Effects of non-perfect thermal sources in turbulent thermal convection

Phys. of Fluids, 16(6), pp. 1965-1979, (2004).

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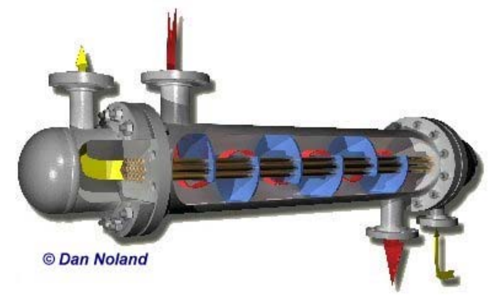
CASPUR

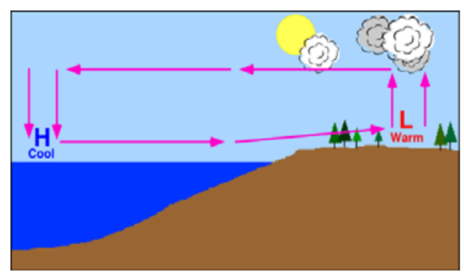
Motivation

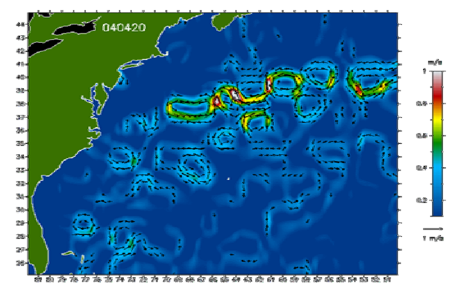
Heat transfer mediated by a fluid takes place in countless phenomena in industrial and natural systems, for example ....

....in cooling problems
(from CPUs to industrial plants)

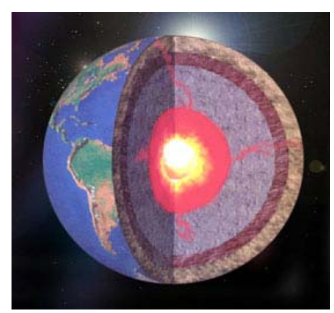


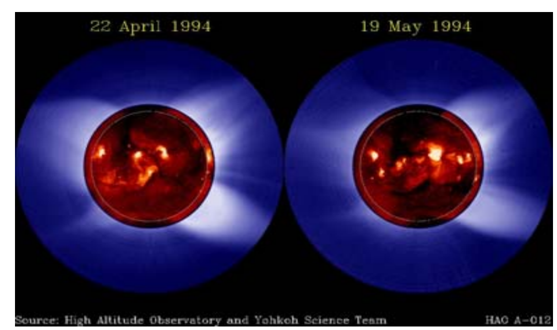
… in the motions of atmosphere and oceans driven by temperature differences





… in planets liquid core and stars convection



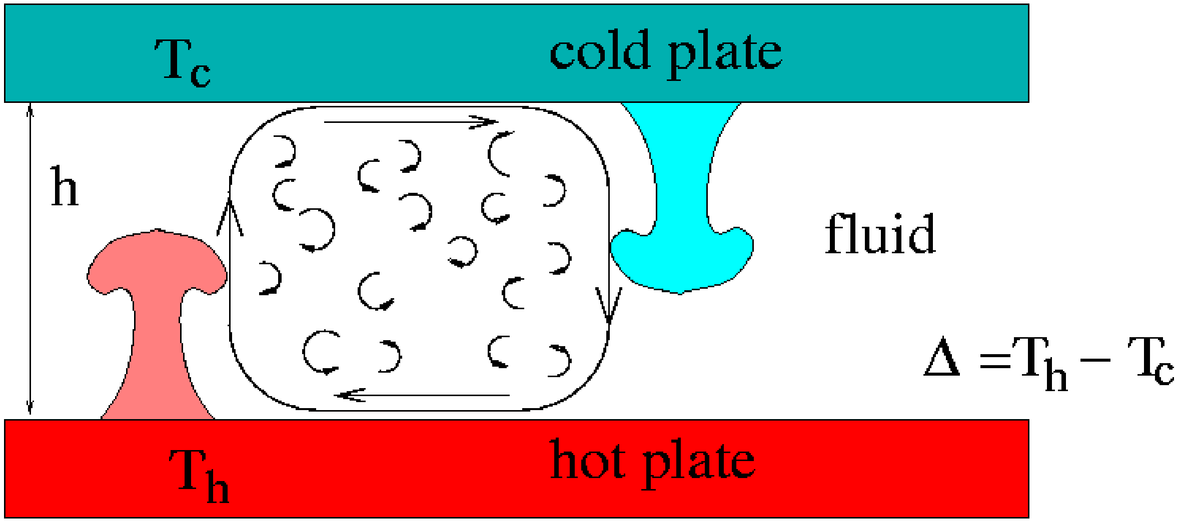


Interesting *per se* owing to rich and complex physics

The Rayleigh-Bénard problem

Fluid layer of depth *h* heated from below and cooled from above

Thermal expansion causes hot fluid to rise and cold fluid to sink
(unstable thermal stratification) *`only few exceptions'*



Rayleigh (1916)
Bénard (1900)

A flow is established if Δ *exceeds a stability threshold.*

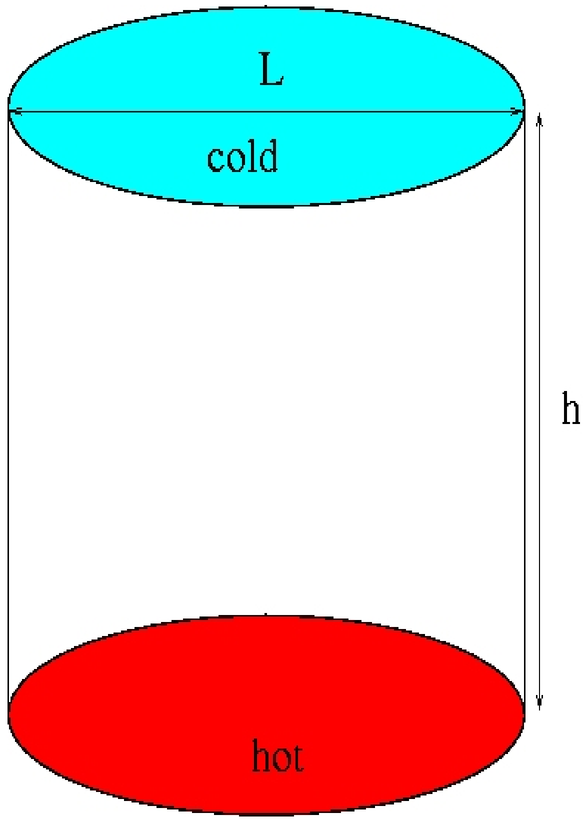
The *ideal* R-B flow is horizontally infinite but any *real* system is laterally bounded

The problem

- Navier Stokes equations with Boussinesq approximation:

$$\frac{D\mathbf{u}}{Dt} = -\nabla p + \theta \hat{x} + \left(\frac{Pr}{Ra}\right)^{\frac{1}{2}} \nabla^2 \mathbf{u}, \qquad \nabla \cdot \mathbf{u} = 0 \qquad \frac{D\theta}{Dt} = \frac{1}{(Pr Ra)^{\frac{1}{2}}} \nabla^2 \theta$$

- Main parameters:
  - **Rayleigh Number:** $Ra = \frac{g \alpha \Delta h^3}{\nu k}$
  - **Prandtl number:** $Pr = \frac{\nu}{k}$
  - **Aspect ratio:** $\Gamma = \frac{L}{h}$



➢Only three control parameters

Experimental Evidence

- Laboratory experiments (*apparently*) identical do not give the same results: *cyl. cell $\Gamma=0.5$ various Pr and Ra ranges*
  - Wu & Libchaber (1992)
  - Goldstein & Tokuda (1979)
  - Chavanne et al. (2001), Roche et al. (2001), Chillà & Castaing (2003)
  - Niemela et al. (2000)
  - Naert et al. (1997)

