

Optimal Vortex Formation as a Unifying Principle in Biological Propulsion

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Vortex Rings

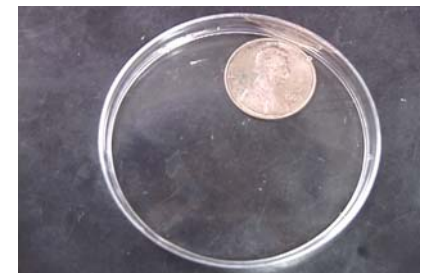
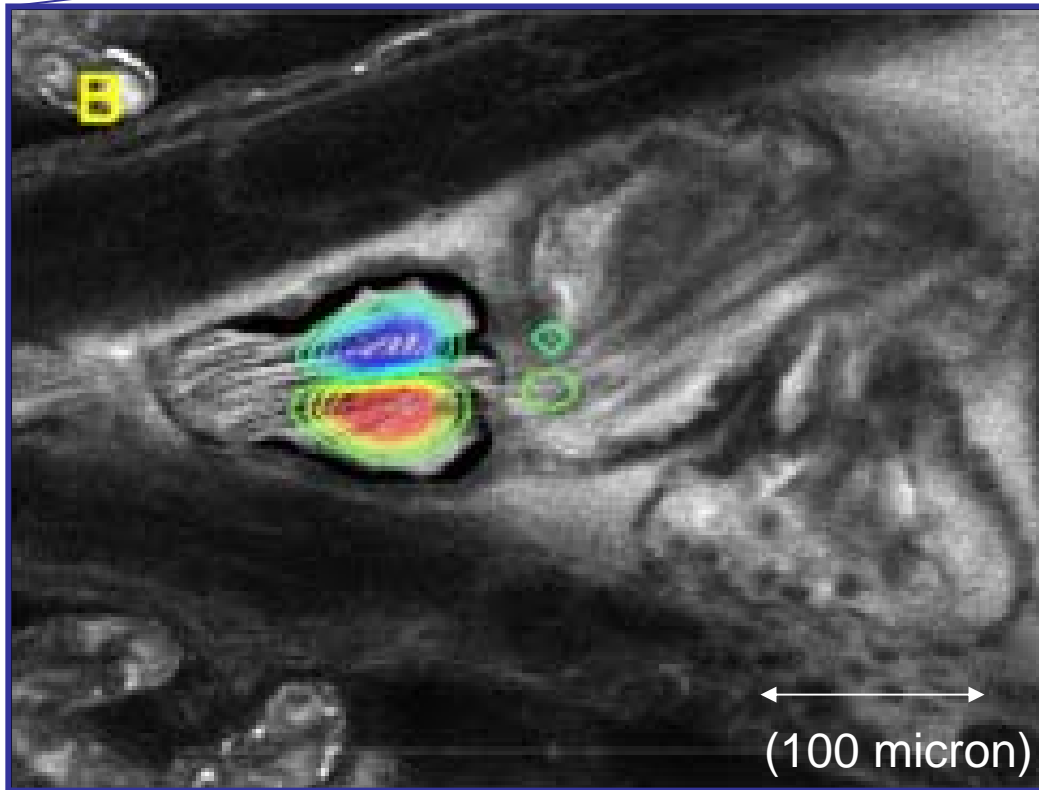


A Secondary Eruption of Mt. St. Helens, June 1980 [Photo by Robert P. VanNatta]

Zebrafish (*Danio rerio*) Embryo



1.5 mm long



Previous Work on Vortex Rings

- Early work: Helmholtz (1858), Kelvin (1869)
- Existence: Hill (1894), Fraenkel (1972), Norbury (1973), and others
- Formation: Saffman (1975, 1978), Pullin (1979), Didden (1979), Glezer (1987)
- Evolution and Turbulent rings: Maxworthy (1972, 1974, 1977), Glezer (1987), and others

Fully-Pulsed Jets:

- Bremhorst *et al.* (e.g., 1979, 1990, and 2000)
- Weihs (1977)

Classical View

Vortex ring "...formation is a problem of **vortex sheet dynamics**, the steady state is a problem of **existence**, their duration is a problem of **stability**, and if there are several we have a problem of vortex **interactions**."

-- *P.G. Saffman (1981), emphasis added*

“New” Motivations

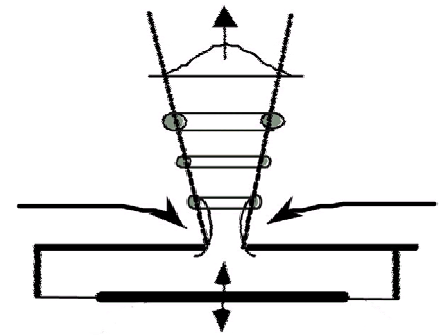
Understanding complicated biological flows:

- Aquatic Propulsion
- Cardiac Flows



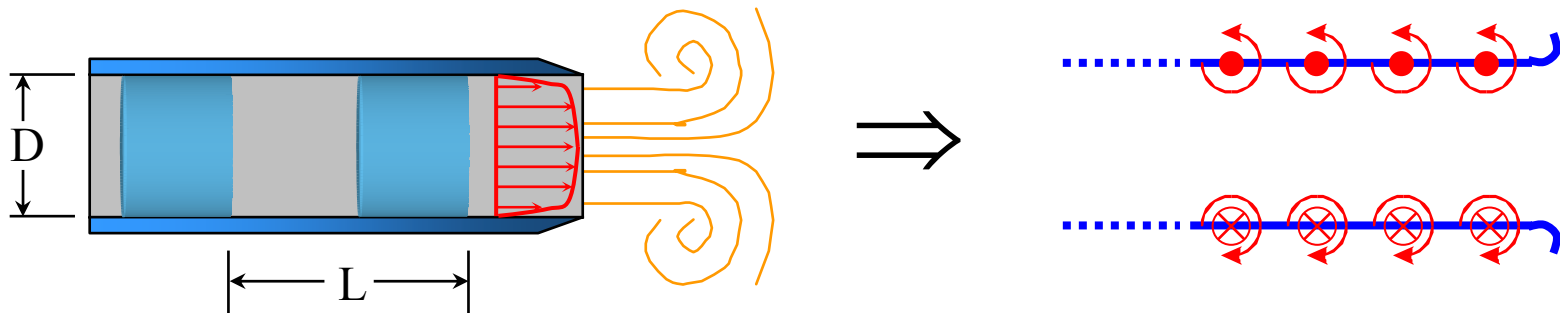
Practical Applications:

- Hydropropulsion /Aeropropulsion
- Micro jet thrusters
- Multi-scale Stirring and Mixing



Canonical Vortex Ring Generator

Vortex rings can be easily generated using a piston-cylinder mechanism to produce a starting jet.

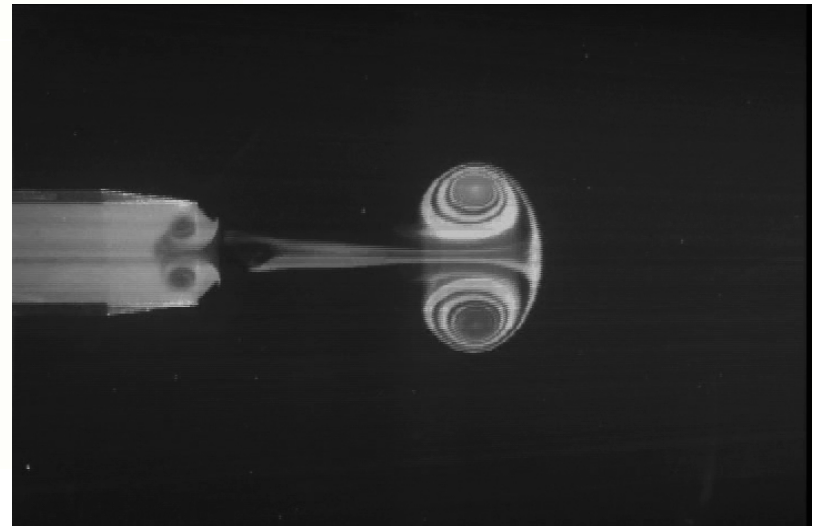
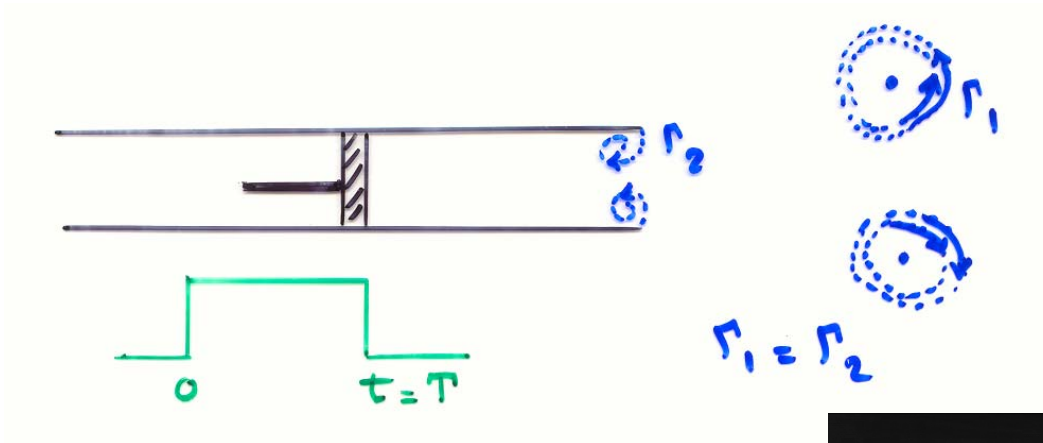


