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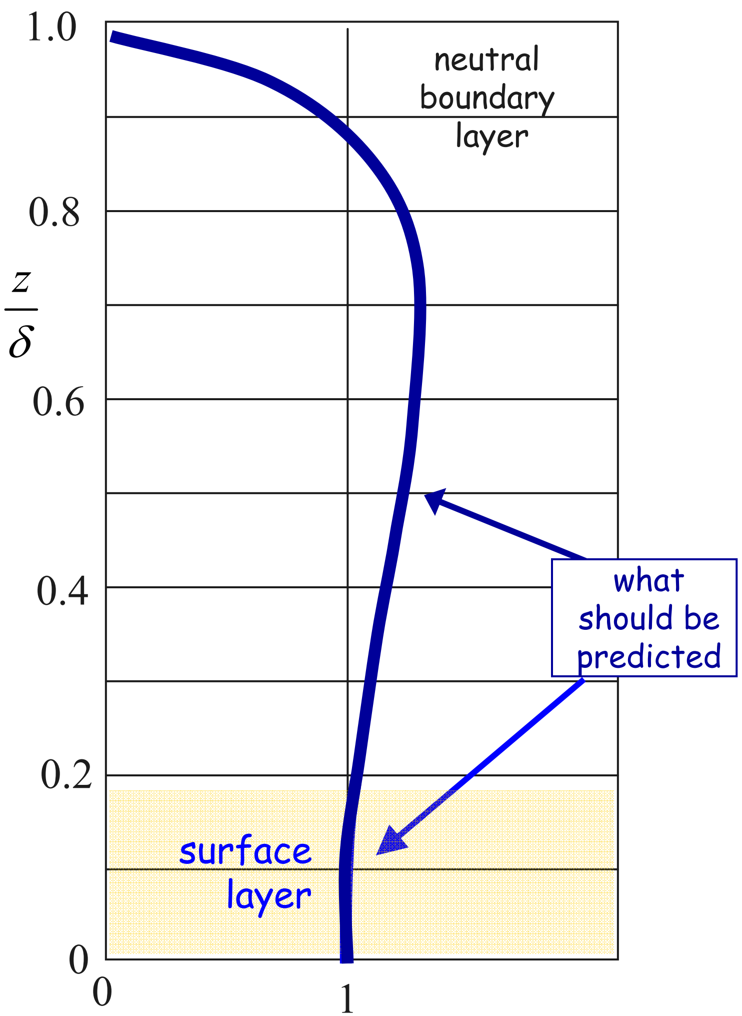
Designing Large-Eddy Simulation of High Reynolds Number Wall-Bounded Flows*

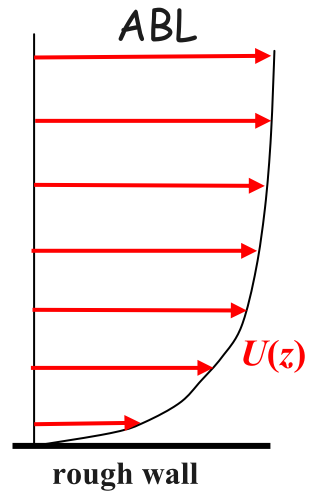
James G. Brasseur & Tie Wei
Pennsylvania State University

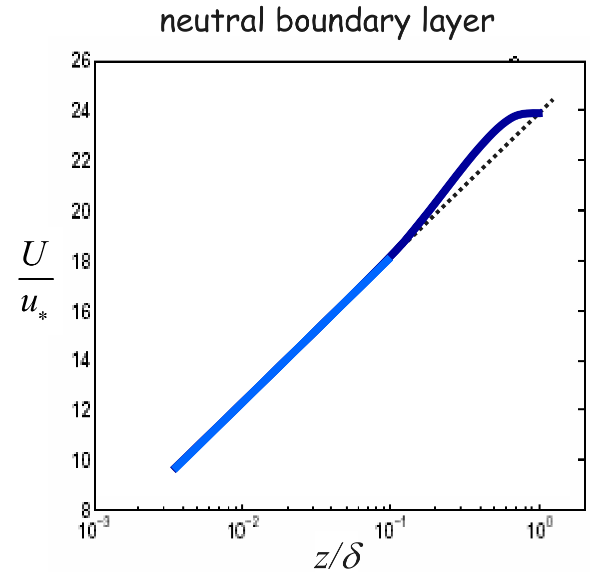
*supported by the Army Research Office

Fundamental Errors in LES Prediction of the High Reynolds Number Boundary Layer



$$\phi_m = \frac{\kappa z}{u_*}\frac{dU}{dz}$$



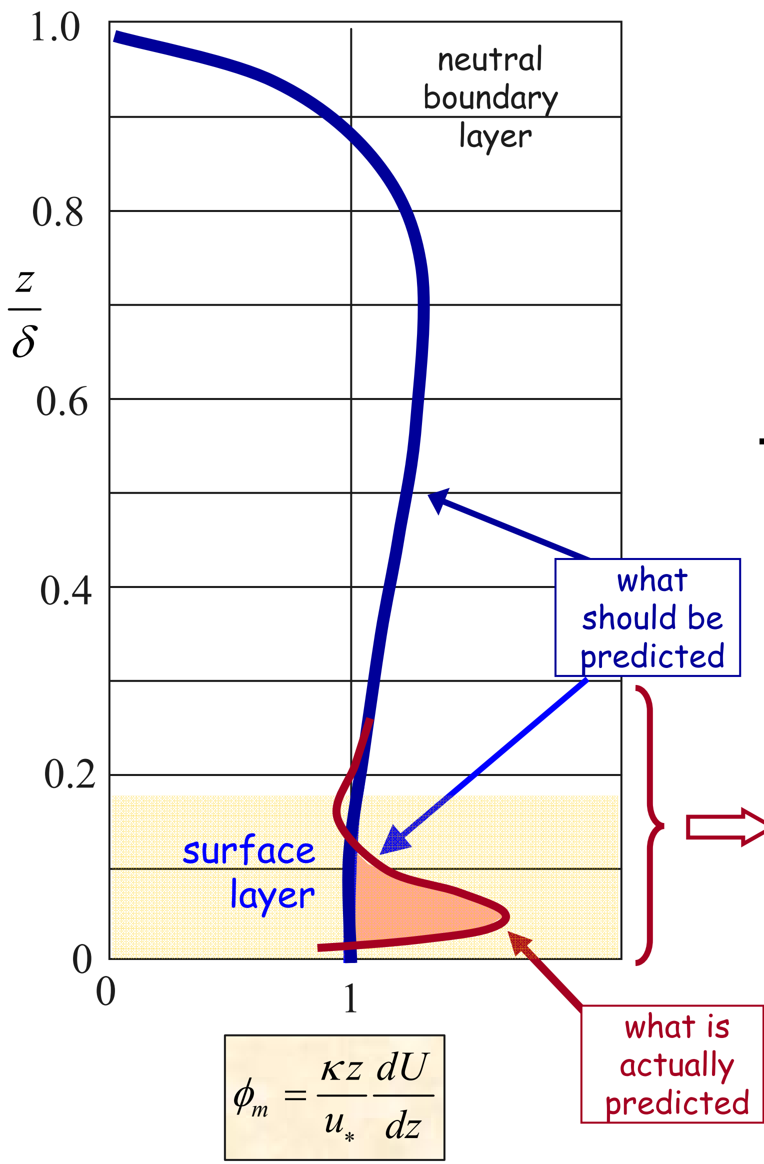


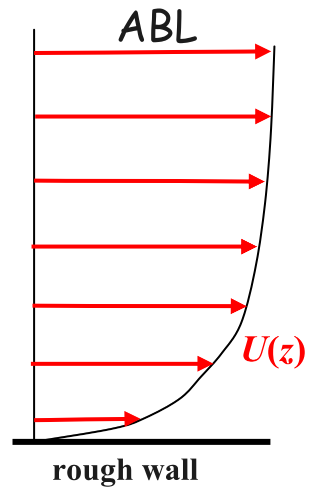
LES of high Reynolds number boundary layers

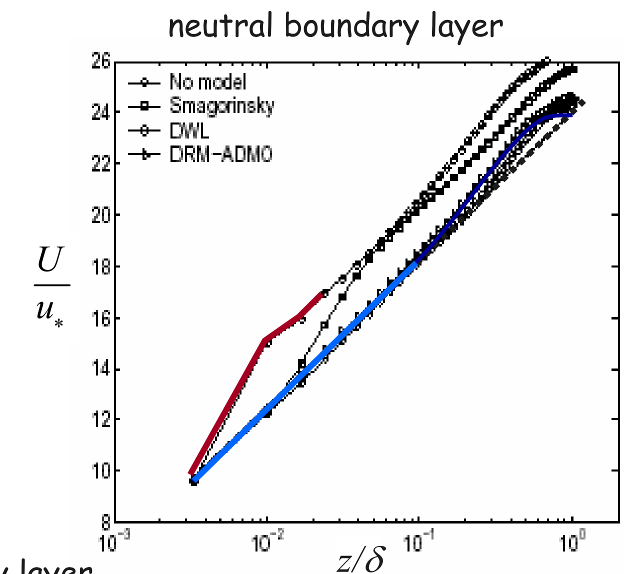
⇒ the viscous sublayer is unresolvable or nonexistent

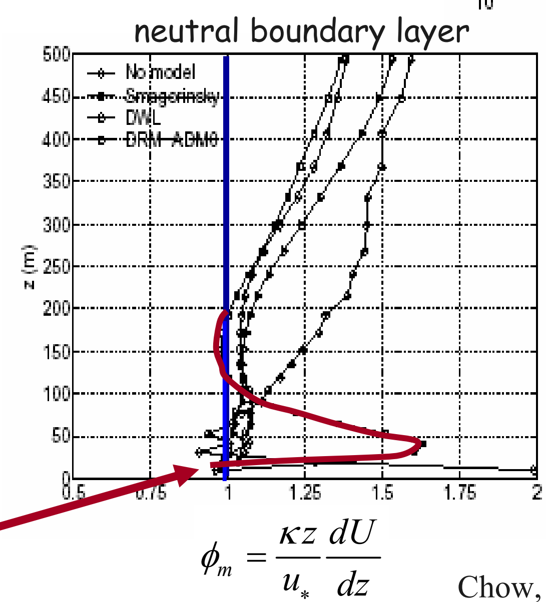
⇒ plus units not useful

Fundamental Errors in LES Prediction of the High Reynolds Number Boundary Layer







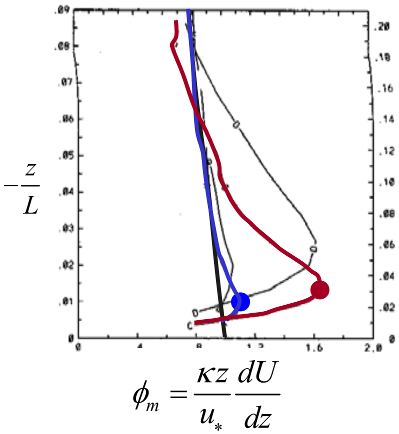


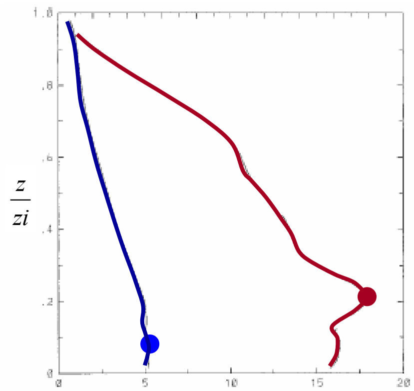
Two Issues:

1. Law-of-the-Wall is not predicted
2. Overshoot

Chow, Street, Xue, Ferziger 2005 *JAS 62*

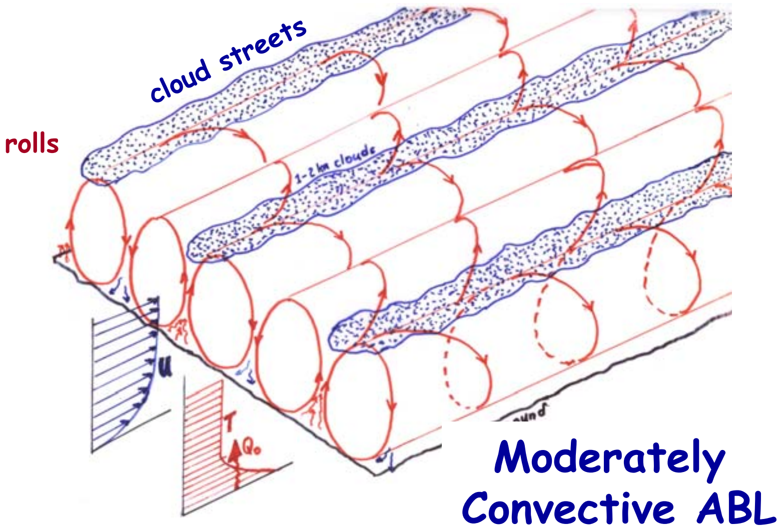
The Importance of the Overshoot





Over-prediction of mean shear and TURBULENCE PRODUCTiON in the surface layer produces poor predictions throughout the ABL of

- thermal eddying structure (e.g., rolls)
- vertical transport, dispersion and eddy structure of momentum, temperature, humidity, contaminants, toxins, …
- correlations, turbulent kinetic energies,…
- cloud cover, CO_2 transport, radiation, …