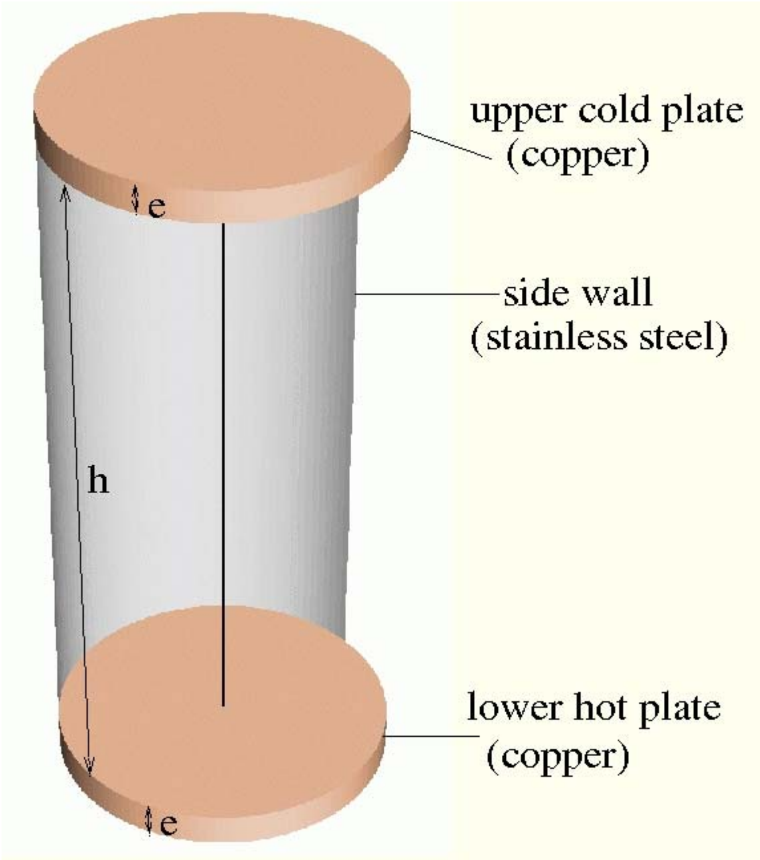
- *Different temperature boundary conditions?* Chaumat et al. (2002)
  - Pr number variation? Ahlers & Xu (2001), Xia et al. (2002)
  - Departure from the Boussinesq approximation?
  - Cell shape? Daya & Ecke (2001)

A Rayleigh-Bénard cell

Working fluid: *water, air, liquid metals (mercury, sodium), pressurized gas, silicon oils, cryogenic pressurized gaseous helium.*

Side wall: *stainless steel, plexiglas (high mechanical properties, poor heat conduction)*

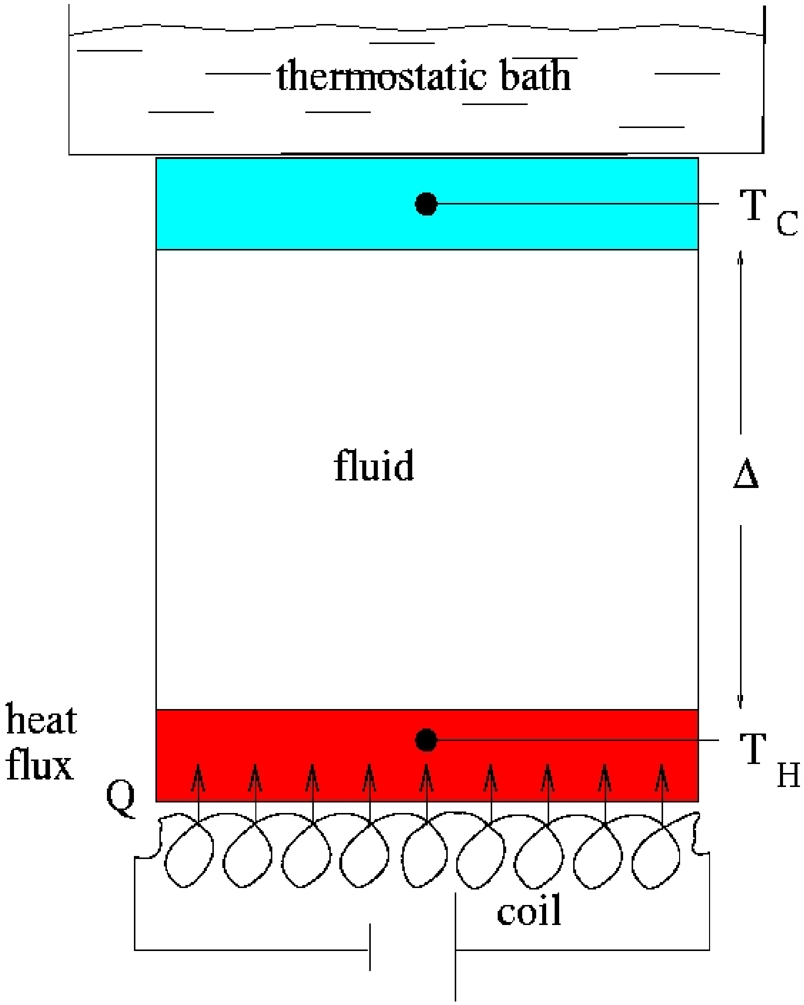
Plates: *copper, brass aluminium sapphire oxygen free pure copper (high mechanical properties, very good heat conduction)*



The arrangement is such to minimize the heat leakage through the sidewall. There are corrections for the sidewall (important only at small **Ra**)

Ahlers 2001, Roche et al. (2001), Verzicco (2002), Niemela & Sreenivasan (2003)

Important assumptions



Upper and lower plates are different

$\Delta = T_H - T_C \ \forall \ Ra$

(Δ possibly computed by extrapolations)

Uniform temperature at the fluid/plate interface

$(\rho\ C\ \lambda)_{plate} >> (\rho\ C_p\ \lambda)_{fluid}$

(Schlicting 2000, p.507)

In thermal convection, however, $\lambda_{eff} = Nu\ \lambda_{fluid}$ and since $Nu = a\ Ra^{\beta}$ eventually $(\rho\ C\ \lambda)_{plate} \approx (\rho\ C_p\ Nu\ \lambda)_{fluid}$

Electrical analogy

"Ohm law" for each component

Δ_T = $[2R_p+R_f]Q$, $2Rp = R_{pl}+R_{pu}$

if $Nu = a\ Ra^{\beta}$ being $\mathbf{R}_f = h/(Nu\lambda_f S)$

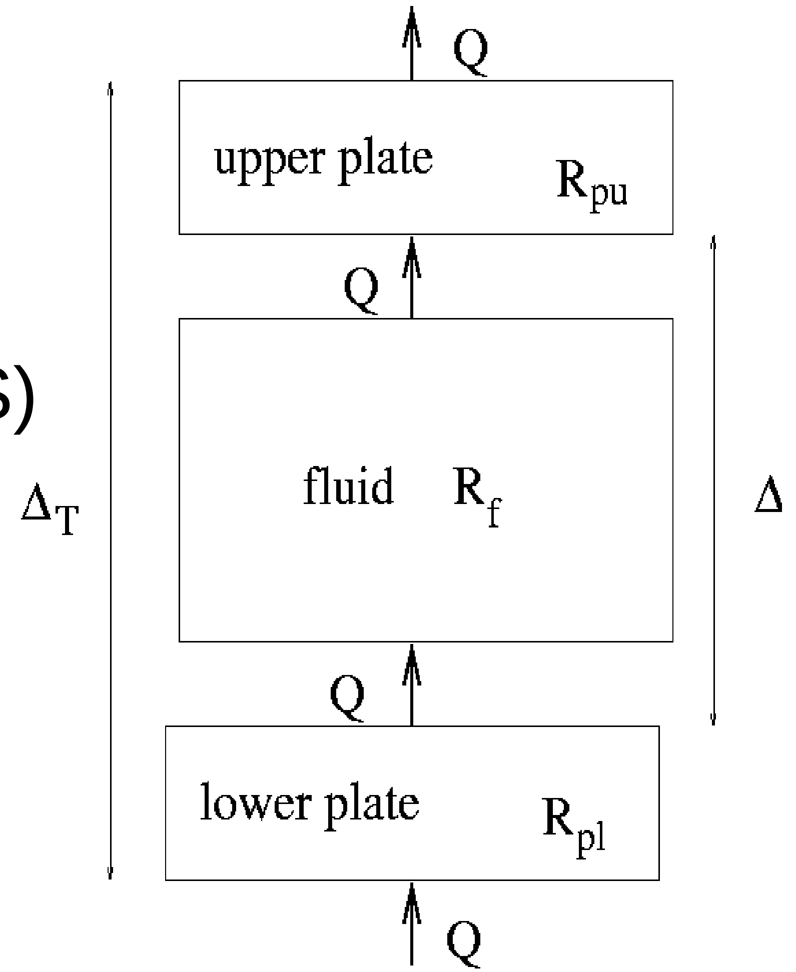
(S is the wet plate/fluid interface) then

$R_f = A/\Delta^{\beta}$ (with $A=h/[aS\lambda_f(g\alpha h^3/\nu\kappa)^{\beta}]$)

$\Delta_T = [2R_p + A/\Delta^{\beta}]Q$

As $Ra \rightarrow \infty$ also $\Delta \rightarrow \infty$

and $\Delta_T = 2R_pQ$



Regardless of the plate properties, eventually they become the bottleneck!

