Magnetic field reversals in turbulent dynamos

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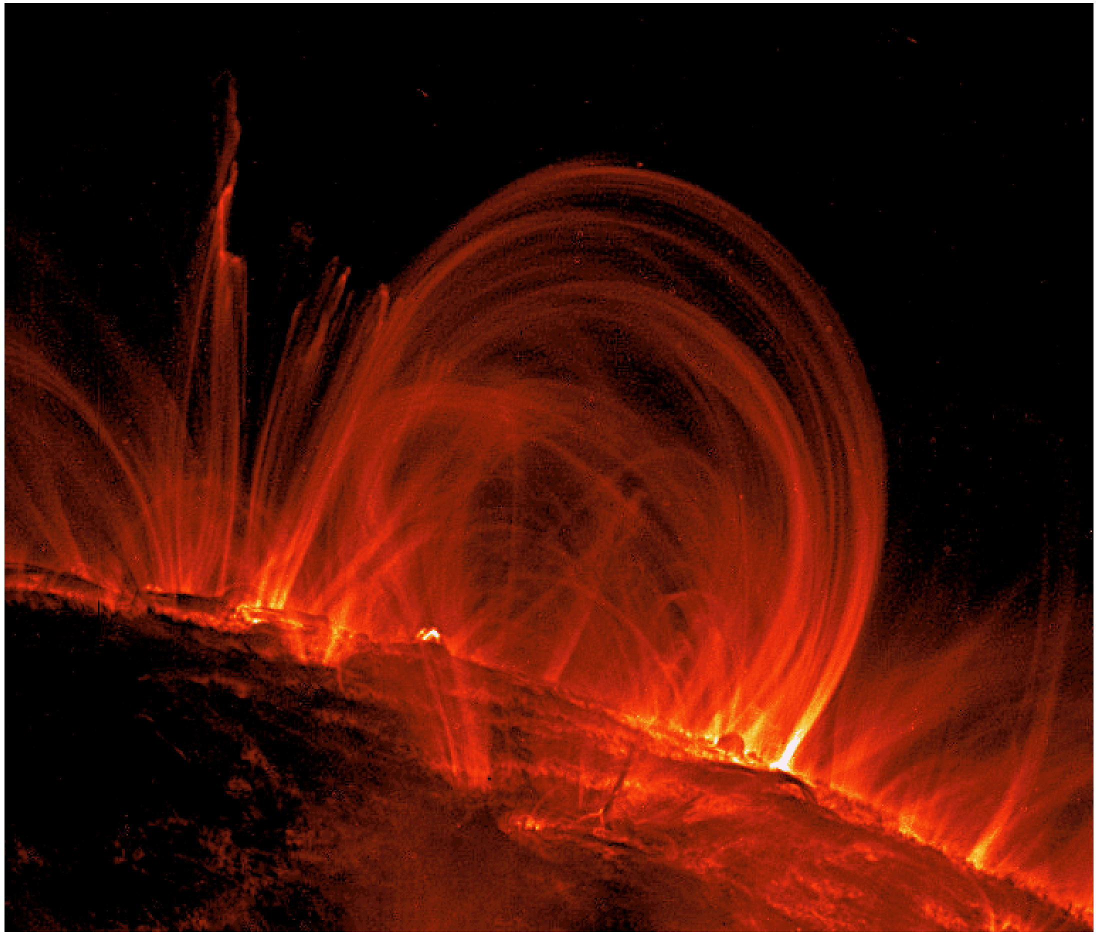
LPS-ENS-Paris

APS, San Antonio, november 23, 2008

Cosmic magnetic fields

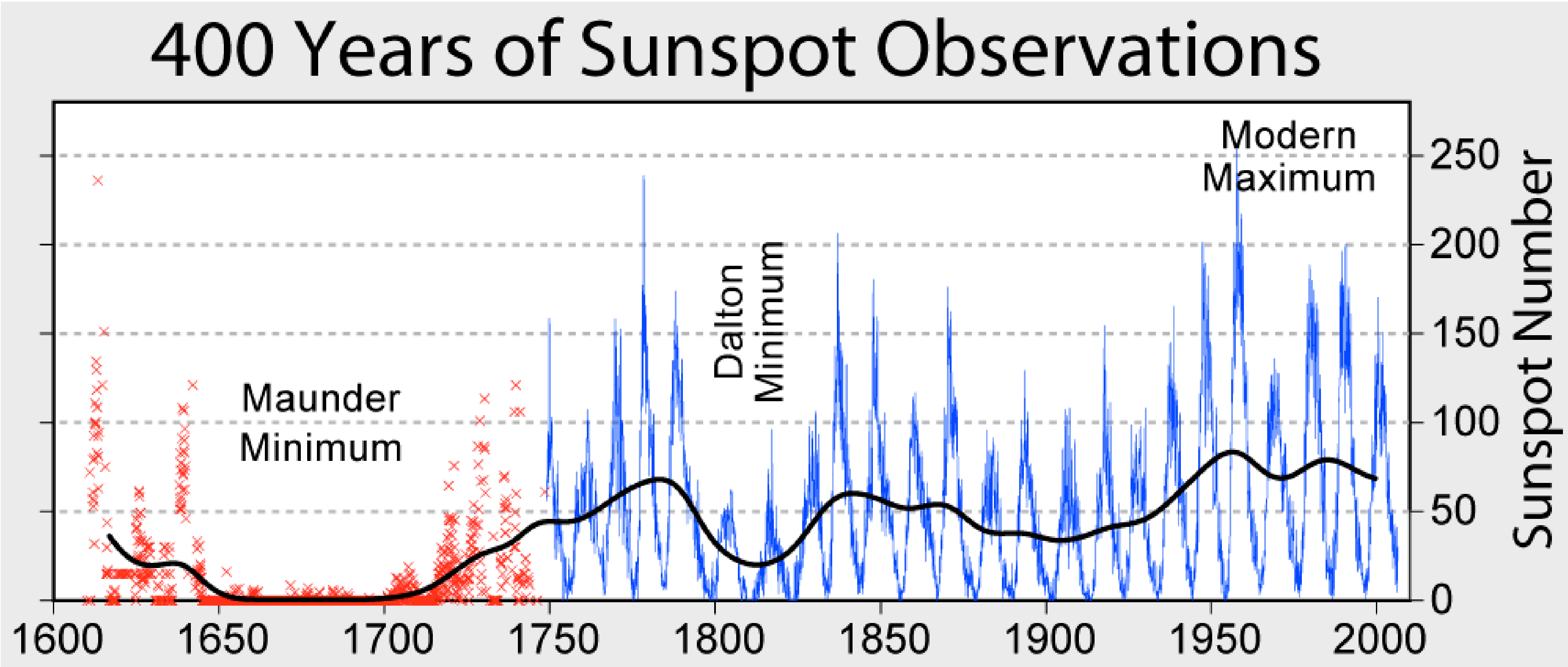


Coronal loops
Credit: M. Aschwanden et al. (LMSAL, TRACE, NASA)

- Earth 0.5 G
- Sun 1 G
  (Hale, 1908) 10^3 G
- Neutrons stars 10^{10} - 10^{13} G
- Galaxy 10^{-6} G
  (Fermi, Teller, ~ 1950)

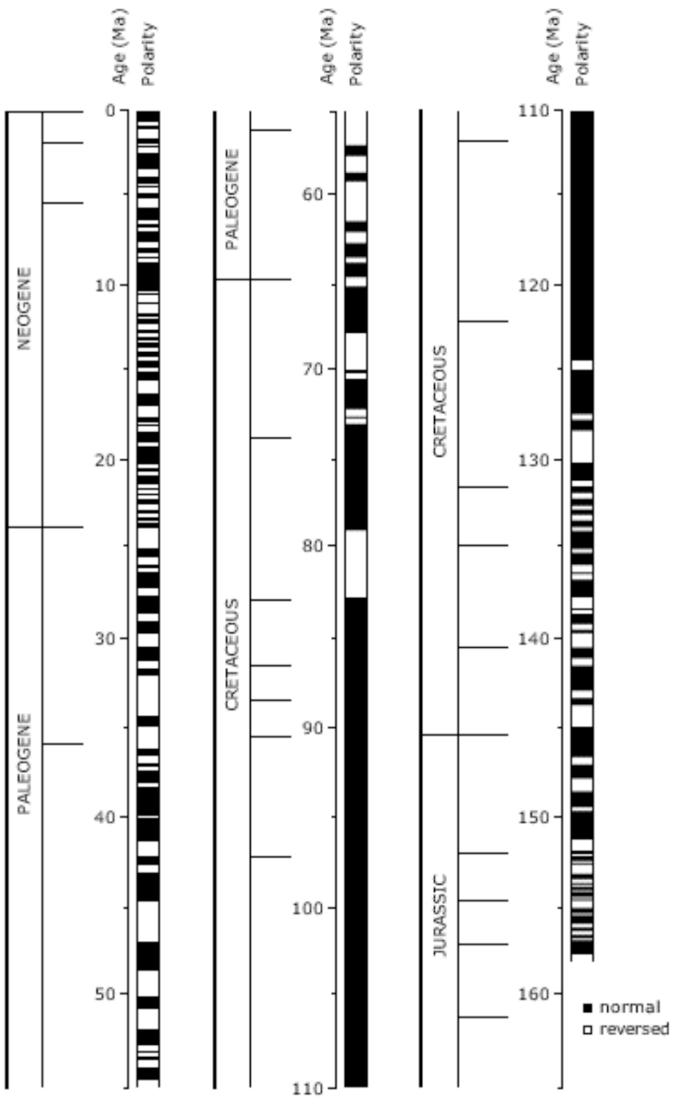
Magnetic field generated by the motion of an electrically conducting fluid

Oscillations of the solar magnetic field

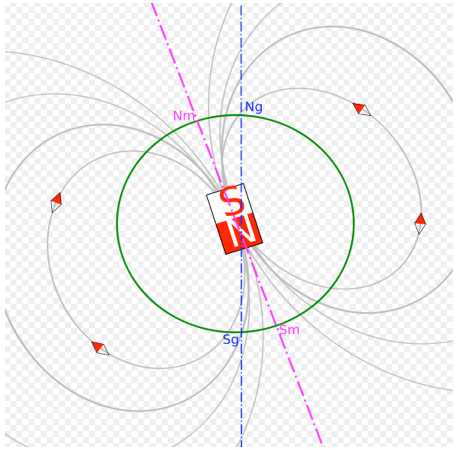


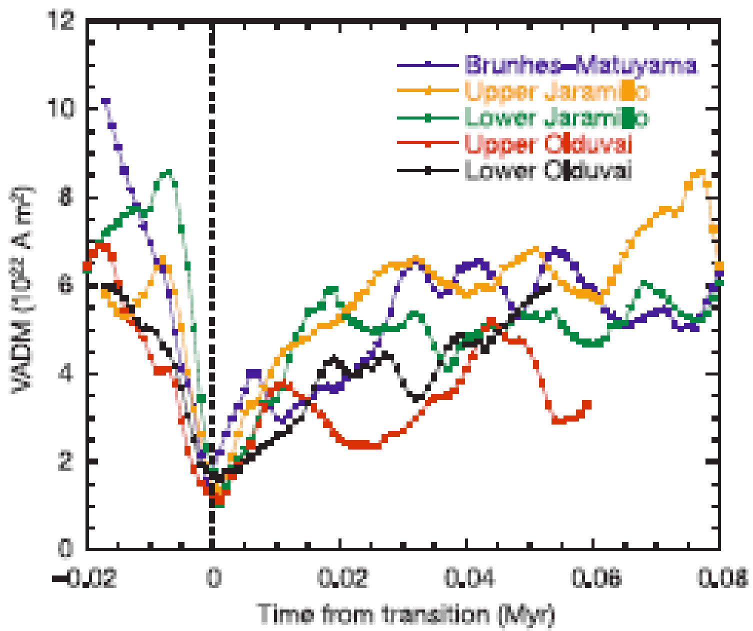
Hoyt et al., Solar Physics

Reversals of the magnetic field of the Earth



Lowrie (1997), "Fundamentals of Geophysics"