Cloud Streets

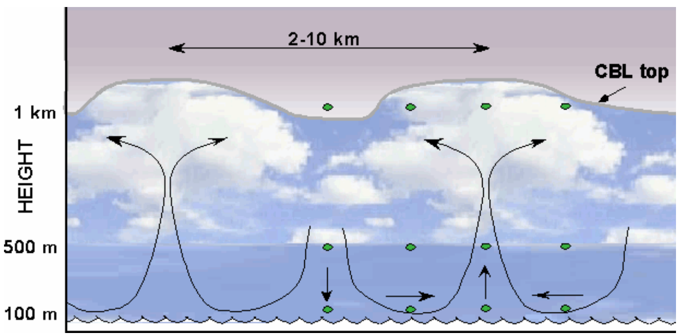
(on top of the rolls, or "very large structures")

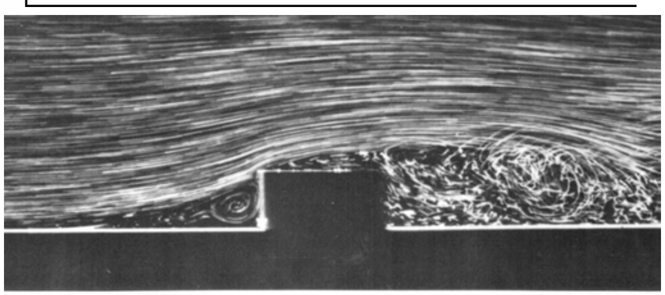
Khanna & Brasseur 1998, J. Atmos. Sci.. 55: 3135

16-year History of the Overshoot in LES of the ABL

Relevant to any LES of boundary layers where the viscous sublayer is unresolved or nonexistent

… enhanced with direct exchange between inner and outer boundary layer:

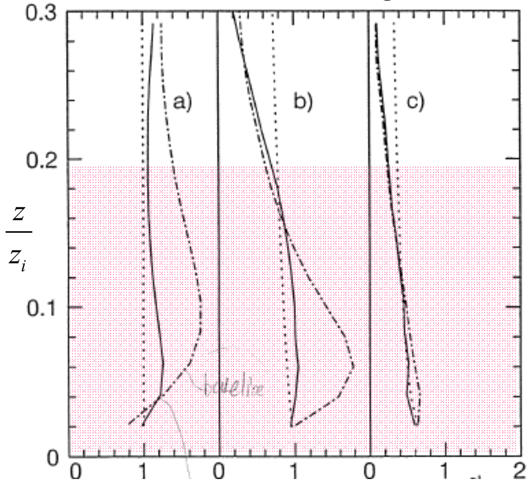




1. **Mason & Thomson 1992, *JFM* 242.**
2. Sullivan, McWilliams & Moeng 1994, *BLM* 71.
3. Andren, Brown, Graf, Mason, Moeng, Nieuwstadt & Schumann 1994 *QJR Meteor Soc* 120 (comparison of 4 codes: Mason, Moeng, Neiustadt, Schumann).
4. Khanna & Brasseur 1997, *JFM* 345.
5. Kosovic 1997, *JFM* 336.
6. Khanna & Brasseur 1998, *JAS* 55.
7. Juneja & Brasseur 1999 *Phys Fluids* 11.
8. Port-Agel, Meneveau & Parlange 2000, *JFM* 415.
9. Zhou, Brasseur & Juneja 2001 *Phys Fluids* 13.
10. Ding, Arya, Li 2001, *Environ Fluid Mech* 1.
11. Reselsperger, Mahé & Carlotti 2001, *BLM* 101.
12. Esau 2004 *Environ Fluid Mech* 4.
13. Chow, Street, Xue & Ferziger 2005, *JAS* 62
14. Anderson, Basu & Letchford 2007, *Environ Fluid Mech* 7.
15. Drobinski, Carlotti, Redelsperger, Banta, Masson & Newson 2007, *JAS* 64.
16. Moeng, Dudhia, Klemp & Sullivan 2007 Monthly Weather Rev 135.

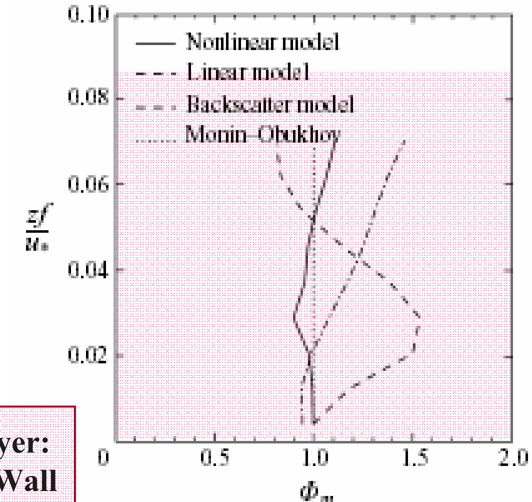
Observation 1: The Overshoot is Sensitive to the SFS Stress Model

Sullivan, McWilliams, Moeng 1994, *BLM* 71

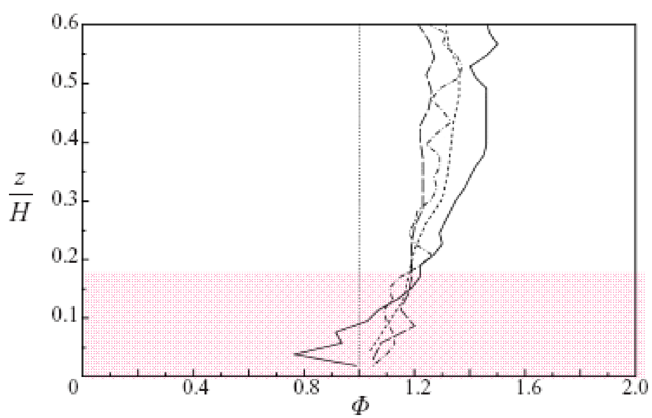


$$\phi_m = \frac{\kappa z}{u_*}\frac{dU}{dz}$$

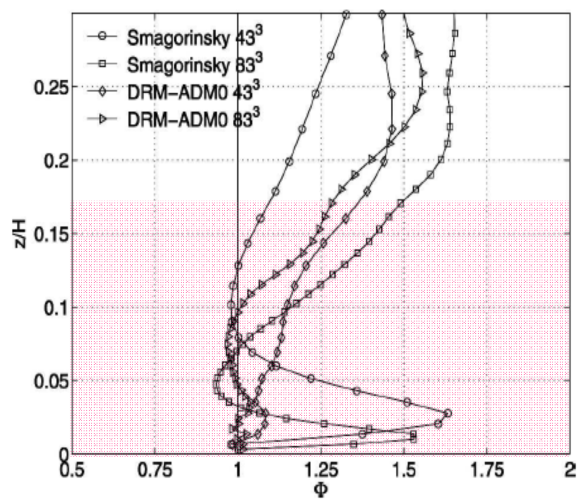
Surface Layer: Law of the Wall Region



Kosovic 1997, *JFM* 336



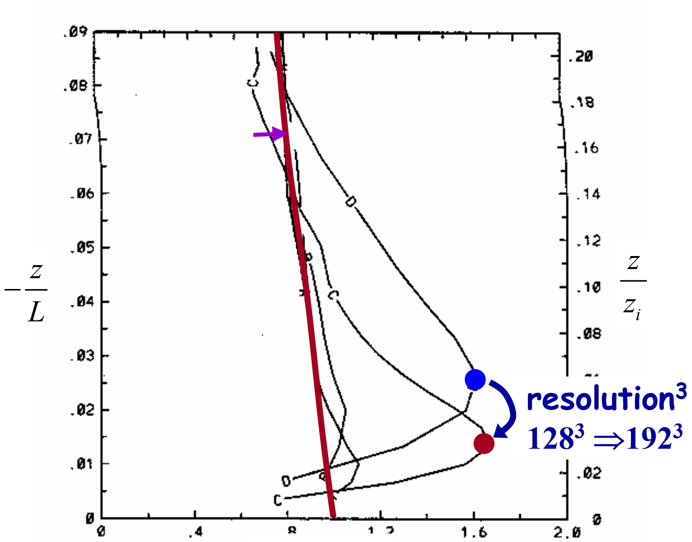
Port-Agel, Meneveau & Parlange 2000, JFM 415



Chow, Street, Xue & Ferziger 2005, JAS 62

Observation 2:
Lack of Grid Independence

Observation 3: The Overshoot is Tied to the Grid

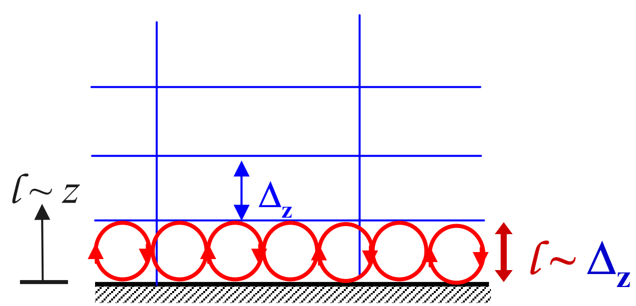


Khanna & Brasseur 1997, *JFM* 345

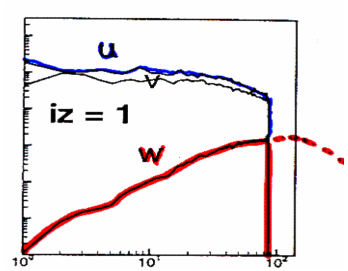
$$\phi_m = \frac{\kappa z}{u_*}\frac{dU}{dz}$$

Increasing grid resolution only moves the overshoot closer to the surface.

Inherent under-resolution at the first grid level

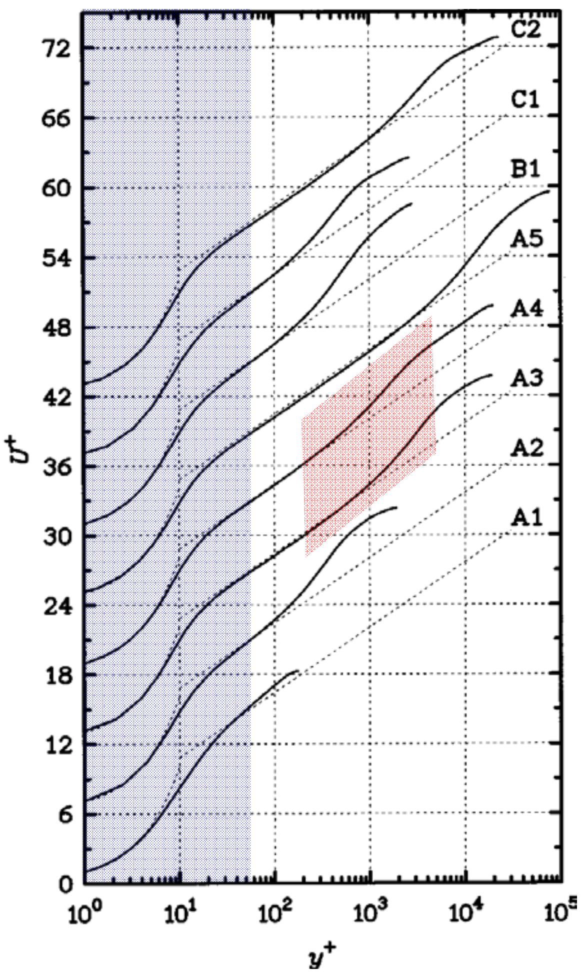


Juneja & Brasseur 1999, *Phys. Fluids* 11

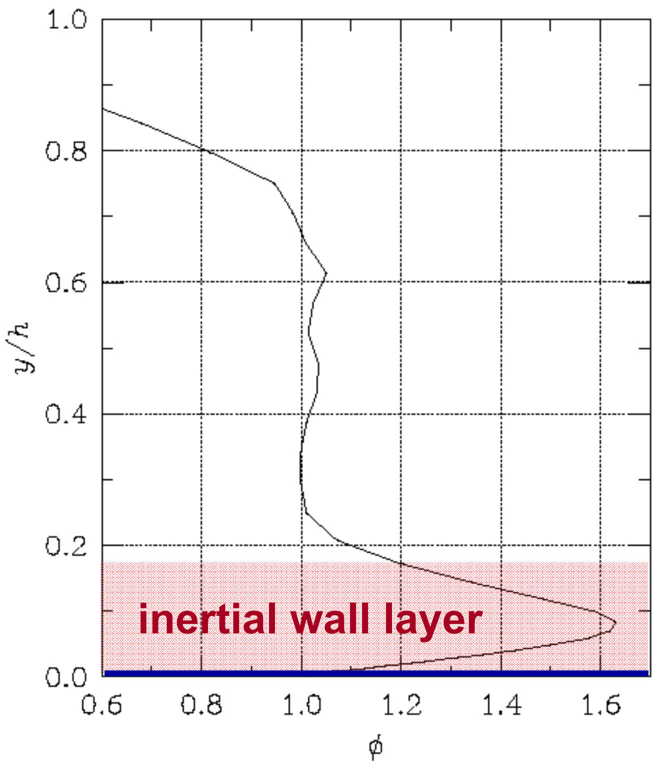


Khanna & Brasseur 1997, *JFM* 345

Observation 4: Inertial Law-of-the-Wall is also not Captured with Smooth Wall BLs

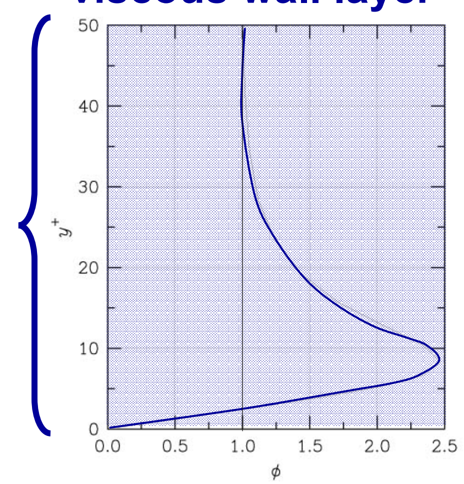


DES: Nikitin, Nicoud, Wasistho, Squires, Spalart. An approach to wall modeling in large-eddy simulation. Phys Fluids 12, 1629-1632, 2000.