Simplified problem

- Governing equations:

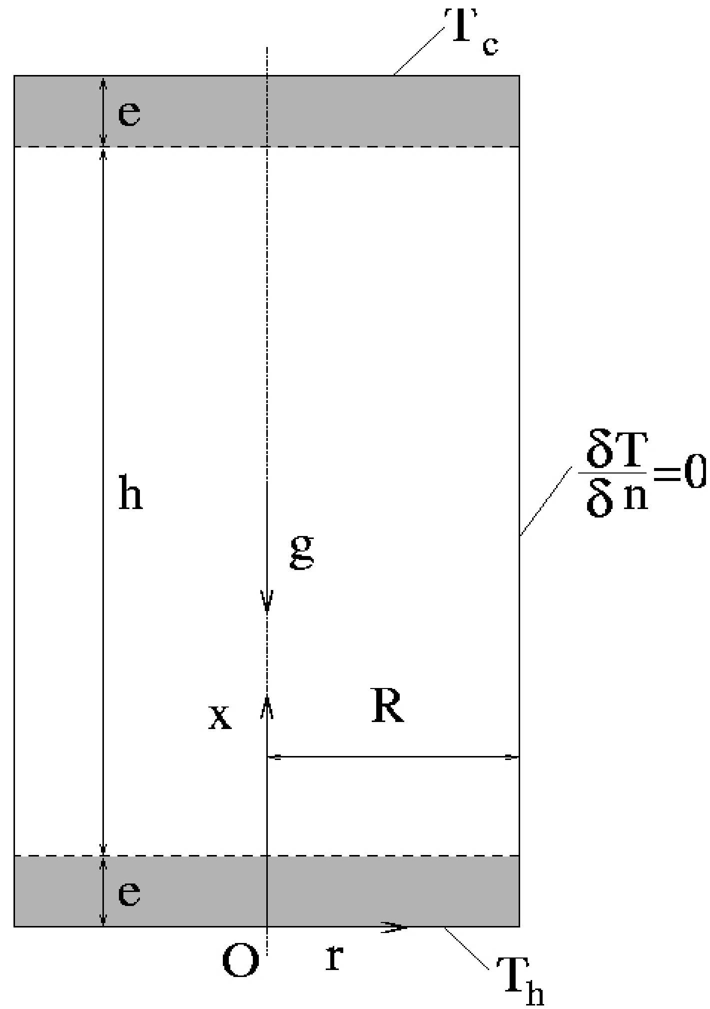
$$\frac{D\mathbf{u}}{Dt} = -\nabla p + \theta\hat{x} + \left(\frac{Pr}{Ra}\right)^{\frac{1}{2}} \nabla^2 \mathbf{u} \qquad \nabla \cdot \mathbf{u} = 0$$

in the fluid domain

$$\frac{D\theta}{Dt} = \frac{1}{(PrRa)^{\frac{1}{2}}} \frac{\rho_f C_{pf}}{\rho C} \nabla \cdot \left(\frac{\lambda}{\lambda_f} \nabla\theta\right)$$

in the fluid and within the plates

- DNS of the 3D equations in cylindrical coordinates `fluid' with variable thermal properties.
- Immersed Boundary method.
- 2^{nd} order finite-difference in space and time *Verzicco & Orlandi (1996), Fadlun et al. (2000)*
- Identical cold and hot plates
- Constant temperature on the dry plate surface



Note that the limit for $e \to 0$ is different from experiments

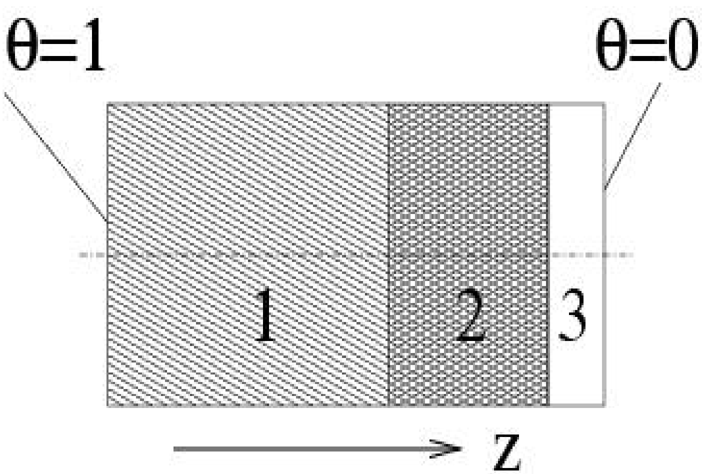
Validation

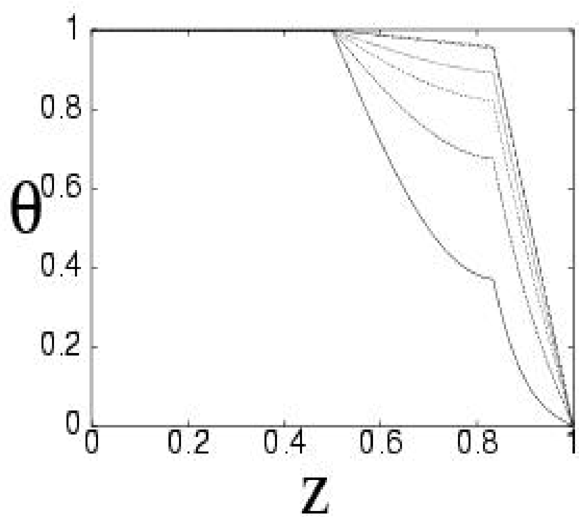
Three materials with different thermal properties

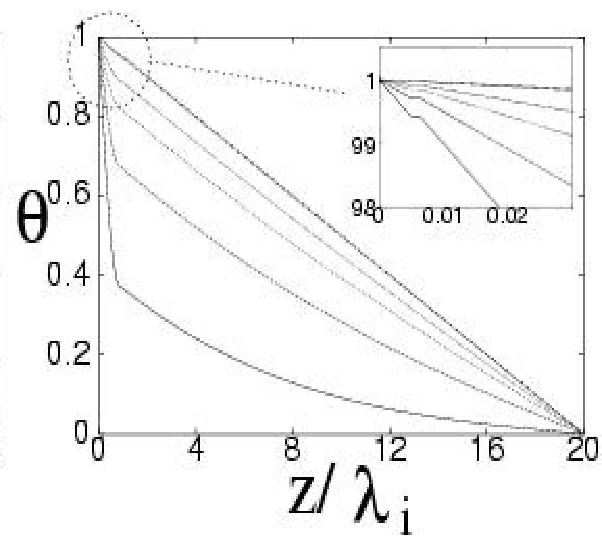
- 1 Copper
- 2 Stainless Steel
- 3 Gaseous Helium

Steady state conductive solution:

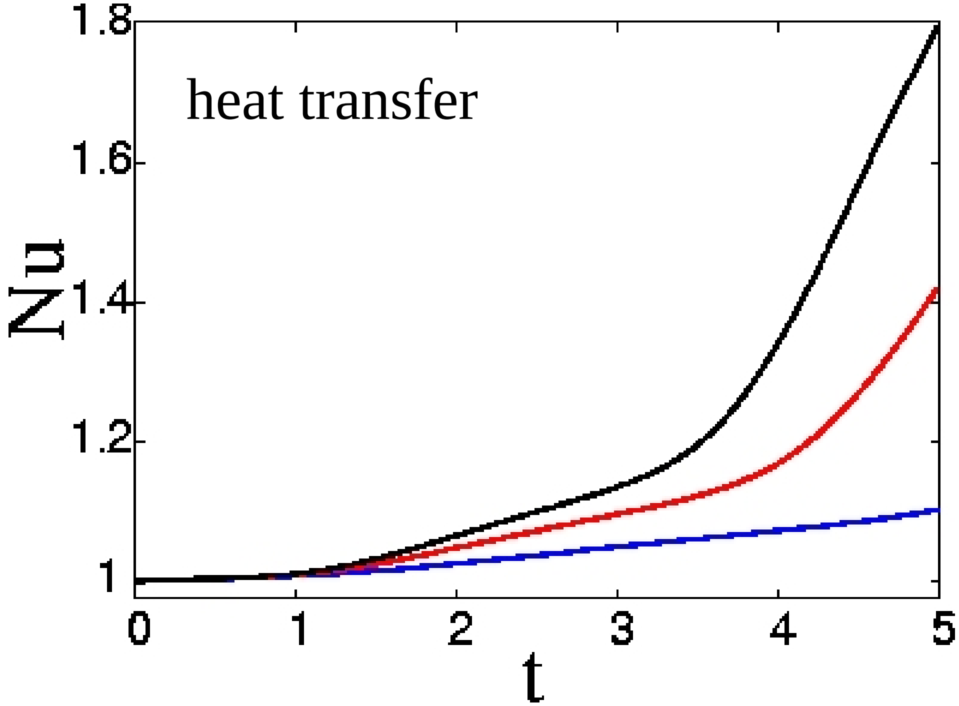
- Temperature profile with discontinuous derivatives at the material interfaces
- Linear temperature profile as function of “z/λ_i”



