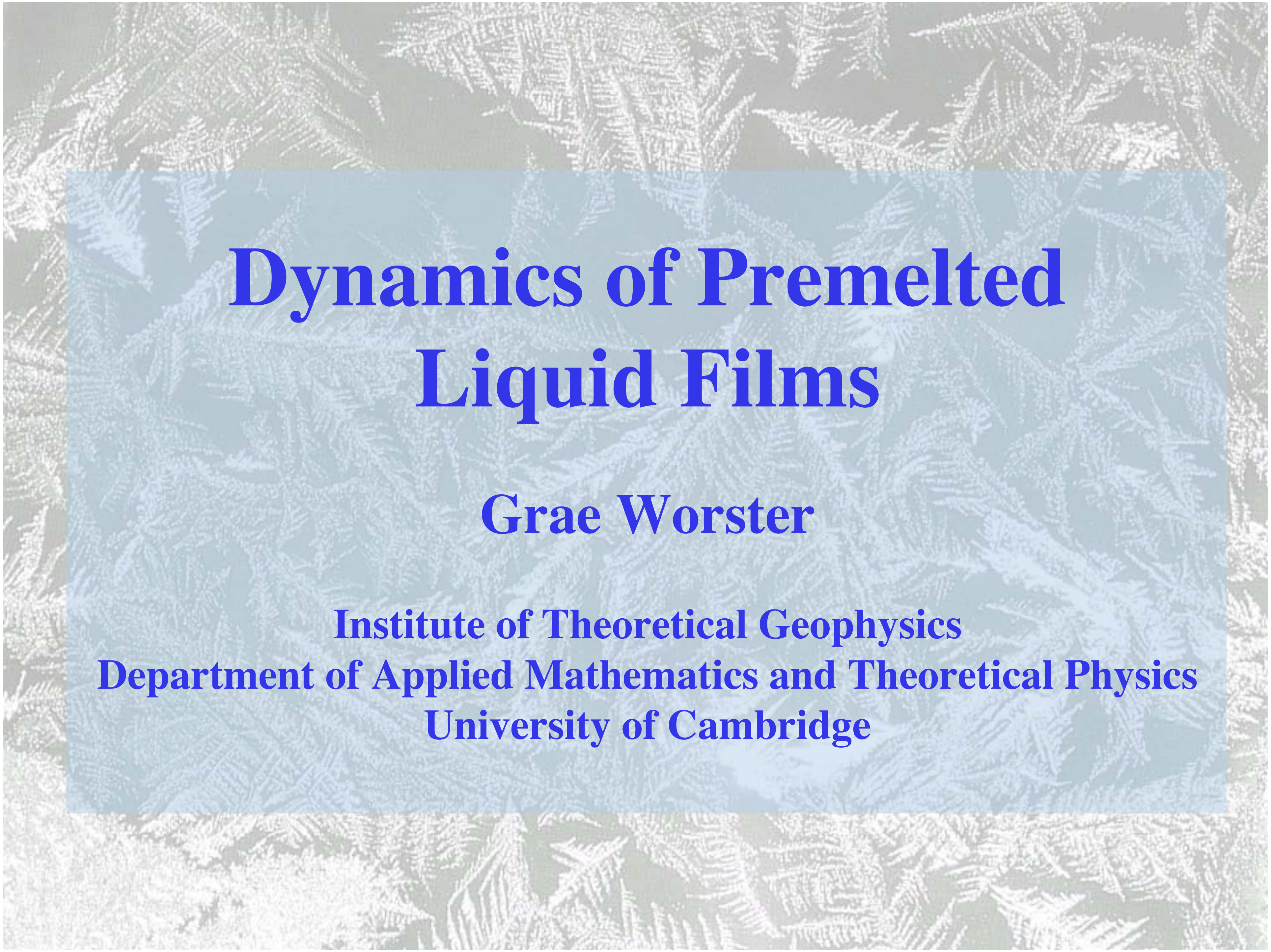

Dynamics of Premelted Liquid Films

Grae Worster

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Department of Applied Mathematics and Theoretical Physics
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Collaborators

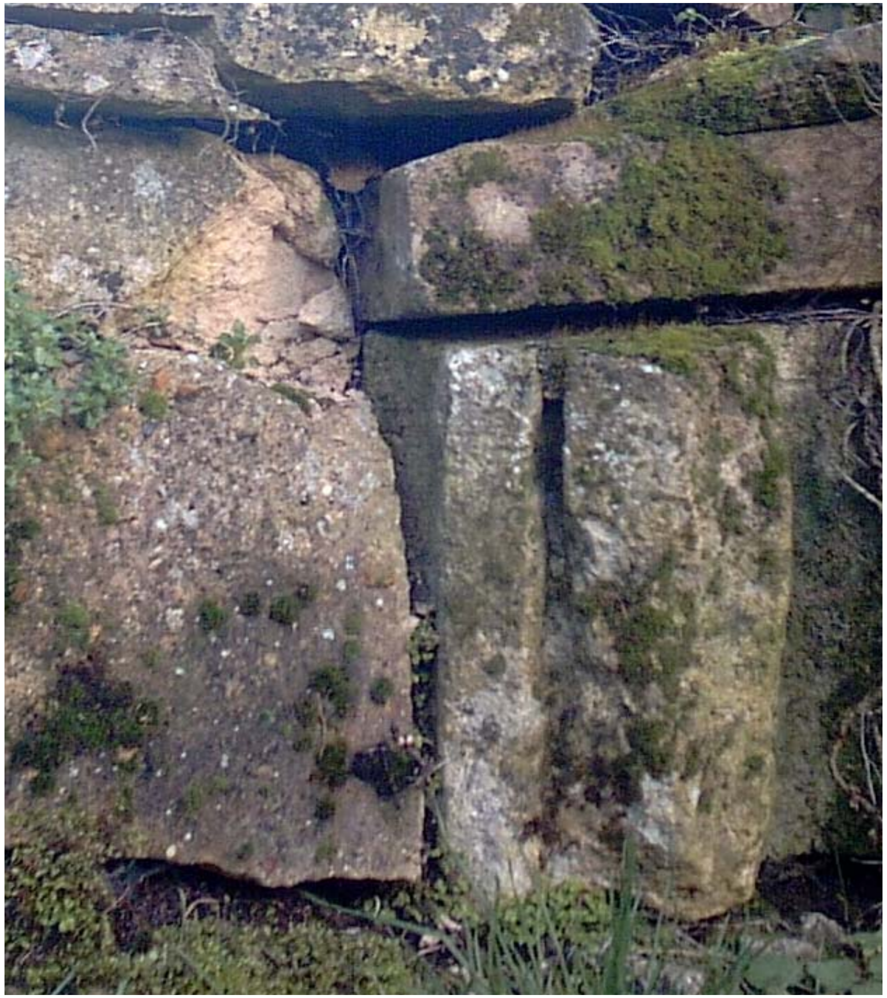
John Wettlaufer

Greg Dash	Stephen Peppin
Larry Wilen	Robert Style
Alan Rempel	Mark Hallworth

Some Effects of Frost

Something there is
that doesn't like a wall,
That sends the frozen
ground swell under it
And spills the upper boulders
in the sun …

Robert Frost



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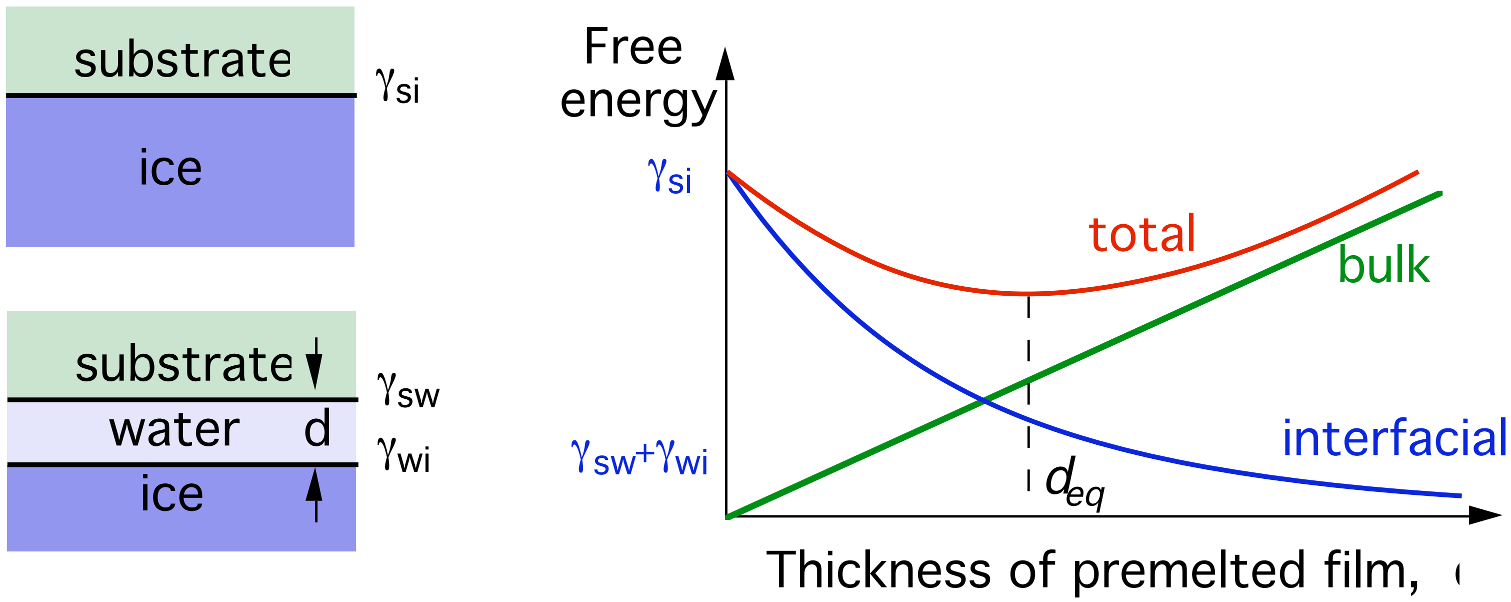
The Forces Responsible…

for frost heave are the same
long-range intermolecular forces
that underlie surface tension…