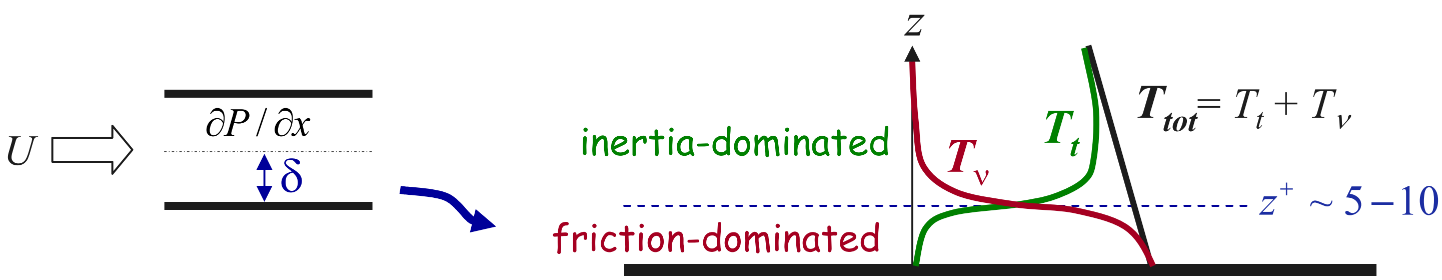


…from Philippe Spalart

The "Log-Layer Mismatch"



Observation 5: The (true) Viscous Overshoot in the Smooth-Wall Channel Flow



stationary, fully developed, mean:

$$\frac{\partial P}{\partial x} = \frac{\rho u_*^2}{\delta} = \frac{\partial T_{tot}}{\partial z} = \frac{\partial T_t}{\partial z} + \frac{\partial T_\nu}{\partial z}$$

$$T_\nu = \mu \frac{\partial U}{\partial z}$$

$$T_t \equiv -\rho \langle u'w' \rangle$$

$$T_t^+ \equiv \frac{T_t}{\rho u_*^2}$$

$$z^+ \equiv \frac{z}{\ell_\nu}$$

$$\ell_\nu = \nu / u_*$$

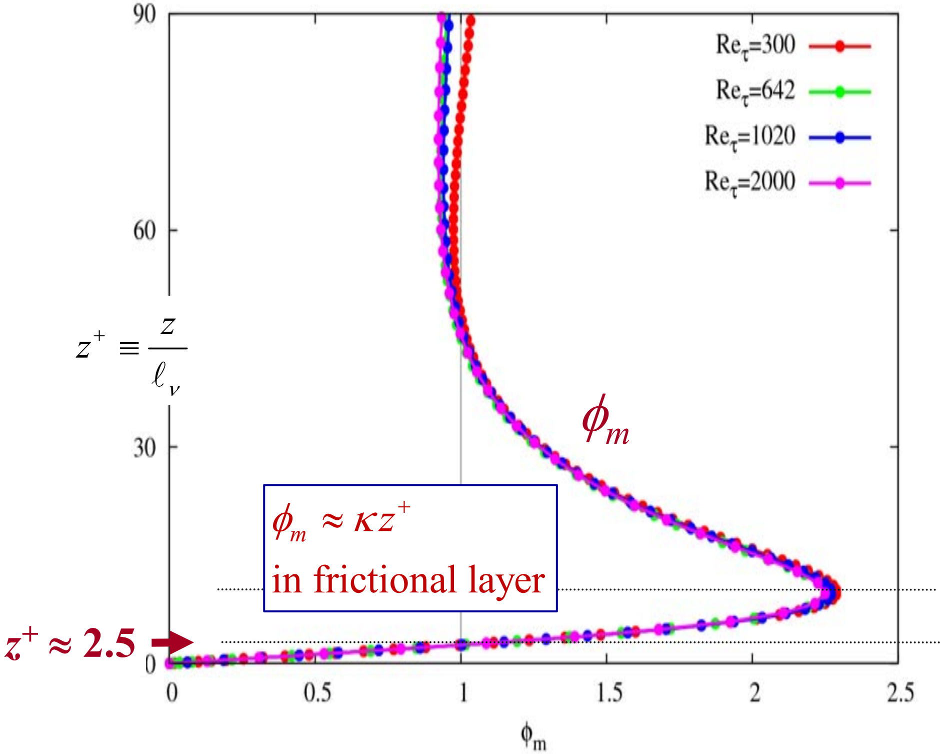
inertial scaling: $\phi_m = \dfrac{\kappa z}{u_*} \dfrac{\partial U}{\partial z} \;\Rightarrow\; \dfrac{T_\nu}{\rho u_*^2} = \left(\dfrac{\nu}{\kappa u_*^2} \right) \phi_m$

integrate $0 \to z$: $\phi_m = \kappa z^+ \left(1 - T_t^+ - \dfrac{z}{\delta} \right) \approx \kappa z^+$ in friction-dominated layer

$\kappa \approx 0.4 \;\Rightarrow\; \phi_m$ exceeds 1 when $z^+ > 2.5$ (!)

Smooth-Wall Channel Flow

DNS data from Iwamoto et al. (*Int. J. Heat and Fluid Flow,* 2002), Hoyas & Jimenez (*Phys. Fluids,* 2006)

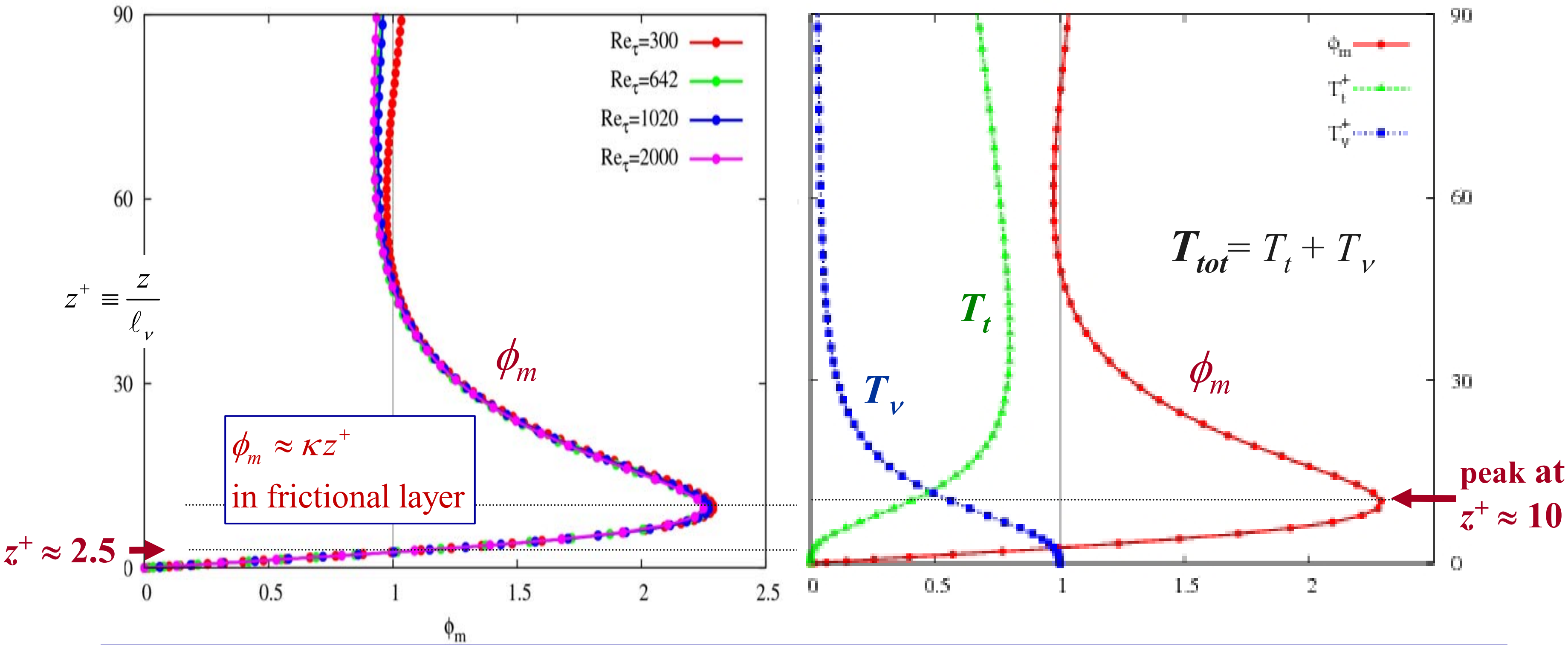


$$\phi_m = \kappa z^+ \left(1 - T_t^+ - \frac{z}{\delta}\right) \approx \kappa z^+$$

in friction-dominated layer

Smooth-Wall Channel Flow

DNS data from Iwamoto et al. (*Int. J. Heat and Fluid Flow,* 2002), Hoyas & Jimenez (*Phys. Fluids,* 2006)

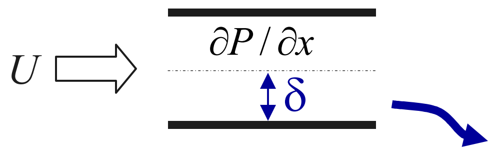


Conclusions

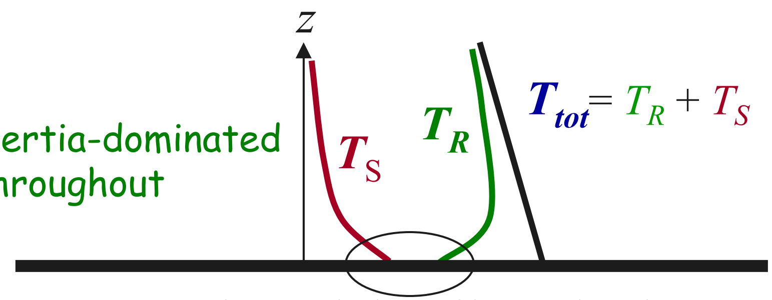
1. In the smooth-wall channel flow the overshoot in ϕ_m is real.
2. The overshoot arises from applying inertial scale z in a frictional layer that has the characteristic *viscous scale* $\ell_\nu = \nu / u_*$

The First Discovery: Scaling

Mean LES of high Re or Rough-Wall Channel Flow



inertia-dominated throughout



under-resolution of integral scales at first few grid levels

$$\frac{\partial u_i^r}{\partial t} + \frac{\partial (u_i^r u_j^r)^{\hat{r}}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \hat{p}^r}{\partial x_i} + \frac{\partial \tau_{ij}^{SFS}}{\partial x_j}$$

$$T_R \equiv -\rho\left\langle u^{r\prime} w^{r\prime} \right\rangle$$

$$T_S = -\rho\left\langle \tau_{13}^{SFS} \right\rangle$$

stationary, fully developed, mean:

$$\frac{\partial P}{\partial x} = \frac{\rho u_*^2}{\delta} = \frac{\partial T_{tot}}{\partial z} = \frac{\partial T_R}{\partial z} + \frac{\partial T_S}{\partial z}$$

Extract Viscous Content of ANY SGS Model:

define "LES viscosity" for strong shear flow

$$\nu_{les}(z) \equiv \frac{T_S(z)}{2\left\langle S_{13}^r \right\rangle}, \quad \nu_{LES} \equiv \nu_{les}(z_1)$$

IF $\tau_{ij}^{SFS} \equiv -2\nu_t S_{ij}^r$,

we find $\nu_{LES} \approx \langle \nu_t \rangle_1$

$$T_R^+ \equiv \frac{T_R}{\rho u_*^2}$$

$$z_{LES}^+ \equiv \frac{z}{\nu_{LES} / u_*}$$

$$\tilde{\nu}_{les} \equiv \frac{\nu_{les}(z)}{\nu_{LES}}$$

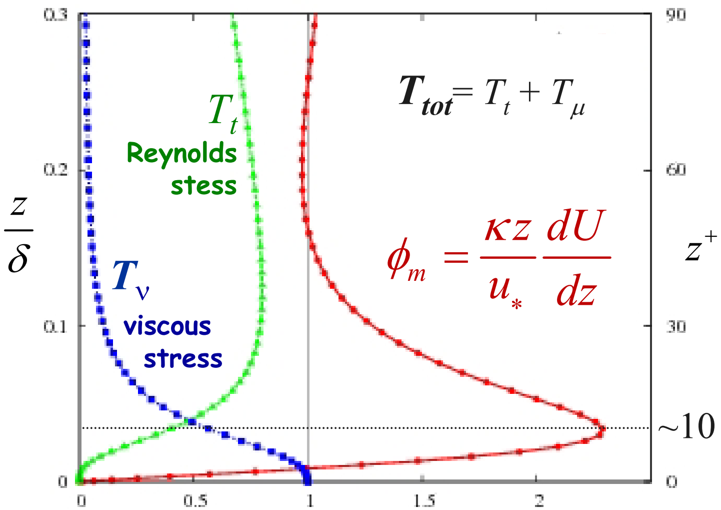
Inertial Scaling

… integrate 0 → z:

$$\phi_m = \frac{\kappa z_{LES}^+}{\tilde{\nu}_{les}(z)}\left(1 - T_R^+ - \frac{z}{\delta}\right) \approx \kappa z_{LES}^+\left(1 - T_R^+\right) \text{ near the surface}$$