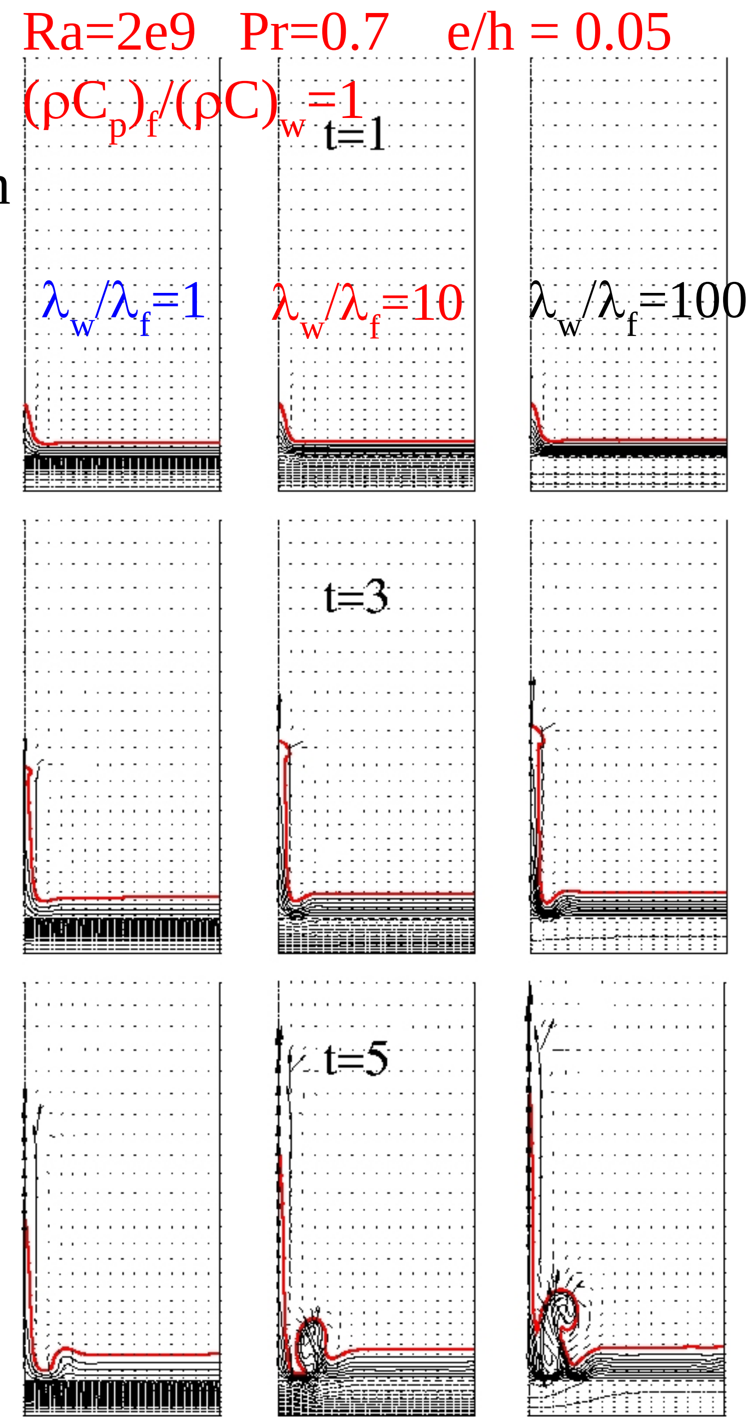




A single hot plume



plume evolution



The plate cools down below the plume and limits the heat transfer

Temperature drop within the plates

Change in the plume dynamics

Time scales

- Time between two successive emissions of plumes

(Castaing et al., 1989)

$\tau_f^* \approx \frac{\delta_\theta^2}{k_f}$ non dimensional $\tau_f \approx \frac{(RaPr)^{1/2}}{4Nu^2}$

τ_f **decreases** with *Ra*
(since *Nu* increases faster than $Ra^{1/4}$)

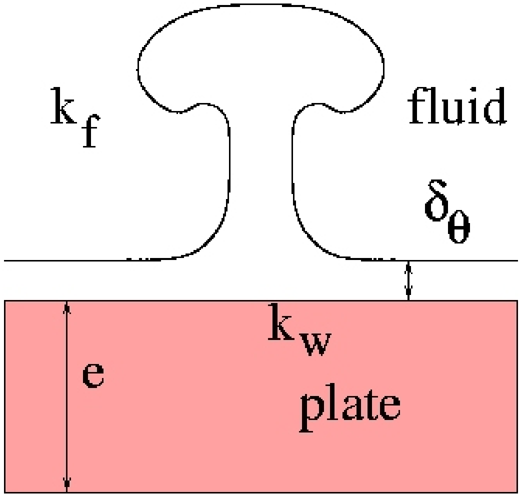
- Time scale of the plate

$\tau_w^* \approx \frac{e^2}{k_w}$ non dimensional $\tau_w \approx (RaPr)^{1/2}\left(\frac{e}{h}\right)^2 \frac{k_f}{k_w}$

τ_w **increases** with *Ra*

- The plate should be 'fast' enough to provide two consecutive plumes with the adequate heat flux

$$\frac{\tau_w}{\tau_f} \ll 1$$



- ...however... $\frac{\tau_w}{\tau_f} \approx 4Nu^2\left(\frac{e}{h}\right)^2 \frac{\lambda_f}{\lambda_w}\frac{(\rho C)_w}{(\rho C_p)_f}$

- <u>*Every* plate becomes eventually not enough conductive!</u>

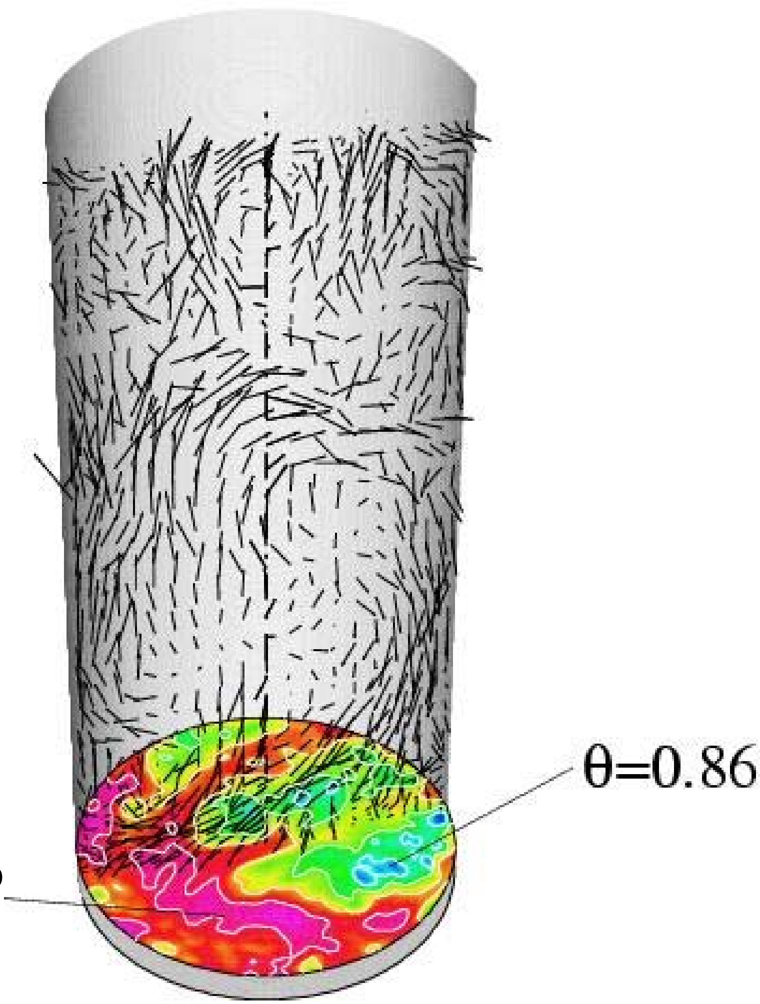
Flow parameters

$$\frac{D\theta}{Dt} = \frac{1}{(PrRa)^{\frac{1}{2}}} \frac{\rho_f C_{pf}}{\rho C} \nabla \cdot \left(\frac{\lambda}{\lambda_f} \nabla\theta \right)$$

Ra (Rayleigh number) ***Pr*** (Prandtl number)

e/h (plate thickness) ***Γ*** (aspect ratio)

$(\rho C_p)_f/(\rho C)_w$ (spec. heats) λ_f/λ_w (thermal cond.)