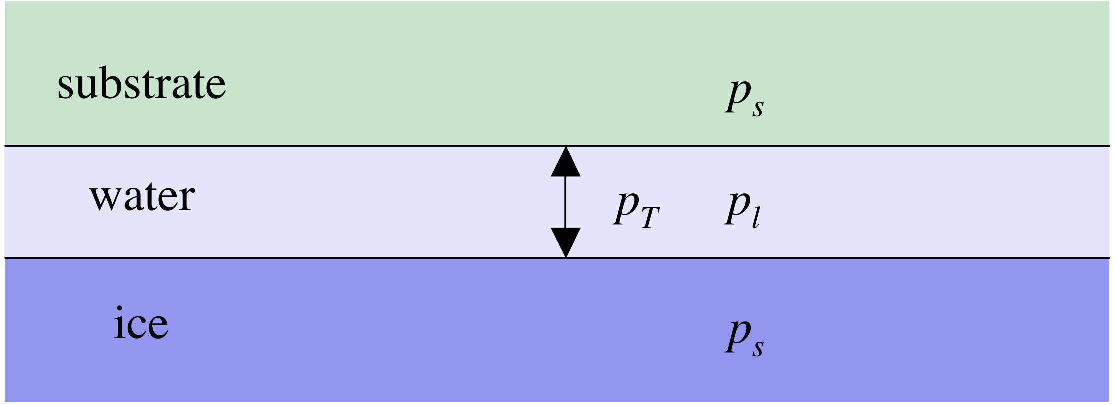
Photograph:
John Bush

and also cause most solids close to their melting points to be
molten at their surfaces.

Thermodynamics of Interfacial Premelting



Dynamics of Interfacial Premelting



substrate p_s

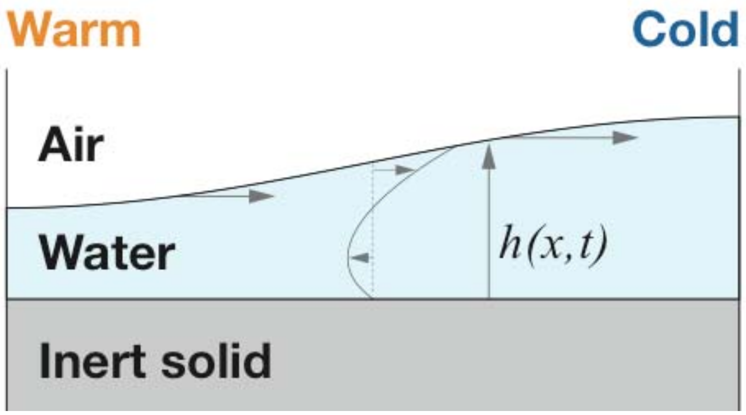
water p_T p_l

ice p_s

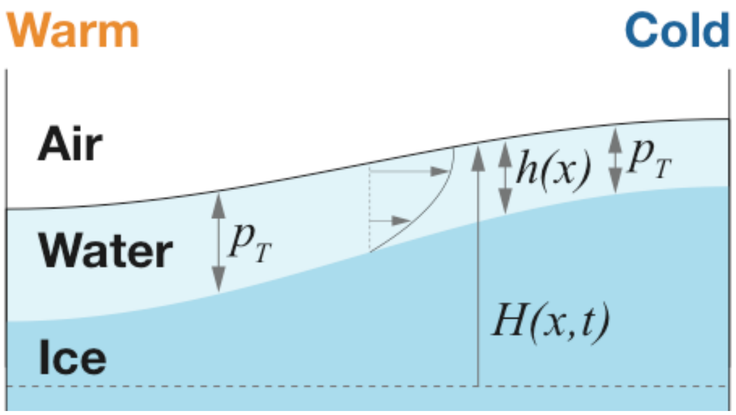
$$\rho L\frac{T_m - T}{T_m} \underset{\text{phase equilibrium}}{=} p_s - p_l \underset{\text{force balance}}{=} p_T$$

For van-der-Waals forces $p_T = \dfrac{A}{6\pi d^3} \Rightarrow d \propto \left(\dfrac{T_m - T}{T_m}\right)^{-1/3}$

Marangoni versus Thermomolecular Flow



Film thickness determined
dynamically

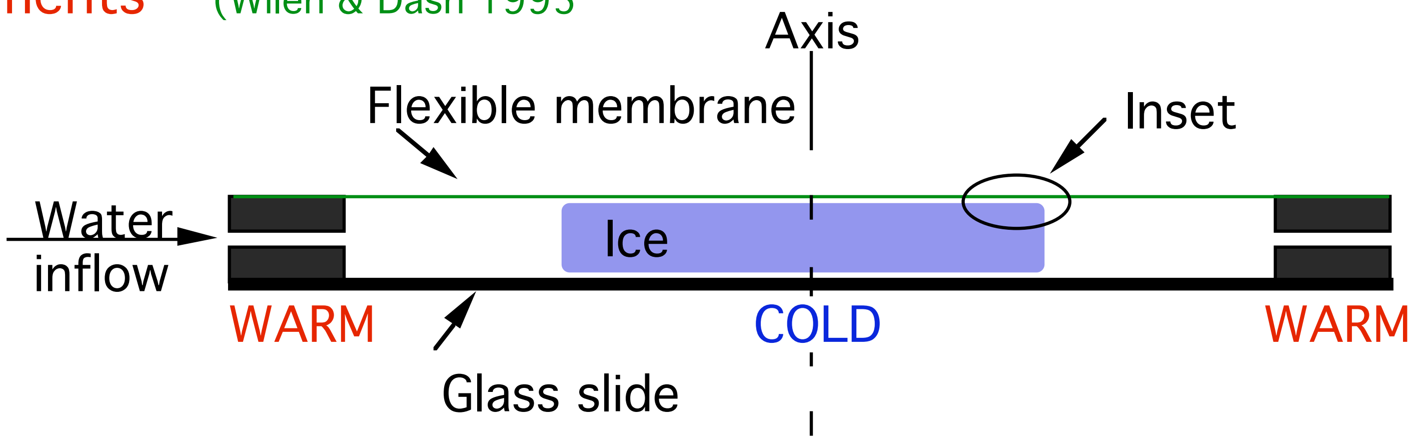


Film thickness determined
thermodynamically

Flow of Premelted Liquid

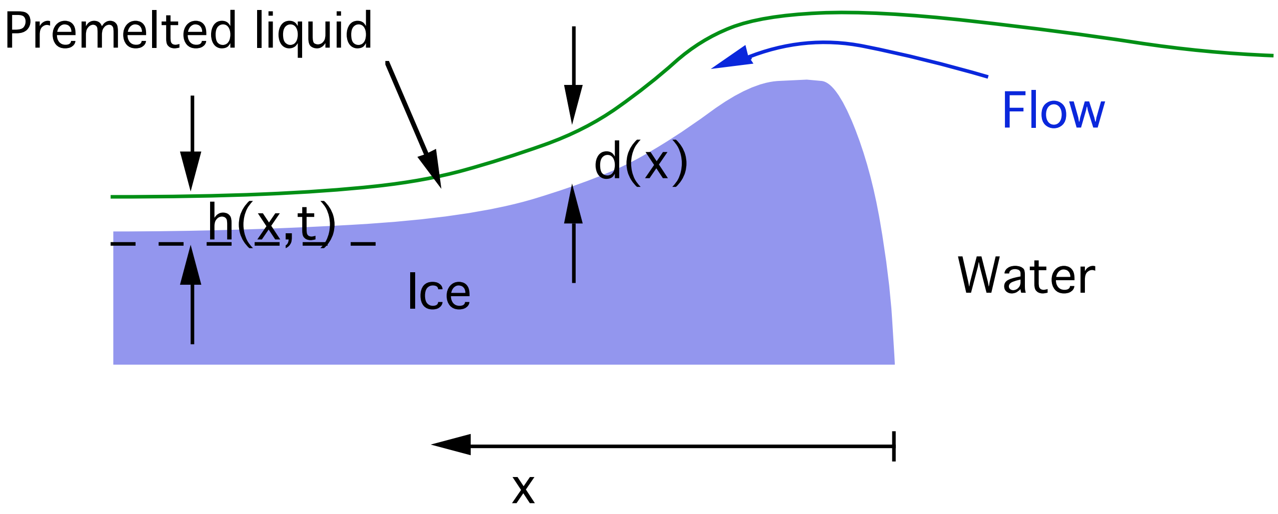
Experiments

(Wilen & Dash 1995



Lubrication Theory

(Wettlaufer & Worster 1995,9



Lubrication Theory

Lubrication theory gives volumetric flow rate in the premelted film to be

$$Q = -\frac{d^{\beta}}{12\mu}\frac{\partial p}{\partial x} = -\frac{\lambda^3 T_m}{12\mu G x}\frac{\partial p}{\partial x}$$

where the pressure driving the flow is

$$p = -\gamma h_{xx} - \frac{\rho L}{T_m}\left(T_m - T\right)$$

Elastic
wall stress

Thermo-molecular
pressure

Conservation of mass gives

$$\frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = 0 \quad \Rightarrow \quad \frac{\partial h}{\partial t} + D\frac{\partial}{\partial x}\left[\frac{1}{x}\left(\frac{\partial^3 h}{\partial x^3}\right) + \beta\right] = 0$$

where $D = \frac{1}{12}\frac{\lambda^3 T_m}{G}\frac{\gamma}{\mu}$ and $\beta = \frac{\rho L G}{\gamma T_m}$