Valet et al., Nature (2005)

MHD equations and dimensionless numbers

$$\frac{\partial \mathrm{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} = -\nabla \left(\frac{\mathrm{p}}{\rho} + \frac{\mathbf{B}^2}{2\mu_0 \rho} \right) + \nu \nabla^2 \mathbf{v} + \frac{1}{\mu_0 \rho} (\mathbf{B} \cdot \nabla) \mathbf{B}.$$

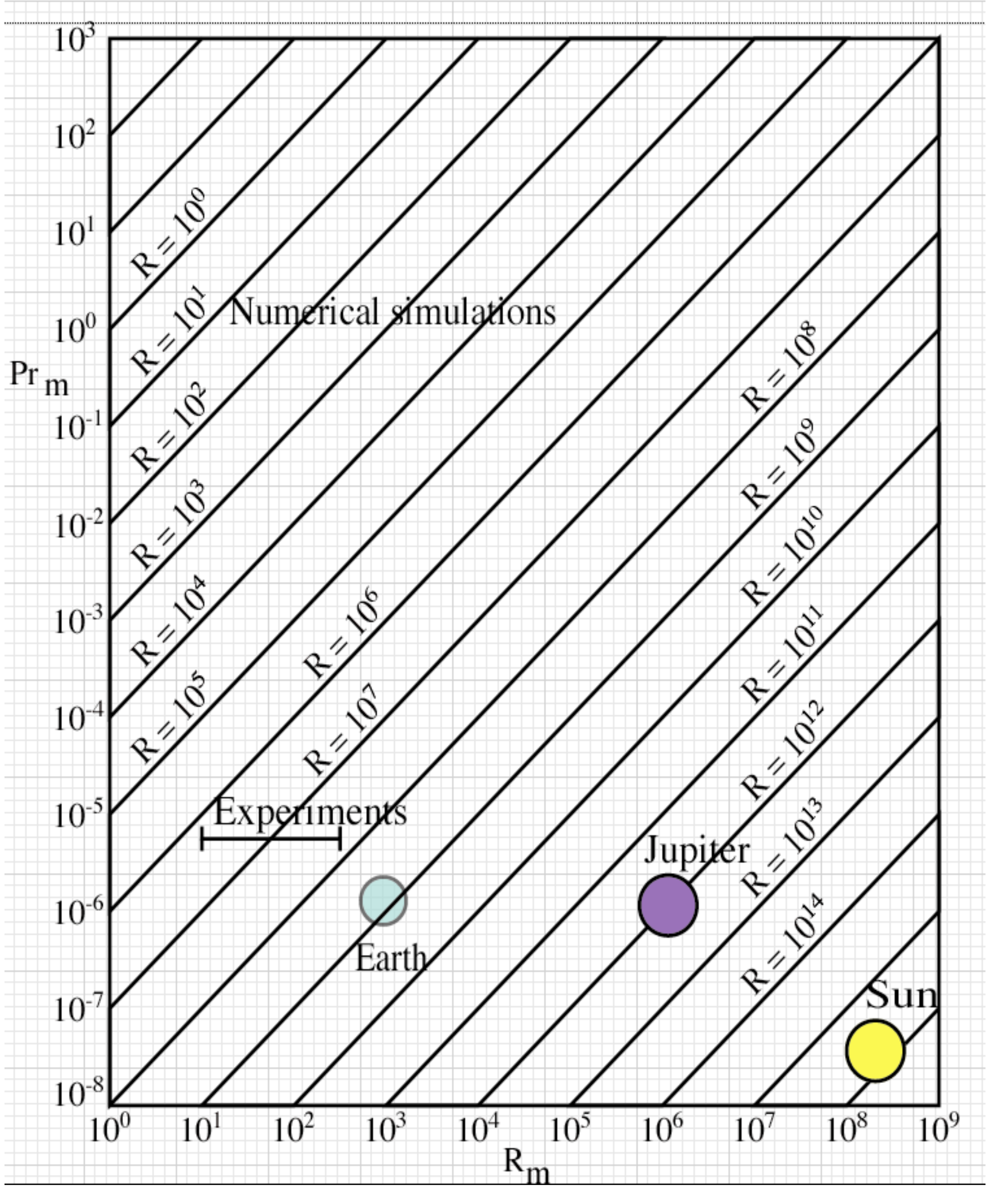
- fluid density: ρ
- kin. viscosity: ν
- velocity : V
- domain size: L
- mag. permeability: μ_0
- elec. conductivity : σ

$Re = VL / \nu$

$R_m = \mu_0 \sigma VL$

$P_m = \mu_0 \sigma \nu$

Experiments, numerical simulations and the universe



$Rm = \mu_0 \sigma L V$

$Pm = \mu_0 \sigma \nu$

Power $P \propto \rho L^2 V^3$ needed to drive a turbulent flow

$\Rightarrow \; Rm \propto \mu_0 \sigma \left(PL/\rho\right)^{1/3}$

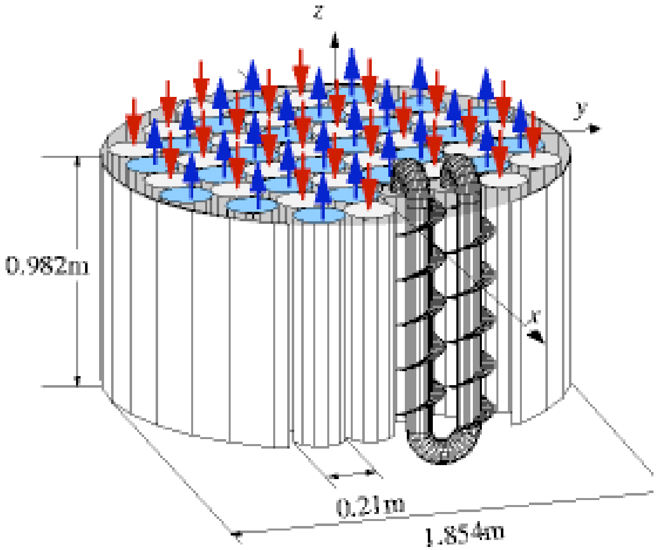
Using liquid sodium, 100 kW for Rm = 50 with L = 1 m

Karlsruhe and Riga experiment (2001)

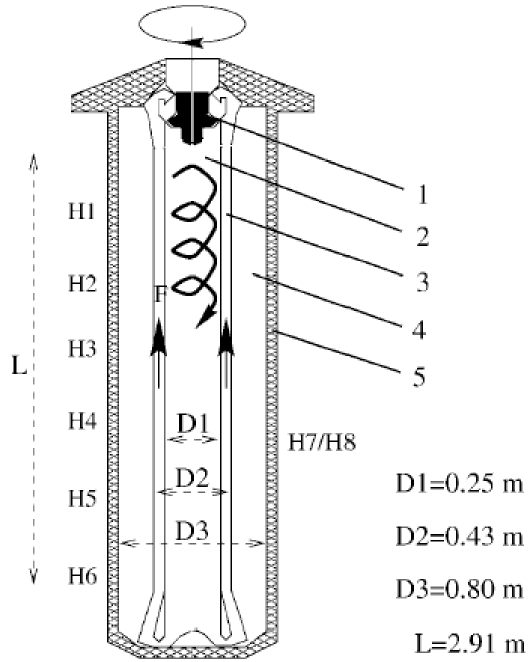
avoid large scale turbulent fluctuations using geometrical constraints

$$\mathbf{V}(\mathbf{r},\mathbf{t}) = \langle \mathbf{V} \rangle(\mathbf{r}) + \tilde{\mathbf{v}}(\mathbf{r},\mathbf{t})$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times [\langle \mathbf{V} \rangle(\mathbf{r}) \times \mathbf{B}] + \nabla \times [\tilde{\mathbf{v}}(\mathbf{r},\mathbf{t}) \times \mathbf{B}] + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$



Stieglitz & Müller, Phys. Fluids 13, 561 (2001)



Gailitis et al., PRL 86, 3024 (2001)

Find a magnetic field generated by the mean flow

No secondary instabilities with large scale dynamics of B

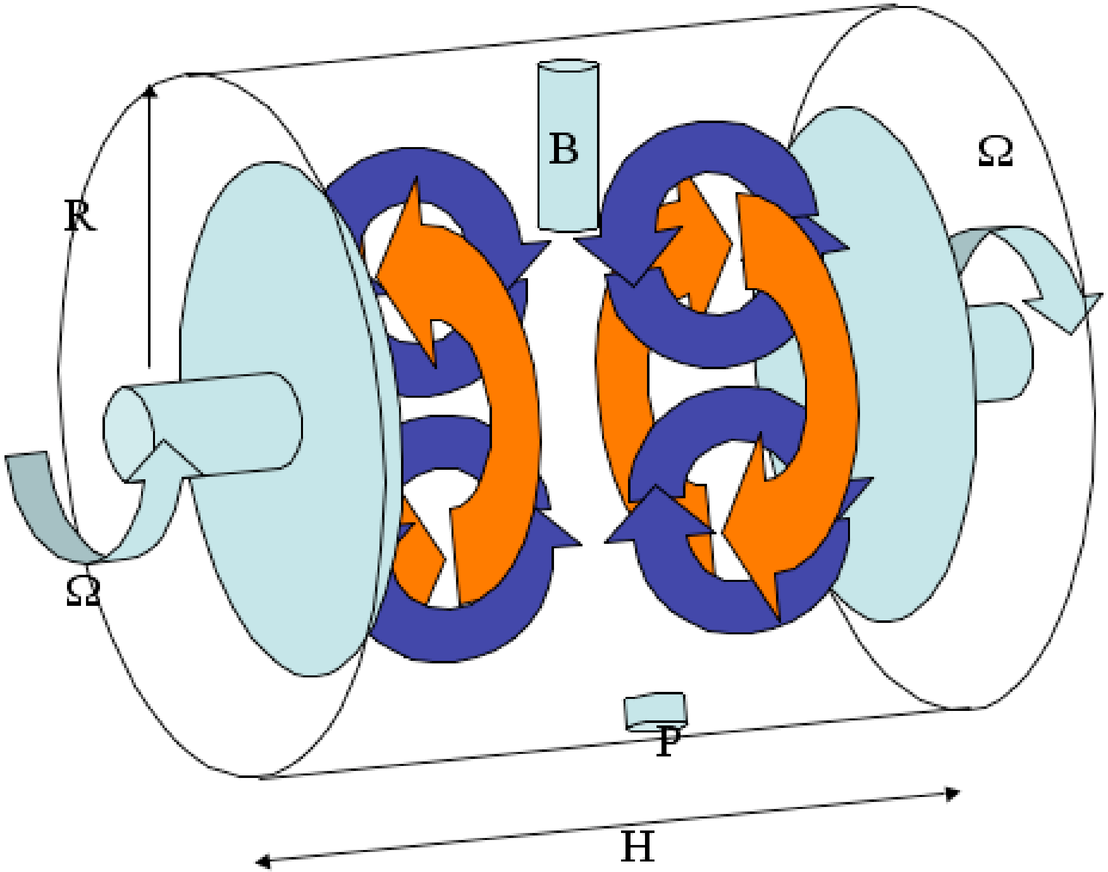
Madison and Maryland experiments

Diameter 1 m, Power 150 kW
Forest et al.