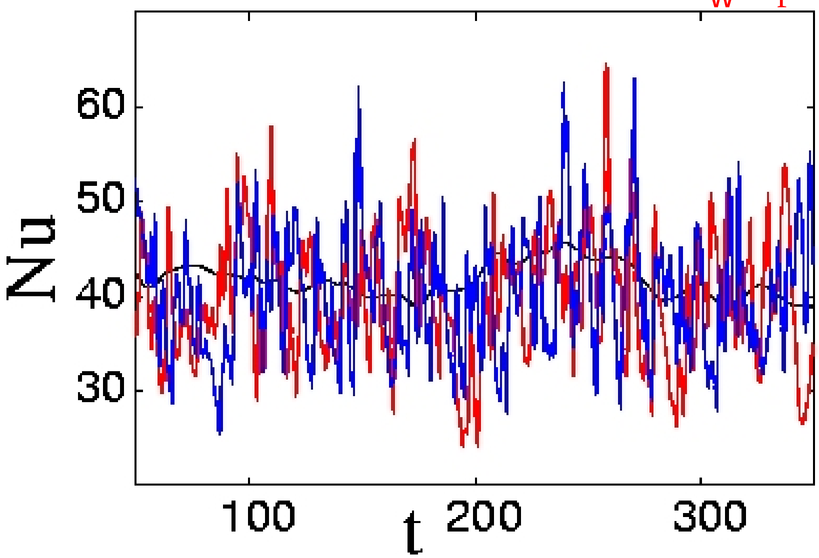


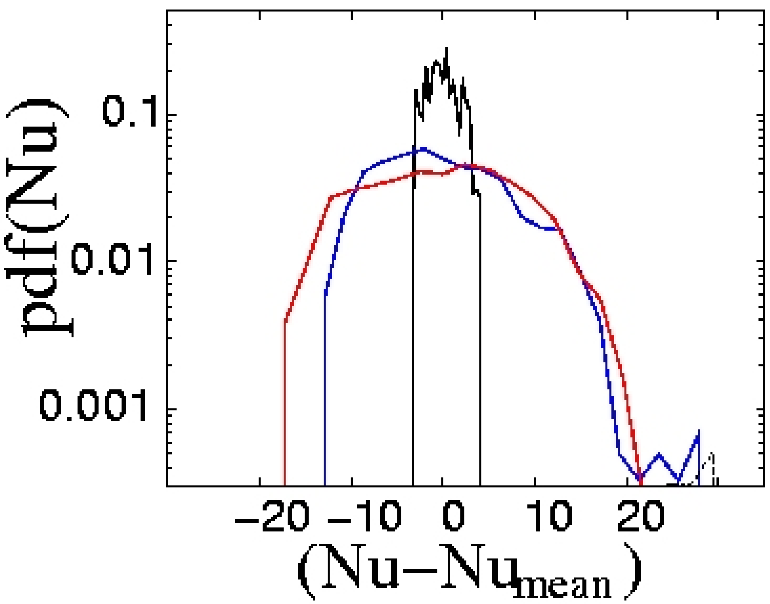
- Analysis limited to: **Γ=0.5 Pr=0.7**

- Parameter space scanned by many *fast* axisymmetric simulations and findings verified by *few* large 3D simulations

Effect of $(\rho C_p)_f/(\rho C)_w$ (specific heats)

Ra=2e8 Pr=0.7 e/h = 0.05 λ_w/λ_f=50





$(\rho C_p)_f/(\rho C)_w$=0.01 ***Nu=40.19±1.2***

$(\rho C_p)_f/(\rho C)_w$=1 ***Nu=39.69±2.1***

$(\rho C_p)_f/(\rho C)_w$=100 ***Nu=39.13±2.1***

very limited (negligible) effect

cryogenic gaseous helium/copper

$(\rho C_p)_f/(\rho C)_w$=3.58

air/copper

$(\rho C_p)_f/(\rho C)_w$=0.00045

water/brass

$(\rho C_p)_f/(\rho C)_w$=1.26

One-dimensional temperature equation for the plate

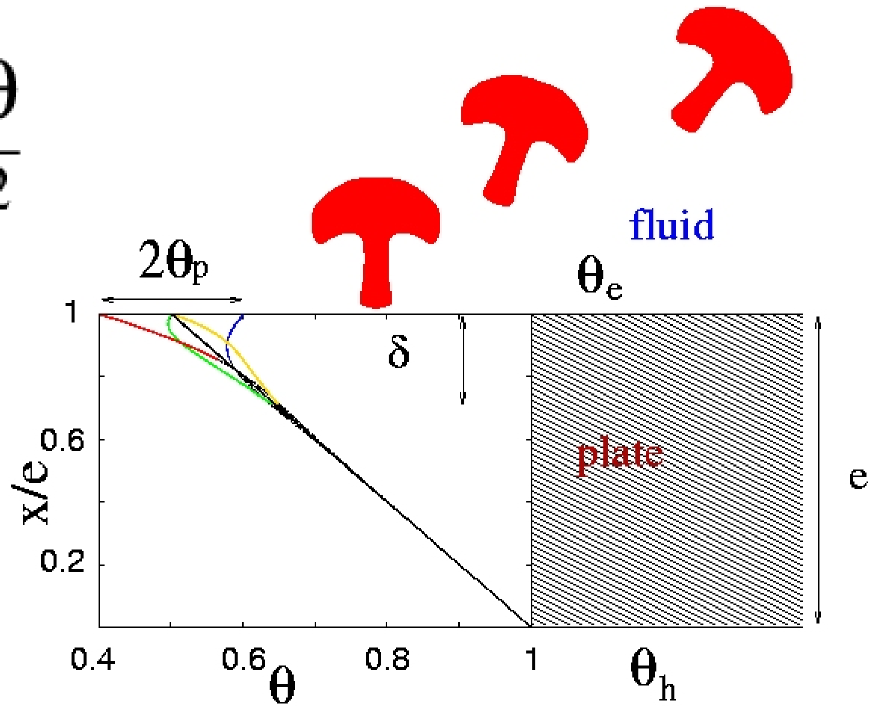
$$\frac{\partial\theta}{\partial t} = \frac{1}{Pe}\frac{\partial^2\theta}{\partial x^2}$$

$$Pe = \sqrt{PrRa}\,k_f/k_w$$

$$\tau \approx \sqrt{RaPr}/(4Nu^2)$$

$$\omega = 2\pi/\tau$$

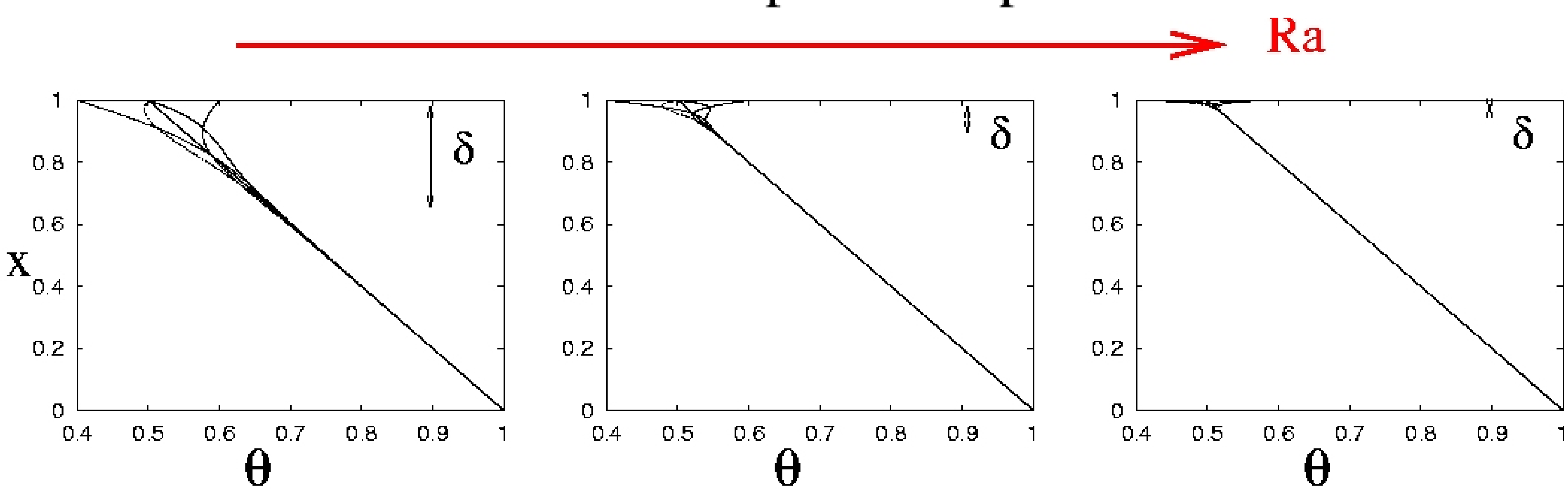
$$\theta(e,t) = \theta_e + \theta_p \exp^{i\omega t}$$



$$\theta(x,t) = \theta_h + (\theta_e - \theta_h)\frac{x}{e} + \theta_p \frac{\exp^{-i\beta x} - \exp^{i\beta x}}{\exp^{-i\beta e} - \exp^{i\beta e}} \exp^{i\omega t}$$