Similarity Solution and Comparison with Experiments

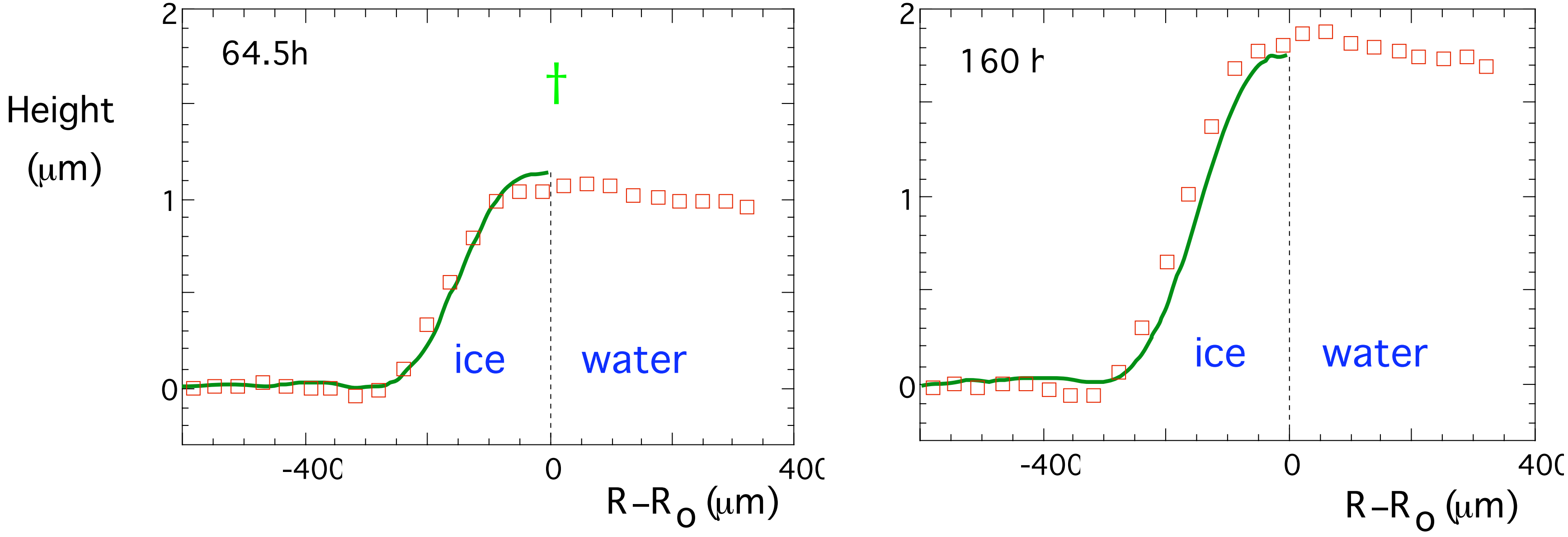
$h = \beta (Dt)^{3/5} f(\eta)$ with $\eta = \frac{x}{(Dt)^{1/5}}$

$$f'''' - \frac{f''' + 1}{\eta} - \frac{1}{5}\eta^2 f' + \frac{3}{5}\eta f = 0$$

† †

$f' = f'' = 0 \quad (\eta = 0), \qquad f \to 0 \quad (\eta \to \infty)$

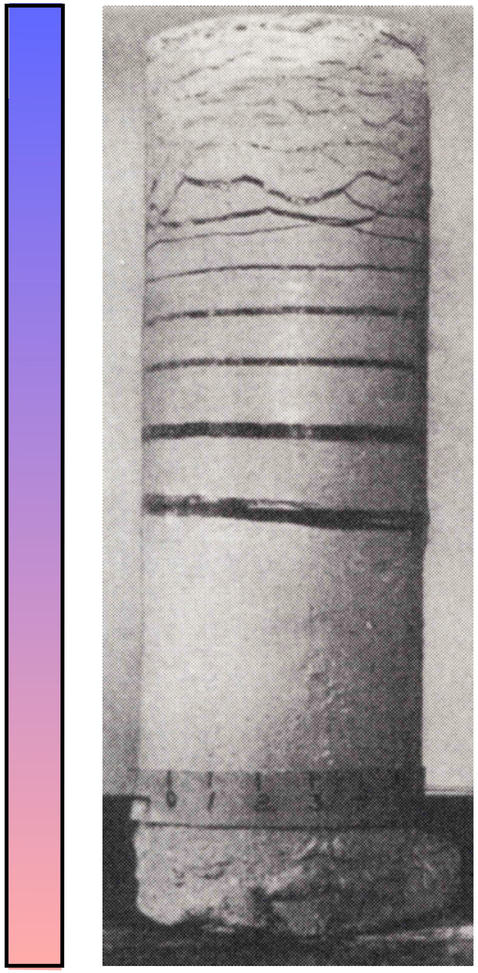
†



Multiple Ice Lenses

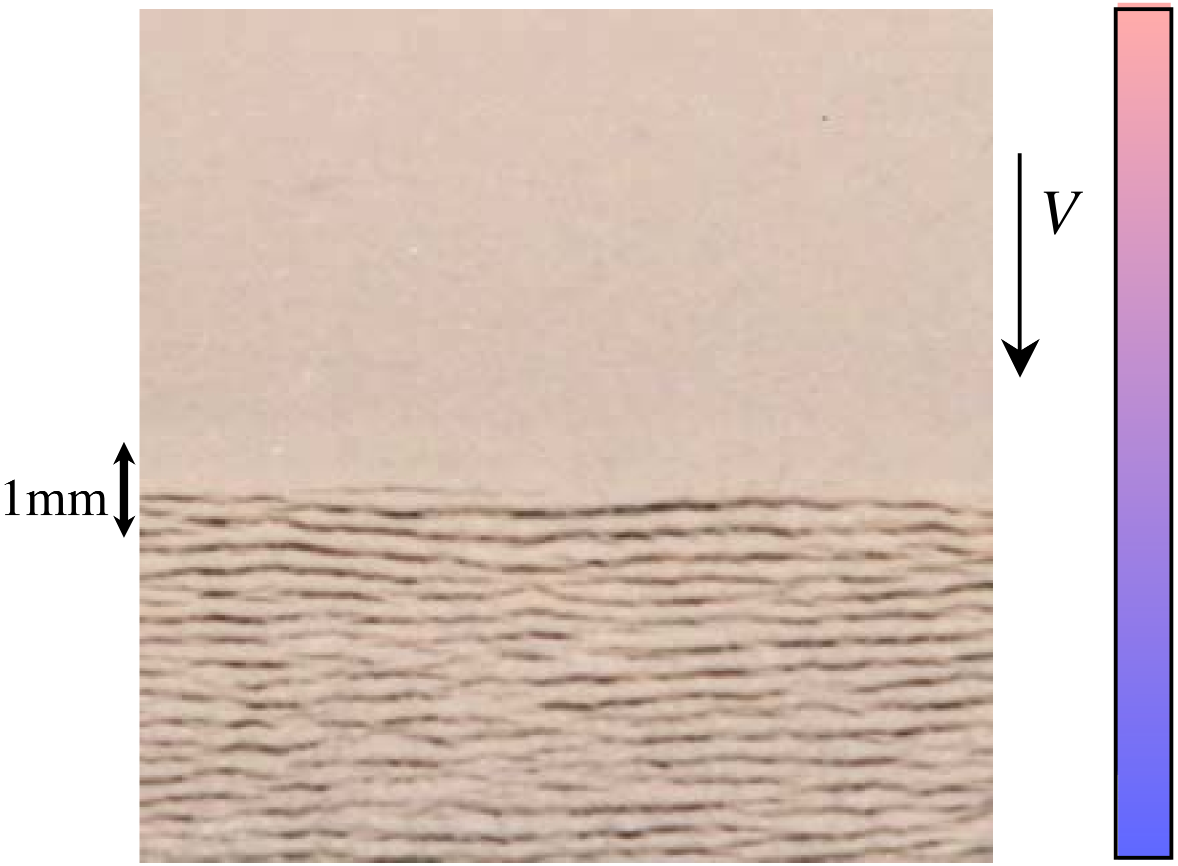
Taber (1930)

cold



warm

warm



cold

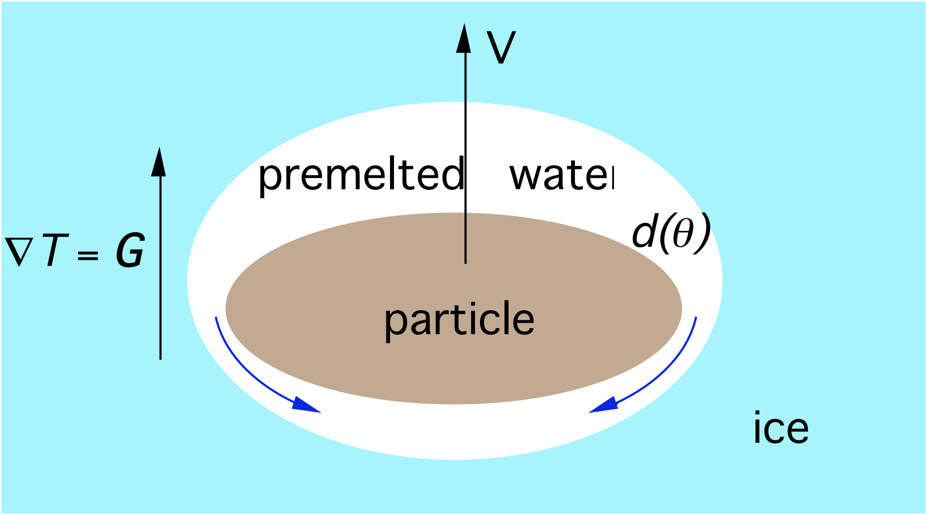
Peppin 2005

Thermodynamic Buoyancy

Rempel, Wettlaufer & W
PRL 2001

Film thickness determined by interfacial pre-melting and curvature

$$\rho L \frac{T_m - T}{T_m} = \frac{A}{6\pi d^3} + \gamma_{sl} \nabla \cdot \boldsymbol{n}$$