Parameters:

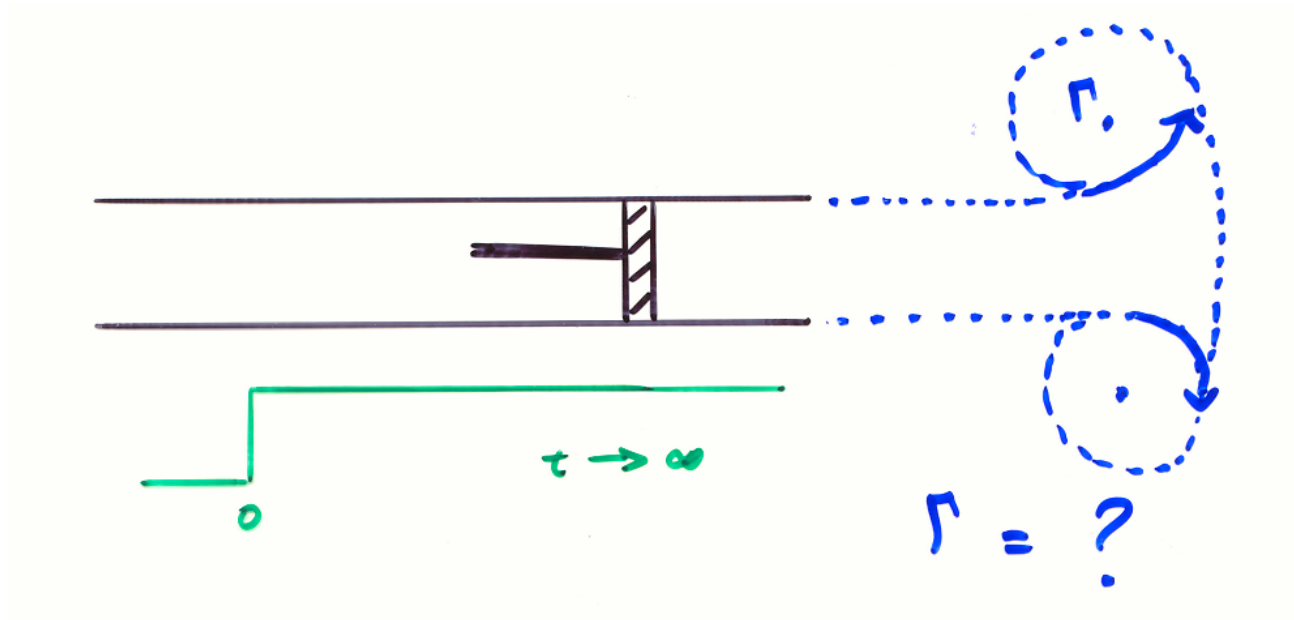
- a) **Time history of piston velocity**
- b) **L/D**
- c) Reynolds Number
- d) Orifice/nozzle Geometry

Can be viewed as the roll up of a half-"infinite" cylindrical vortex sheet.

Vortex Ring Formation



Vortex Ring Formation



Vortex Ring Experiments

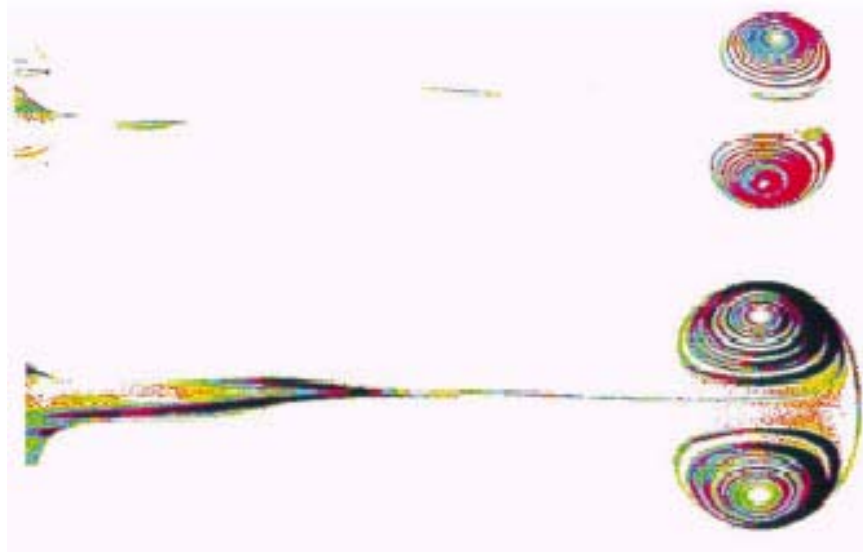
Glezer (1981)	L/D<1
Didden (1979)	L/D<2
Kwon and Bernal (1989)	L/D<4
Auerbach (1980)	L/D<1
Schatzle (1987)	L/D<1
Weigand and Gharib (1994)	L/D<1
Maxworthy (1977)	L/D<3
Sallet (1974)	L/D<1

Can we generate arbitrarily large vortex rings?