Diameter 3 m, Power 1 MW
Lathrop et al.

A « turbulent » dynamo ?
Motivations for the von Karman flow

- **Strong turbulence**
- **Differential rotation**
- **Helicity**
- **« Analogy » B - Ω**
- **Global rotation**



An instability from a fully turbulent regime

The VKS collaboration

CEA-Saclay

S. Aumaître, A. Chiffaudel, B. Dubrulle, F. Daviaud,
L. Marié, R. Monchaux, F. Ravelet

ENS-Lyon

G. Verhille, M. Bourgoin, P. Odier,
J.-F. Pinton, N. Plihon, R. Volk

ENS-Paris

M. Berhanu, B. Gallet, C. Gissinger, S. Fauve,
N. Mordant, F. Pétrélis

VKS 2 experiment

• Liquid sodium:150 l

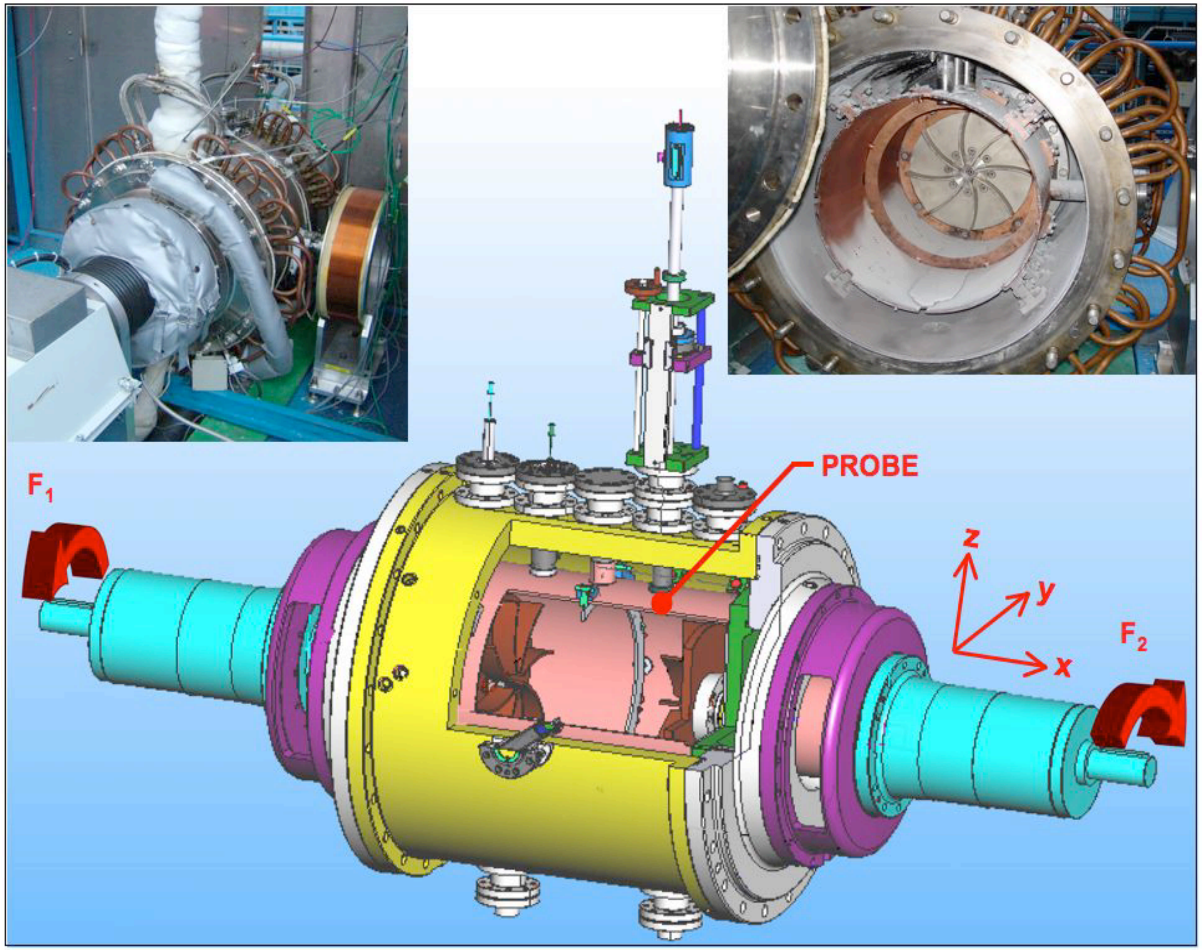
• Power: 300 kW

• Temperature control

• Measurments :

- power
- pressure
- magnetic field

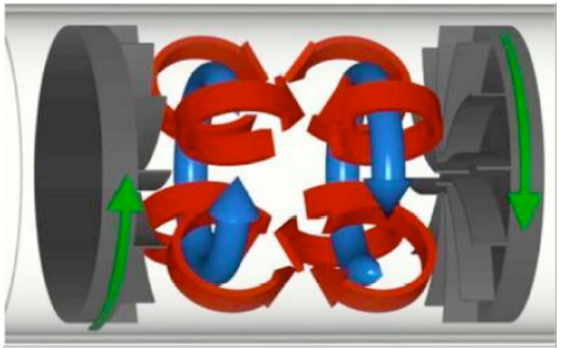
Iron impellers



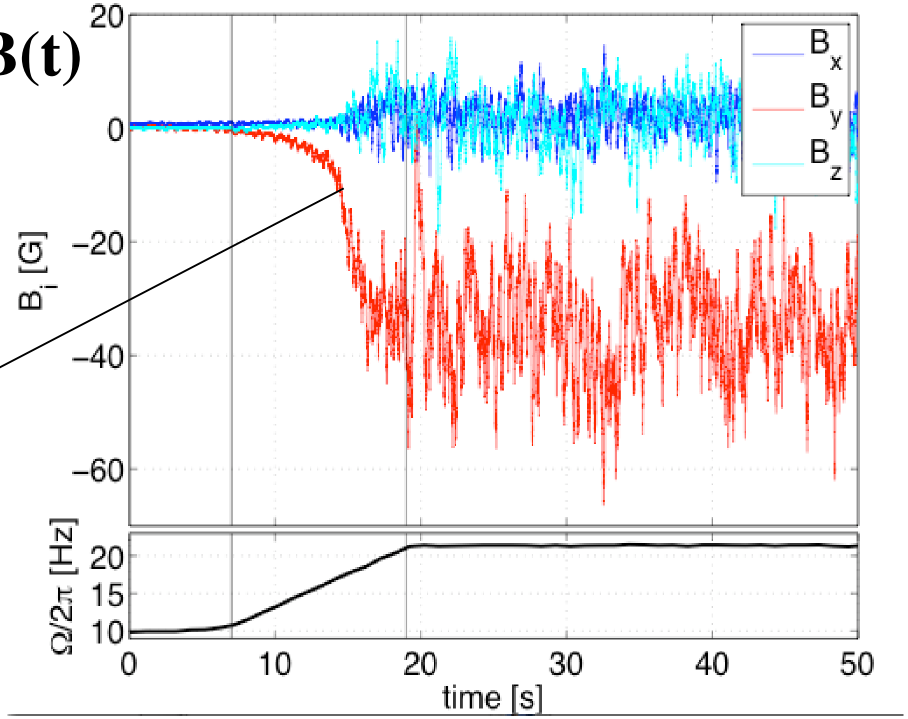
Dynamo with counter-rotating impellers

Monchaux et al., PRL 98, 044502 (2007)

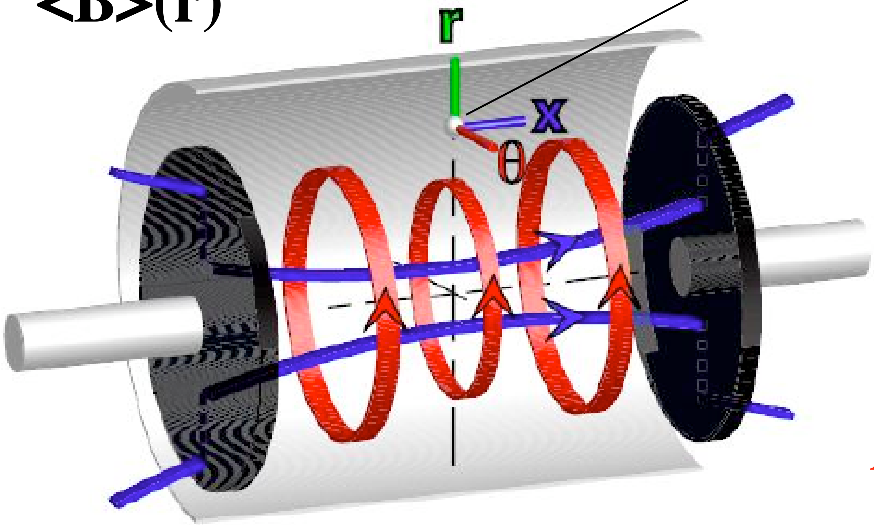
<V>(r)



B(t)