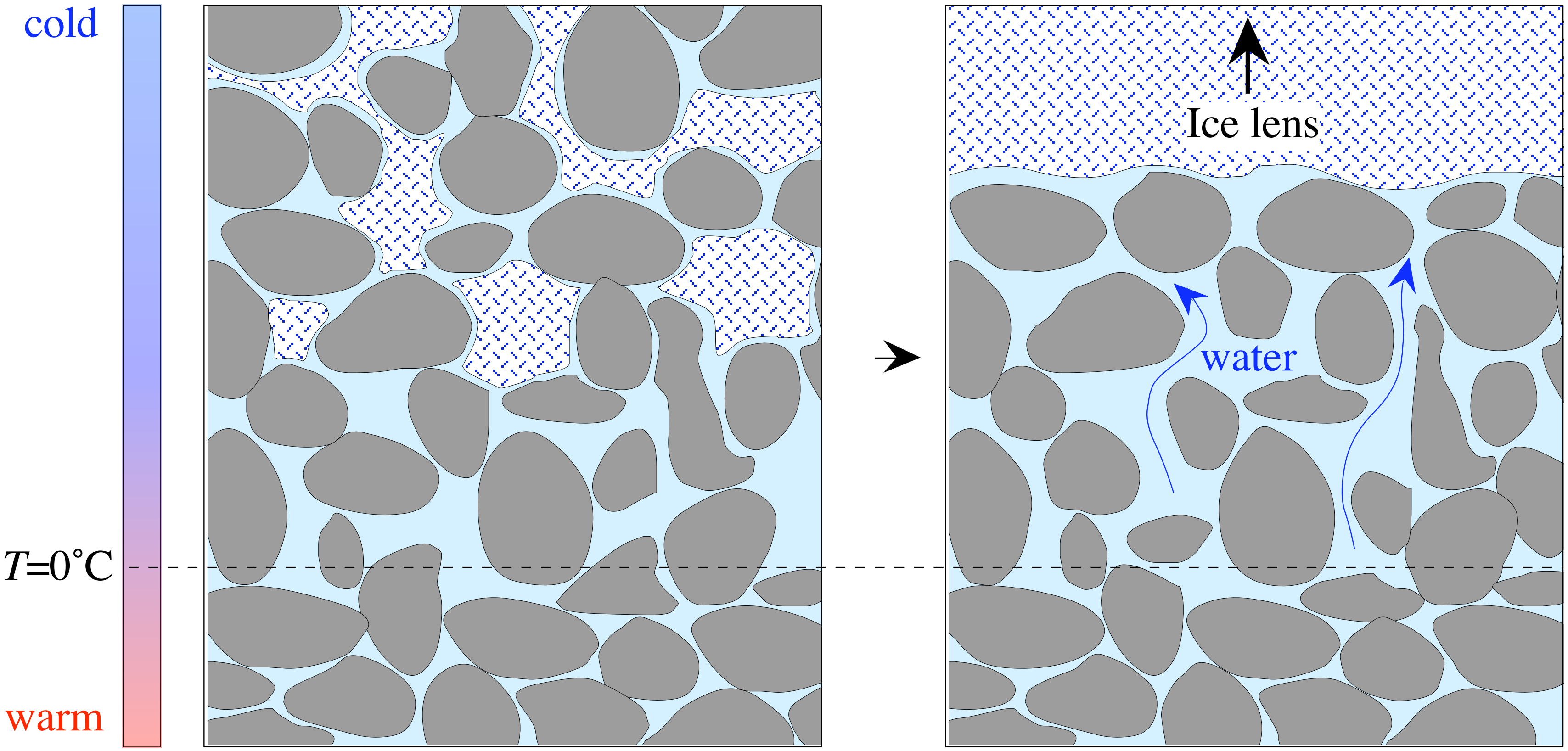


Net force on particle is $$\boldsymbol{F} = \int_S p_T \boldsymbol{n}\, dS = m_s \frac{L}{T_m} \langle \nabla T \rangle$$

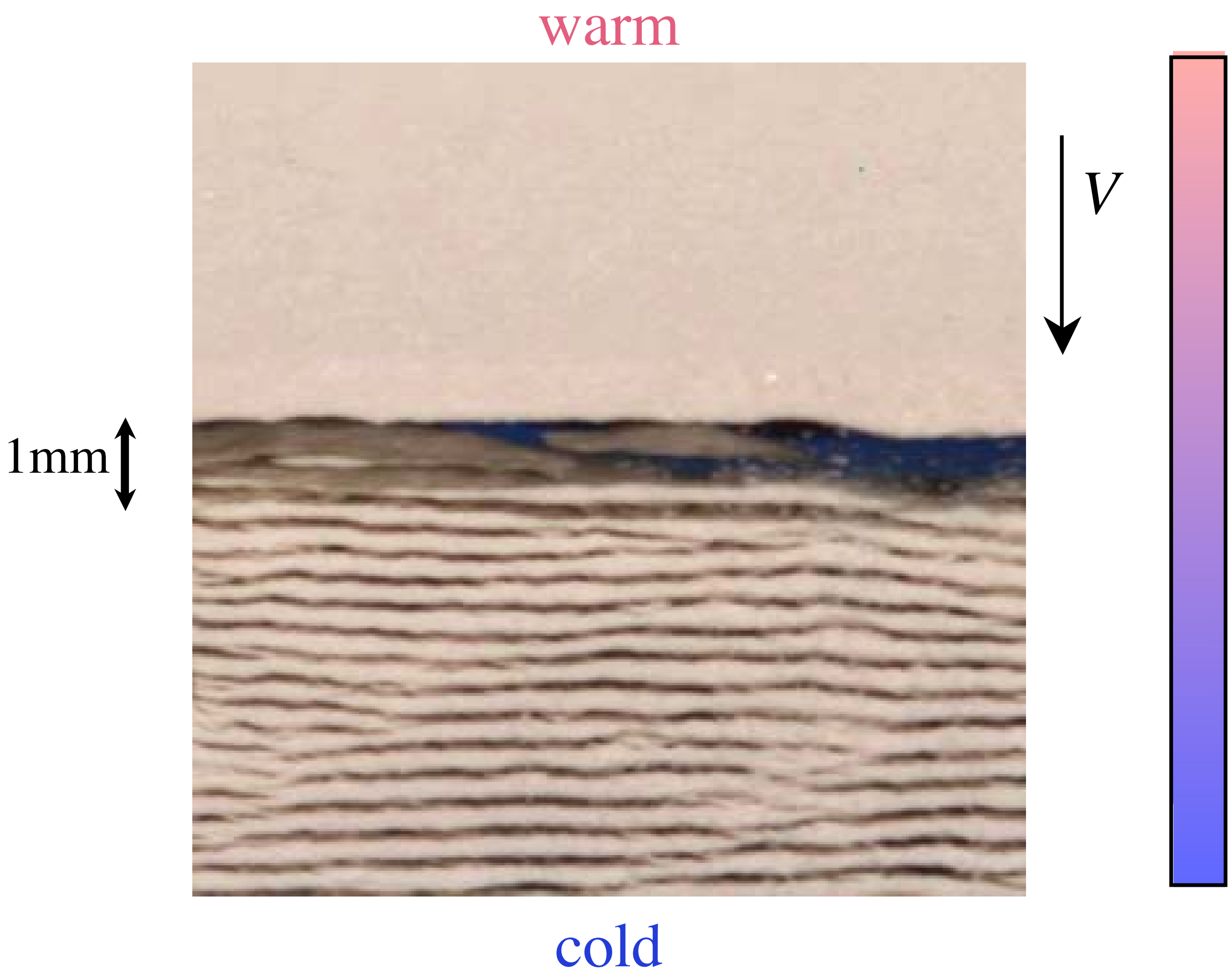
Where m_s is the mass of ice displaced by the particle.

cf Archimedes

Freezing of soil - formation of ice lenses



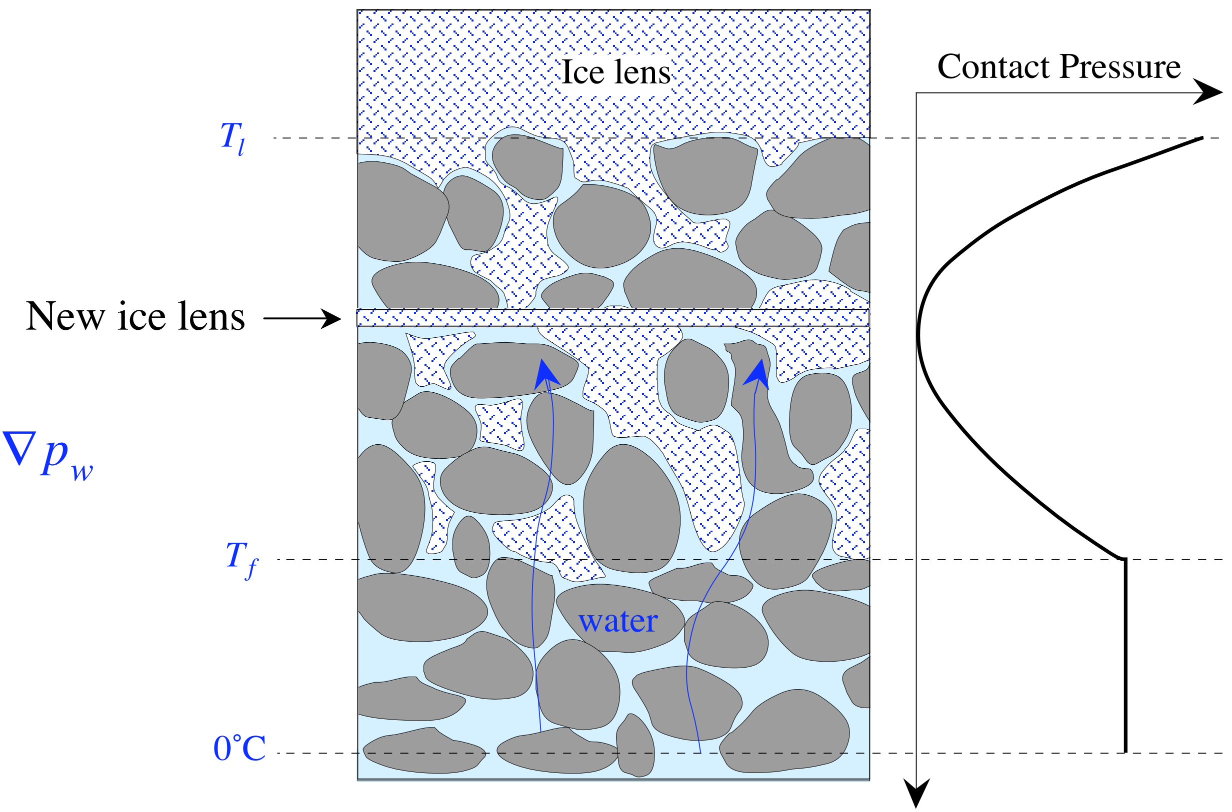
Single Ice Lens - Complete Particle Rejection



Kaolinite. 60% by weight. Particle size approximately 1 μm.