$$\leftarrow L/D = \left[\int_0^t U(\tau) d\tau \right] / D = 2$$

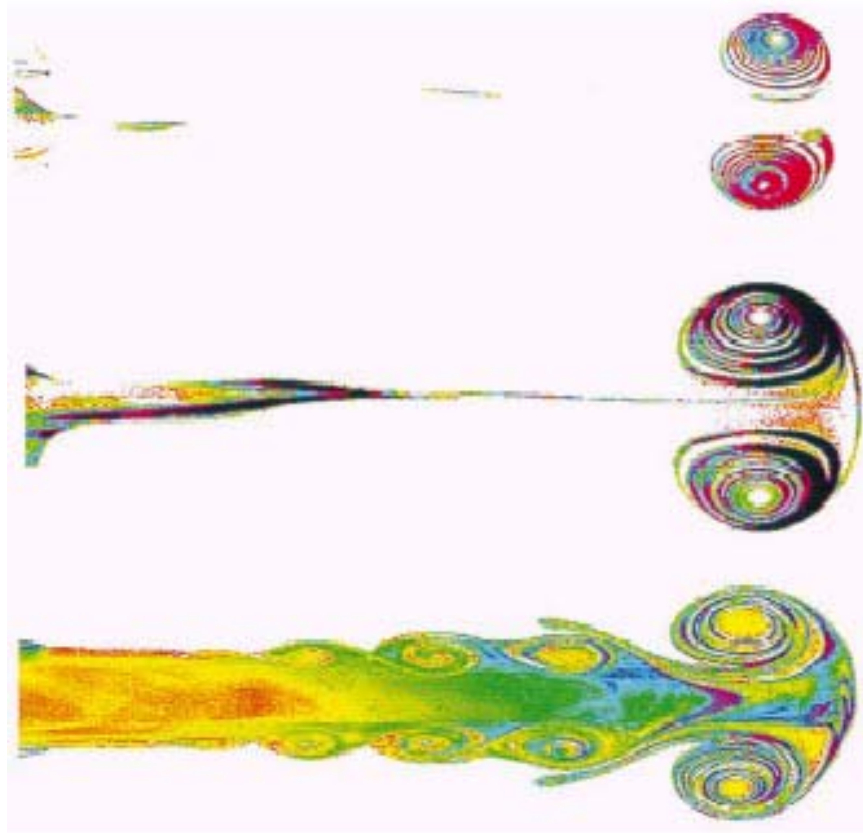
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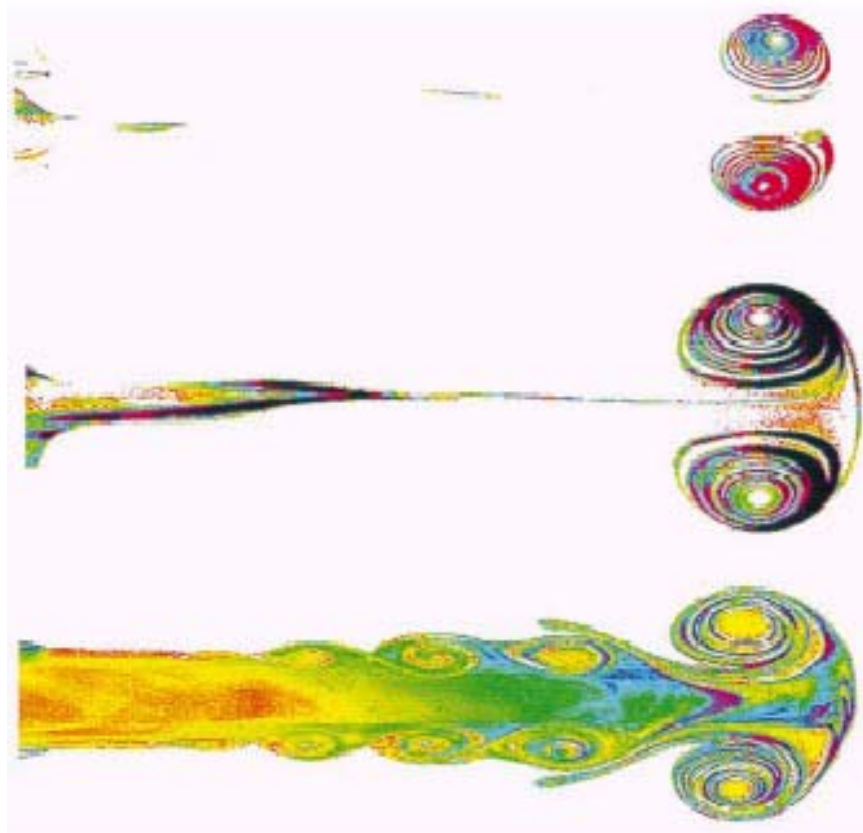


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$$\leftarrow L/D = 14.5$$

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$$L/D \approx 4 \equiv \begin{cases} \text{Vortex ring formation time} \\ \text{Vortex pinch-off time} \end{cases}$$

$L/D = 2$ vs. 4



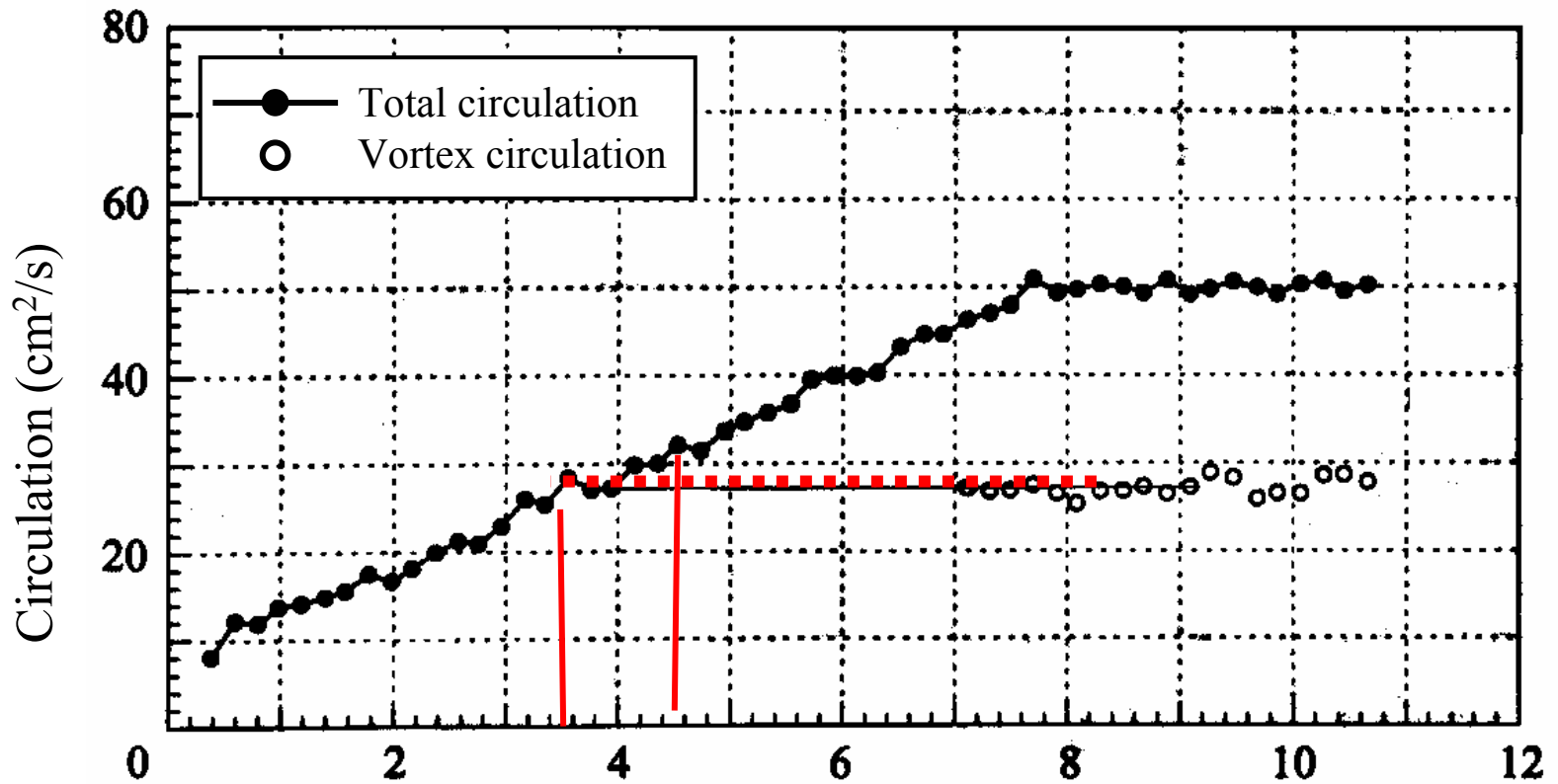
$L/D = 2$



$L/D > 4$

The Formation Number (F)

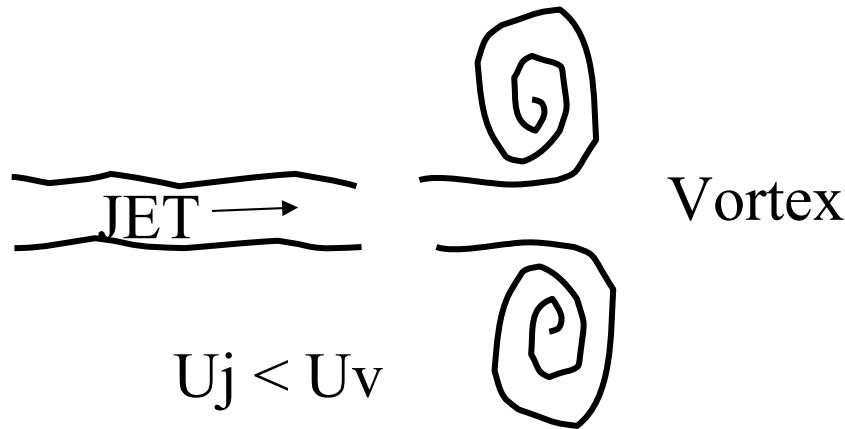
$$\text{Formation Time} \leftrightarrow \bar{U}_p t / D = L/D$$