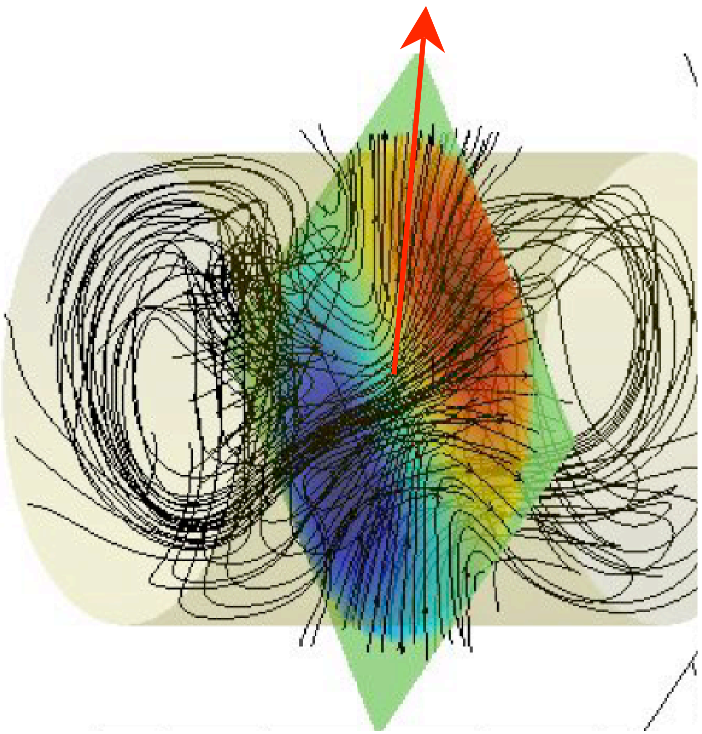


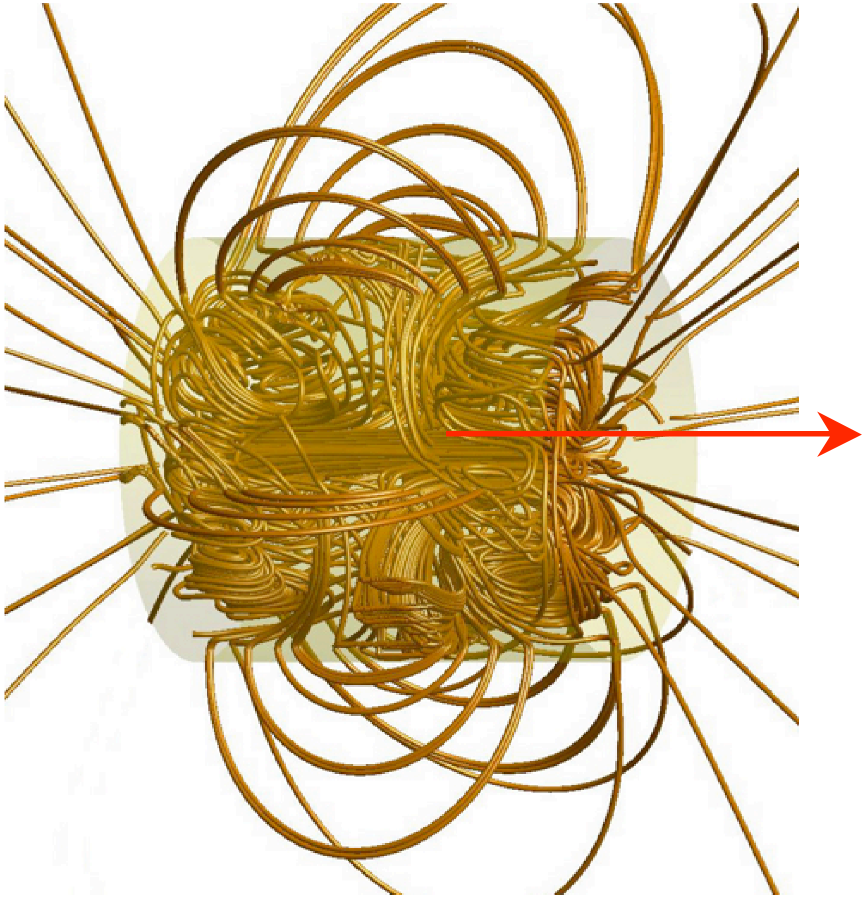
<B>(r)



A problem with Cowling theorem?

Geometry of the generated mean magnetic field using a numerical model C. Gissinger, E. Dormy



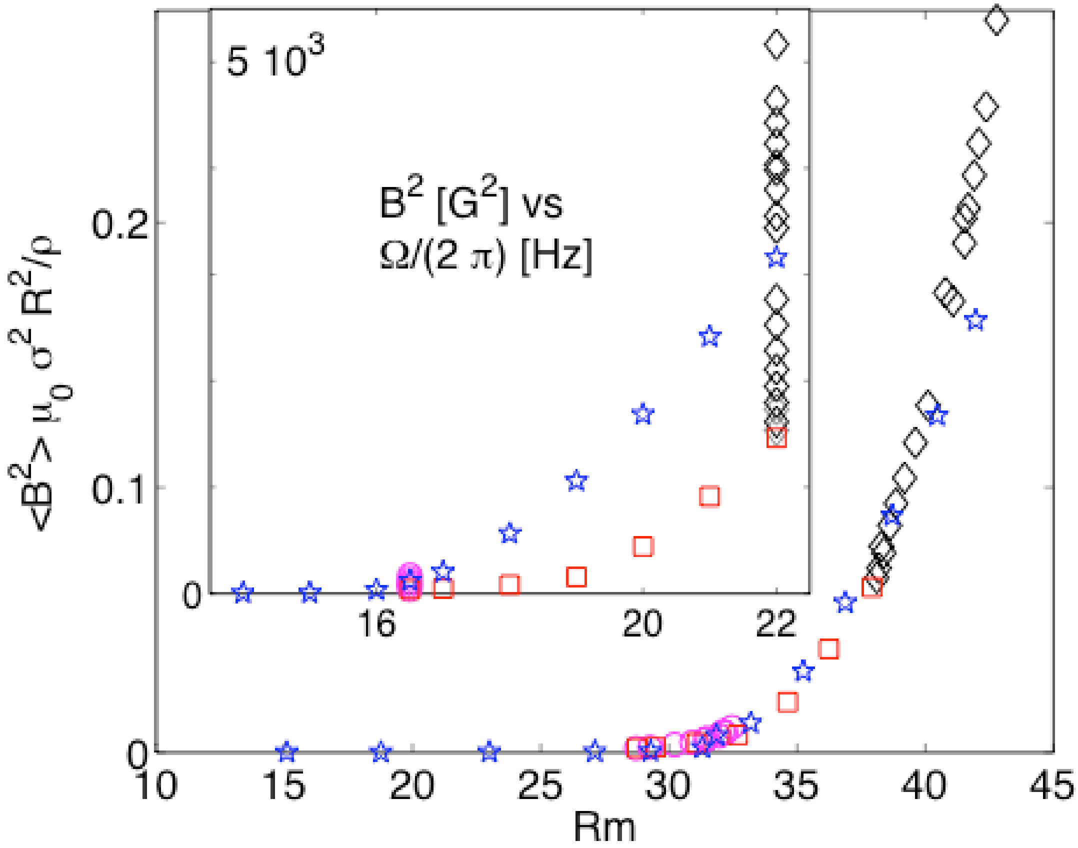


Mean flow alone: equatorial Dipole; the magnetic field should break axisymmetry (Cowling, 1934)

Flow with non axisymmetric velocity fluctuations: axial dipole

The VKS dynamo is not generated by the mean flow alone

Magnetic energy density



Small Re

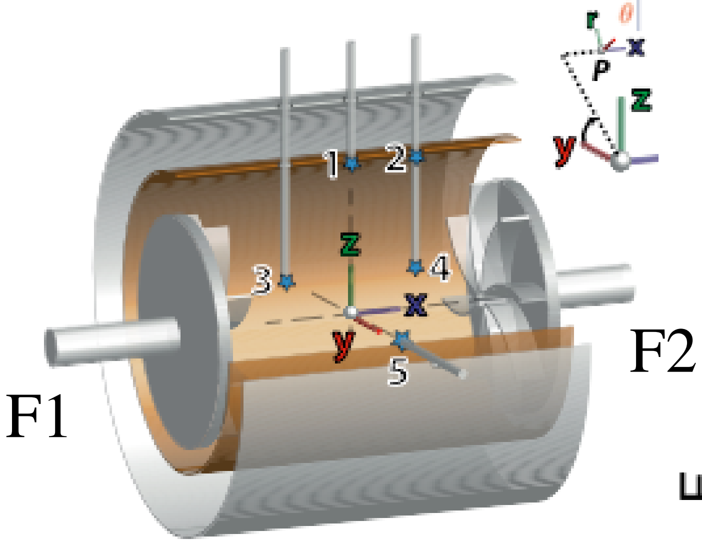
$$B^2 \propto \frac{\rho\nu}{\sigma L^2}\,(R_m - R_{mc})$$