Single Ice Lenses in Nature – Needle Ice

Dynamics of the Lenses and Frozen Fringe



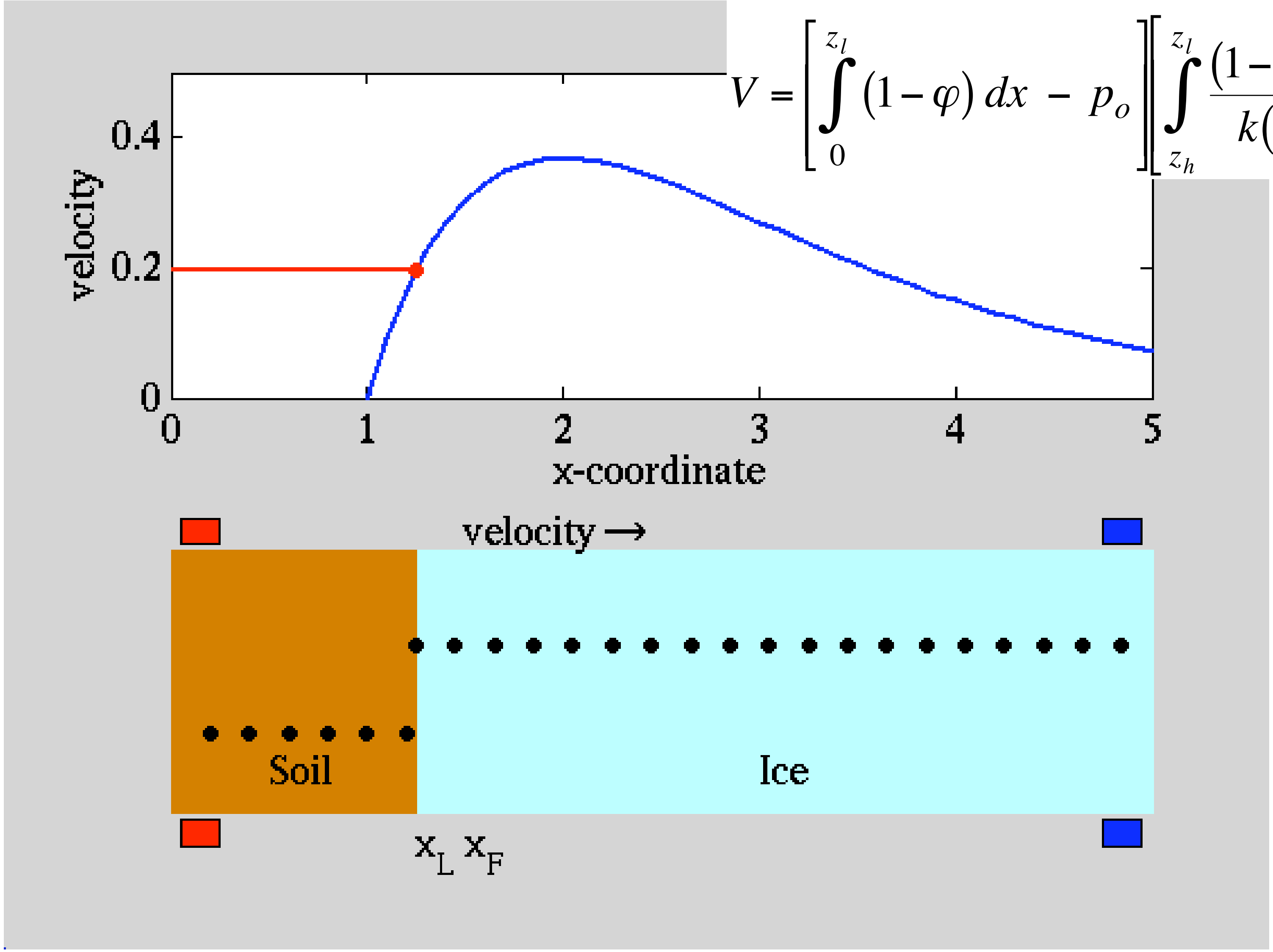
$$\mathbf{u} = -k(\varphi)\nabla p_w$$

Net vertical inter-particle force is

$$P_p = P_o - \frac{\rho L}{T_m}\nabla T\left[\int_0^z (1-\varphi(\eta))\,d\eta \;-\; z(1-\varphi(z))\right] + \mu V\int_{z_h}^{z}\frac{(1-\varphi)^2}{k(\varphi)}\,d\eta$$

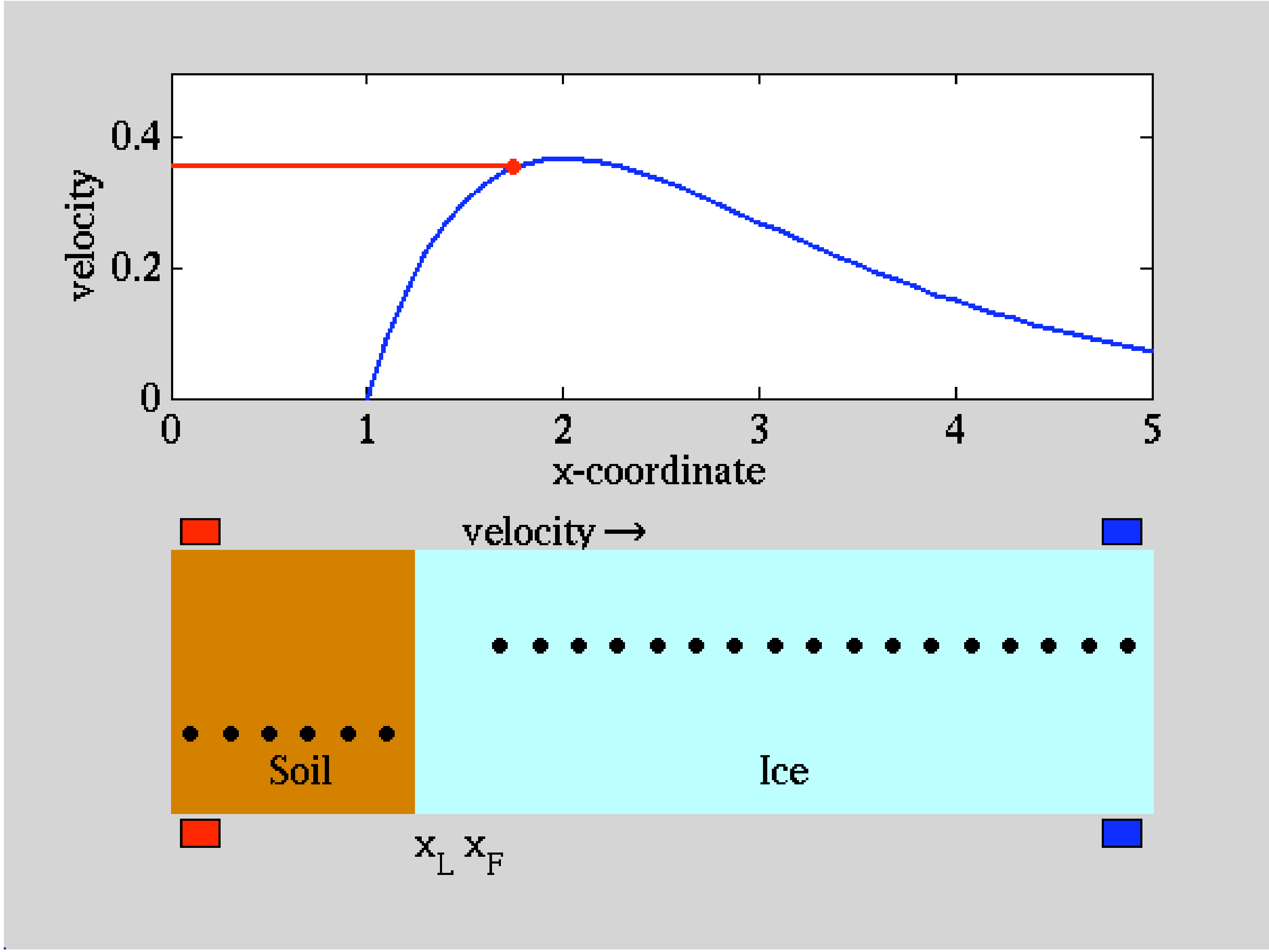
Overburden Thermodynamic buoyancy Viscous drag

Calculations of ice-lens dynamics



$$V = \left[\int_0^{z_l} (1-\varphi)\, dx \;-\; p_o\right]\left[\int_{z_h}^{z_l} \frac{(1-\varphi)^2}{k(\varphi)}\, dx\right]^{-1}$$