The First Discovery:
A Spurious Frictional Surface Layer

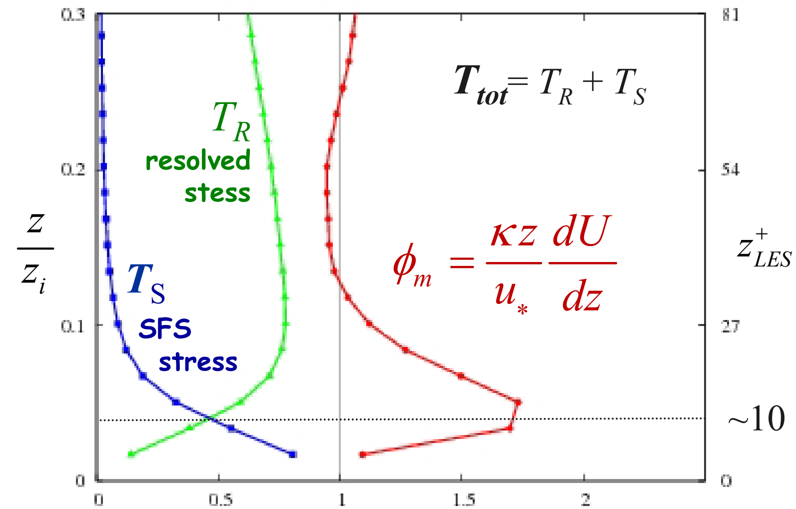
DNS: Smooth-wall Channel Flow



$\phi_m \approx \kappa z^+ =$ in friction-dominated layer

$z^+ = \frac{z}{\nu / u_*}$ where ν parameterizes real friction

LES: Rough-wall ABL



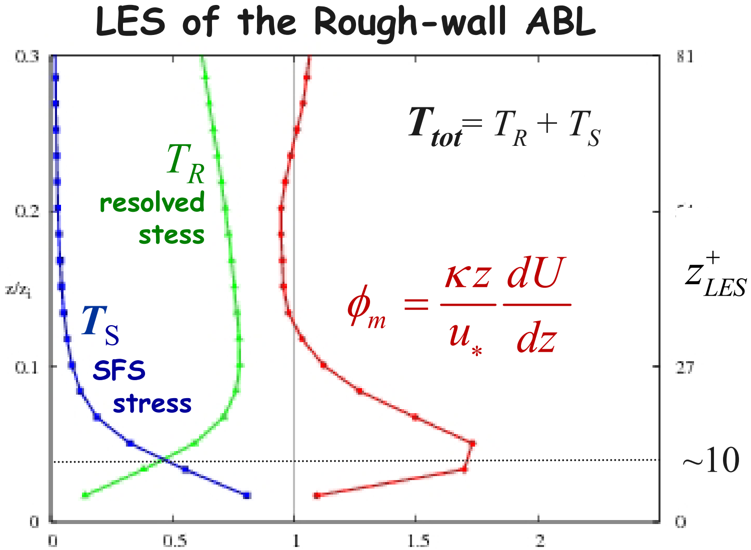
$\phi_m \approx \kappa z^+_{LES} =$ near the first grid level

$z^+_{LES} = \frac{z}{\nu_{LES} / u_*}$ where ν_{LES} parameterizes friction in the (inertial) SFS stress

Conclusion

The overshoot in ϕ_m arises from
applying an inertial scaling to a numerical LES "viscous" layer

The First Discovery:
A Criterion to Suppress the Overshoot



$$z^+_{LES} = \frac{z}{\ell_{\nu_{LES}}}$$

a numerical LES "viscous" scale

$$\ell_{\nu_{LES}} \equiv \frac{\nu_{LES}}{u_*}$$

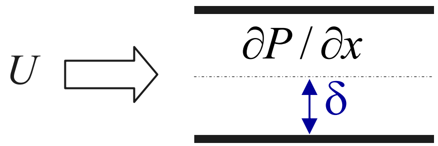
ν_{LES} is a "numerical-LES viscosity"

The overshoot arises from "numerical-LES friction" at the surface akin to the real frictional layer on smooth walls

$$\Re \equiv \left(\frac{T_R}{T_S}\right)_{z_1} > \Re^* \sim O(1)$$

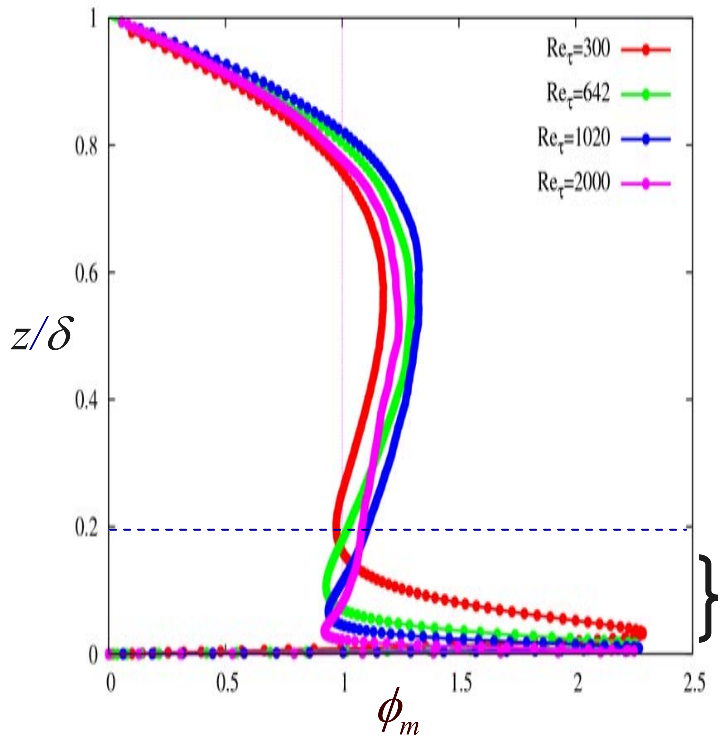
⇒ To eliminate the overshoot the ratio T_R/T_S must exceed a critical value ~ O(1) at the first grid level.

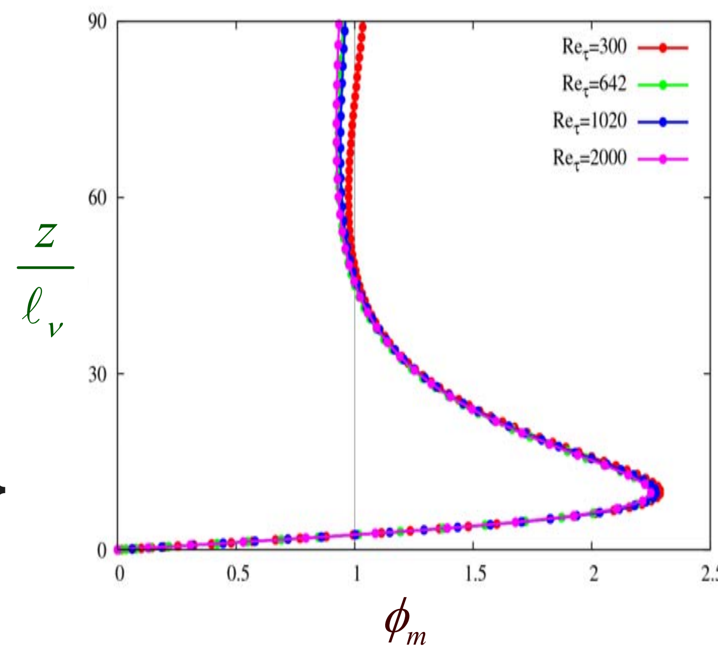
The Second Discovery:
MORE IS NEEDED TO PREDICT LAW-OF-THE-WALL!



$$\mathrm{Re}_\tau \equiv \frac{u_* \delta}{\nu} = \frac{\delta}{\ell_\nu}, \quad \text{where} \quad \ell_\nu = \nu / u_*$$

$\Rightarrow$ $\mathrm{Re}_\tau > \mathrm{Re}_\tau^*$ to support an inertial surface layer





DNS data from Iwamoto et al. (*Int. J. Heat and Fluid Flow,* 2002), Hoyas & Jimenez (*Phys. Fluids,* 2006)

A Second Requirement: Relative Inertia to LES Friction in the Simulation

define $$\mathrm{Re}_{LES} \equiv \frac{u_* \delta}{\nu_{LES}} = \frac{\delta}{\ell_{\nu_{LES}}}$$ LES Reynolds Number

$$\ell_{\nu_{LES}} = \nu_{LES} / u_*$$ ~ a nonphysical length scale

$$\Rightarrow \mathrm{Re}_{LES} > \mathrm{Re}^*_{LES}$$ to support an inertial surface layer

What sets Re_{LES}? Consider Smag: $\tau_{ij}^{SFS} \equiv -2\nu_t S_{ij}^r, \quad \nu_t = (C_s\Delta)^2 \,|\, S \,|$

$$\Rightarrow \quad \nu_{LES} \approx \langle \nu_t \rangle |_1 \approx 2(C_s\Delta)^2 \langle s_{13}^r \rangle \approx (C_s\Delta)^2 \frac{u_*}{\tilde{\kappa}_1} \frac{1}{\Delta_z}$$

$$\ell_{\nu_{\mathrm{LES}}} \equiv \frac{\nu_{LES}}{u_*} = \left(\frac{C_S^2 (AR)^{4/3}}{\tilde{\kappa}_1} \right) \Delta_z$$

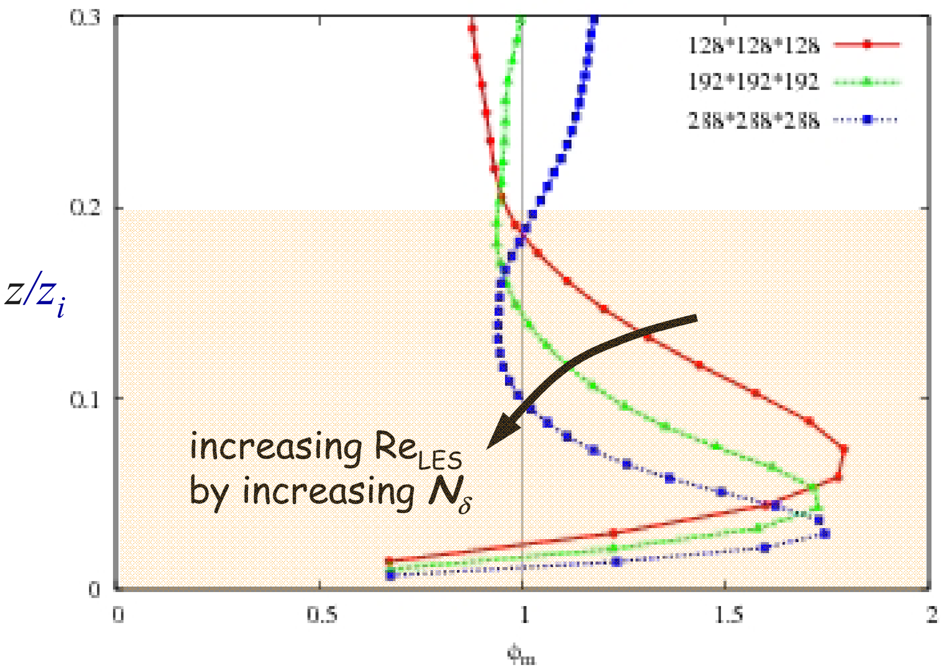
$$\mathrm{Re}_{\mathrm{LES}} \approx \frac{\tilde{\kappa}_1 N_\delta}{C_S^2 (AR)^{4/3}}$$