Models for Vortex Ring Pinch-off

- Gharib *et al.* (1998): invokes Kelvin-Benjamin Variational principle

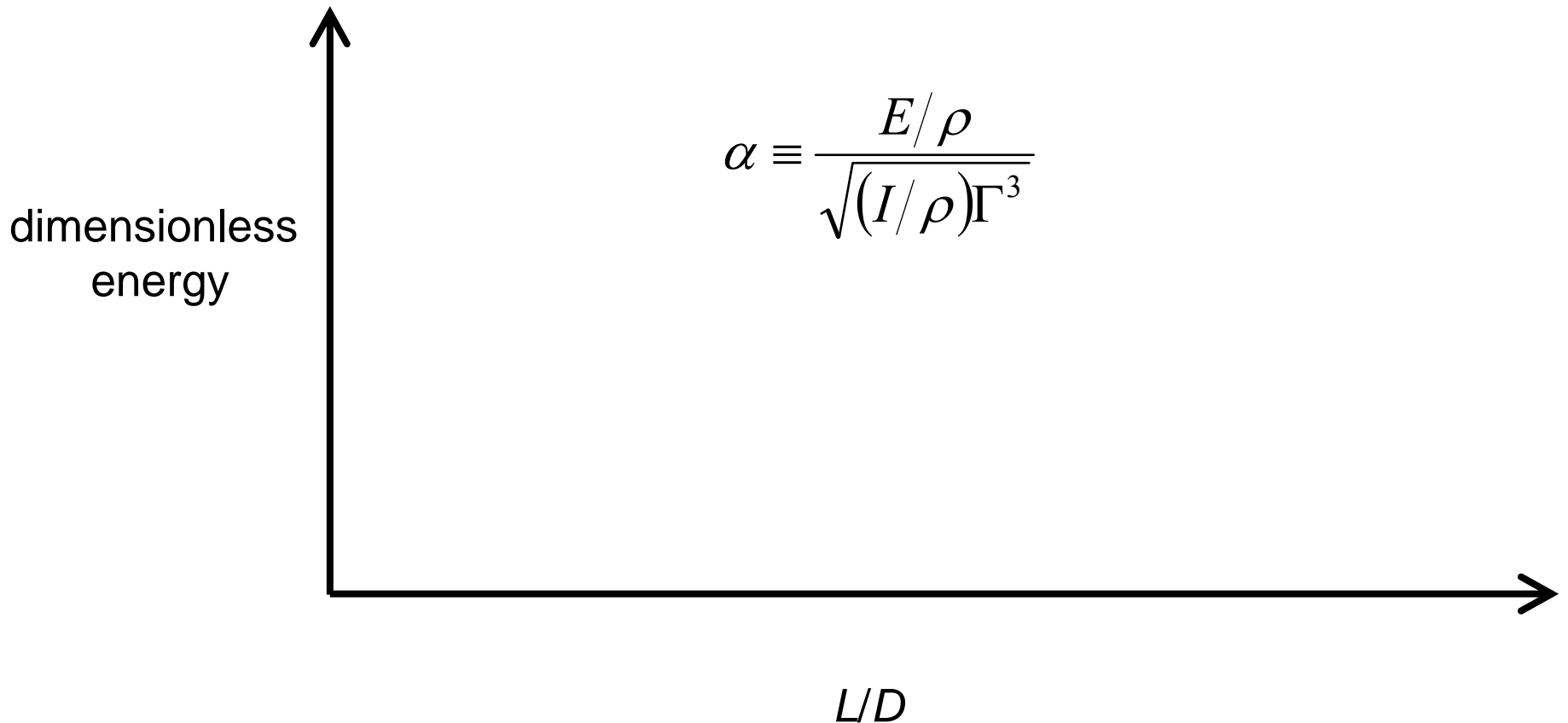
Vortex ring pinch off "...occurs when the source energy falls below that of a steadily translating vortex ring"



M. Shusser and M. Gharib, *Physics of Fluids* (2000)

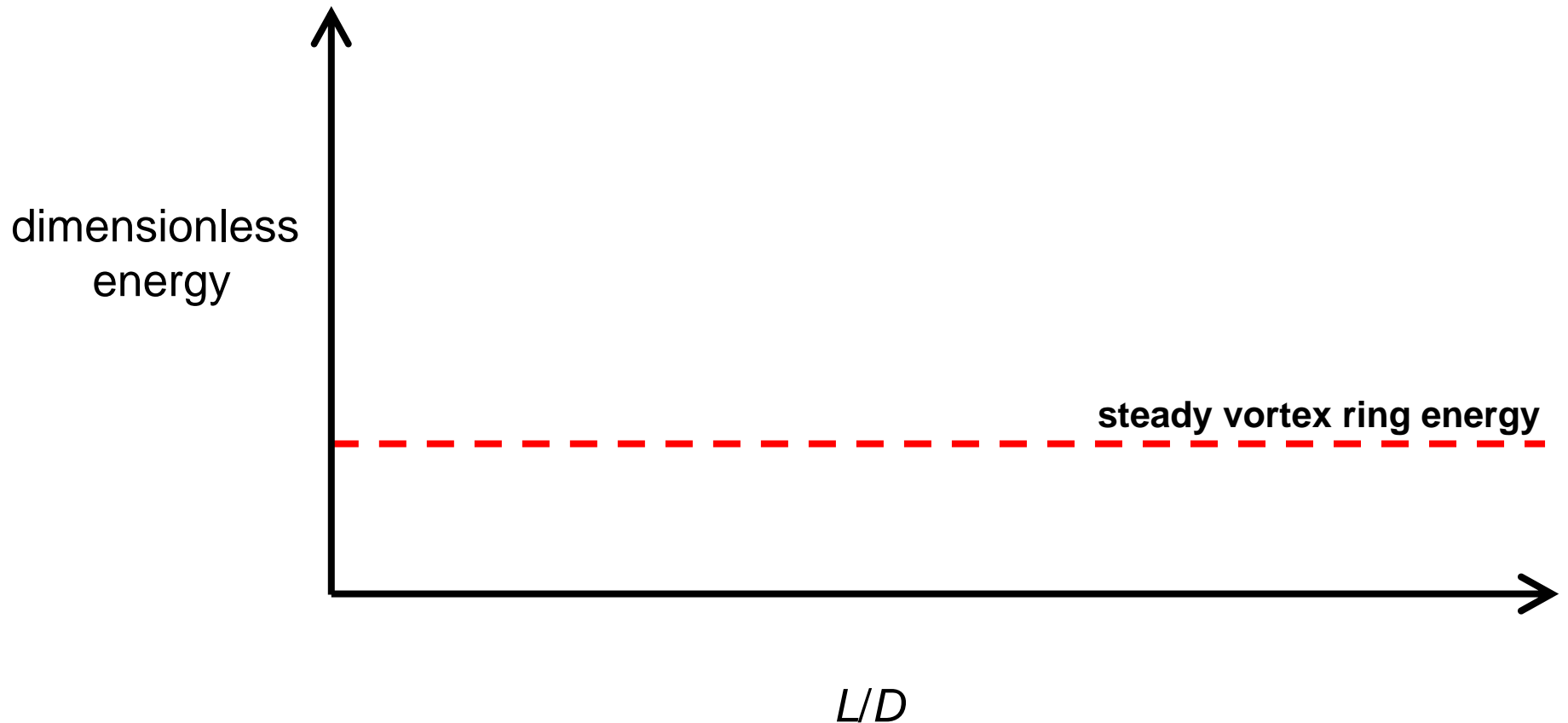
Vortex ring growth is limited by **energy effects**

For vortex ring growth: **vortex generator energy > vortex ring energy**
(W.T. Kelvin, 1875; T.B. Benjamin 1976)



