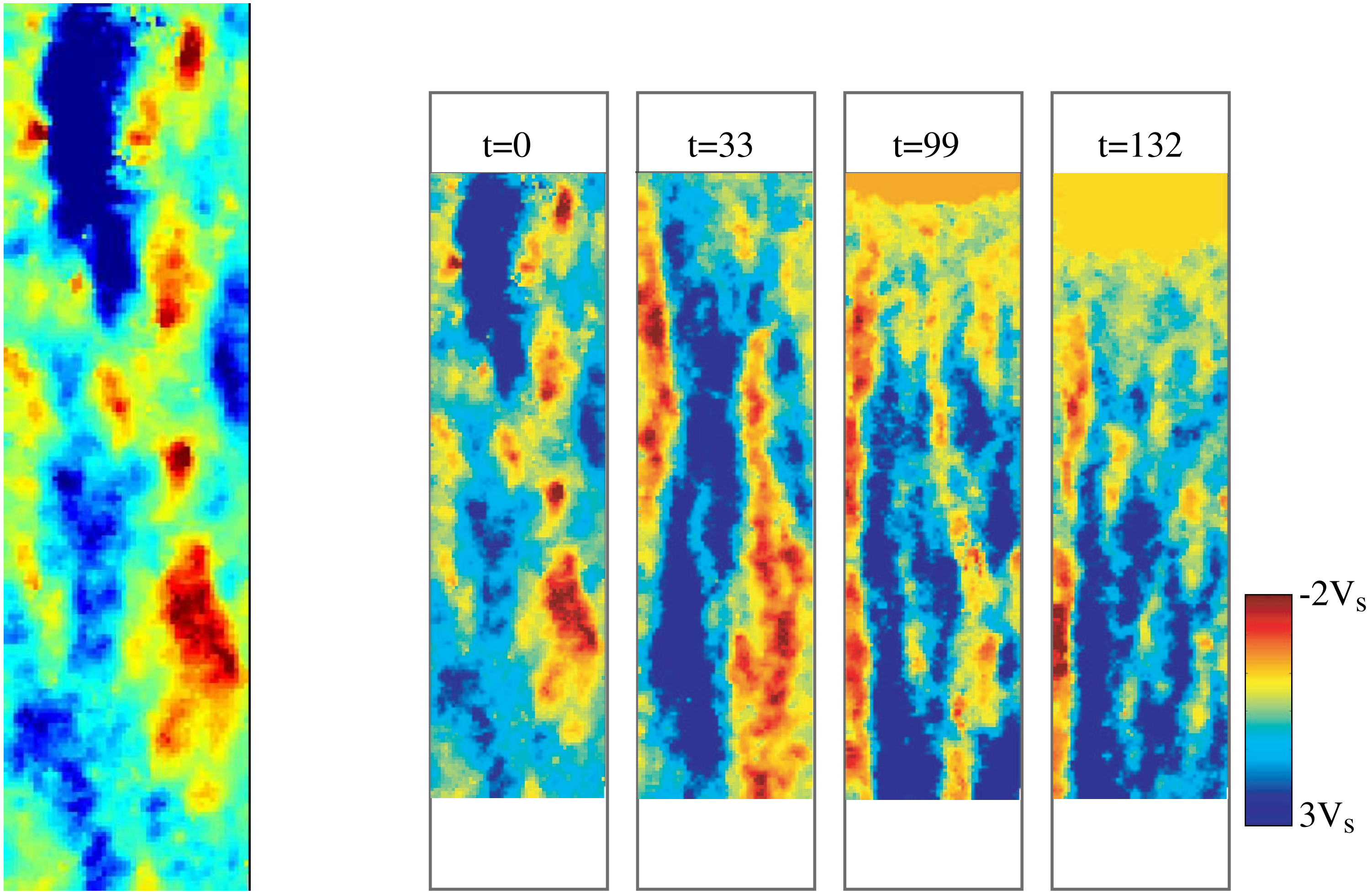
Packet instability → Streamers



Fluorescing fibers within a laser sheet

Metzger, Guazzelli, & Butler 2005

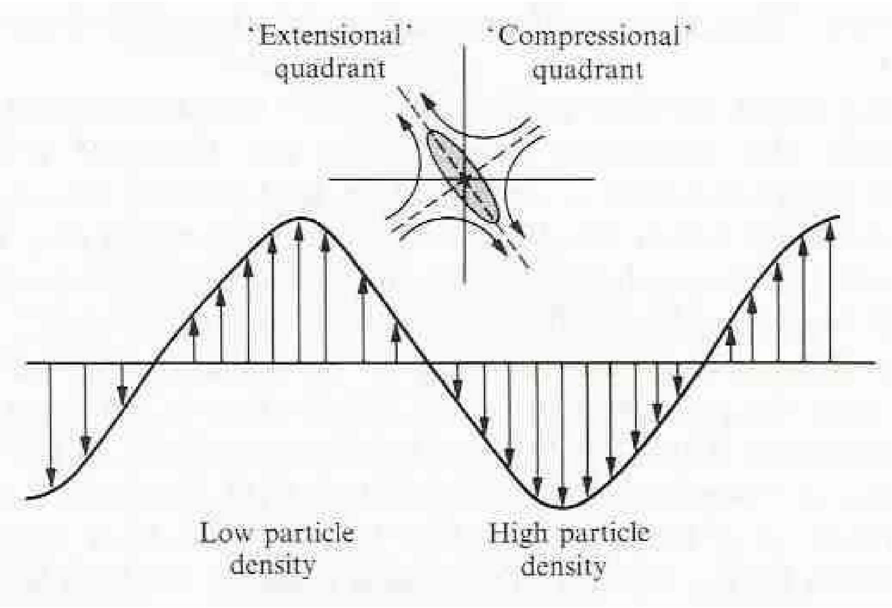
Large-scale streamers

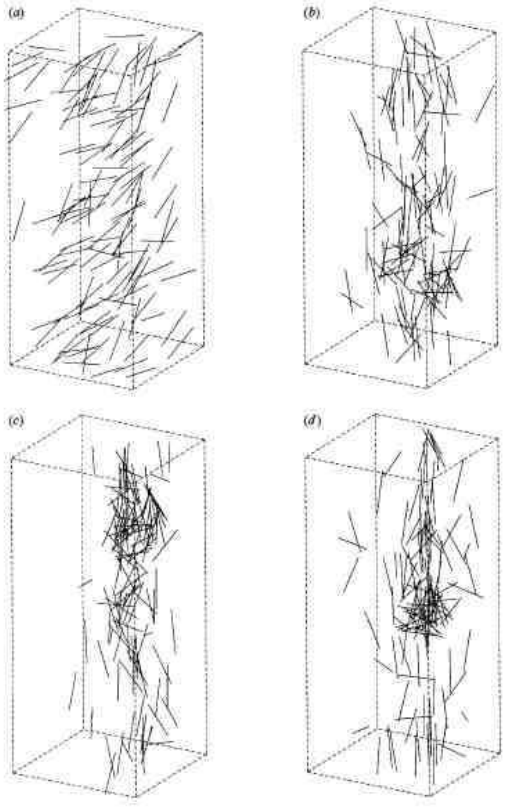


Vertical velocity versus time from PIV measurements

Metzger, Guazzelli, & Butler 2005

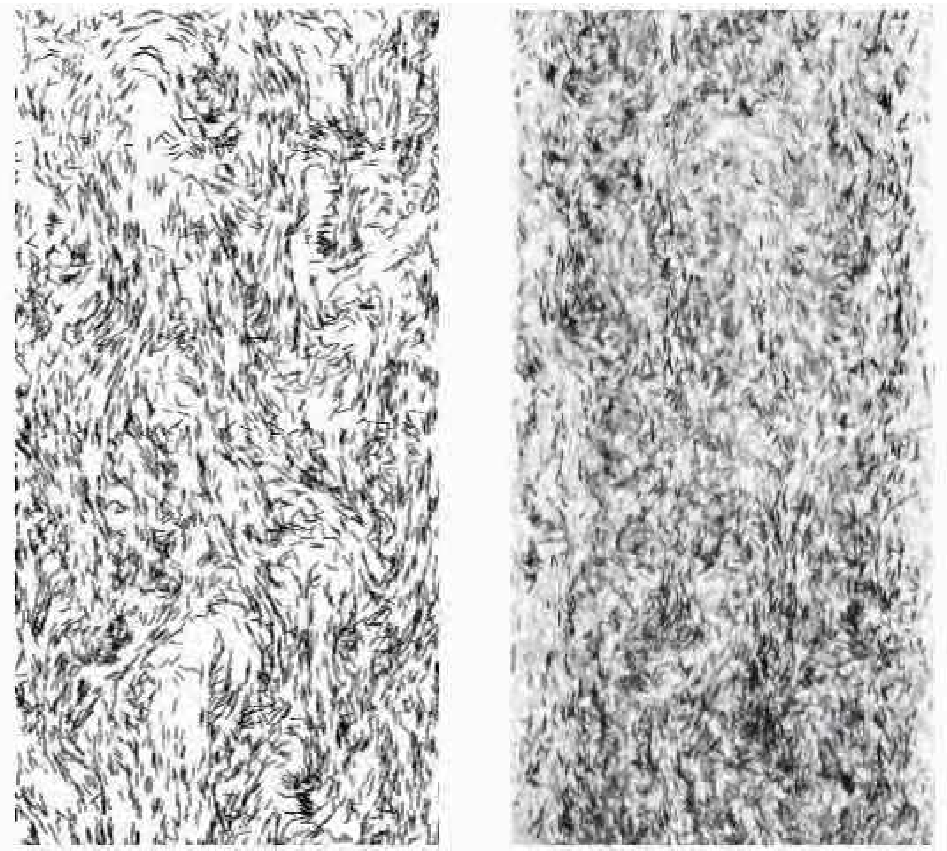
Modeling the instability





Koch & Shaqfeh 1989, Mackaplow & Shaqfeh 1998, Butler & Shaqfeh 2002, Saintillan, Darve, & Shaqfeh 2005

Simulations versus Experiments



Steady state? Wave-length selection?

Saintillan, Shaqfeh, Darve,