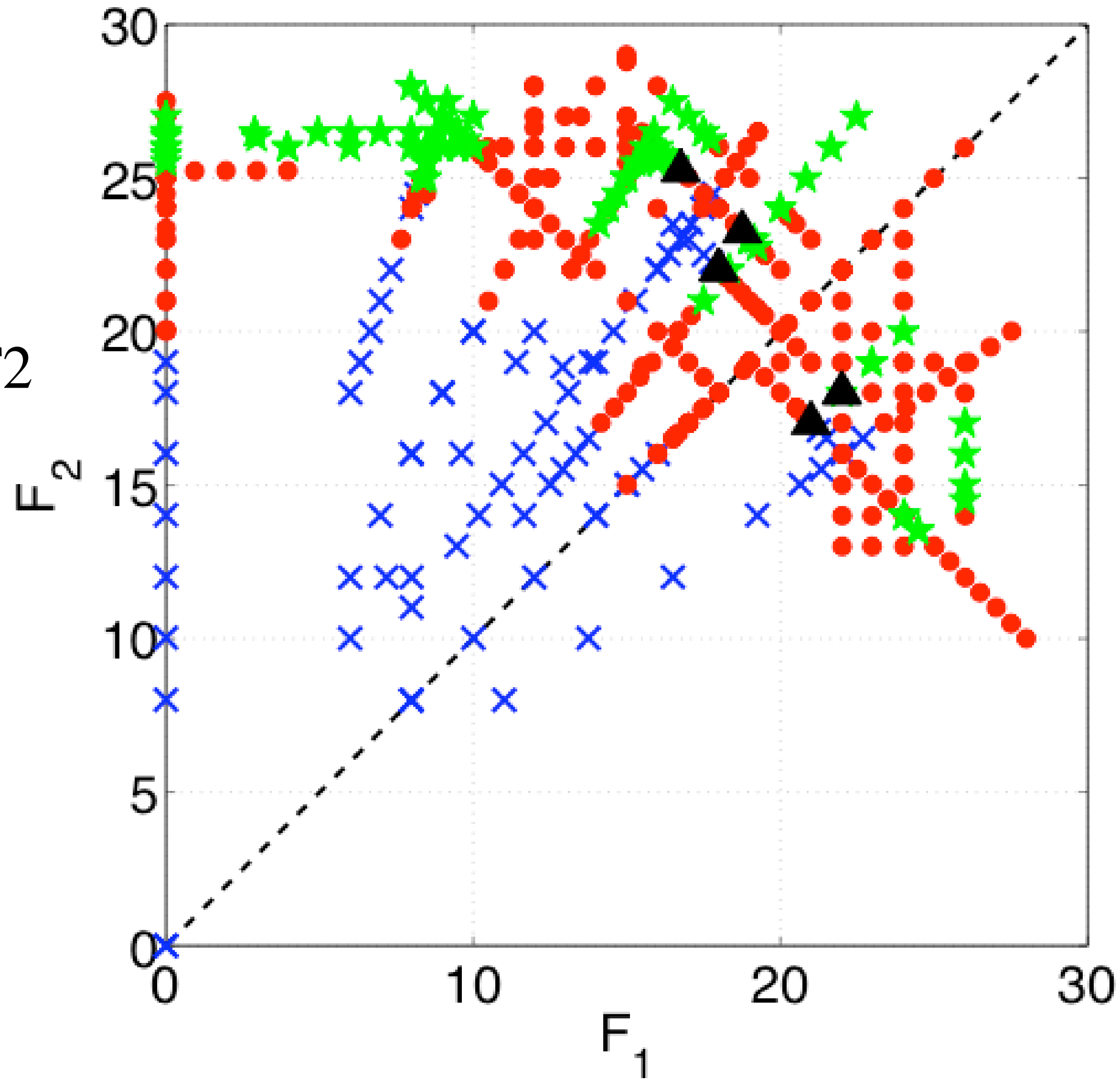
Large Re

$$B^2 \propto \frac{\rho}{\mu_0(\sigma L)^2}\,(R_m - R_m^c)$$

F. Pétrélis and S. Fauve, Eur. Phys. J. B 22, 273 (2001)

Dynamical regimes

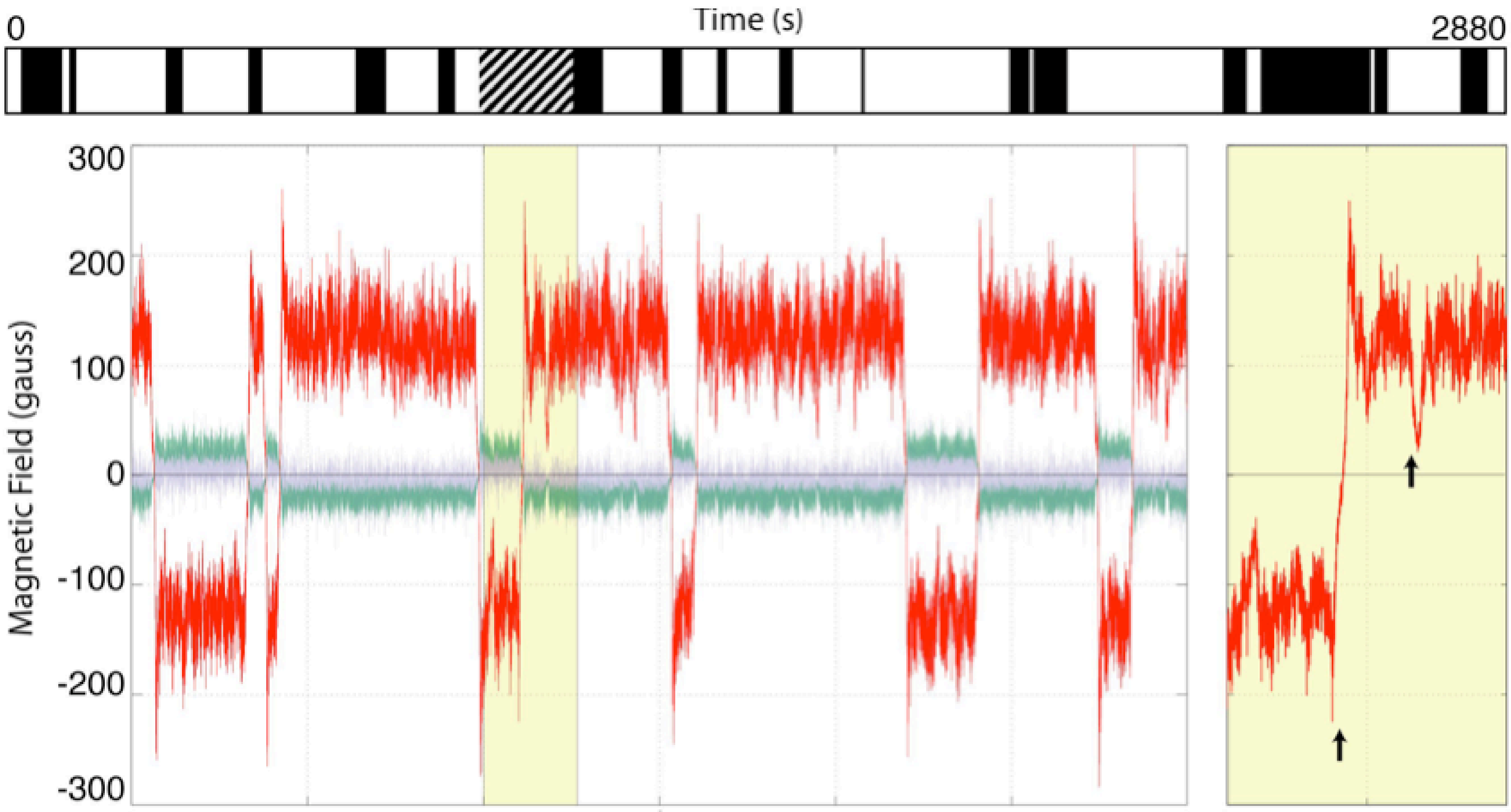




- No dynamo
- Stationary dynamos
- Dynamical regimes

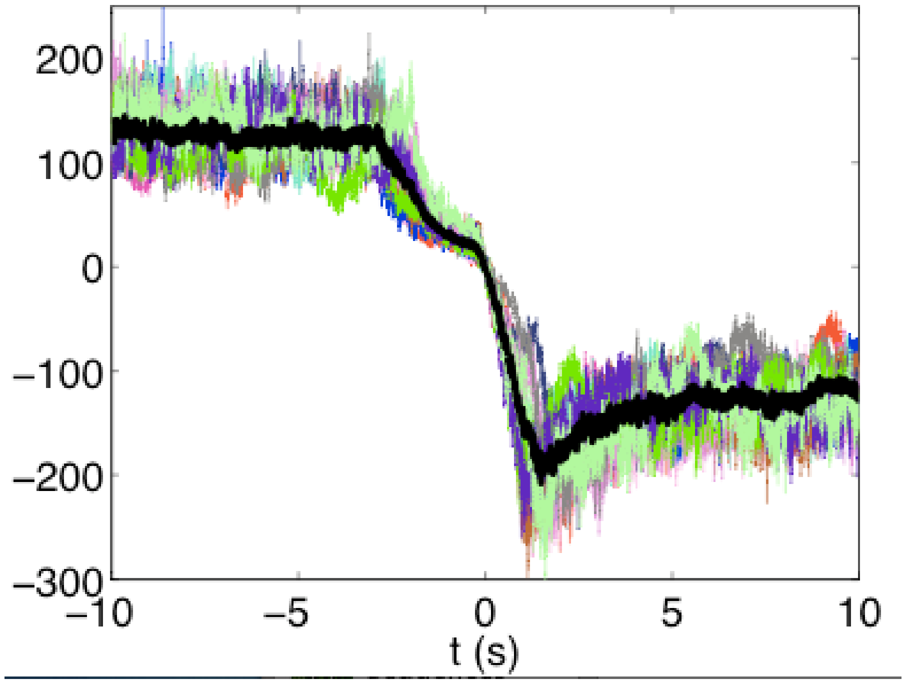
Reversals of the magnetic field

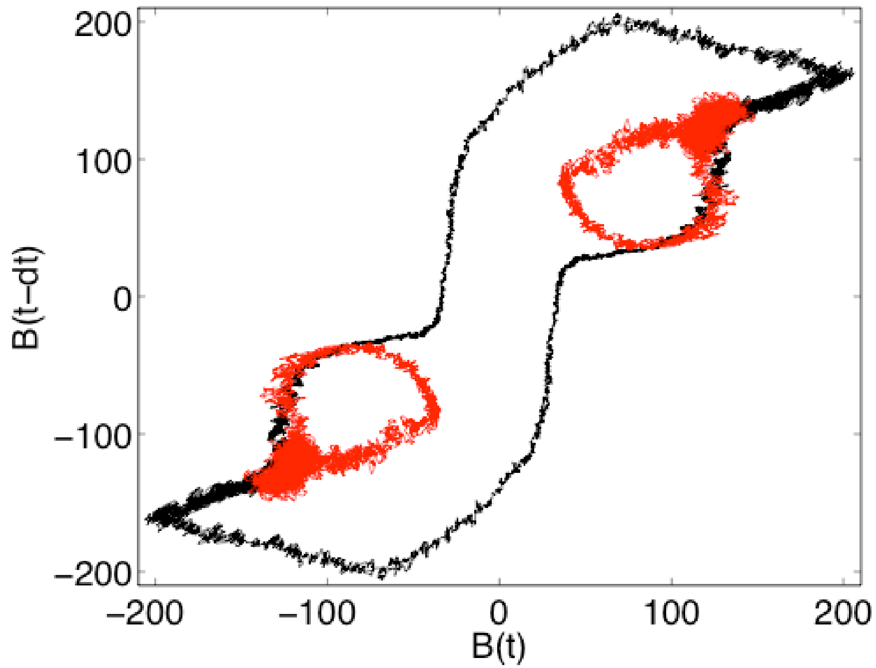
Berhanu et al., EPL 2007



Robustness of the reversal trajectories despite turbulent fluctuations

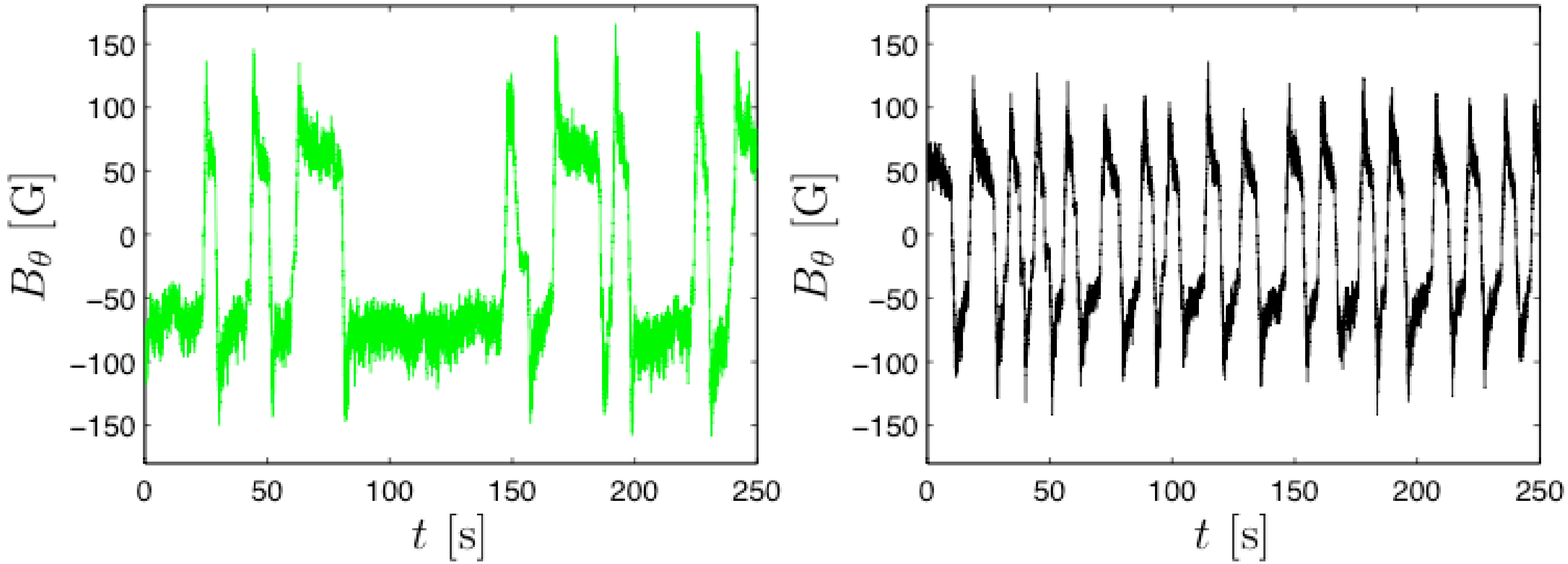
12 superimposed reversals (slow decay, fast recovery, overshoots)





A low dimensional dynamical system despite high Re ($5.\ 10^6$) ?