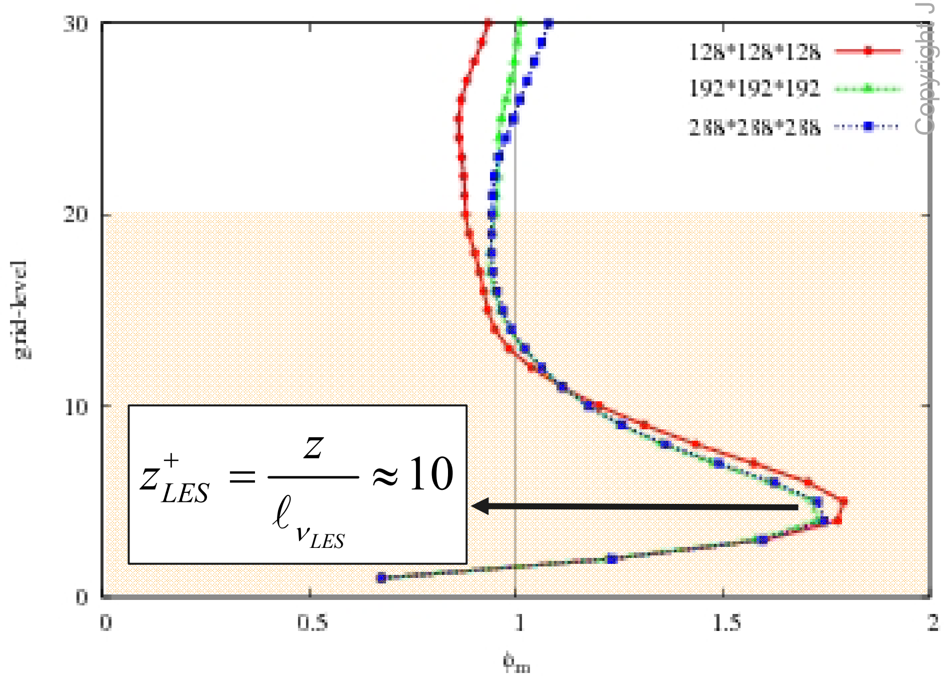
where $N_\delta \equiv \dfrac{\delta}{\Delta_z}$ = vertical resolution of grid

$AR \equiv \dfrac{\Delta_x}{\Delta_z} = \dfrac{\Delta_y}{\Delta_z}$ = aspect ratio of grid

back to observation 3:

Why the Overshoot is Tied to the Grid





$$\ell_{\nu_{\text{LES}}} = \left(\frac{C_S^2 (AR)^{4/3}}{\tilde{\kappa}_1} \right) \Delta_z \propto \Delta_z$$

$\Rightarrow$ overshoot is tied to the grid

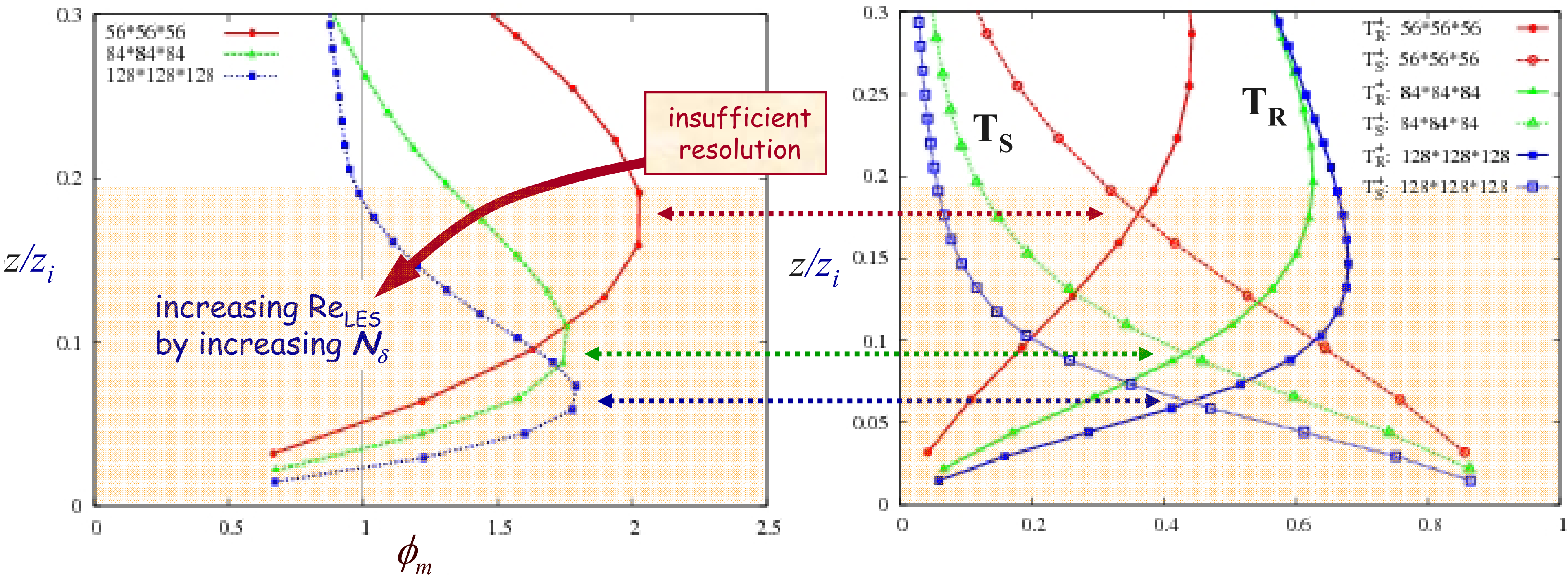
- A Third Requirement -

Increasing Re_{LES} by Increasing Resolution

exact: $\frac{T_R}{T_S} \equiv \Re$

$$Re_{LES} = \frac{N_\delta}{\xi \tilde{\kappa}_1}\left(\Re + 1\right) \Rightarrow Re^*_{LES} = \frac{N^*_\delta}{\xi \tilde{\kappa}_1}\left(\Re^* + 1\right)$$

$\Rightarrow$ suggests a critical vertical resolution, N^*_δ



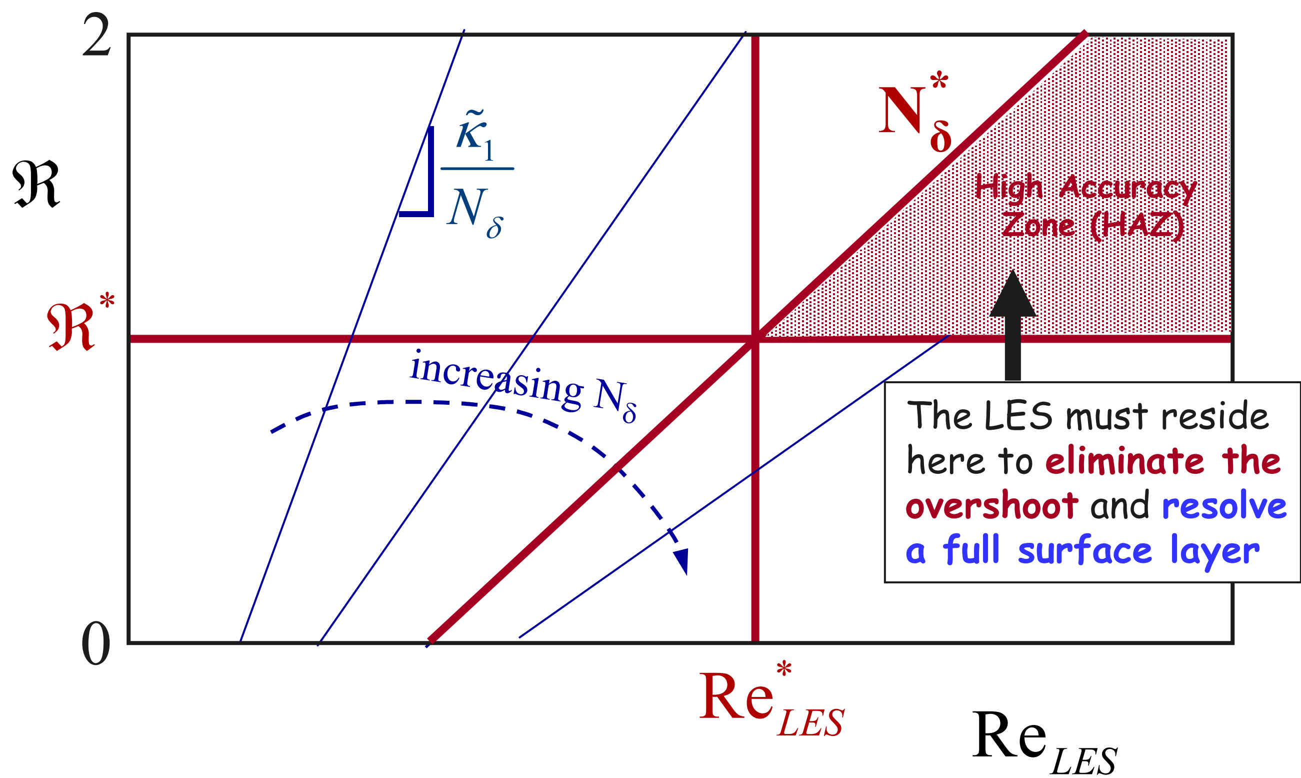
The Third Discovery

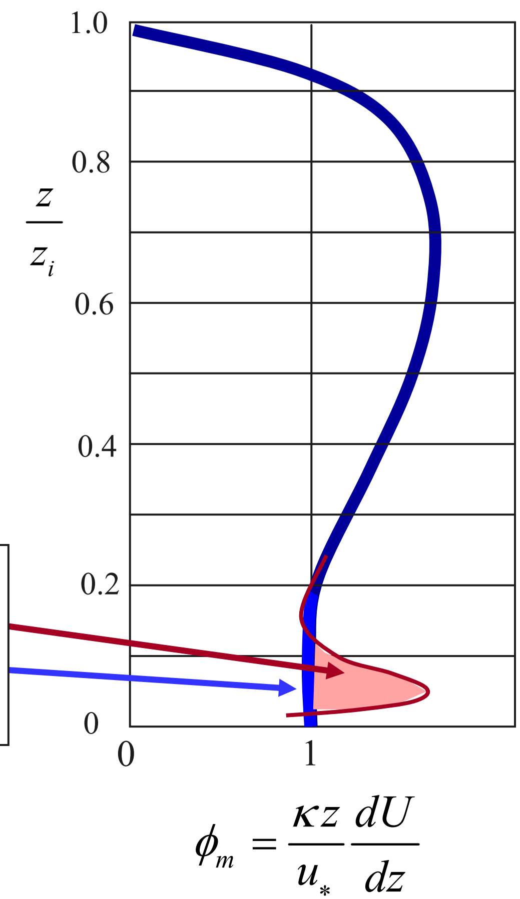
The $\Re - \mathrm{Re}_{LES}$ Parameter Space

$$\Rightarrow \frac{T_R}{T_S} \equiv \Re = \left(\frac{\xi \tilde{\kappa}_1}{N_\delta}\right) \mathrm{Re}_{LES} - 1$$

exact

$$\xi \approx \left(\frac{N_\delta - 1}{N_\delta}\right) \cos\theta \sim 0.9$$





First Tier in Designing High-Accuracy LES

The $\Re - \mathrm{Re}_{LES}$ Parameter Space

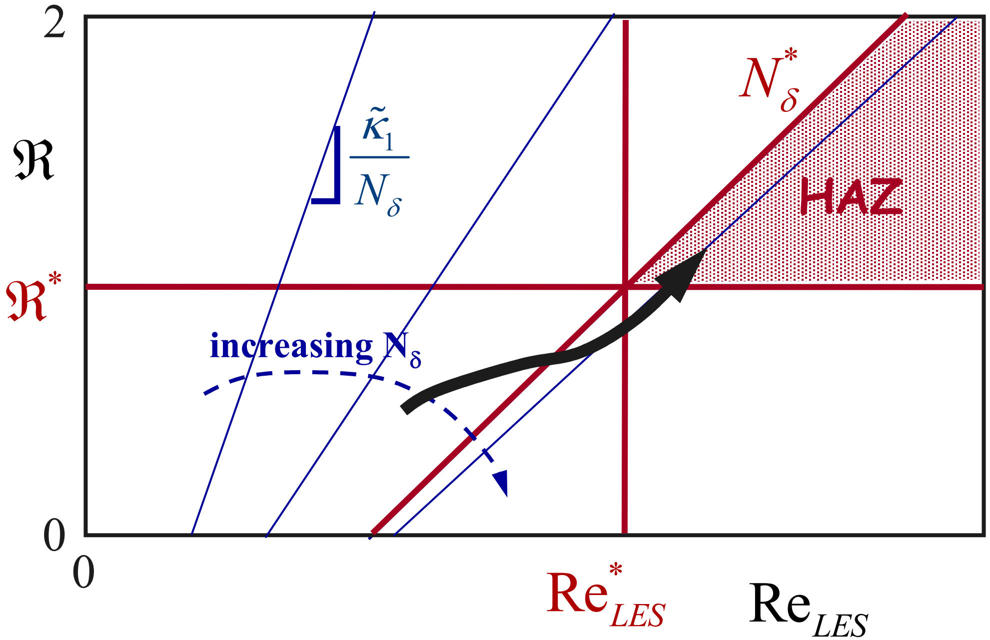
For any SFS stress model:

$$\Re = \left(\frac{\xi \tilde{\kappa}_1}{N_\delta} \right) \mathrm{Re}_{LES} - 1$$

Scaling on Smag:

$$\Re = \frac{T_R}{T_S} = \frac{\xi \tilde{\kappa}_1^2}{C_S^2 (AR)^{4/3}} - 1$$

$$\mathrm{Re}_{LES} = \frac{\delta}{\ell_{\nu_{LES}}} = \tilde{\kappa}_1 \frac{N_\delta}{C_S^2 (AR)^{4/3}}$$