Metzger, Guazzelli, & Butler 2005
see more on Video Entry #17 (Gallery of Fluid Motion)

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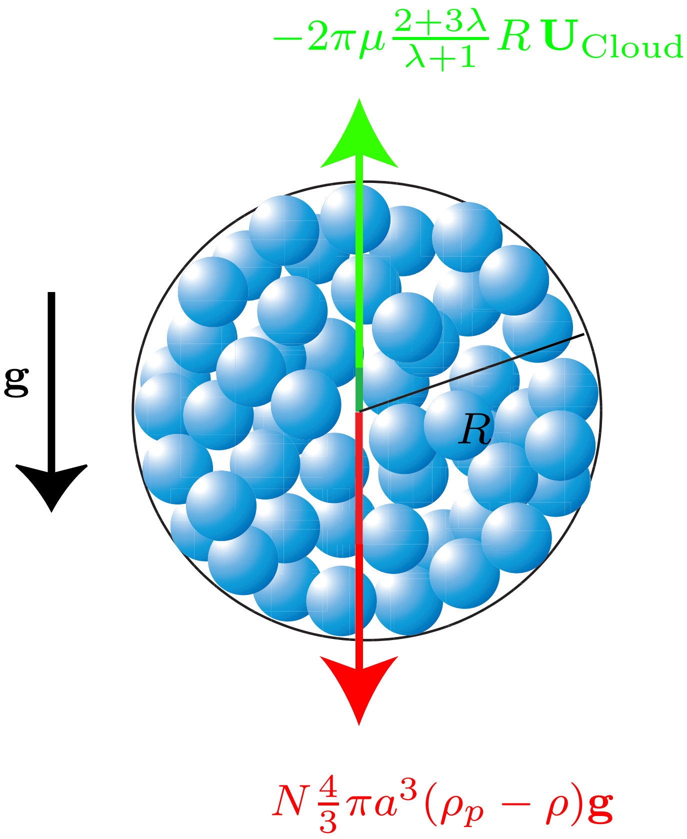
Spherical cloud of spheres



- Drag force (Hadamard, Rybczyński 1911):

$$\mathbf{F}^{\mathbf{h}} = -2\pi\mu\frac{2+3\lambda}{\lambda+1}R\,\mathbf{U}_{\text{Cloud}}$$

with $\lambda = \mu_s/\mu$

- Settling velocity:

$$\mathbf{U}_{\text{Cloud}} = \frac{N\frac{4}{3}\pi a^3(\rho_p-\rho)\mathbf{g}}{2\pi\mu\frac{2+3\lambda}{\lambda+1}R}$$

Suspension mixture = effective fluid of viscosity μ_s

Stability of the cloud?

- *It is important to note that the drop is found to be stable without any surface tension to maintain the spherical shape.* Feuillebois 1984.
- *A sperical blob shape is especially well suited to a study of random particle migration . . . because it maintains essentially constant form.* Nitsche & Batchelor 1997.
- *A single spherical drop does not deform substantially.* Machu *et al.* 2000.
- At creeping flow conditions the suspension drop retains a compact, roughly spherical shape while settling. Bosse *et al.* Gallery of Fluid Motion 2005.

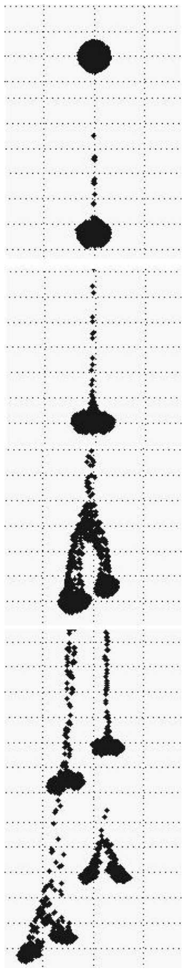
But the cloud is unstable!