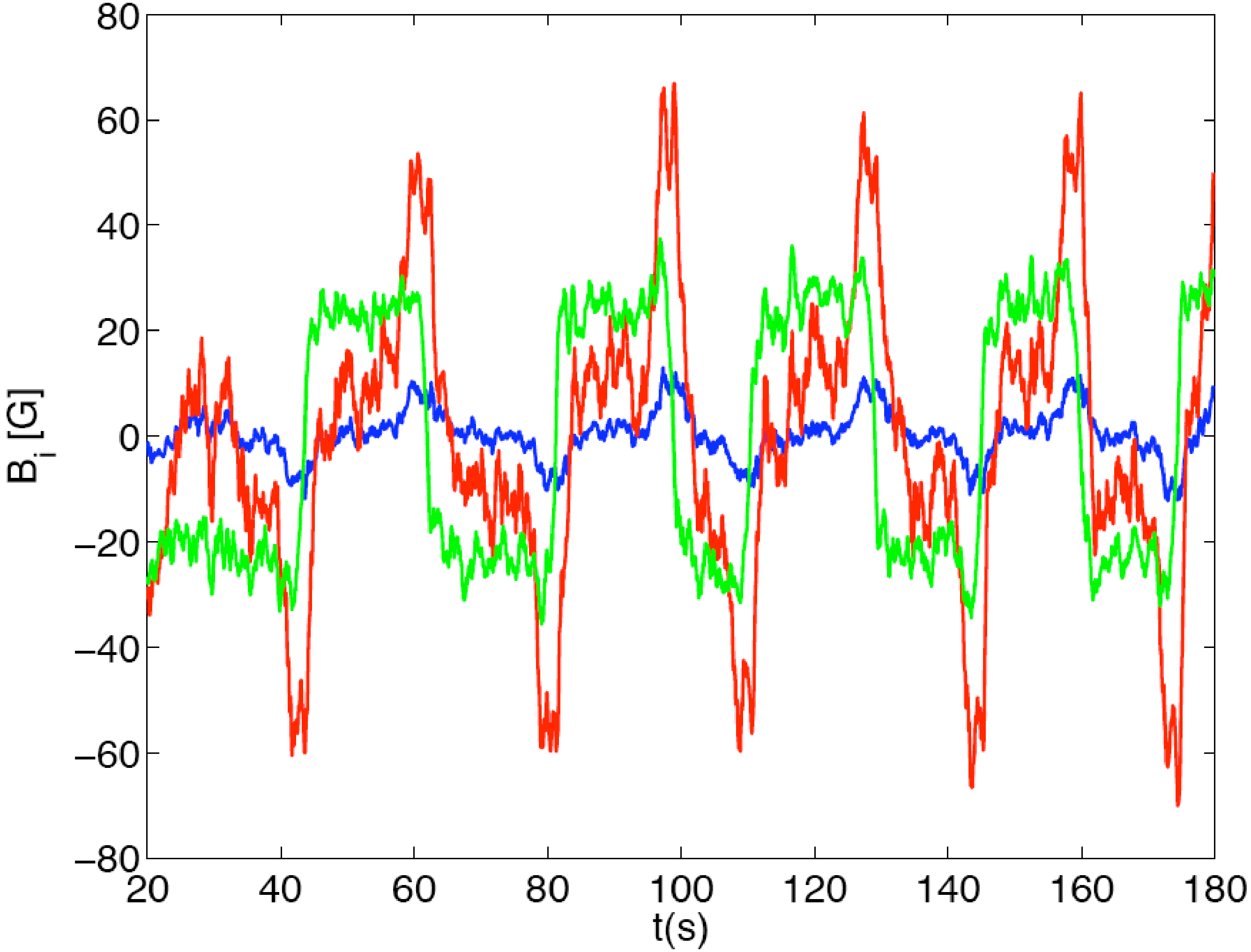
A continuous transition between random and nearly periodic reversals



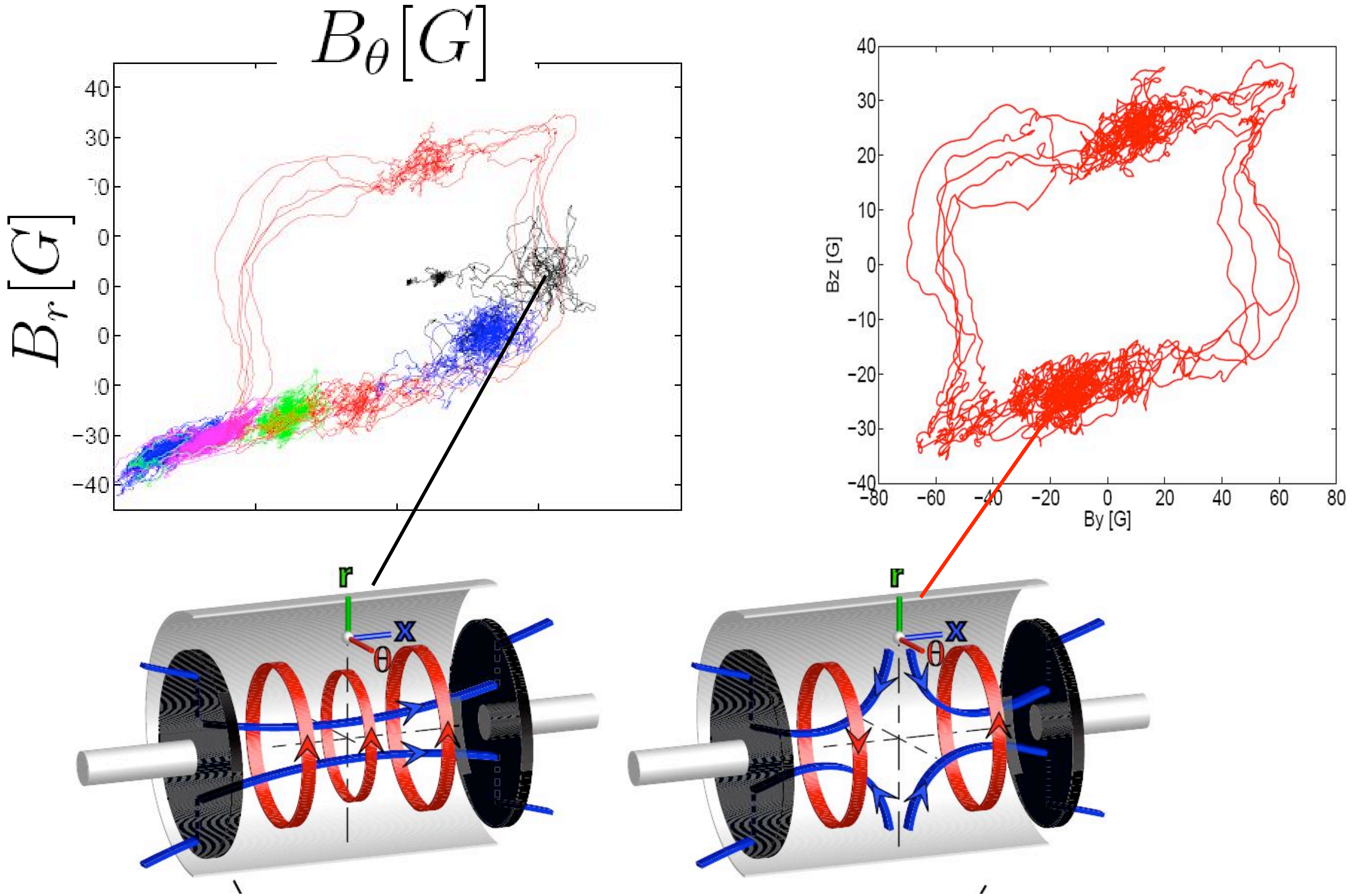
An increase of temperature such that Rm is increased and Re decreased (5%) strongly affects waiting times between successive reversals.

It is thus difficult to imagine that turbulent fluctuations are the dominant mechanism to induce reversals

From stationary to time dependent dynamos: A relaxation oscillator

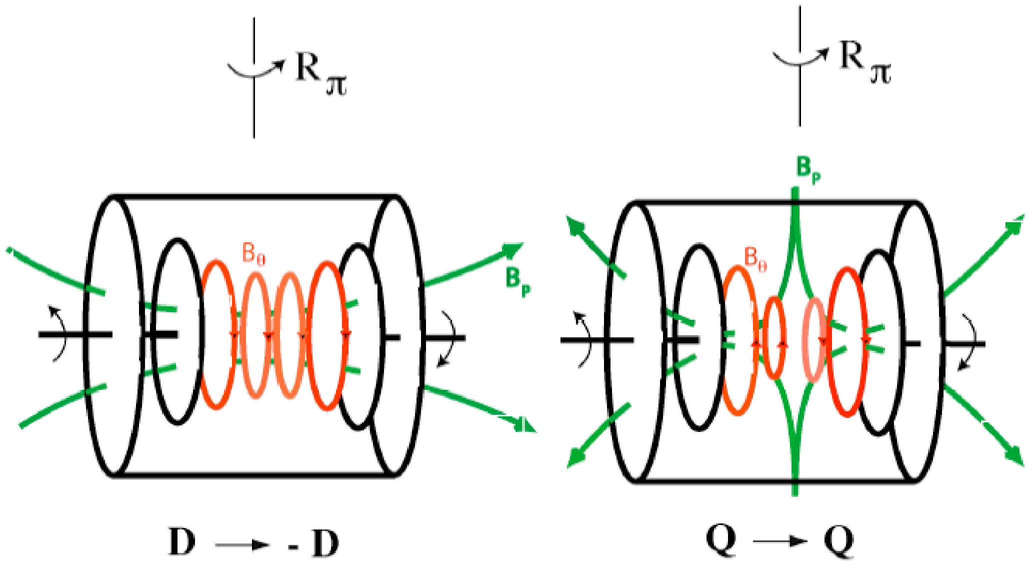


Dipole and quadrupole :excitability



A model for oscillations and reversals

work with François Pétrélis



$$\mathbf{B}(\mathbf{r}, t) = d(t)\, \mathbf{D}(\mathbf{r}) + q(t)\, \mathbf{Q}(\mathbf{r}) + \ldots$$

$$d_t = \alpha\, d + \beta\, q - a_1\, d^3 - a_2\, d^2 q - a_3\, d q^2 - a_4\, q^3$$

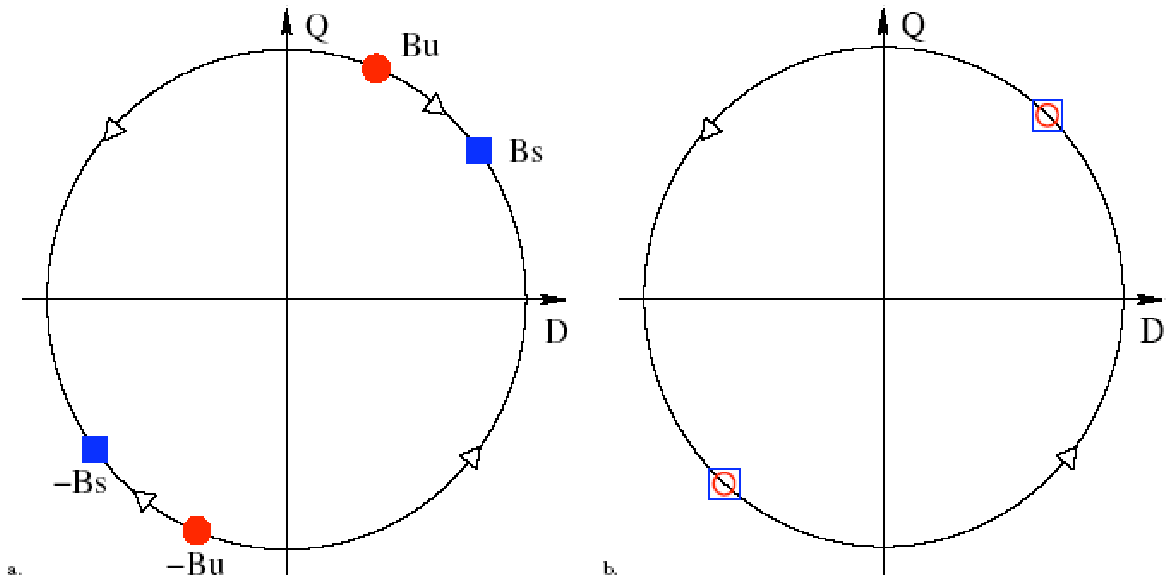
$$q_t = \gamma\, d + \delta\, q - b_1\, d^3 - b_2\, d^2 q - b_3\, d q^2 - b_4\, q^3$$