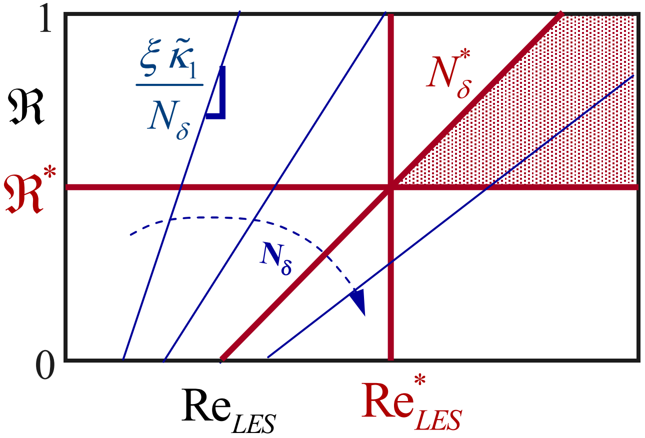


To move the LES into the HAZ:

1/ first adjust N_δ

2/ then adjust AR together with the model constant

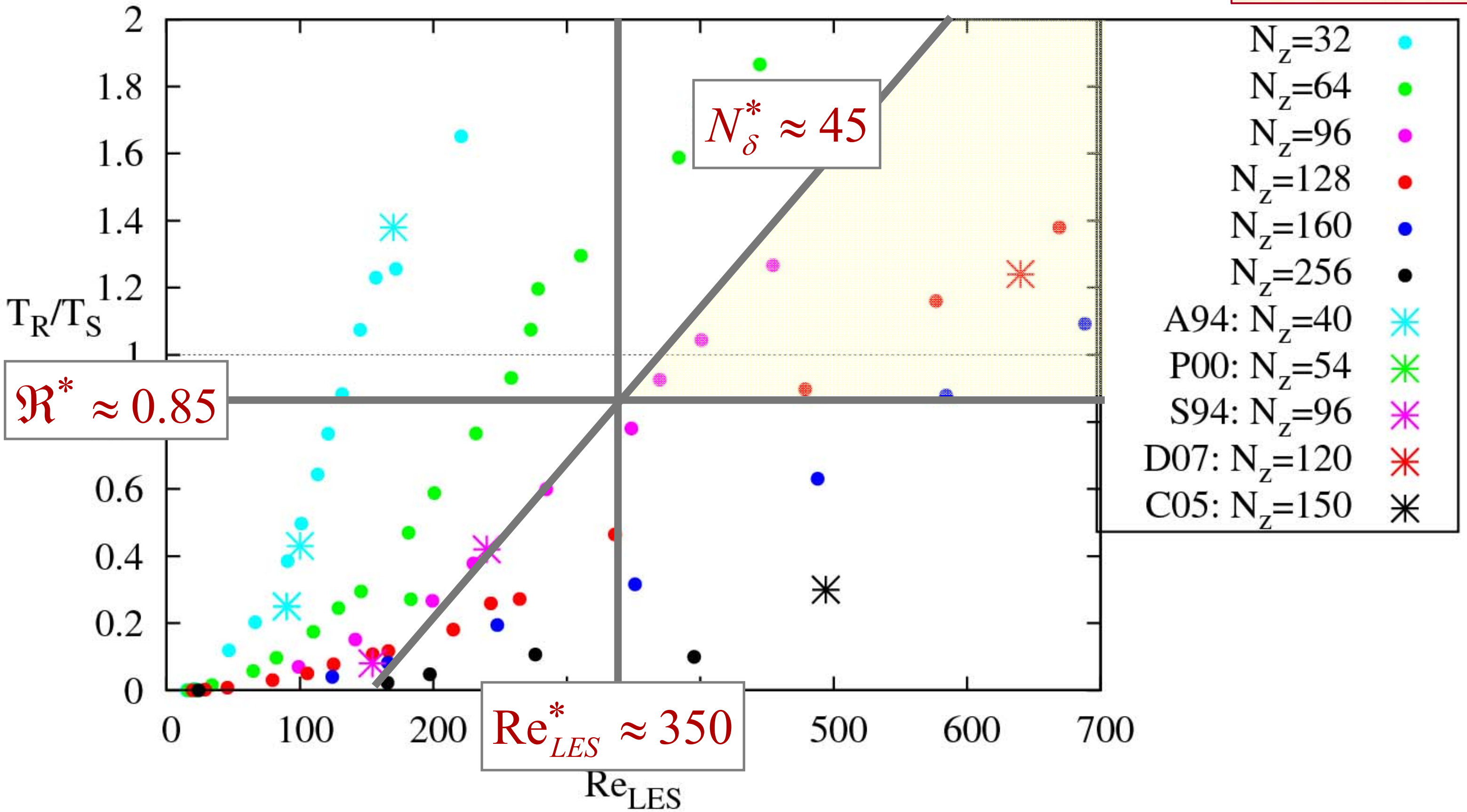
Numerical Experiments



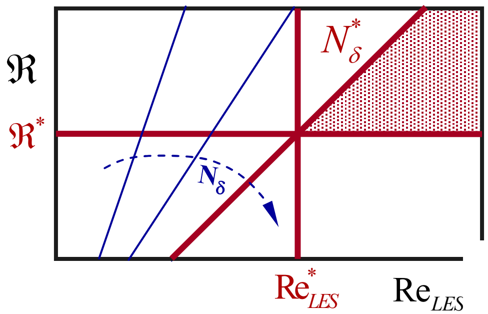
$$\Rightarrow \quad \frac{T_R}{T_S} \equiv \mathfrak{R} = \left(\frac{\xi\tilde{\kappa}_1}{N_\delta}\right) Re_{LES} - 1$$

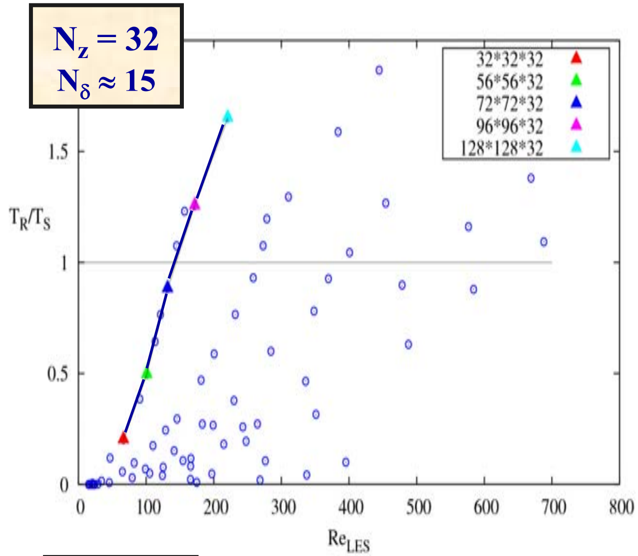
$$Re_{LES} = \tilde{\kappa}_1 \frac{N_\delta}{C_S^2 (AR)^{4/3}}$$

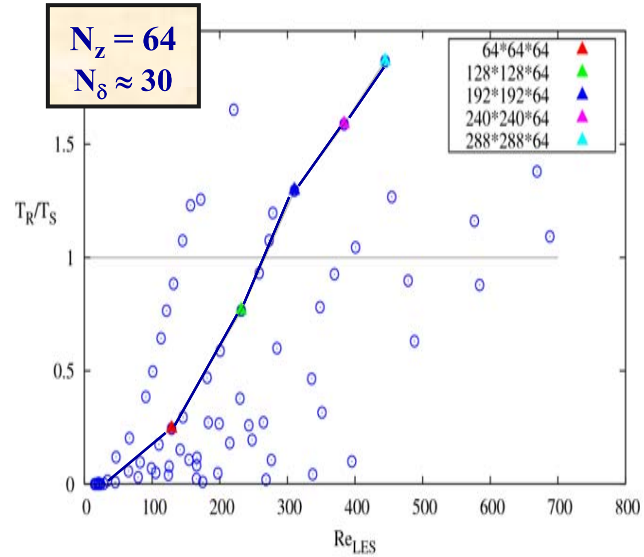
$$\mathfrak{R} = \frac{\xi\tilde{\kappa}_1^2}{C_S^2 (AR)^{4/3}} - 1$$