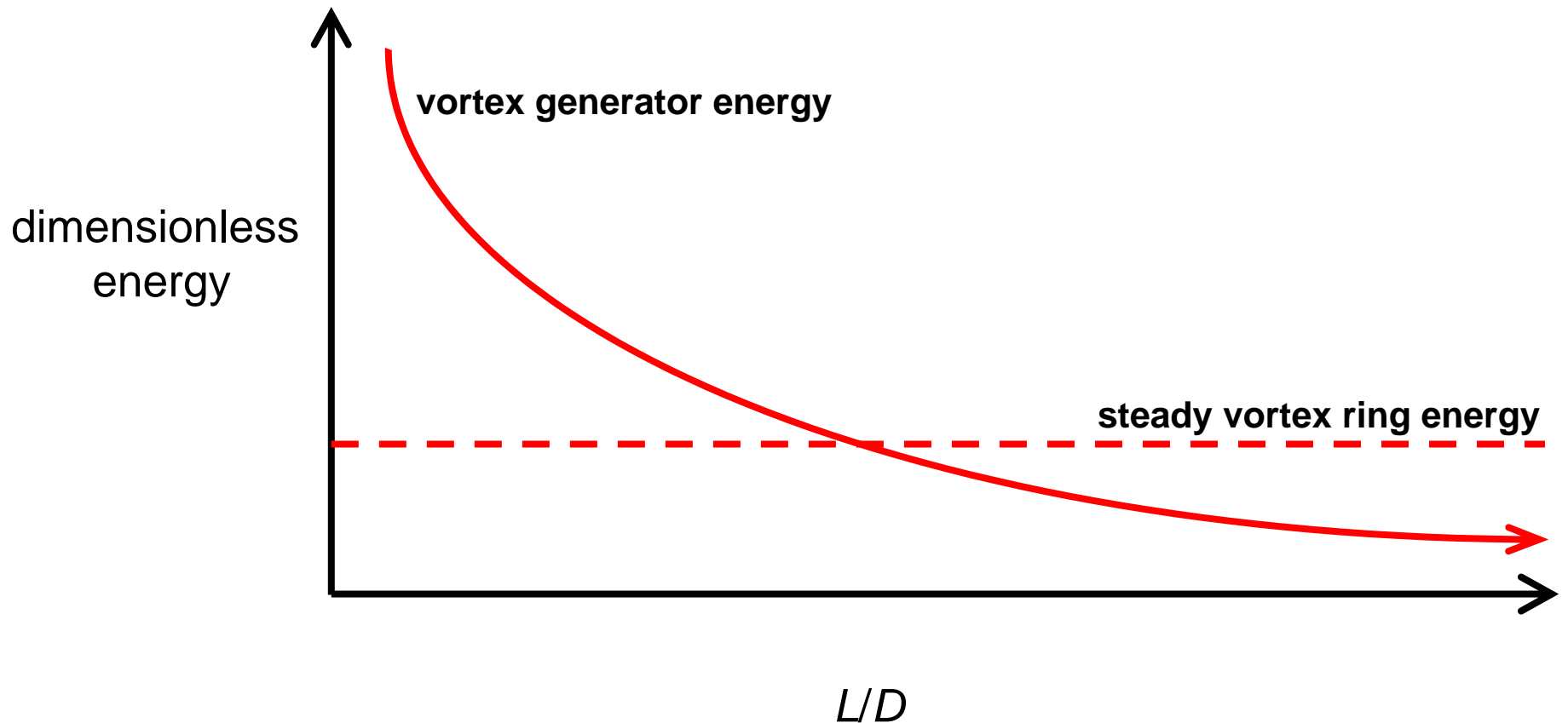
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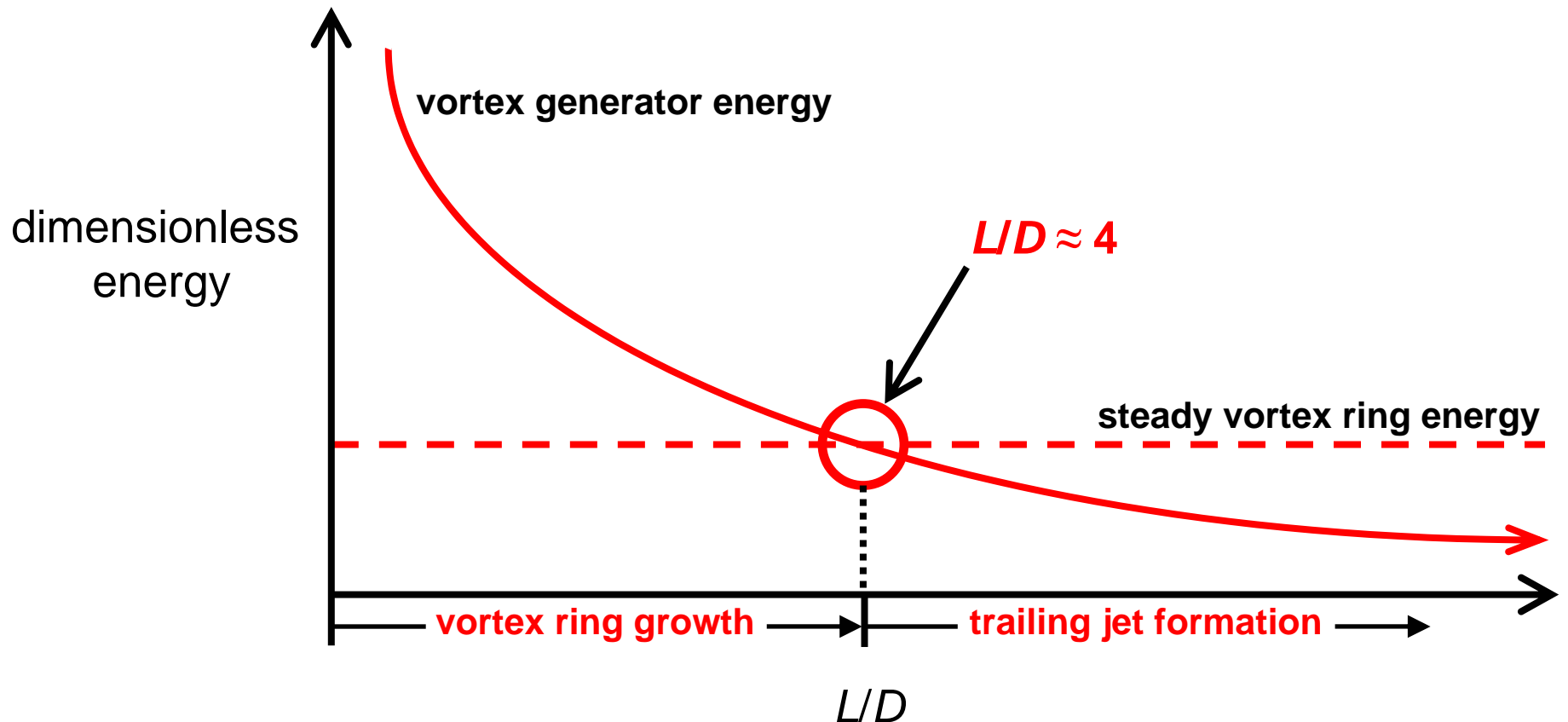
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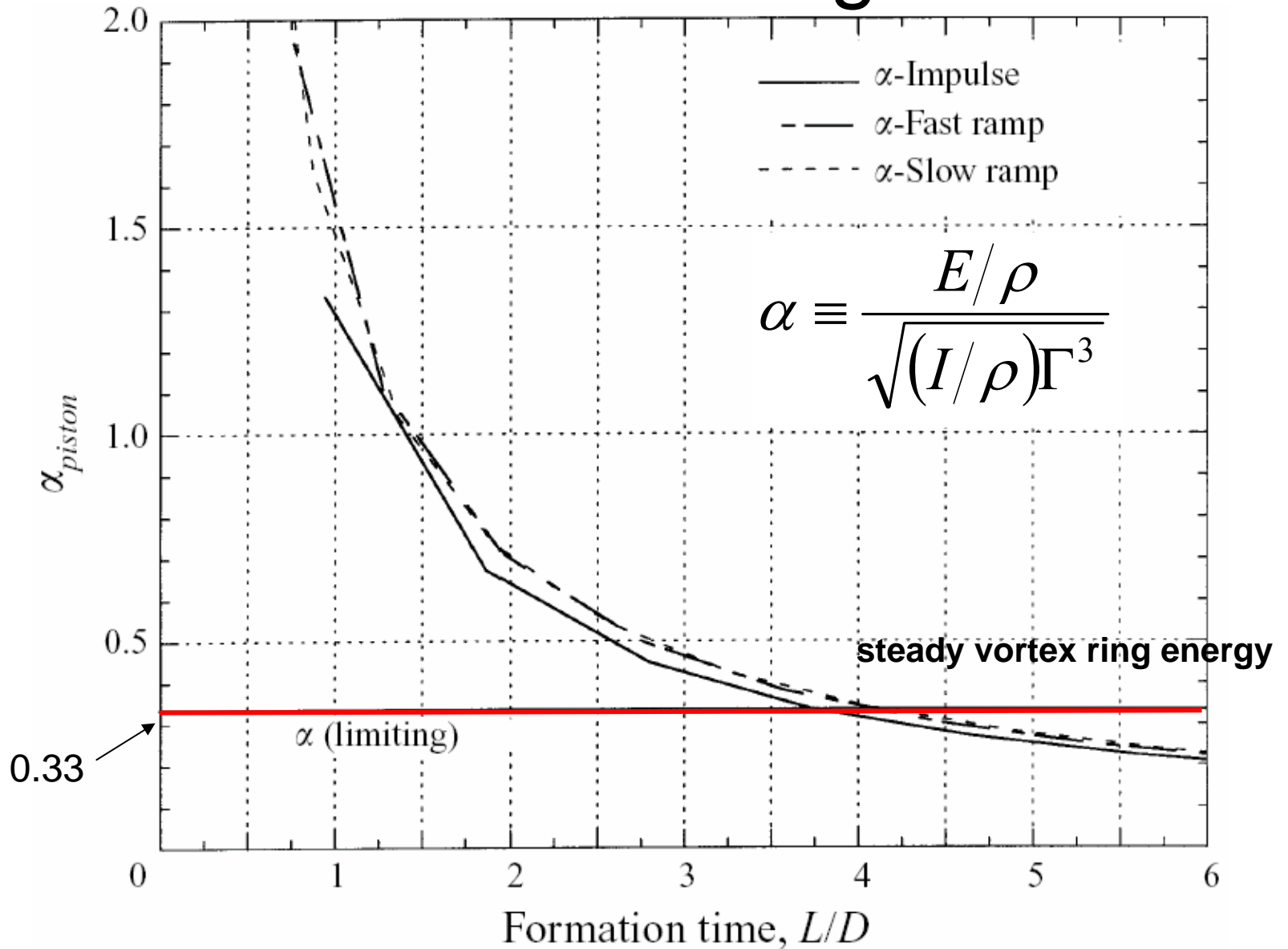