Calculations of ice-lens dynamics



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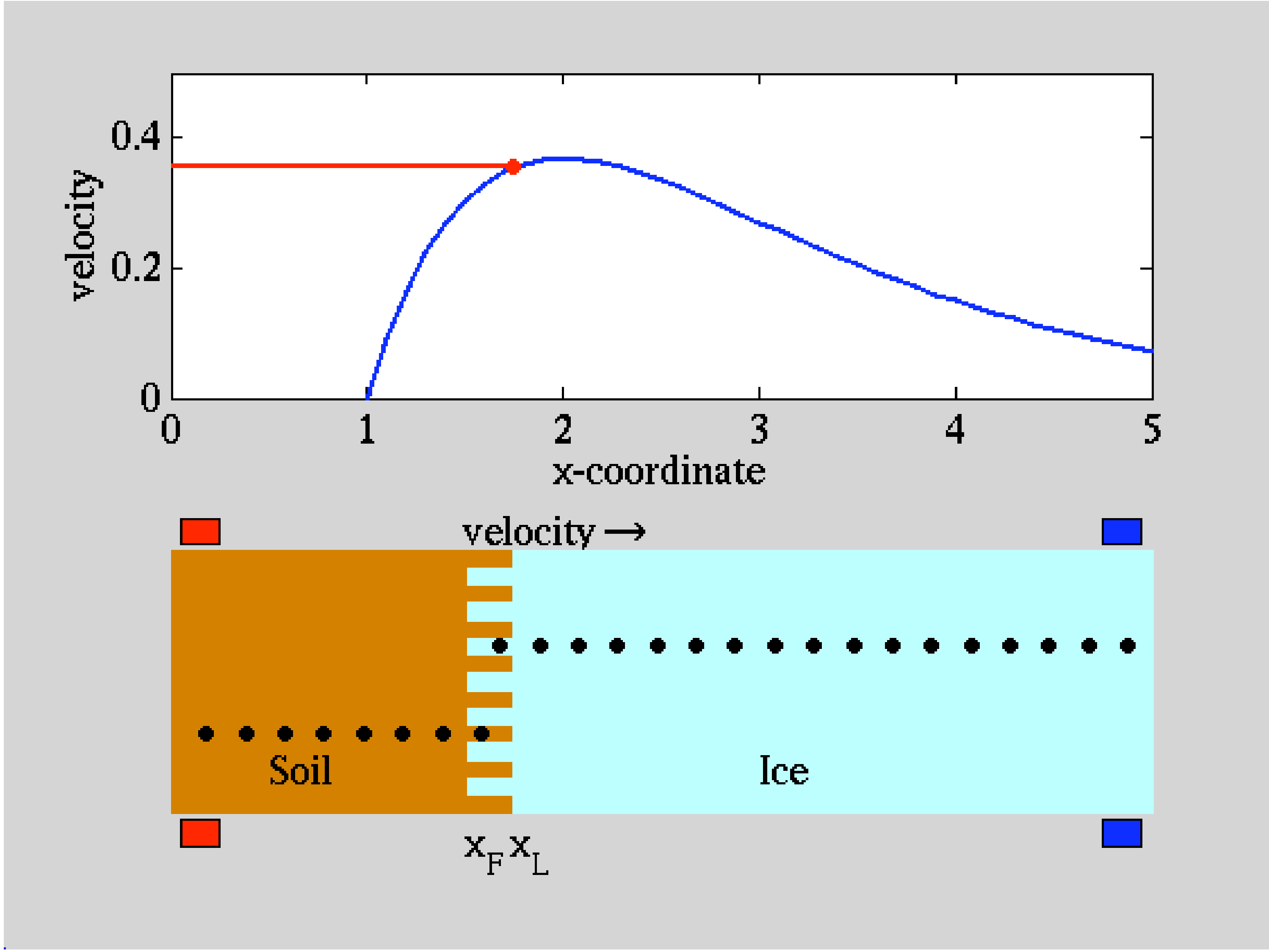

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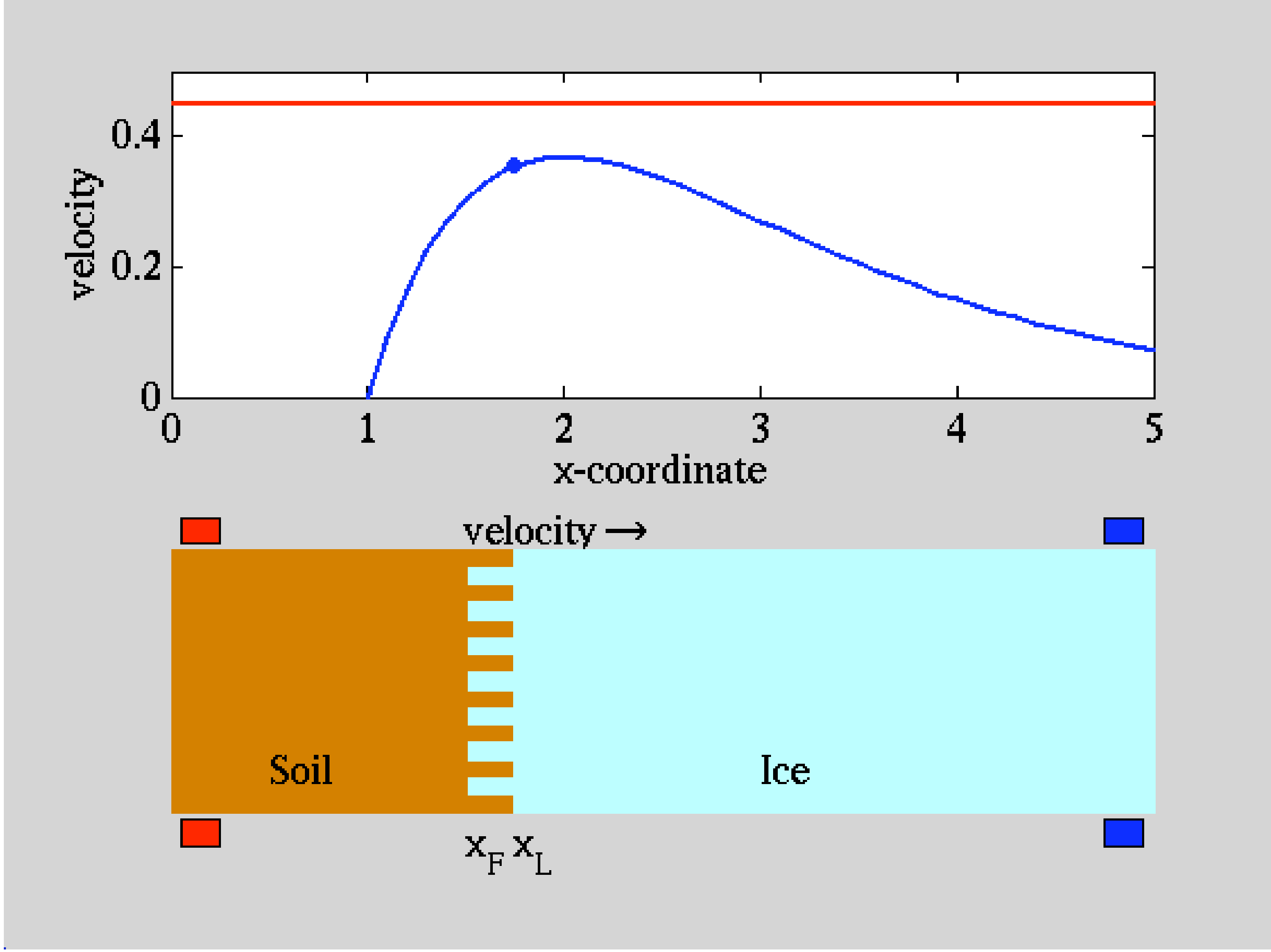

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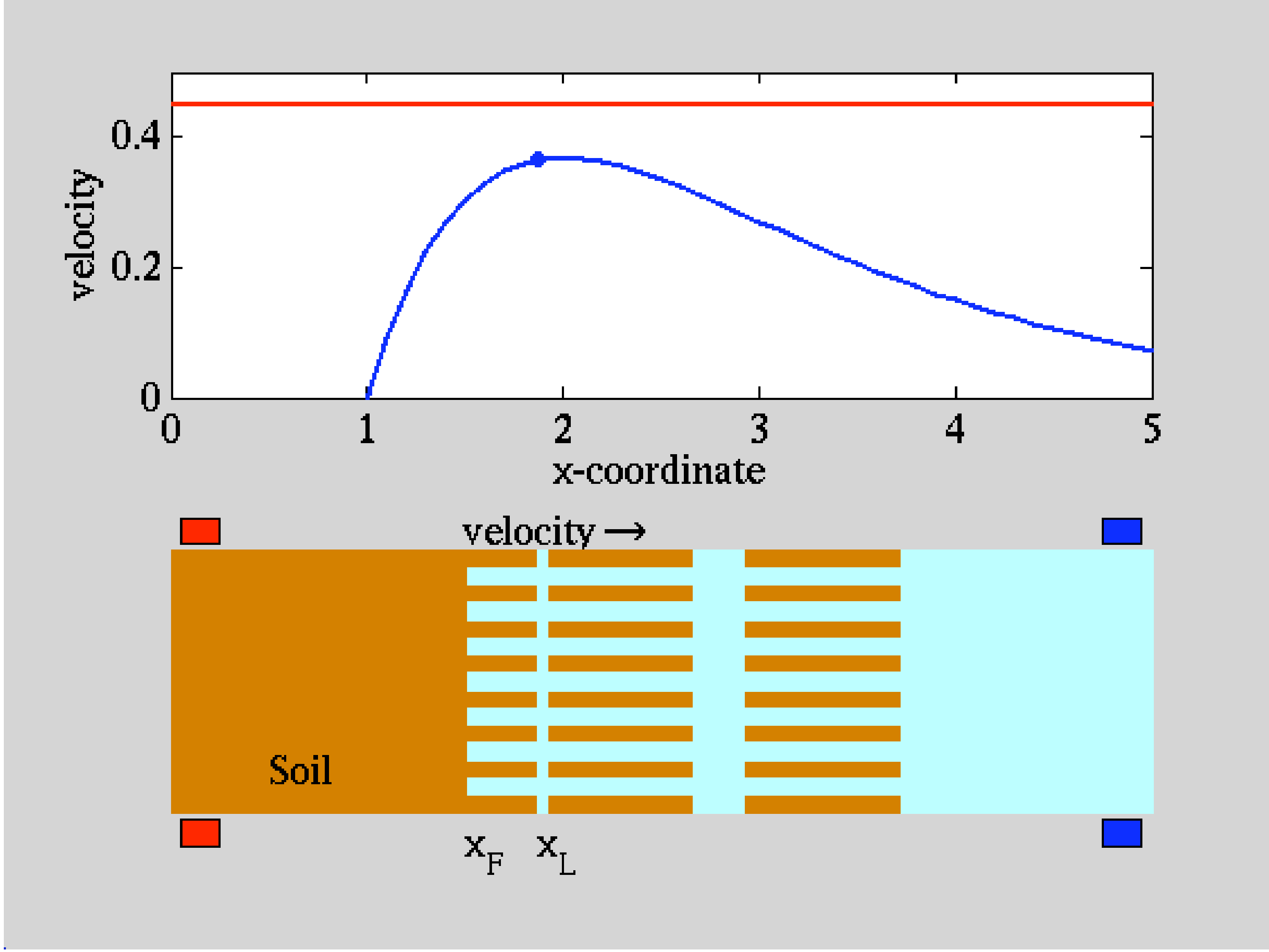