$\delta/e \sim \sqrt{(k_w/k_f)}\,/Nu$

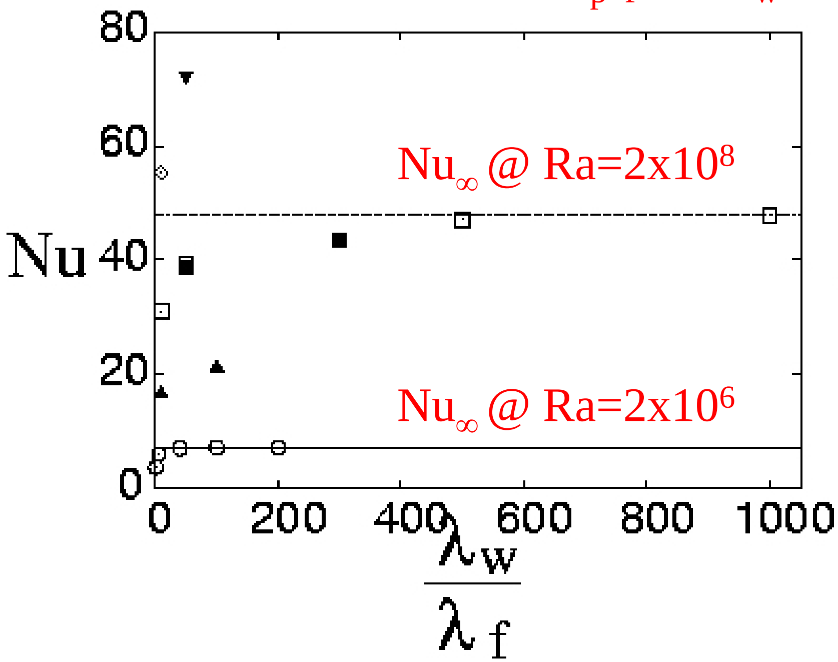
Penetration length

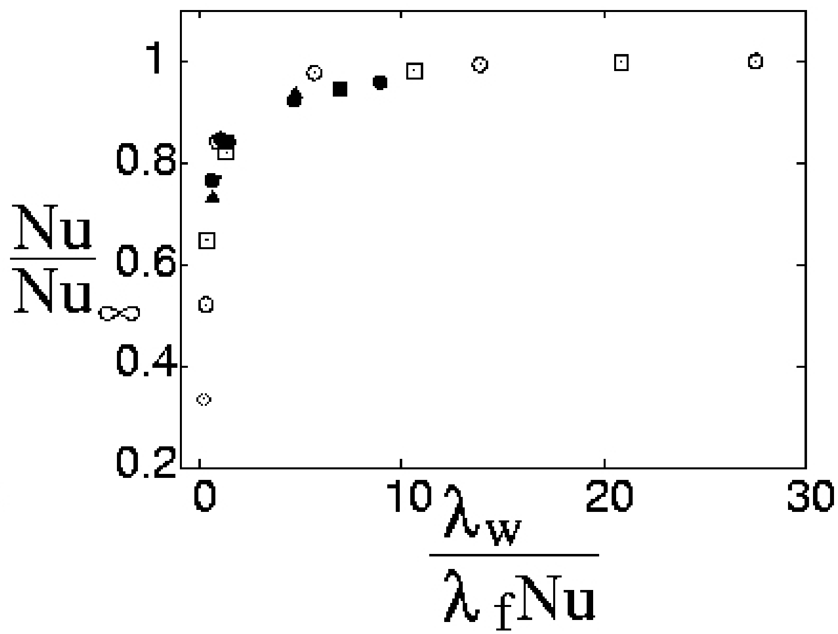


Effect of λ_w/λ_f (thermal conductivities)

solid symbols for 3D flows
open symbols for axisym. flows

Pr=0.7 e/h = 0.05 $(\rho C_p)_f/(\rho C)_w=1$





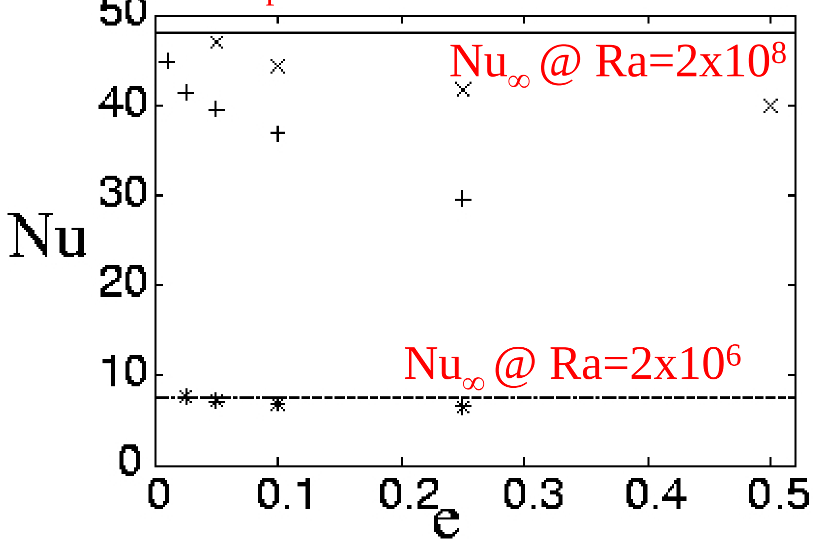
$2x10^6 < Ra < 2x10^{10}$ for $\lambda_w/\lambda_f \to \infty$ $Nu \to Nu_\infty$

Governing parameter $\dfrac{\lambda_w}{\lambda_{eff}} = \dfrac{\lambda_w}{Nu\lambda_f}$

Note: the temperature drop within the plates (which can be easily corrected) is at most 10%Δ at $\lambda_w/\lambda_f=1$. The corresponding Nu decrease is 65% at $Ra=2x10^{10}$ and 48% at $Ra=2x10^6$

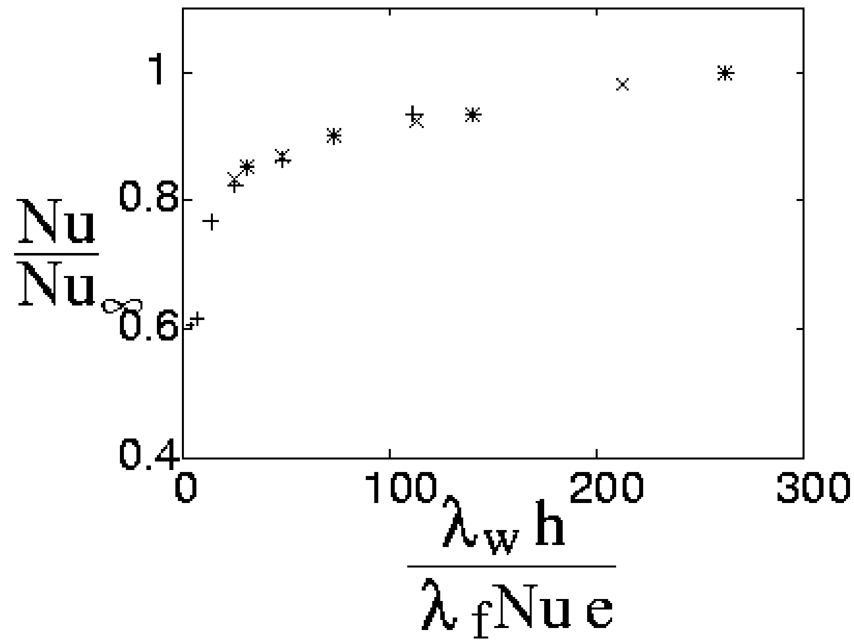
Effect of ***e/h*** (plate thickness)