The broken R symmetry couples dipolar and quadrupolar modes

A limit cycle generated by a saddle-node bifurcation

$A = d + iq = R \exp i(\theta + \theta_0)$

$$\dot{A} = \mu A + \nu \bar{A} + \beta_1 A^3 + \beta_2 A^2 \bar{A} + \beta_3 A \bar{A}^2 + \beta_4 \bar{A}^3$$

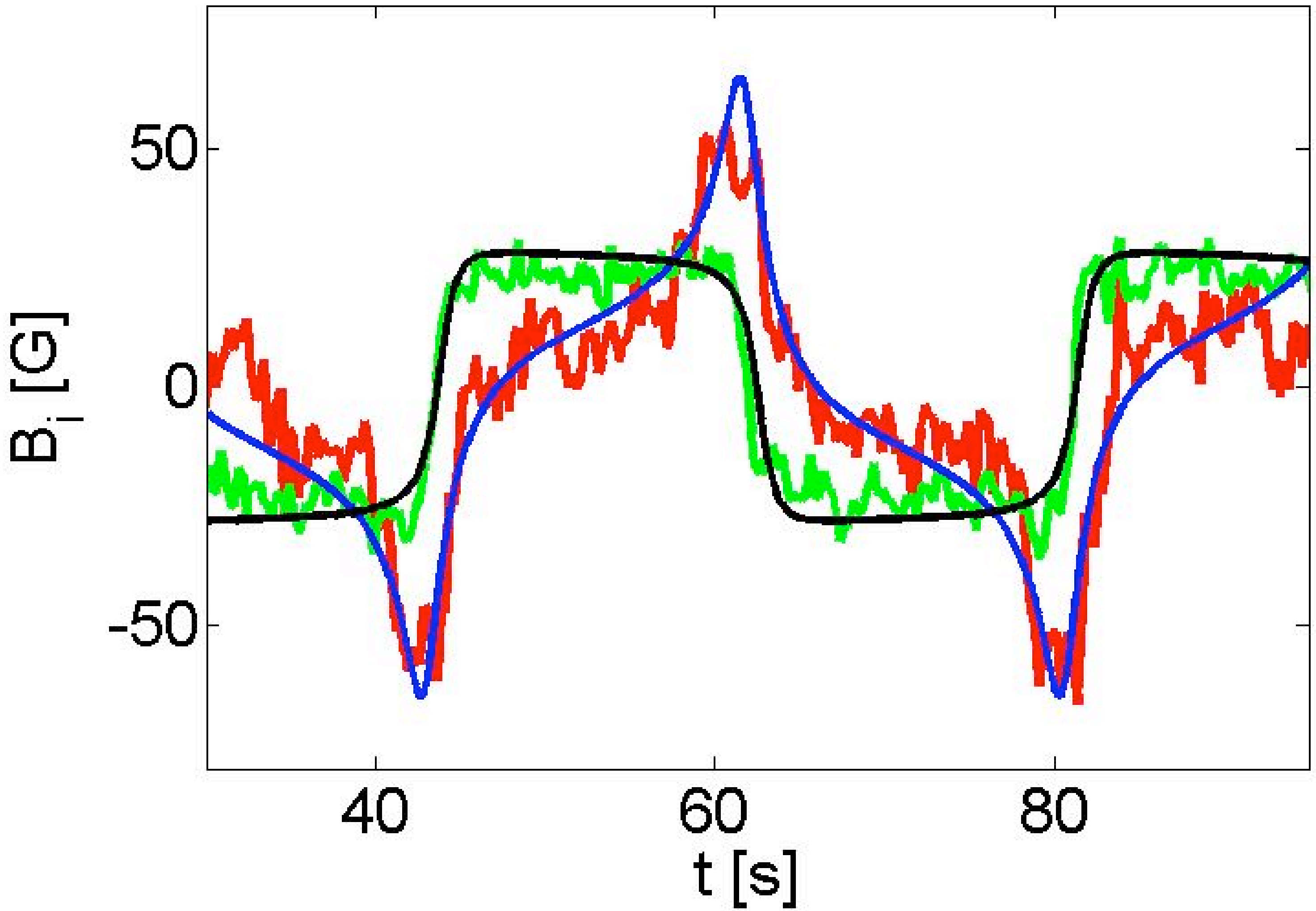


$\theta_t = \nu - \rho \sin 2\theta$ + terms in $\cos 4\theta$, $\sin 4\theta$

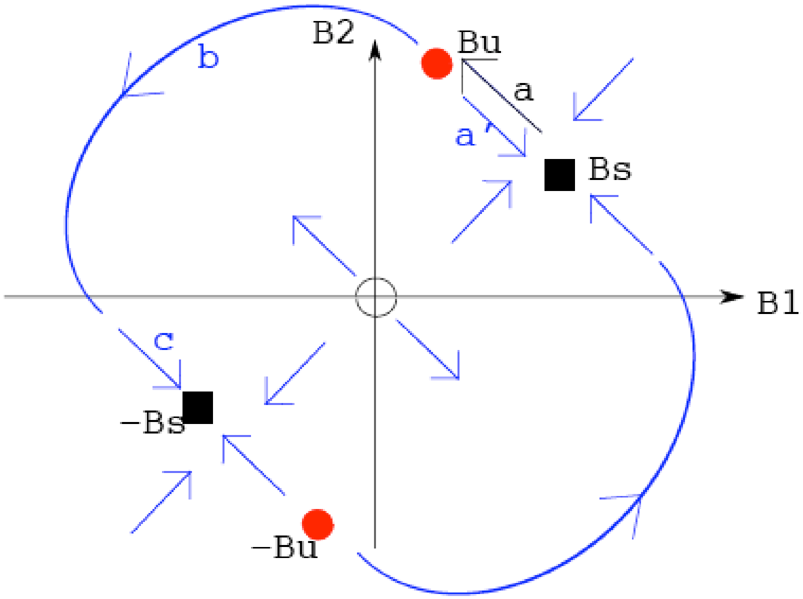
Increasing the asymmetry of the driving (ν) generates a limit cycle

Pétrélis and Fauve, 2008

Oscillation of the magnetic field in the VKS experiment compared to the deterministic model



Generic mechanism for a 2D system with B-> -B symmetry



Connection of B to -B

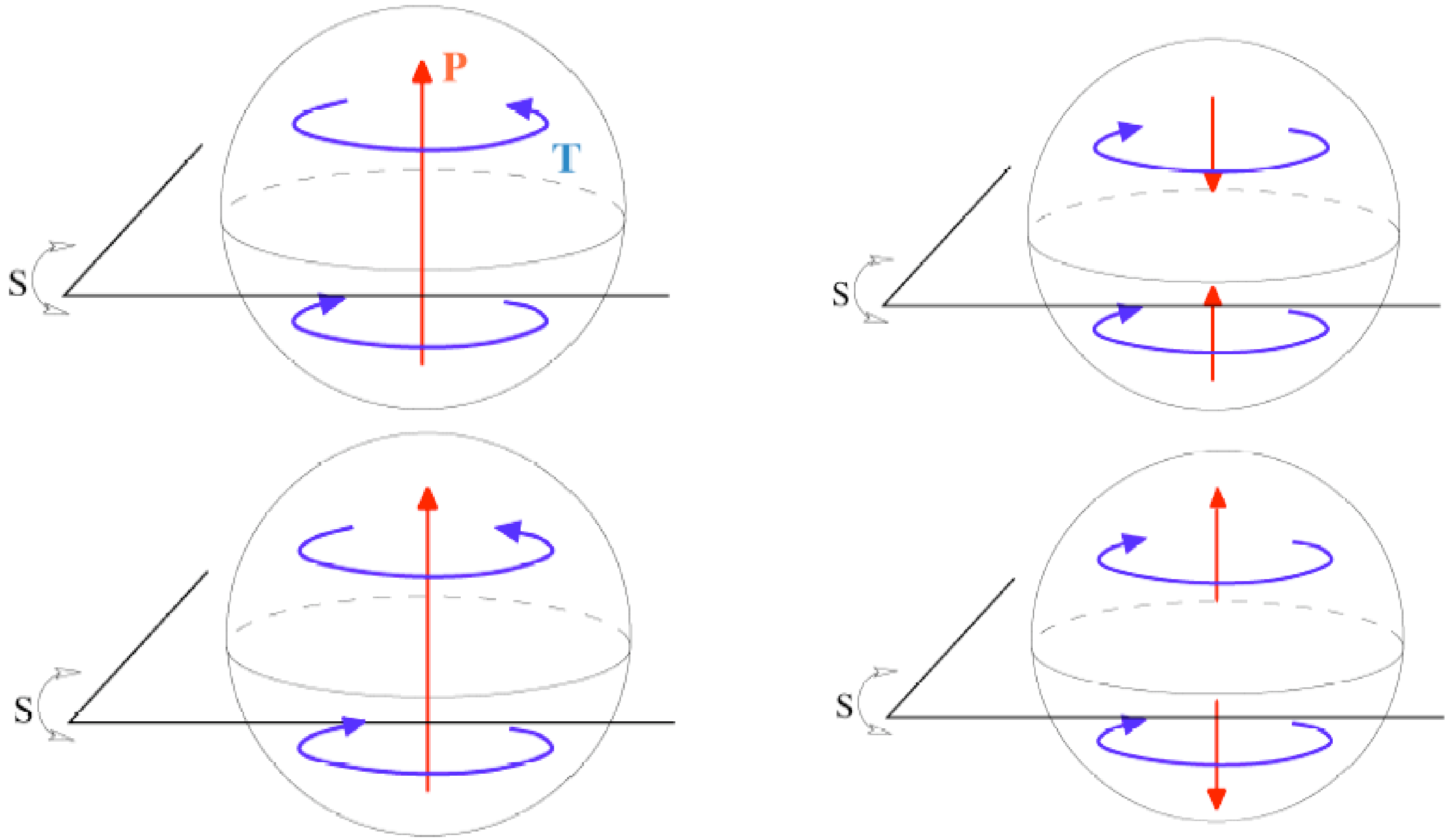
Before the SN bifurcation, fluctuations can generate random reversals

Prediction of

- the shape of reversals versus excursions: slow and fast phases, overshoot or not
- long periods without reversals by slightly changing the parameters
- continuous transition from reversals to oscillations in the presence of fluctuations

Geomagnetic reversals caused by breaking mirror symmetry of core dynamics ?