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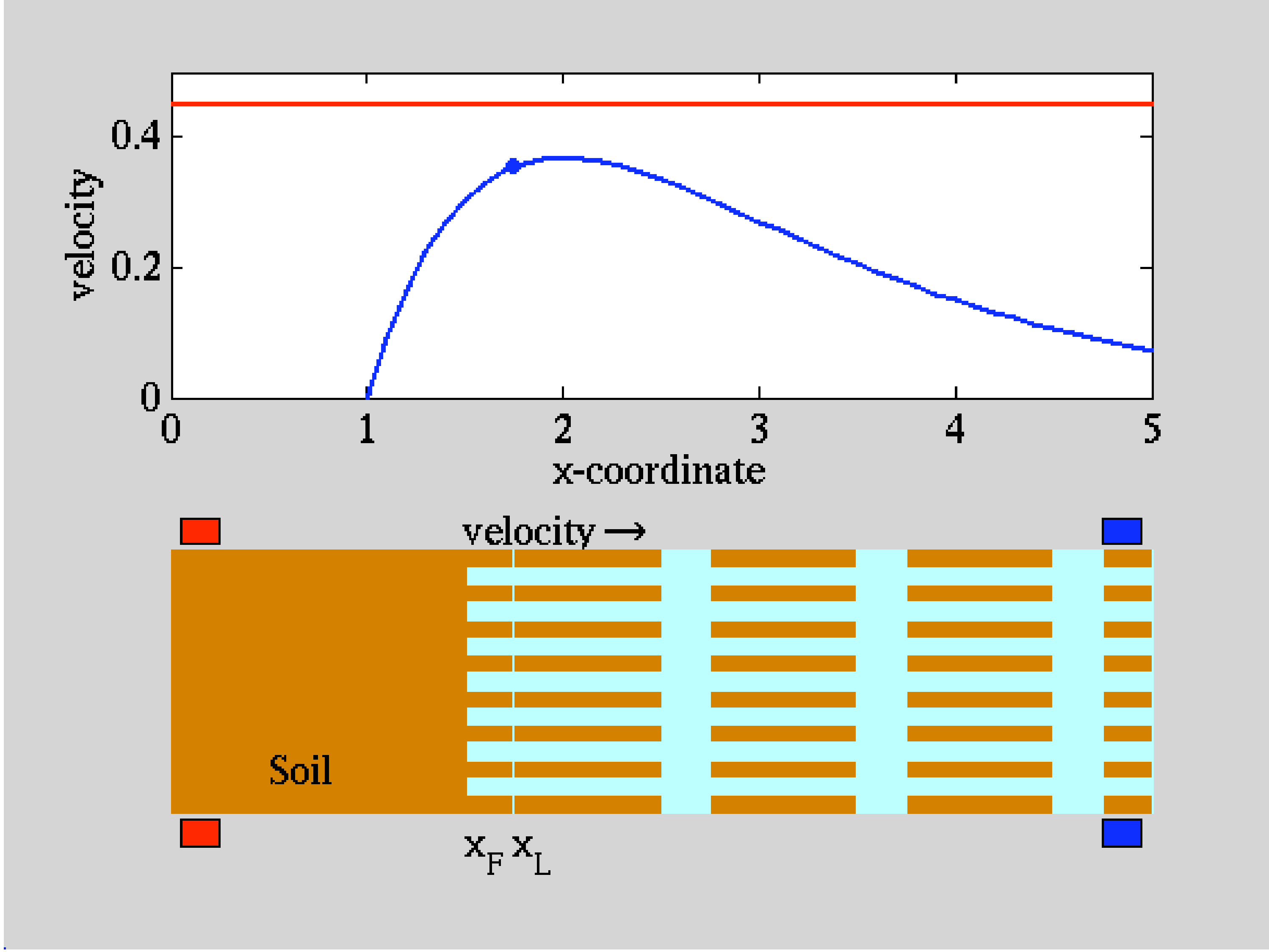
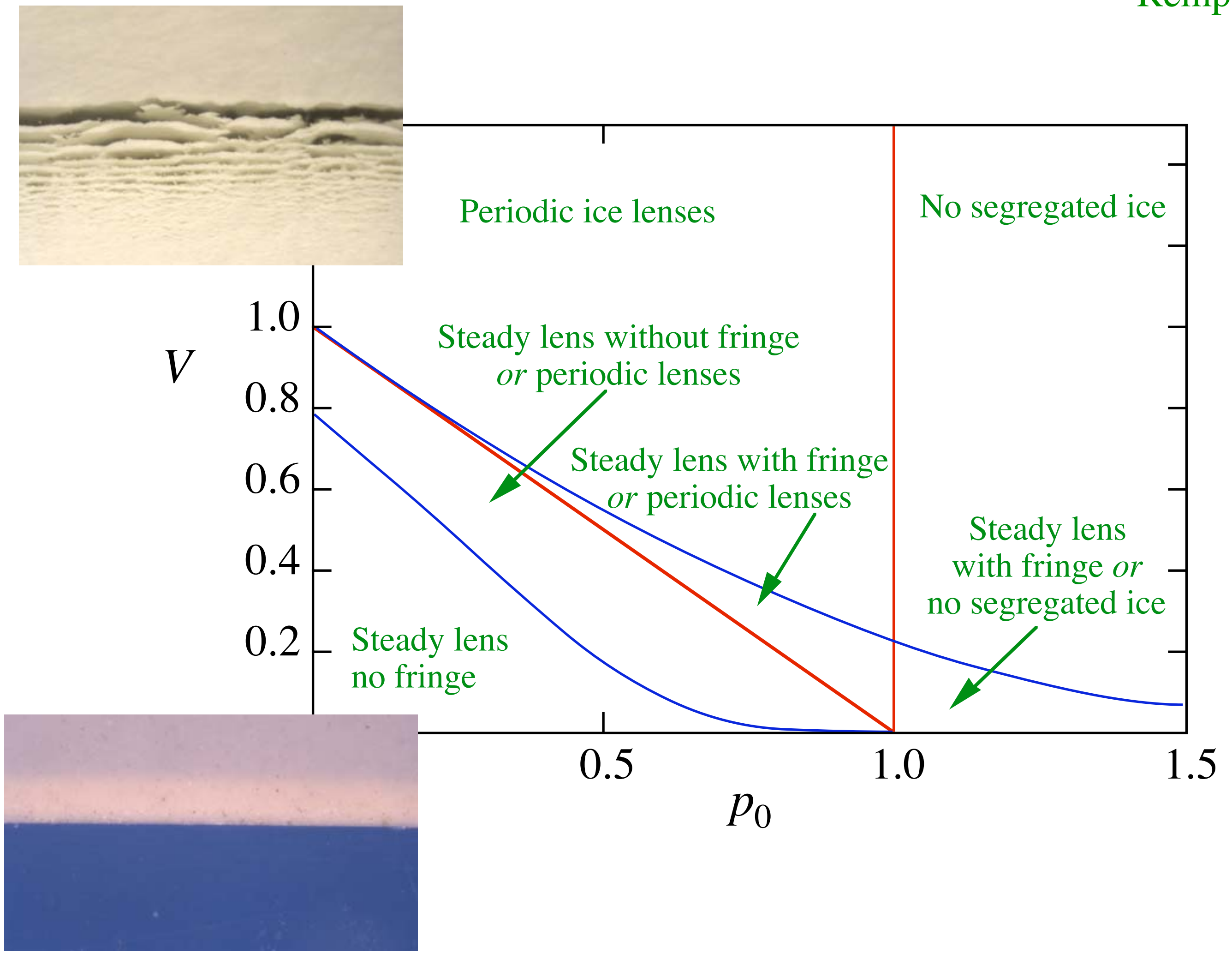
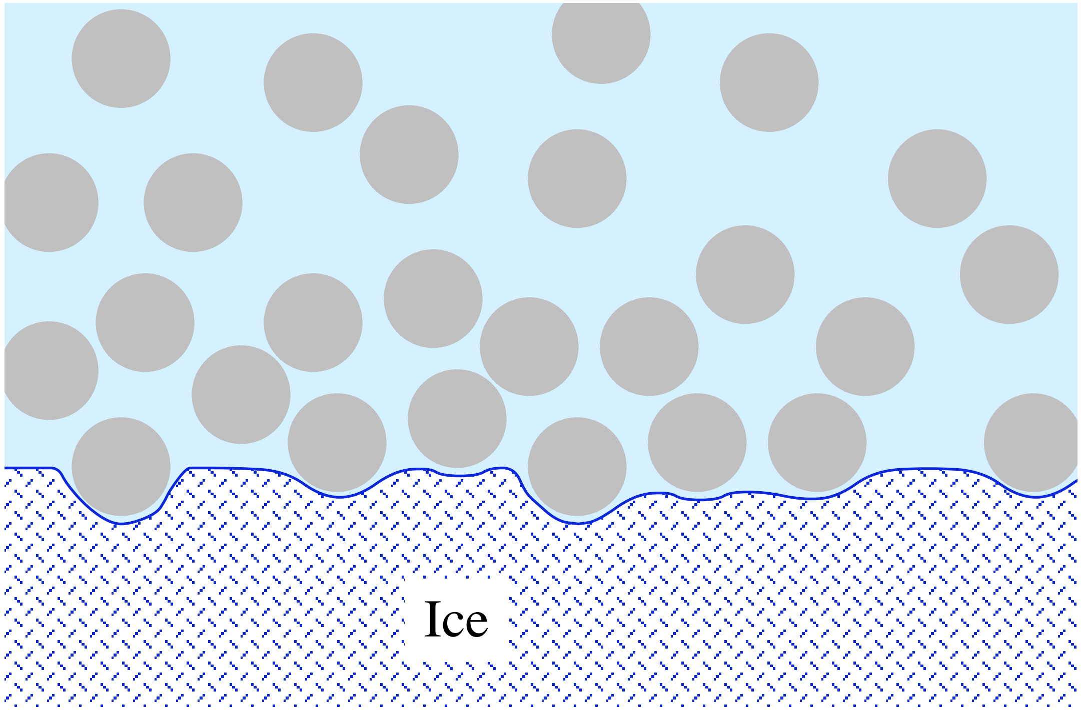