Spherical cloud

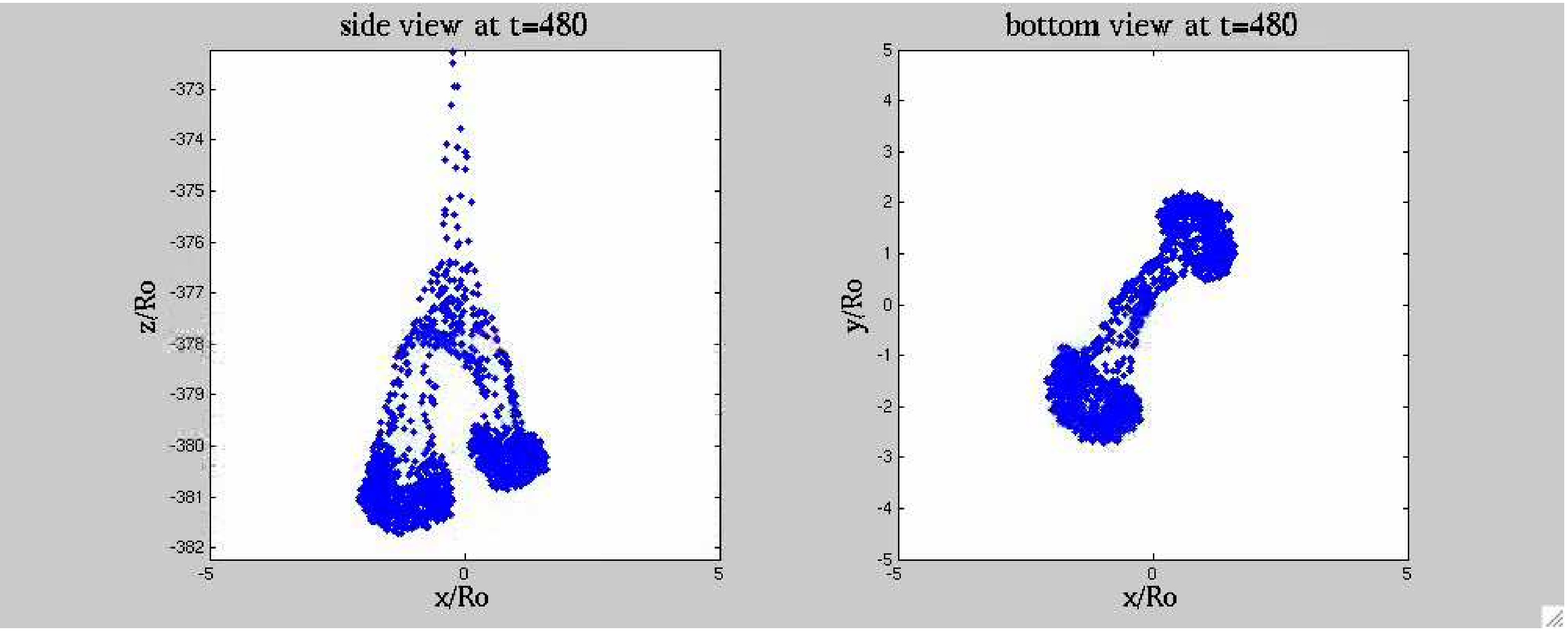
Torus

Break-up

and so on



Evolution of the cloud



Cloud composed of 3000 point-particles

Successive instabilities? Break-up?

Metzger, Ekiel-Jeżewska, & Guazzelli 2005
see more on talk FK.00007

Conclusions

- Long-range nature of the multi-body hydrodynamic interactions
  Coupling between hydrodynamics and suspension microstructure
  → Collective dynamics: swirls, streamers, instabilities
- More open problems
  - Larger concentrations
  - Bidisperse or polydisperse particles
  - Anisotropic particles (platelets)
  - Deformable particles: Saintillan *et al.* 2005
  - Non-Newtonian fluids: Mora, Talini, & Allain 2005
  - Inertia

Collaborations

- B. Herzhaft, H. Nicolai, Y. Peysson (ESPCI Paris) and D. Chehata, B. Metzger (IUSTI Marseille)
- L. Bergougnoux (IUSTI Marseille)
- E. J. Hinch (University of Cambridge)
- M. L. Ekiel-Jeżewska (IPPT-PAN Warsaw)
- J. E. Butler (University of Florida)
- E. Darve, M. B. Mackaplow, D. Saintillan, and E. S. G. Shaqfeh (Stanford University)
- G. M. Homsy (University of California Santa Barbara)