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- Mohseni (1998): combines **Norbury vortex model** and slug model approximation for **ring translational velocity**

$$U_{vortex} = 0.5 U_{piston}$$

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$$\Omega_{jet} = \Omega_{vortex}$$

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However, entrained fluid by the leading vortex can reach over 50 percent of its total volume (Ω_{vortex})

J.O. Dabiri and M. Gharib, *Journal of Fluid Mechanics* (2004)

Physical Implications of Pinch-Off

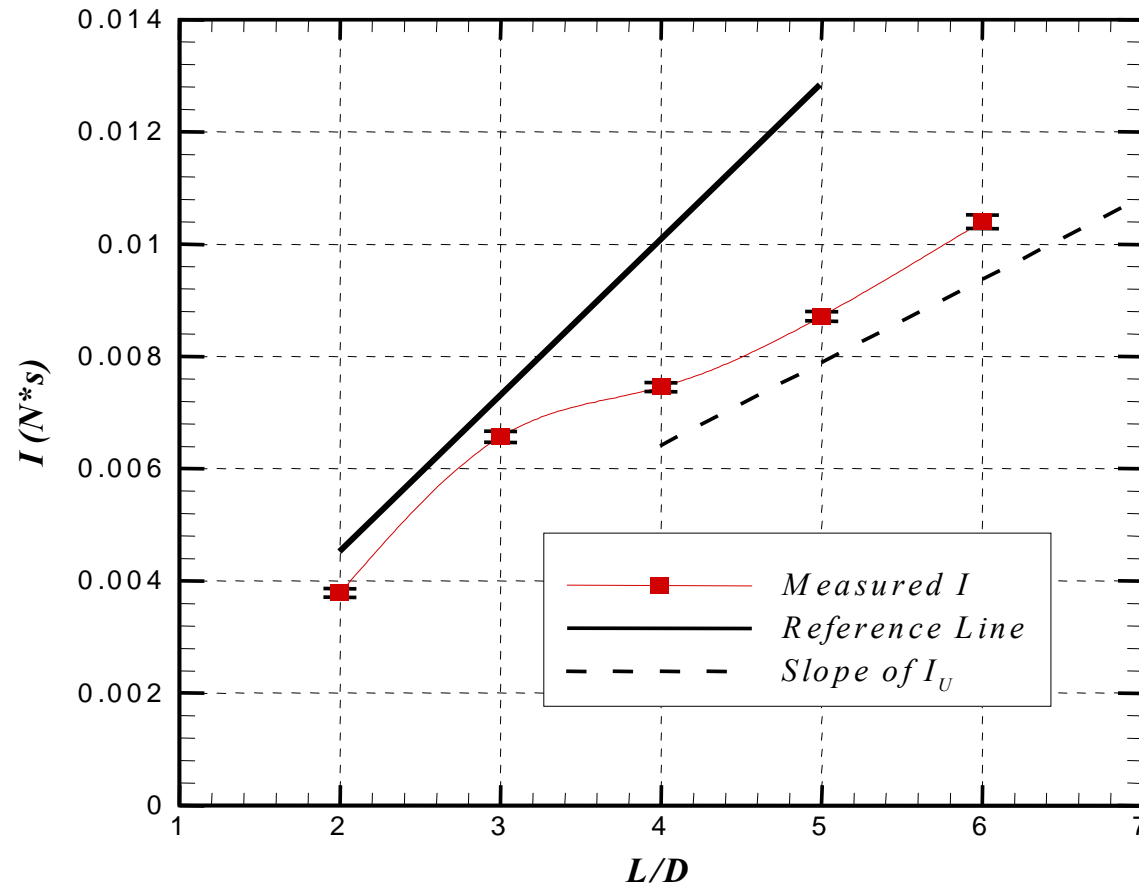
Pinch-Off \Rightarrow Maximum vortex ring strength (energy)

\Rightarrow Maximum fluid entrainment per vortex ring
and maximum vortex ring velocity

\Rightarrow Maximum thrust per pulse

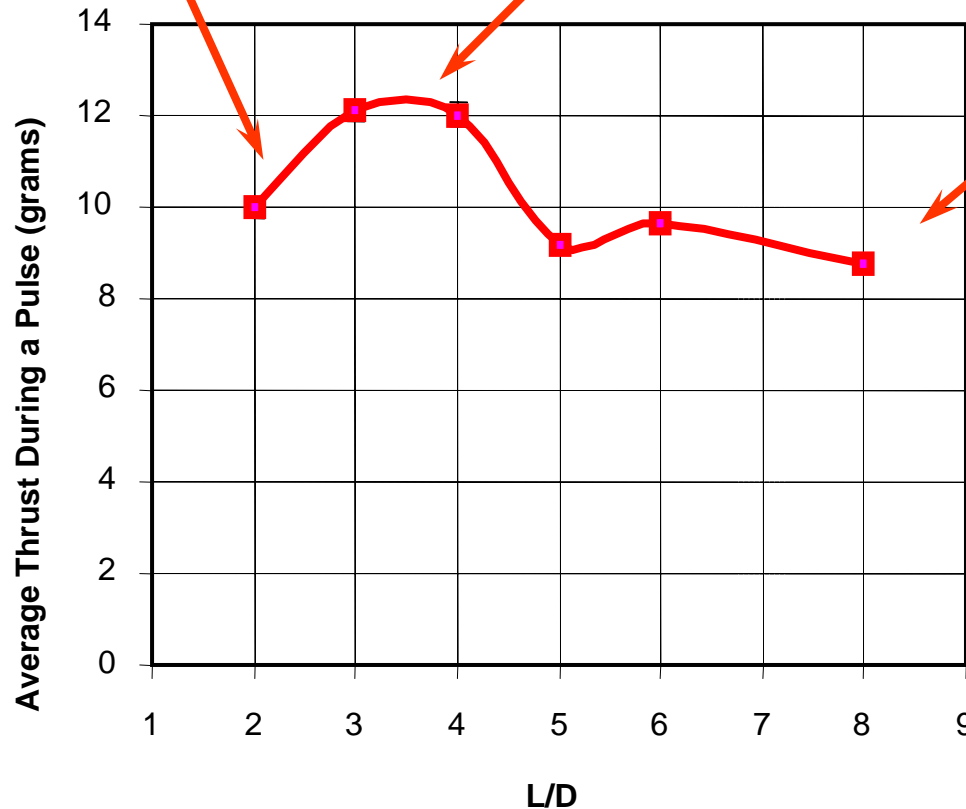
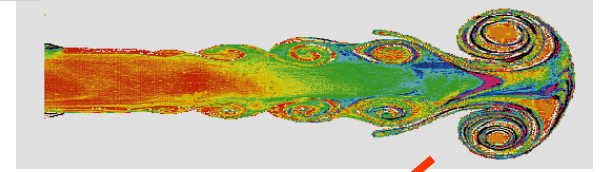
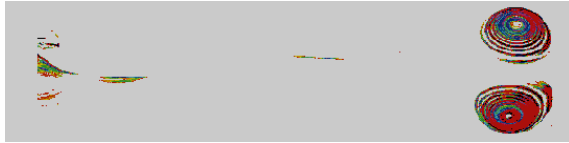
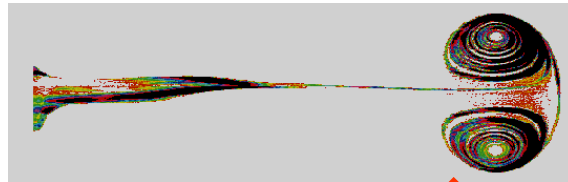
?