Pr=0.7 $(\rho C_p)_f/(\rho C)_w=1$



2x10 ^{6} ***< Ra < 2x10*** ^{8}

50< λ_w/λ_f ***<500***

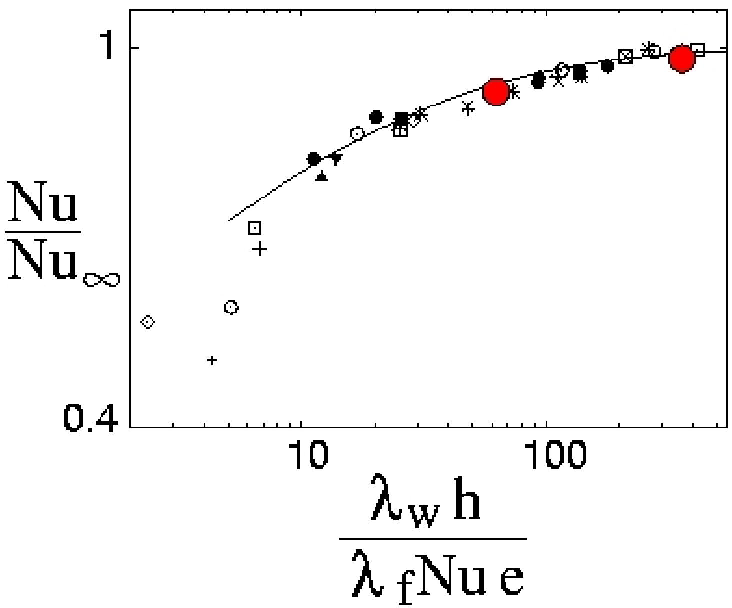


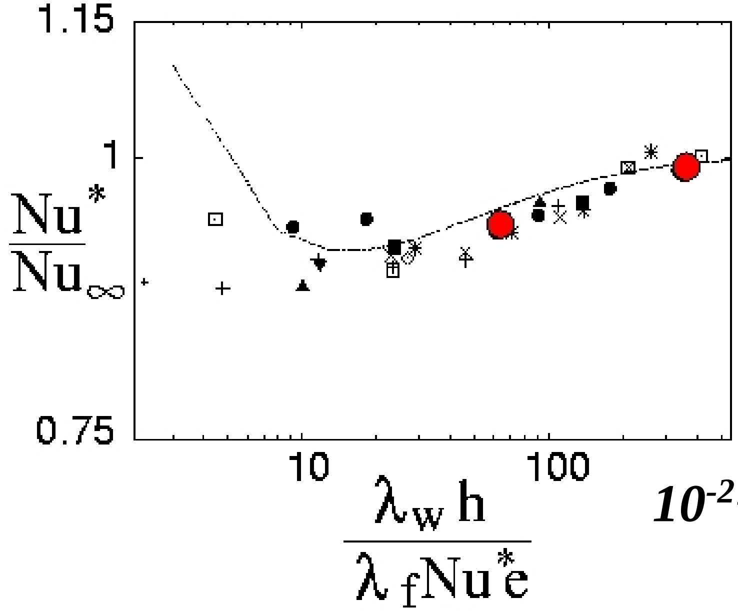
for **e/h→0** **Nu → Nu∞**

Governing parameter $\dfrac{\lambda_w}{\lambda_{eff}}\dfrac{h}{e}=\dfrac{\lambda_w}{Nu\lambda_f}\dfrac{h}{e}$

Note: the limit **e/h** → 0 is different from experiments because the `dry side' of each plate is a surface at constant temperature. In the experiment **e/h=0** is unfeasible since the heat capacity of the plates is needed to homogenize the temperature b.c.

Global correction





scanned parameters:

$1 < \lambda_w/\lambda_f < 10^4$

$0.01 < e/h < 0.5$

$2\times10^6 < Ra < 2\times10^{10}$

$10^{-2} < (\rho C_p)_f/(\rho C)_w < 10^2$

cross-check 3D simulations:	$Ra = 2\times10^8$	$\lambda_w/\lambda_f < 162$	$e/h < 0.08$	$(\rho C_p)_f/(\rho C)_w = 0.5$
	$Ra = 2\times10^7$	$\lambda_w/\lambda_f < 216$	$e/h < 0.035$	$(\rho C_p)_f/(\rho C)_w = 3$

empirical fit:

$$X = \frac{\lambda_w}{Nu\lambda_f}\frac{h}{e} \qquad \frac{Nu}{Nu_\infty} = 1 - \exp[-(X/4)^{0.33}]$$

Nu* corrected by the temperature drop within the plates

$$\frac{Nu^*}{Nu_\infty} = \frac{Nu}{Nu_\infty}\frac{X}{X-2}$$

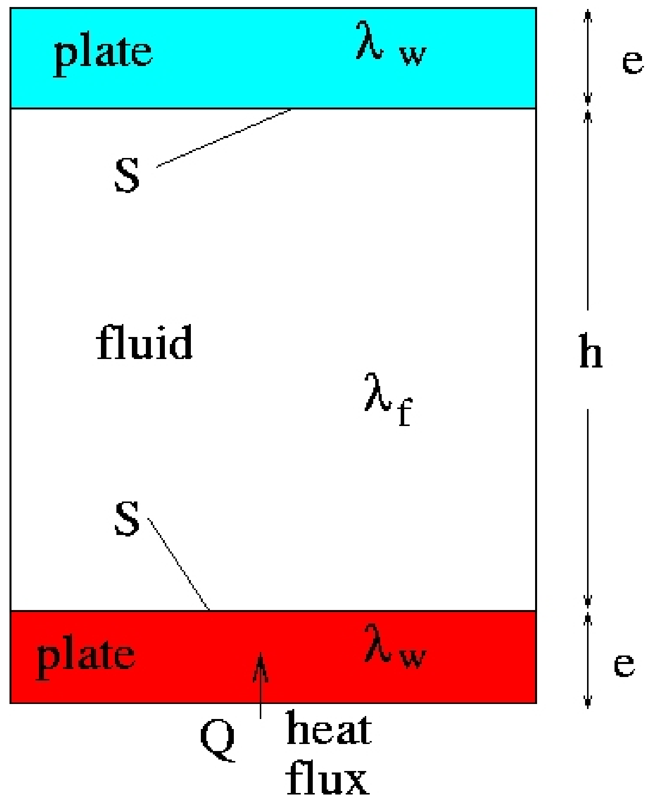
Correction unreliable for ***$X<10$*** (the flow might relaminarize)

Interpretation: thermal resistances

S wet plate/fluid interface

Thermal resistance of the plates $$R_p = \frac{e}{\lambda_w S}$$

Thermal resistance of the fluid layer $$R_f = \frac{h}{Nu \lambda_f S}$$

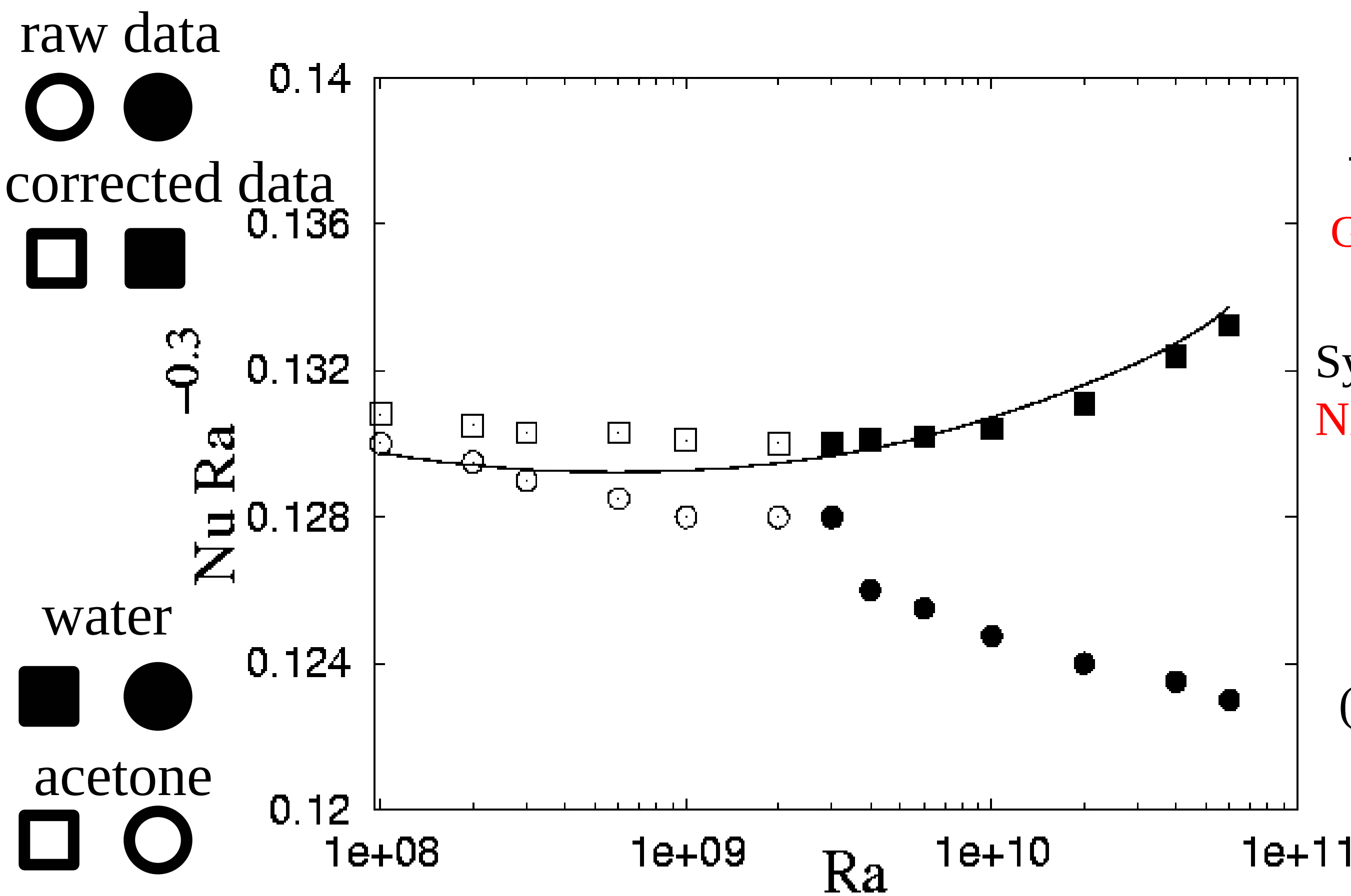


Note that the plates are identical (not true in experiments).