Modes of Behaviour

Rempel, Wettlaufer & W.
JFM 2004



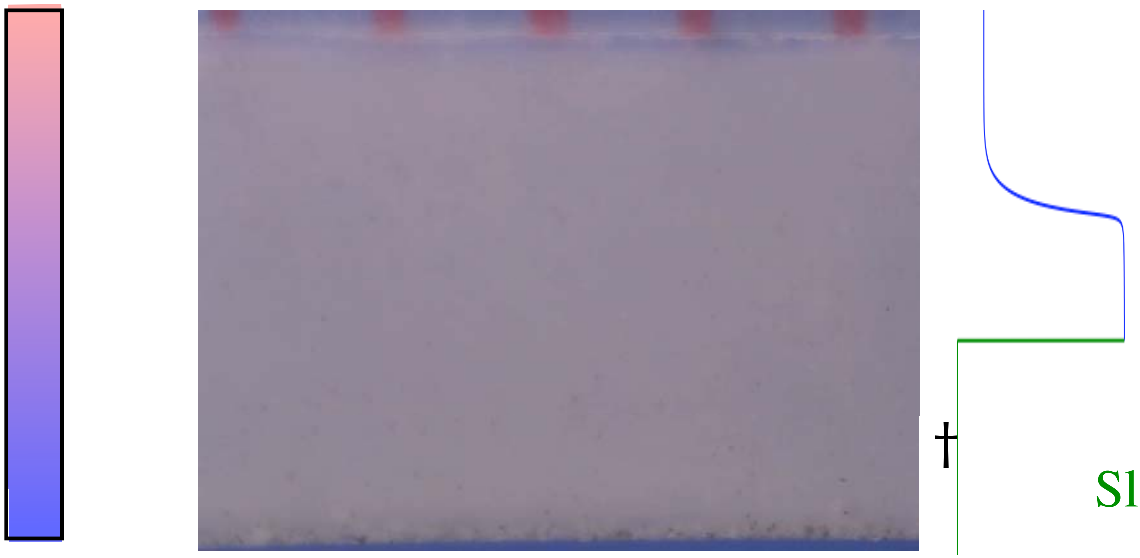
Freezing of a Colloidal Suspension



$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\left(D(C)\frac{\partial C}{\partial z}\right)$$

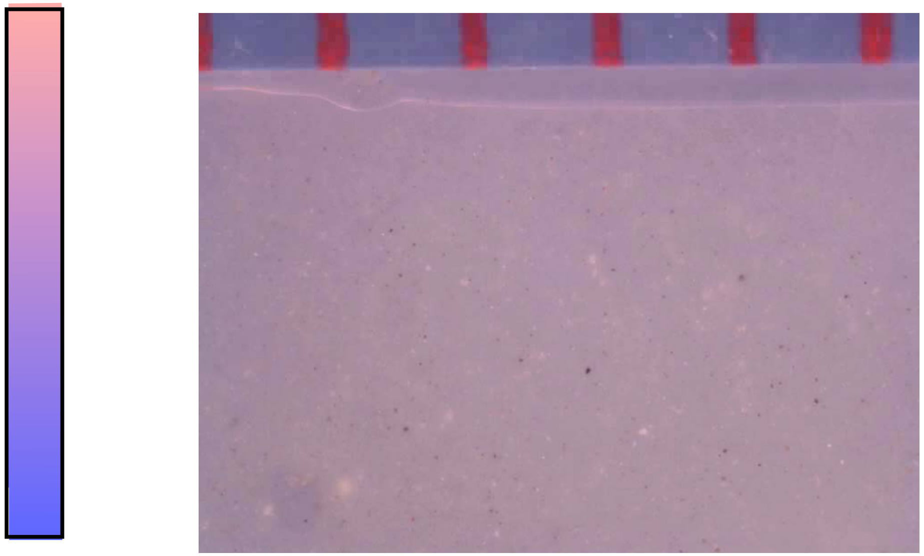
$$T_i = T_i(C)$$

Freezing of a Colloidal Suspension



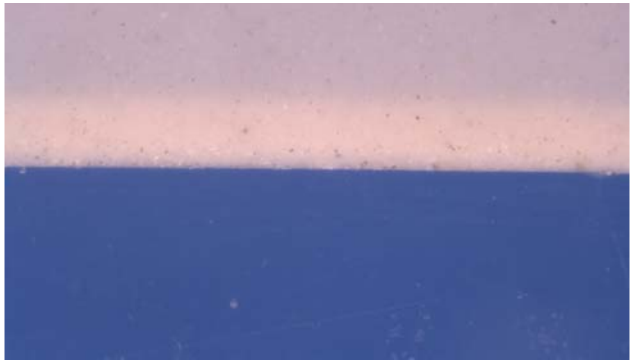
$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial z}\left(D(C)\frac{\partial C}{\partial z} \right)$$

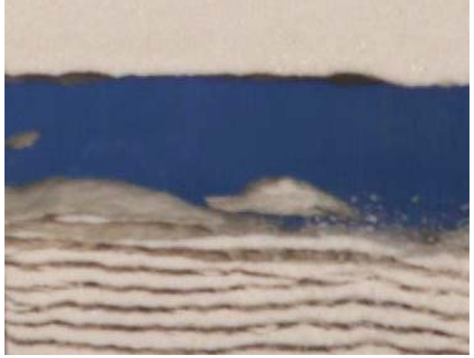
Slow freezing rate

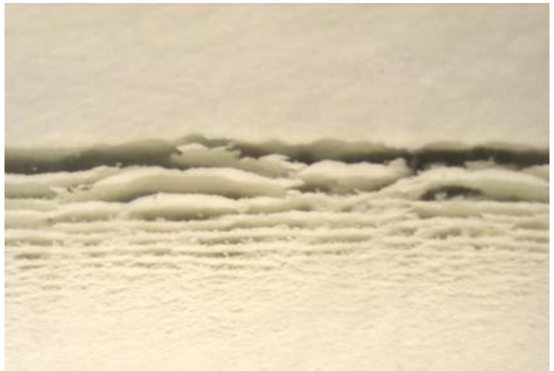


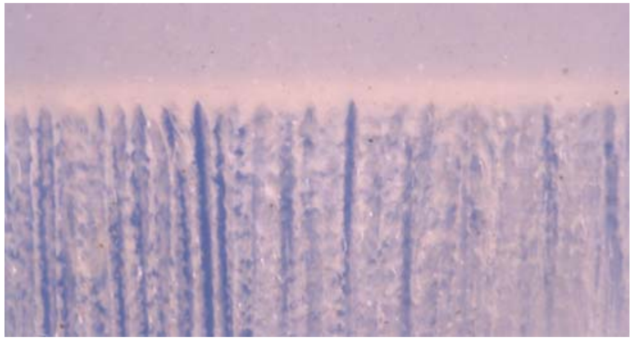
Fast freezing rate

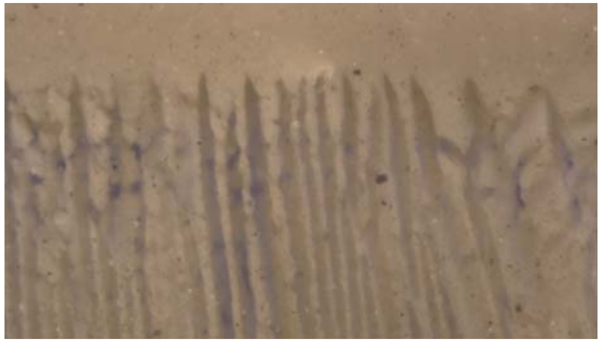
Different Types of Behaviour

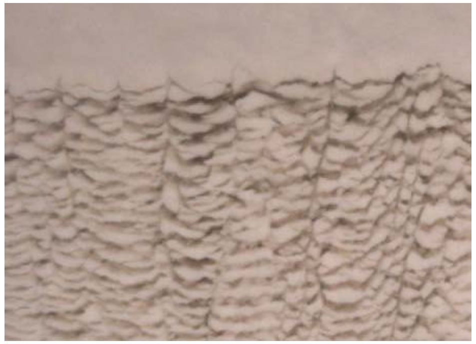












Summary and Conclusions

Long-range intermolecular forces can cause most solids to premelt at their surfaces or at interfaces with other materials

Temperature gradients give rise to gradients in thermo-molecular pressure:
- surface transport;
- thermodynamic buoyancy

Competition between thermodynamic buoyancy and viscous fluid flow determines heaving rates and lens initiation

Interplay between
- morphological instability of lens front,
- nucleation beyond compaction layer and
- thermodynamic buoyancy within compaction layer

may determine a wide range of different behaviours