Total Impulse of Starting Jets

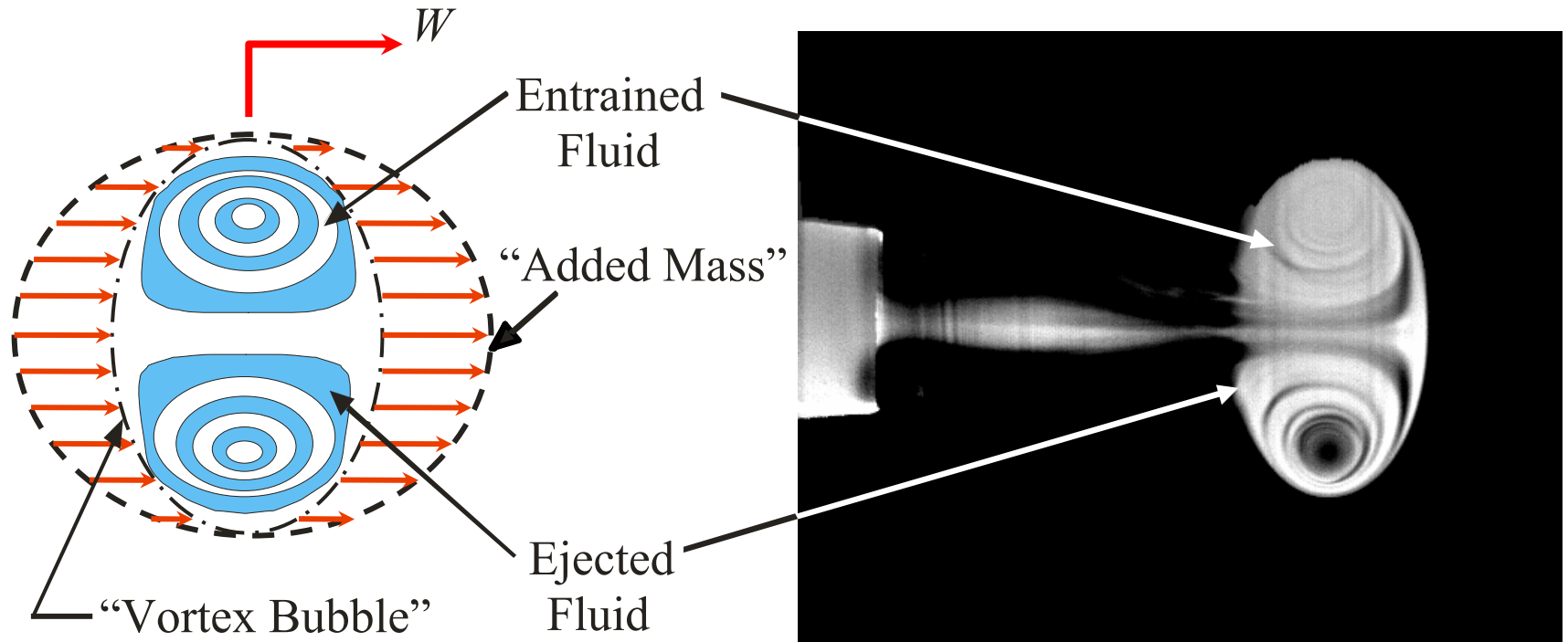


$$I_U = I_U(t_p) \equiv \int_0^{t_p} \int_A u_J^2(r, \tau) dA d\tau$$

Average Force of Starting Jets



Added and Entrained Mass



$$I_U + I_p = (m_{ejected} + m_{entrained} + m_{added})W$$

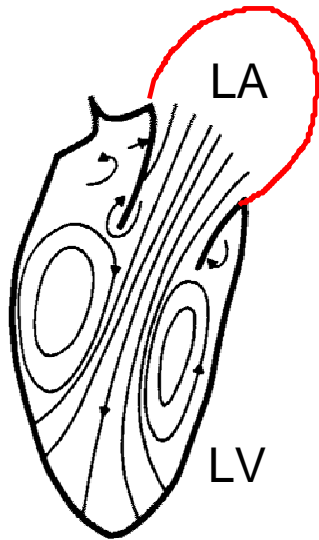
Experiments $\Rightarrow m_{ejected}W < I_U$

[L/D = 2.0, NS Ramp]

Limiting physical processes dictate “optimal”
parameters for vortex ring formation

Experiments demonstrate correlation between vortex ring pinch-off and maximum mass and momentum transfer

Do biological systems exploit vortex ring formation for optimal fluid transport?



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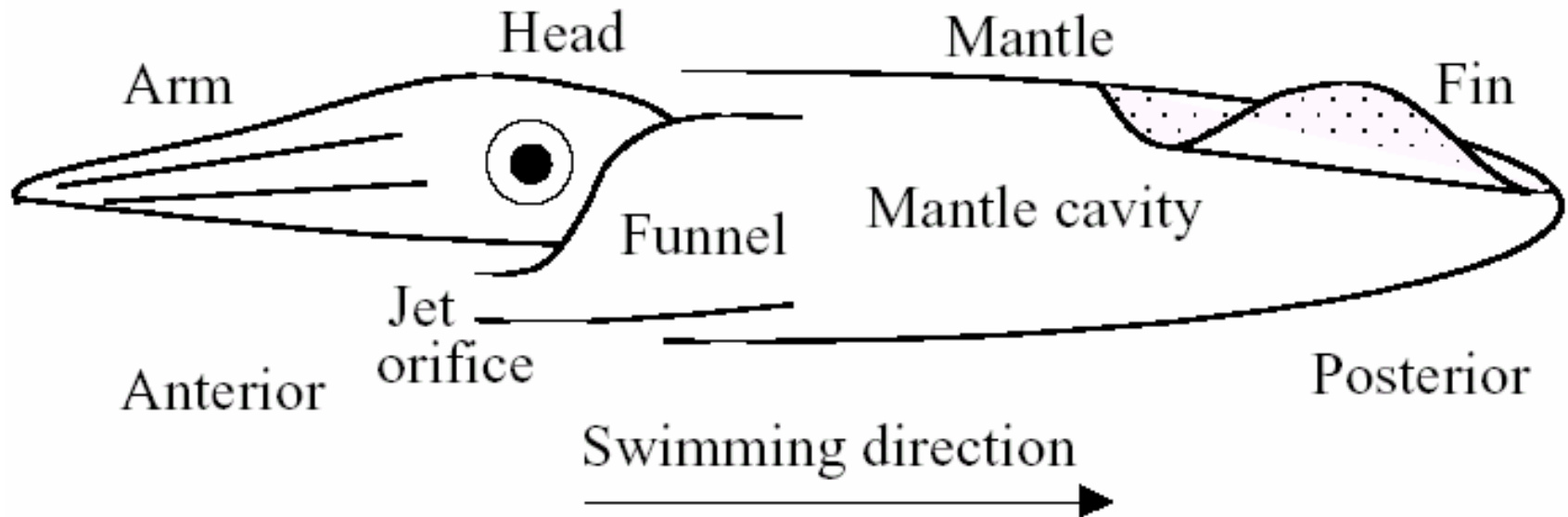