The governing parameter is the ratio of the thermal resistances.

$$\frac{R_f}{R_p} = \frac{h \lambda_w}{Nu \lambda_f e}$$

An example of correction



Theoretical model by
Grossmann & Lohse (2002)

Symbols: experiment by
Nikolaenko & Ahlers (2003)

Γ=1 ***Pr=4***

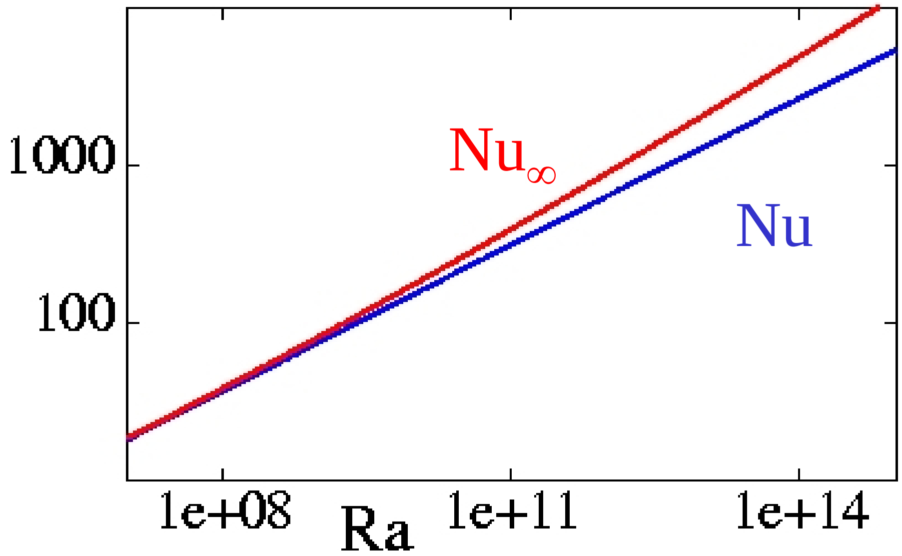
(different from simulations)

lower plate: aluminium with e=3.5 cm

upper plate: aluminium with e=3.34 cm and cooling channels 0.8 cm above the wet surface.

Correction applied with $\boldsymbol{e_{eff}= 0.9cm}$ (equal for both plates)

Some consequences



$Nu = a\,Ra^{\beta}$

$$\beta_\infty = \frac{d \ln Nu_\infty}{d \ln Ra} = \beta + \frac{\exp^{-\frac{C}{Ra^{\beta/3}}}}{1 - \exp^{-\frac{C}{Ra^{\beta/3}}}} \frac{C\beta}{3Ra^{\beta/3}}$$

$$C = [(\lambda_w h)/(4\lambda_f e a)]^{1/3}$$ (constant)

β_∞ always larger than β

If β is constant with Ra β_∞ changes.

The curvature of the Nu vs Ra relation is a signature of the Grossmann & Lohse (2000) model.

A historical note

Suggested by E. Villermaux

Péclet pointed out an `anomaly' in the thermal conductivity of metals λ_w, measured by the heat flux, in experiments of forced convection: He observed that λ_w became independent of ***e*** (metal sample thickness) only for ***e*** below a threshold.

***Ann. de Chim. et Phys.* tome II, 3eme Serie, (1841), 107-115.**

J.C.H. Péclet 1793-1857

In fact, for $e \to 0$ the thermal resistances $\frac{R_f}{R_p} = \frac{h\lambda_w}{Nu\lambda_f e} \to \infty$

and $Nu \to Nu_\infty$ <u>the correction vanishes!</u>

The heat flux becomes independent of the plate thickness.

Successive works

Brown et al. (2005): Experiments in water and acetone (Pr≈4) in cylindrical cells with 0.4<Γ<3: Correction $f(X) = 1-\exp[-(aX)^b]$ with a=0.275 b=0.39 (present values: a=0.25, b=0.33)

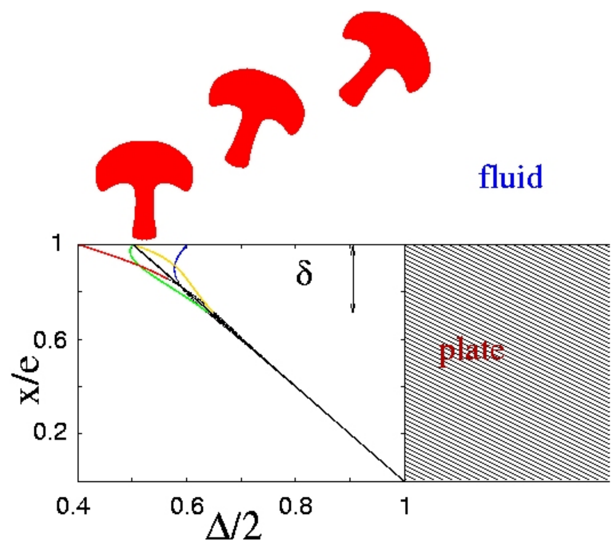
The correction is robust with respect to Pr and Γ

Chillà et al. (2004): Various types of corrections depending on the relative plate thickness (*e/h*). For typical experimental thicknesses the \`thermal impedance‘ is the control parameter.

Chillà-Rastello number

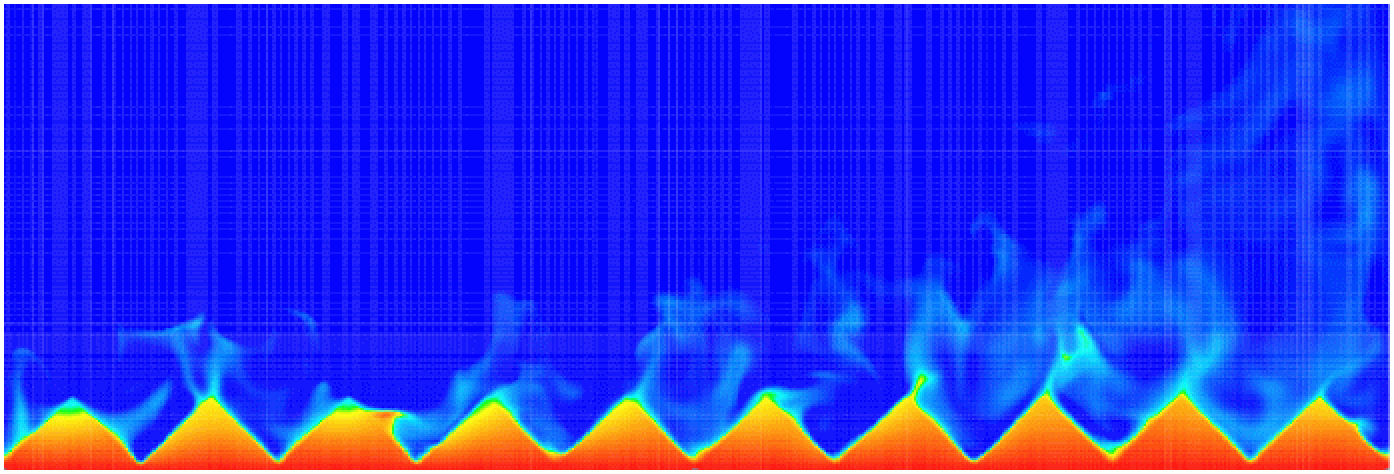
$$Cr = \frac{2\,Nu}{Re\,Pr}\sqrt{\frac{\lambda_p C_p}{\lambda_f C_f}}$$

Only an interfacial skin of depth δ is affected by temperature variations.



Ongoing work

Plate effect for 'non-smooth' plates *(Stringano & Verzicco, 2005)*



Every surface is rough below a given scale

Experiments aiming at investigating extremely high Ra numbers (Kraichnan regime) will eventually have to contend with the surface roughness (when the thermal b.l. thickness becomes comparable with height of the asperities)

The correction is more severe than for flat plates (point effect)

Conclusion

The finite (although very high) thermal conductivity of the plates might limit the heat transfer in thermal convection.

The governing parameter is the ratio of the thermal resistances.

$$\frac{R_f}{R_p} = \frac{h\lambda_w}{Nu\lambda_f e}$$

For $R_f/R_p > 300$ $Nu/Nu_\infty > 0.98$, however, since $Nu = a\,Ra^\beta$ every plate beyond a certain Ra is not adequate.

Working fluid (λ_w), plate features (e, λ_f) and cell geometry (h) should be fitted to the flow regime ($Nu(Ra)$).