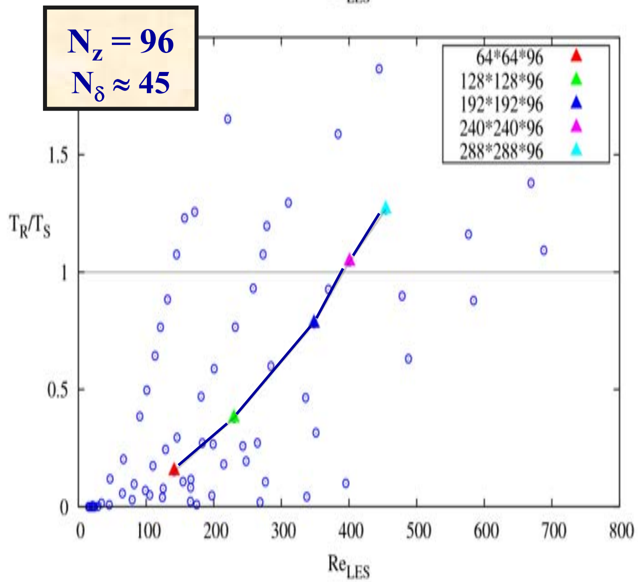


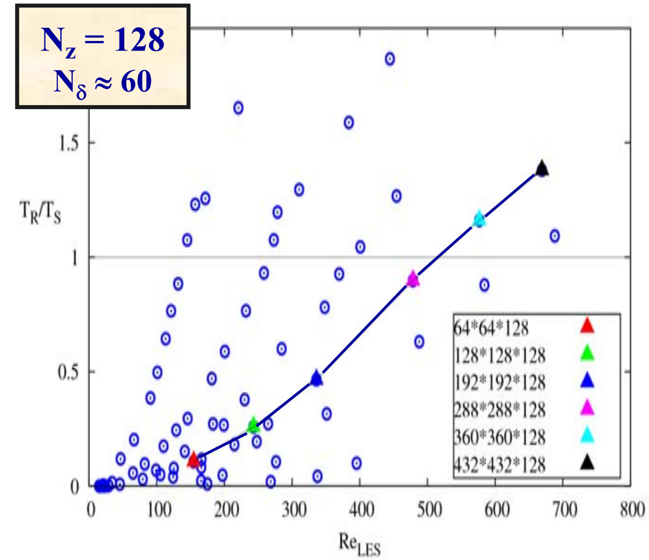
Vertical Resolution: N_δ

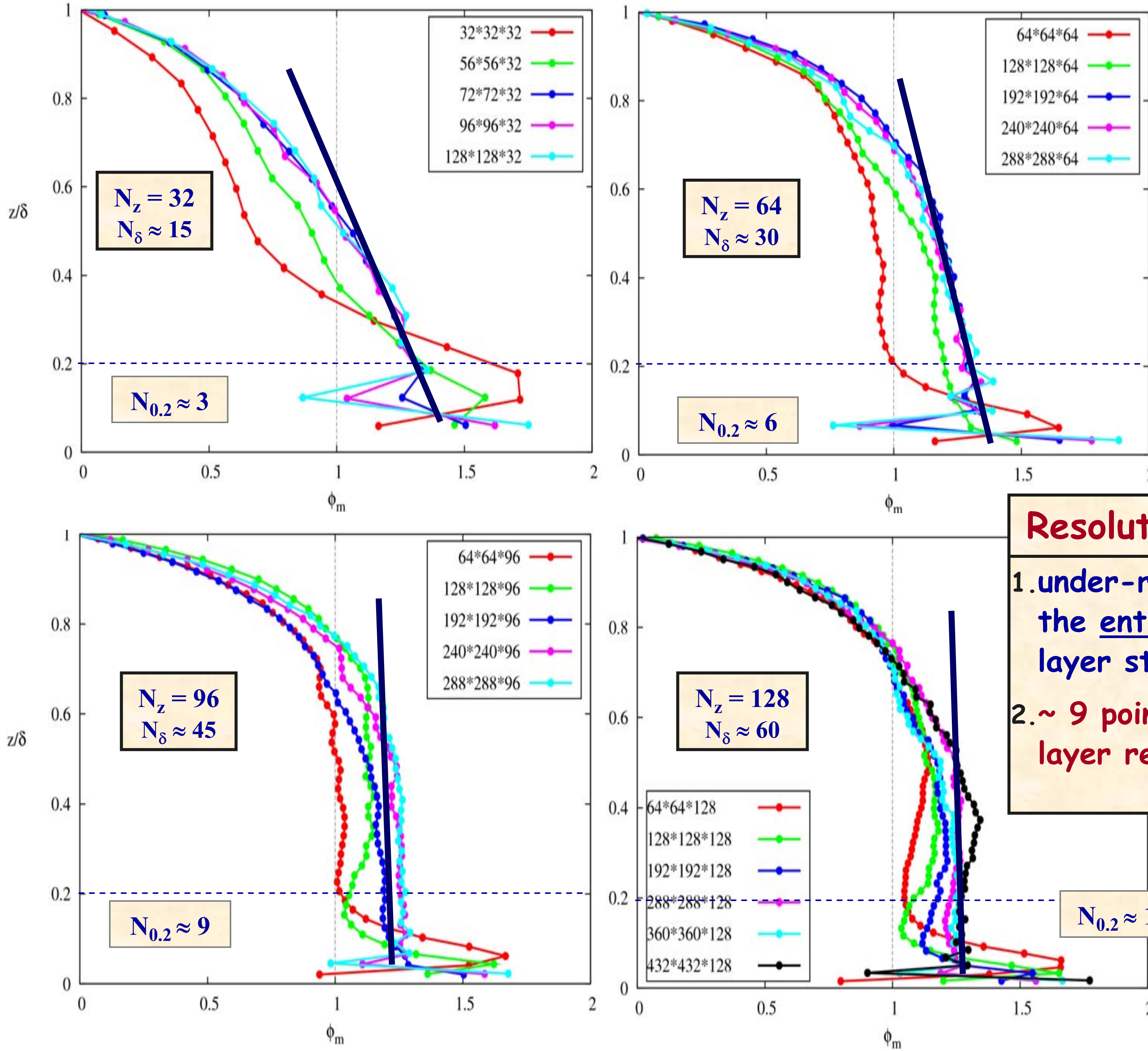










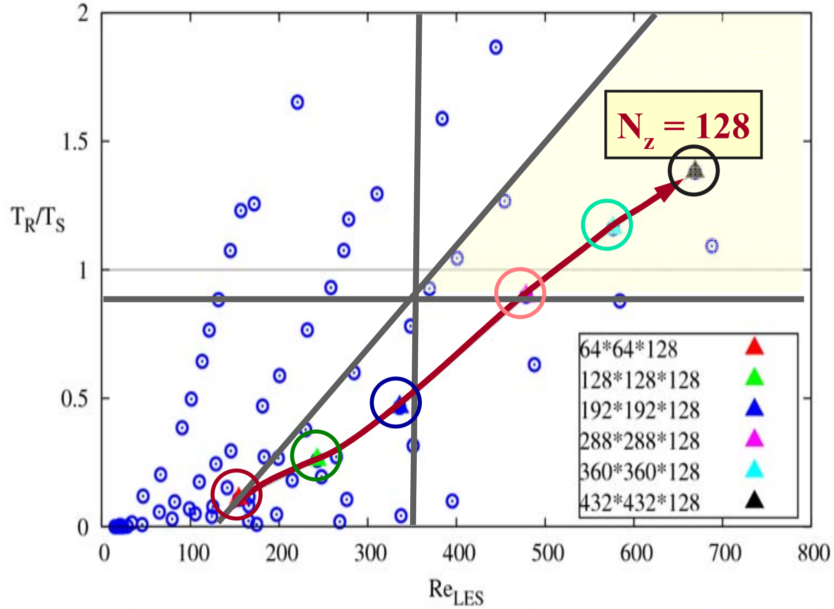


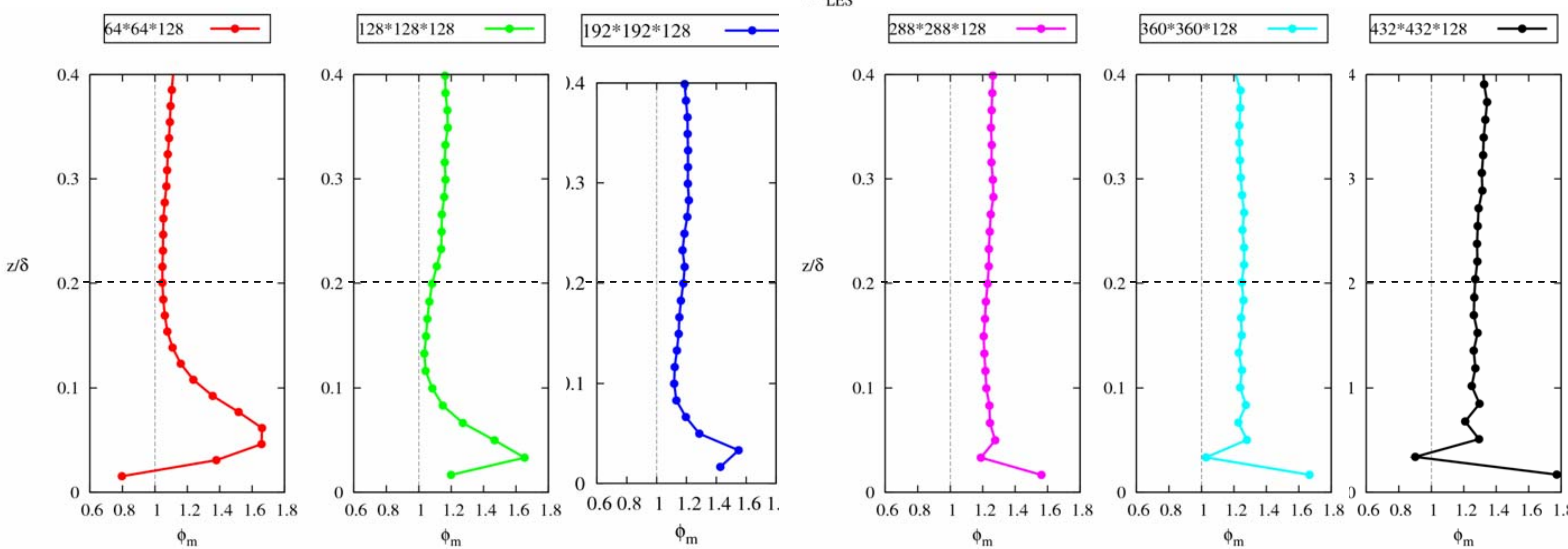
Resolution N_δ

1. under-resolution alters the <u>entire</u> boundary layer structure
2. ~ 9 points in surface layer required

$\Rightarrow$ $N_\delta^* \approx 45$

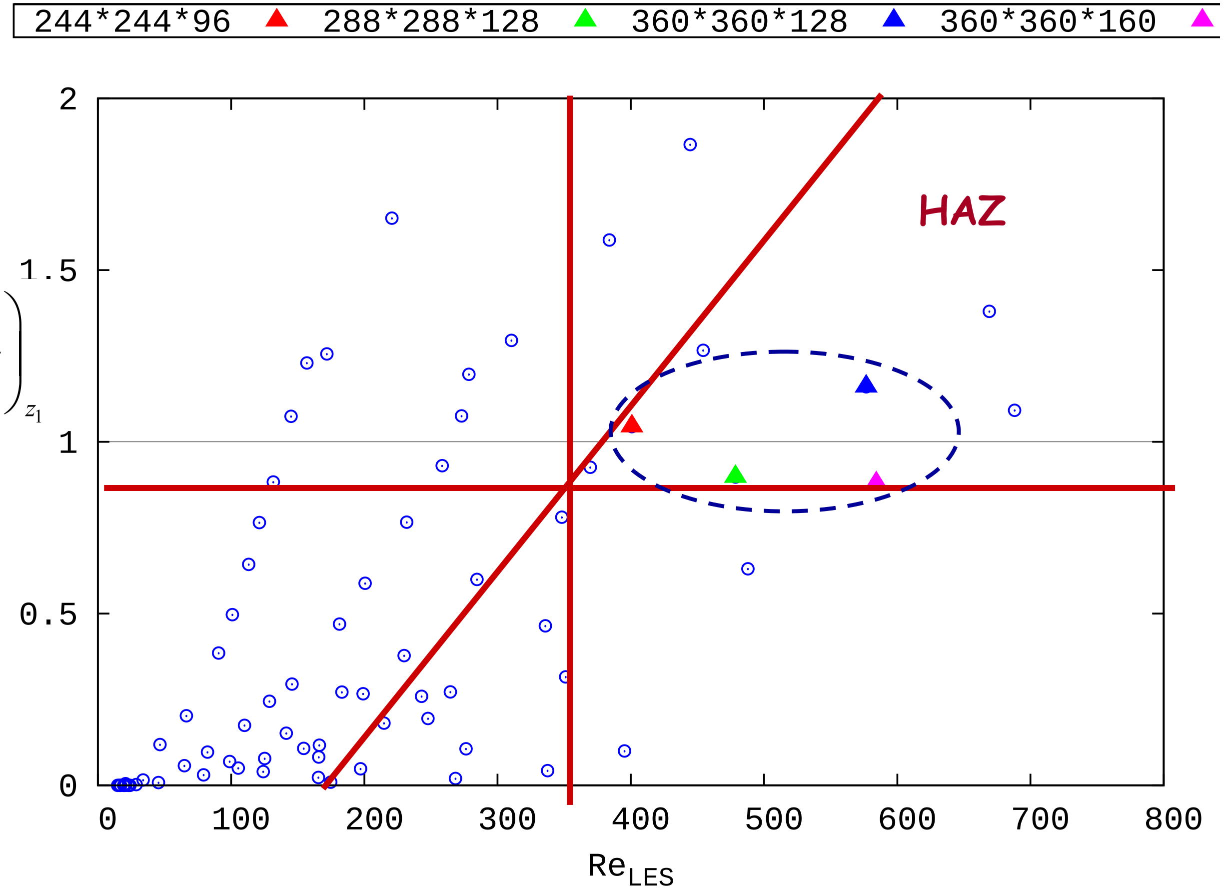
Designing High-Accuracy LES





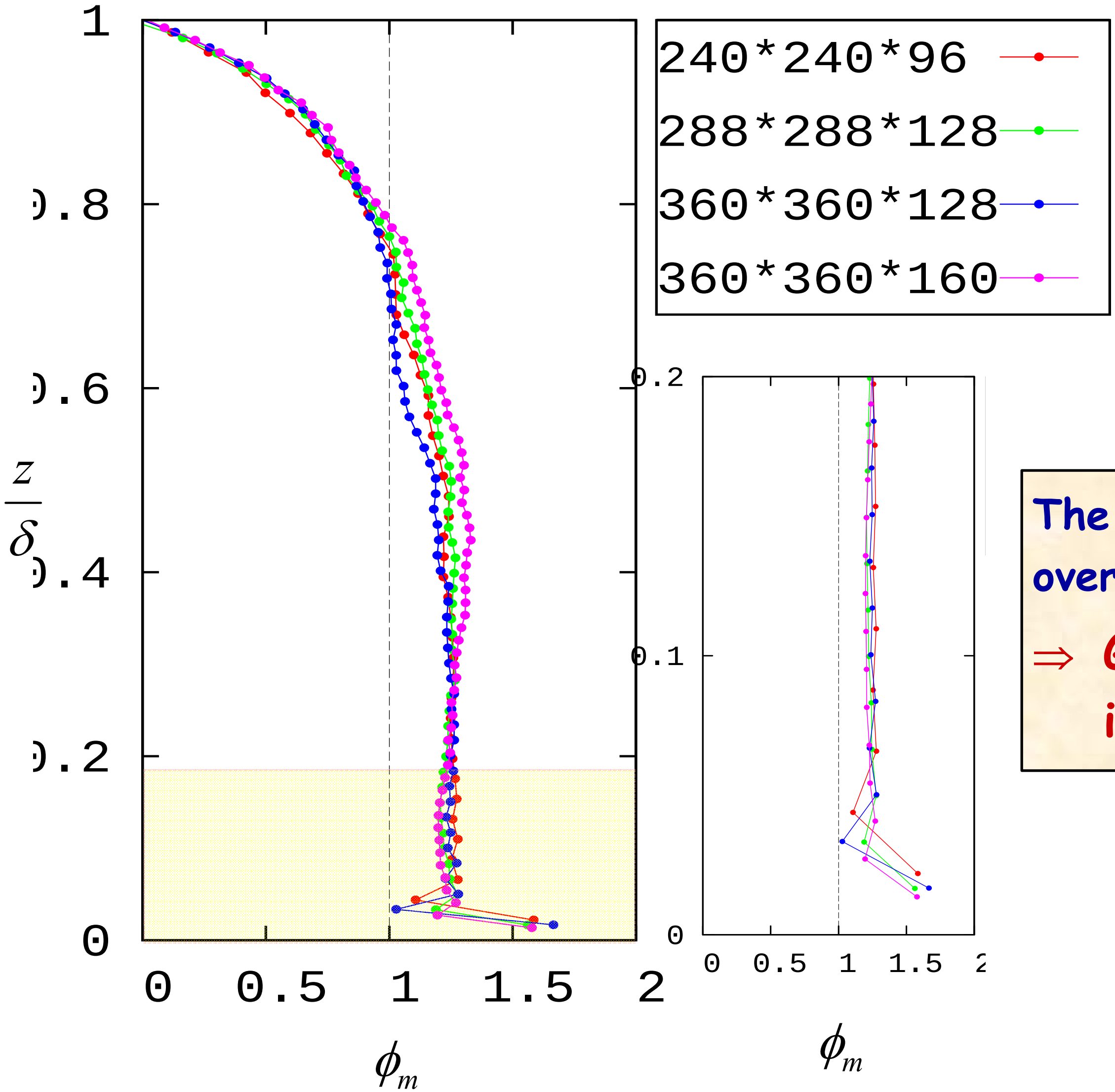
Simulations with High Accuracy

- N_z from 96 to 160
- Smagorinsky model with C_s = 0.10
- Aspect ratio 1.6 to 2.0