F. Pétrélis, S. Fauve, E. Dormy, J. P. Valet (2008)

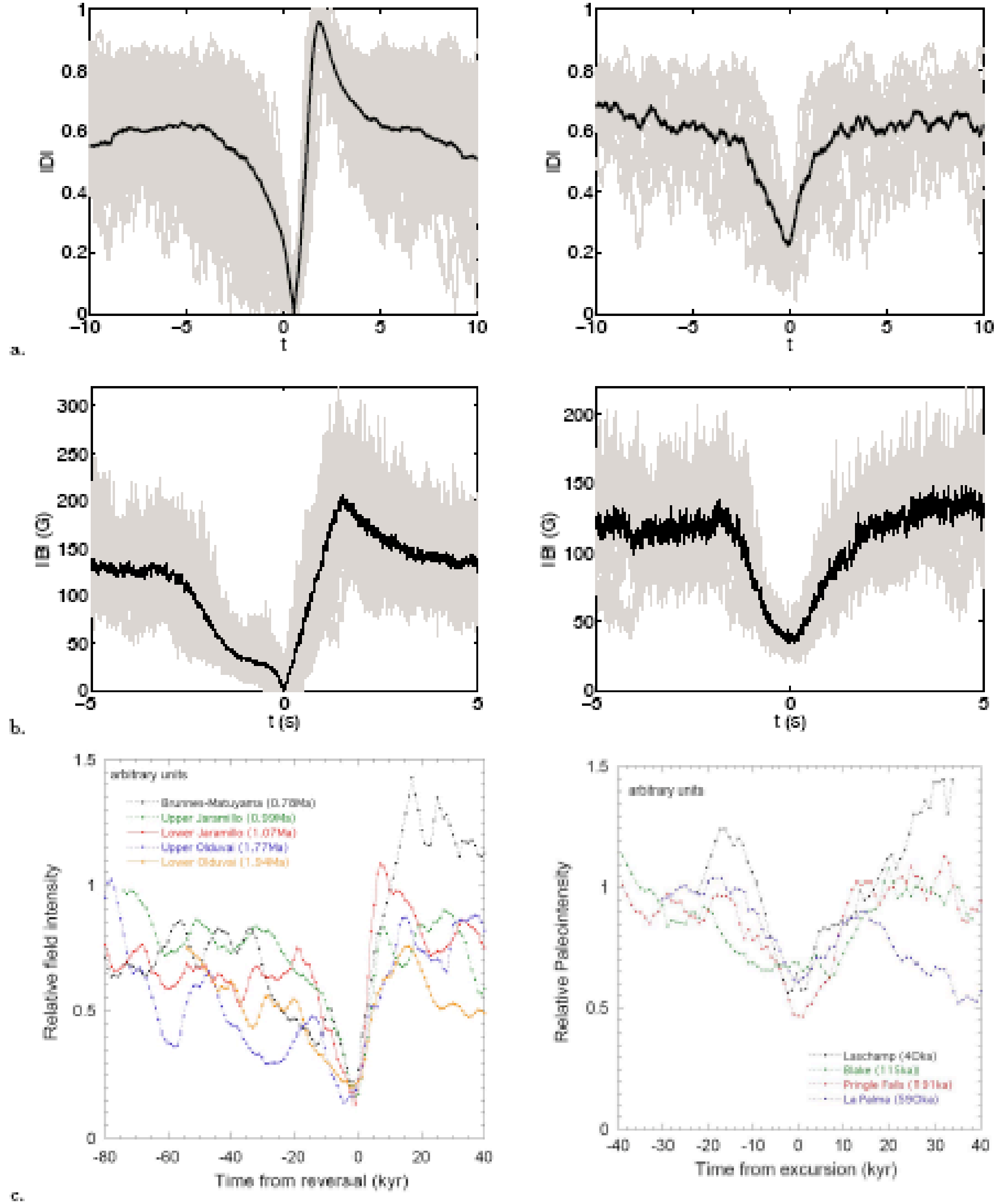


Dipolar modes D -> D

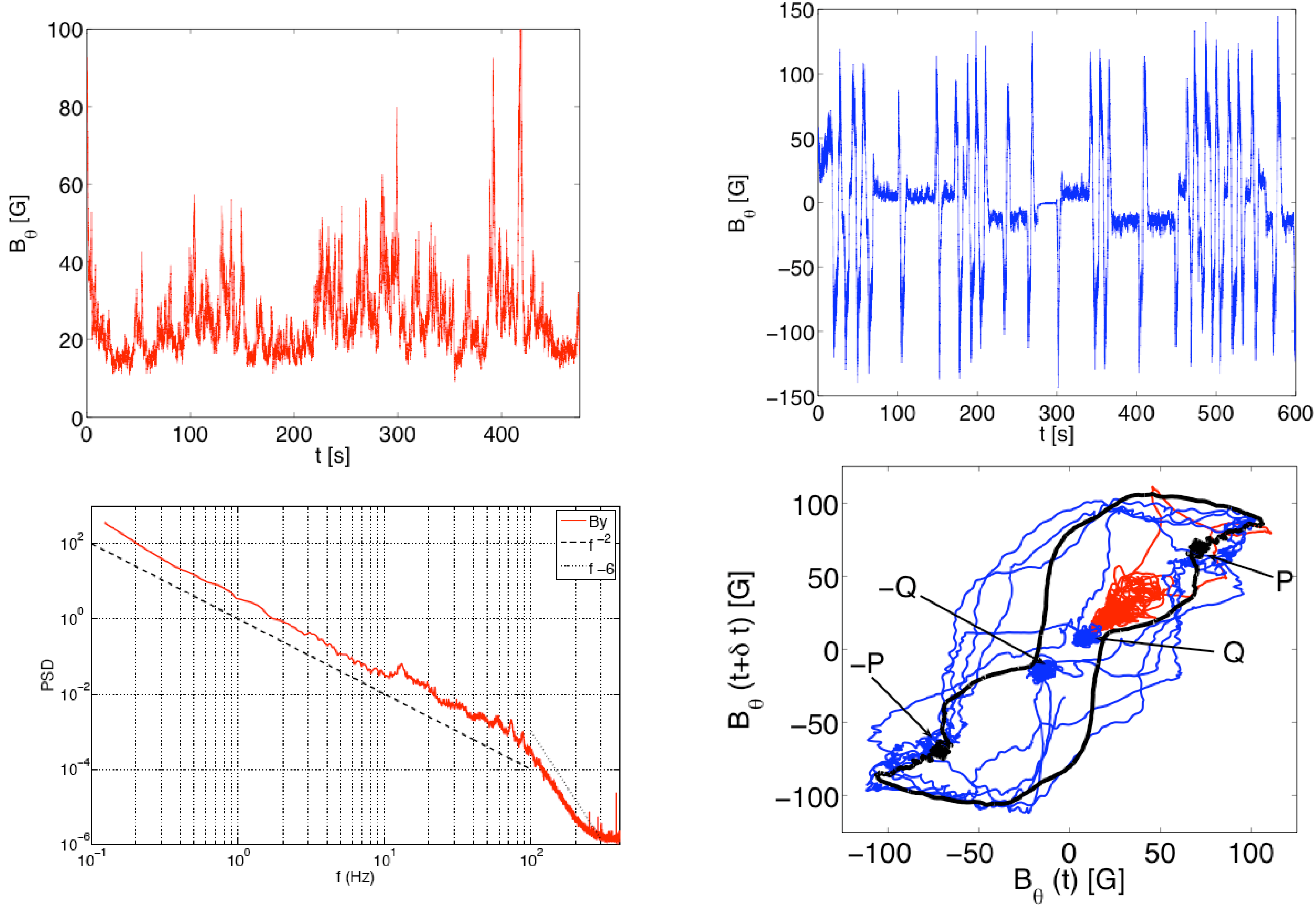
Quadrupolar modes Q -> - Q

Model, VKS experiment and the Earth

$$\theta_t = \nu - \rho \sin 2\theta + \xi(t)$$



Asymmetric and symmetric intermittent bursts



A simple model for all the dynamical regimes of the VKS experiment

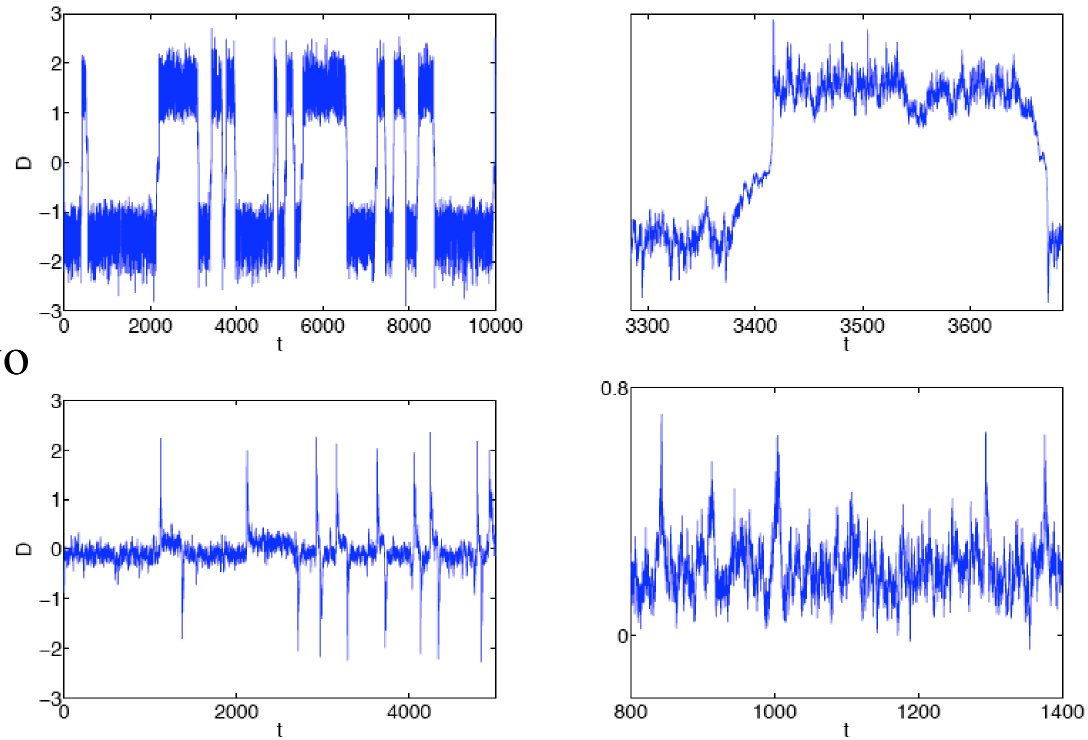
A = d + i q

$$\dot{A} = \mu A + \nu \bar{A} + \beta_1 A^3 + \beta_2 A^2 \bar{A} + \beta_3 A \bar{A}^2 + \beta_4 \bar{A}^3$$

When the higher order terms are taken into account, the dynamics can involve four fixed points

More complex dynamics result
From the interaction between two
Saddle-node bifurcations:

- reversals
- symmetric bursts
- asymmetric bursts



Conclusions

- VKS dynamo not generated by the mean flow alone
- Good agreement for the scaling of the magnetic field
- Many different regimes in a small parameter range
- Large scale dynamics of the field
  - governed by a few modes
  - not smeared out by turbulent fluctuations
- Reversals result from the competition between different modes (no need any external triggering mechanism) and are due to a broken symmetry of the flow
- A similar mechanism can be involved for planetary or stellar time dependent dynamos