$$\Re = \left(\frac{T_R}{T_S} \right)_{z_1}$$

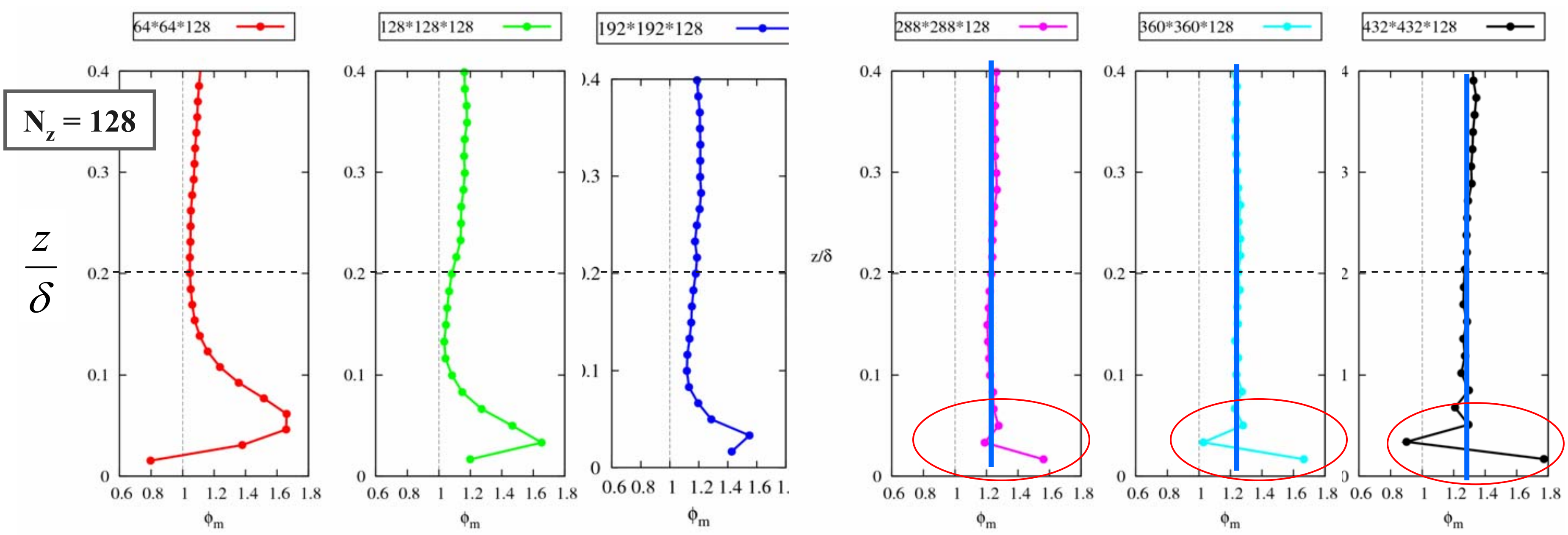
Convergence of LES Over the Entire ABL



The simulations converge well over the entire ABL

⇒ Grid independence is achieved!

This is a necessary process in designing LES … but NOT ALL ISSUES ARE RESOLVED!



These require consideration of

(1) the lower surface BC

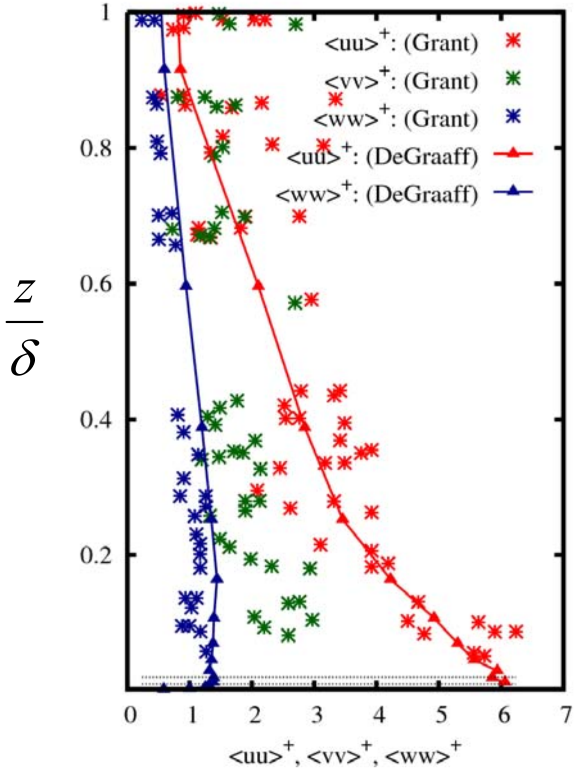
(2) the SFS model

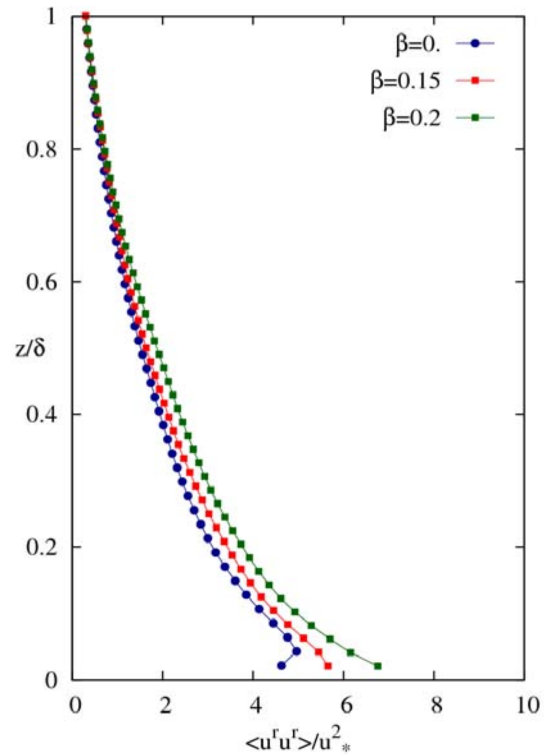
(3) algorithm and numerics

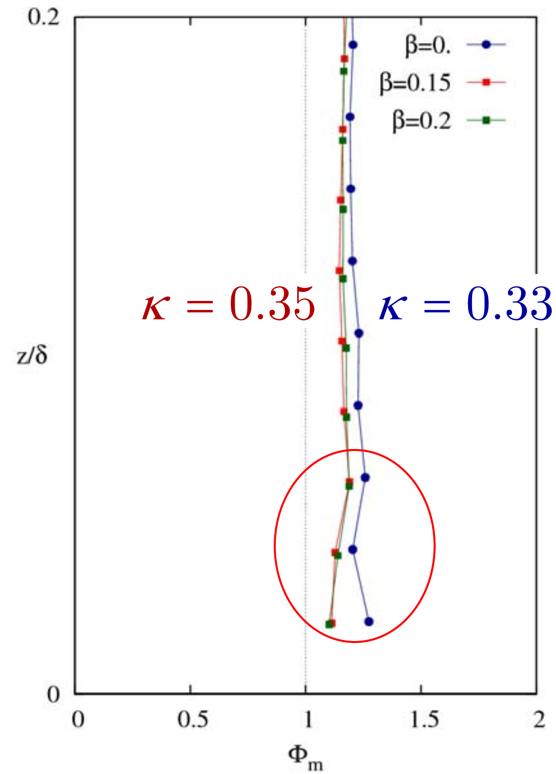
Prediction of the Von Karman constant

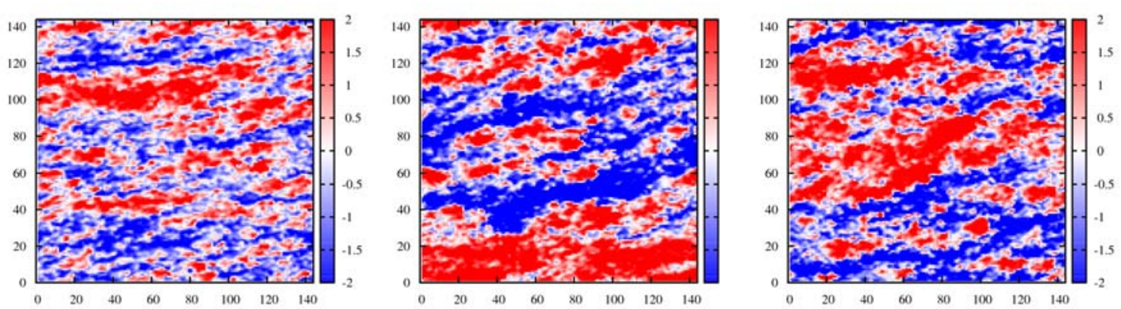
Oscillations in mean gradient at surface

Analysis of the Lower BC on Fluctuating Stress

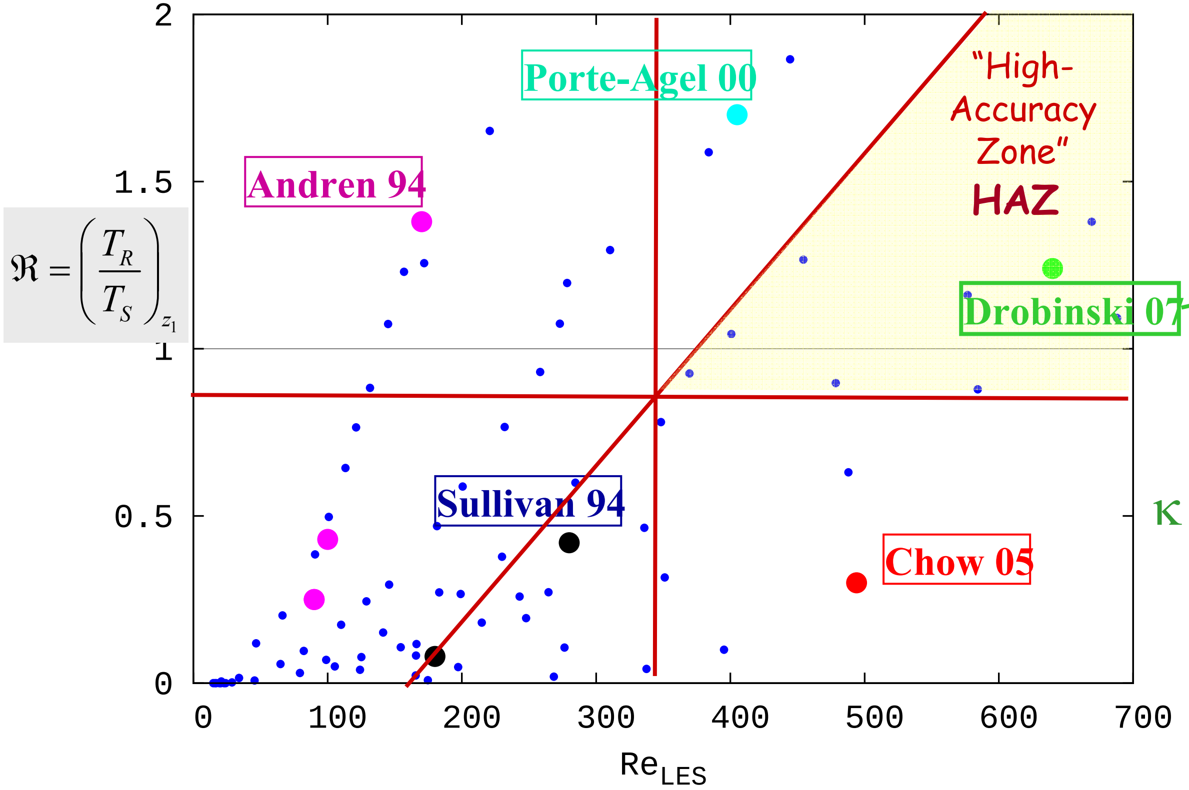


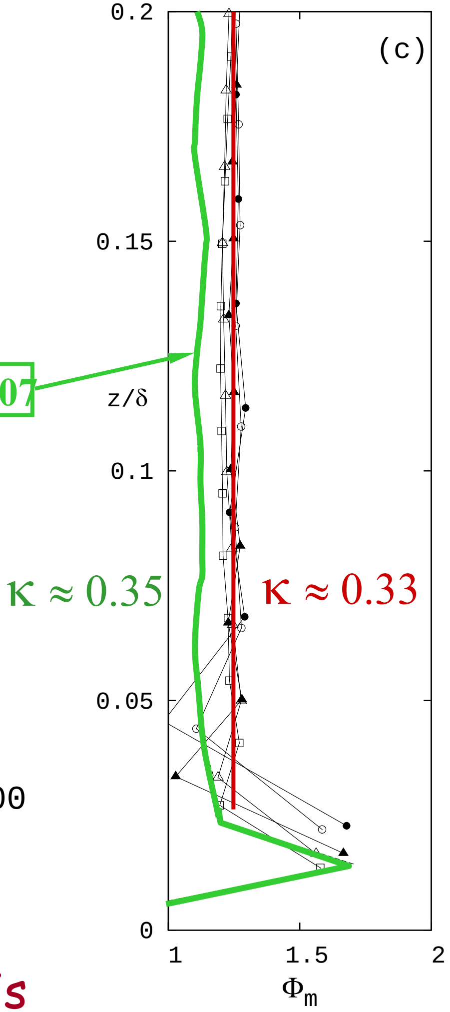






The prediction of the Von Karman Varies with SFS Model





… we are now initiating a systematic analysis

Conclusions

- **Accurate Prediction of Law-of-the-Wall ⇒**
  1. removal of the overshoot in mean gradient
  2. proper scaling in lower 15-20% of boundary layer
- **To Capturing the Law-of-the-Wall, the simulation must be in the "High-Accuracy Zone" (HAZ) by**
  - vertical grid resolution
  - grid aspect ratio
  - friction in the discretized dynamical system: model constant, algorithm
- **Other issues to resolve after LES is in the HAZ:**
  - lower boundary condition
  - details of the closure for SFS stress
  - algorithmic issues: dealiasing, numerical dissipation
- **We find that Adjusting BC and SFS Closure affects:**
  - location of the LES on the HAZ
  - Von Karman constant