B. Lin and M. Gharib (2003)



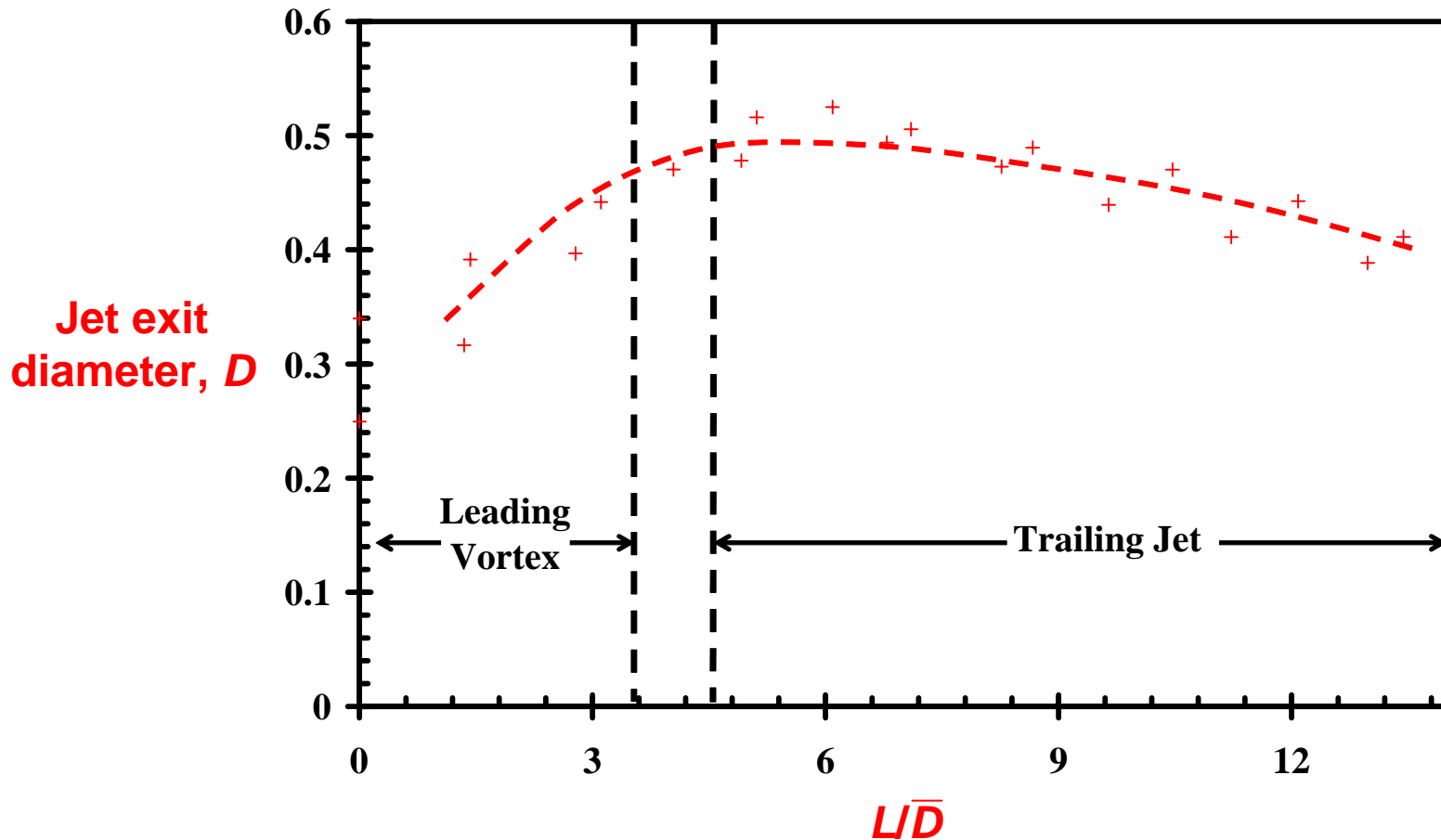
J.O. Dabiri et al. (2004)

*Previous attempts to verify optimal vortex formation in biological systems have been **inconclusive***



$$L/D \rightarrow L/\bar{D} \text{ where } \bar{D} = \frac{1}{t} \int_0^t D(\tau) d\tau$$

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Measurements in the literature...

Squid: $L/\bar{D} = 4-7, 10-40, 34, \mathbf{87}$

Salps: $L/\bar{D} = 3-4, 6.7, 10-20$

Jellyfish: $L/\bar{D} = \mathbf{0.2}-4.4$

Are we missing something?

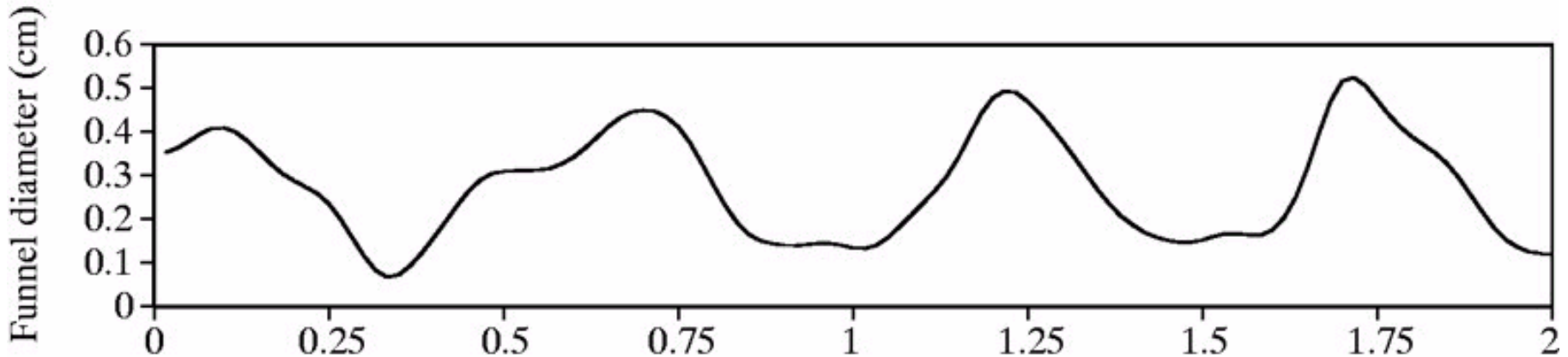
Biological Factors Affecting Pinch-off

- **Co-flow**

Swimming and flying animals generate vortices
in a free-stream flow

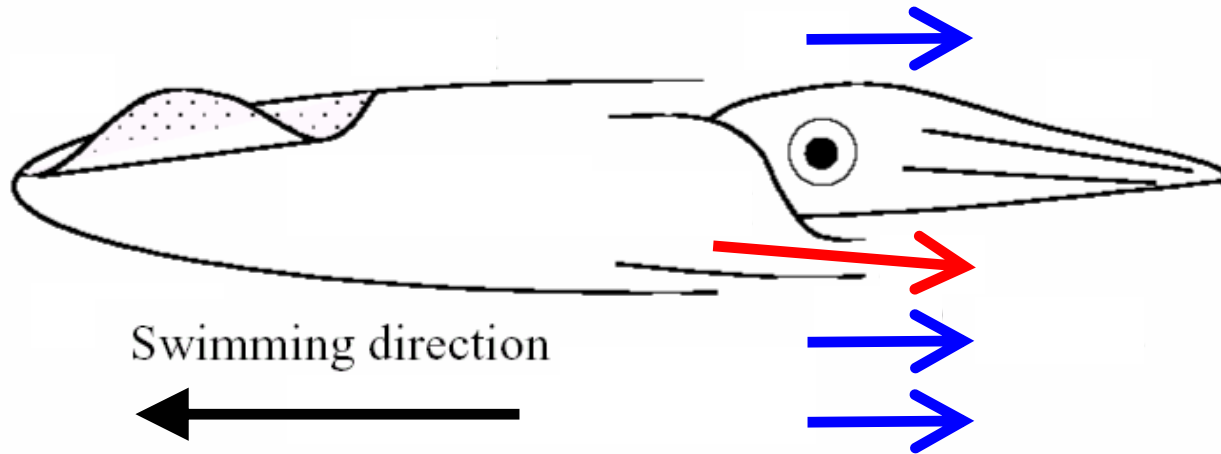
Biological Factors Affecting Pinch-off

- **Co-flow**
Swimming and flying animals generate vortices in a free-stream flow
- **Temporal exit diameter variation**
e.g. squid (Bartol *et al.*, 2001)
Parallels time-dependent flap kinematics in other locomotion modes



*A proper analysis must include
fluid-structure interactions*

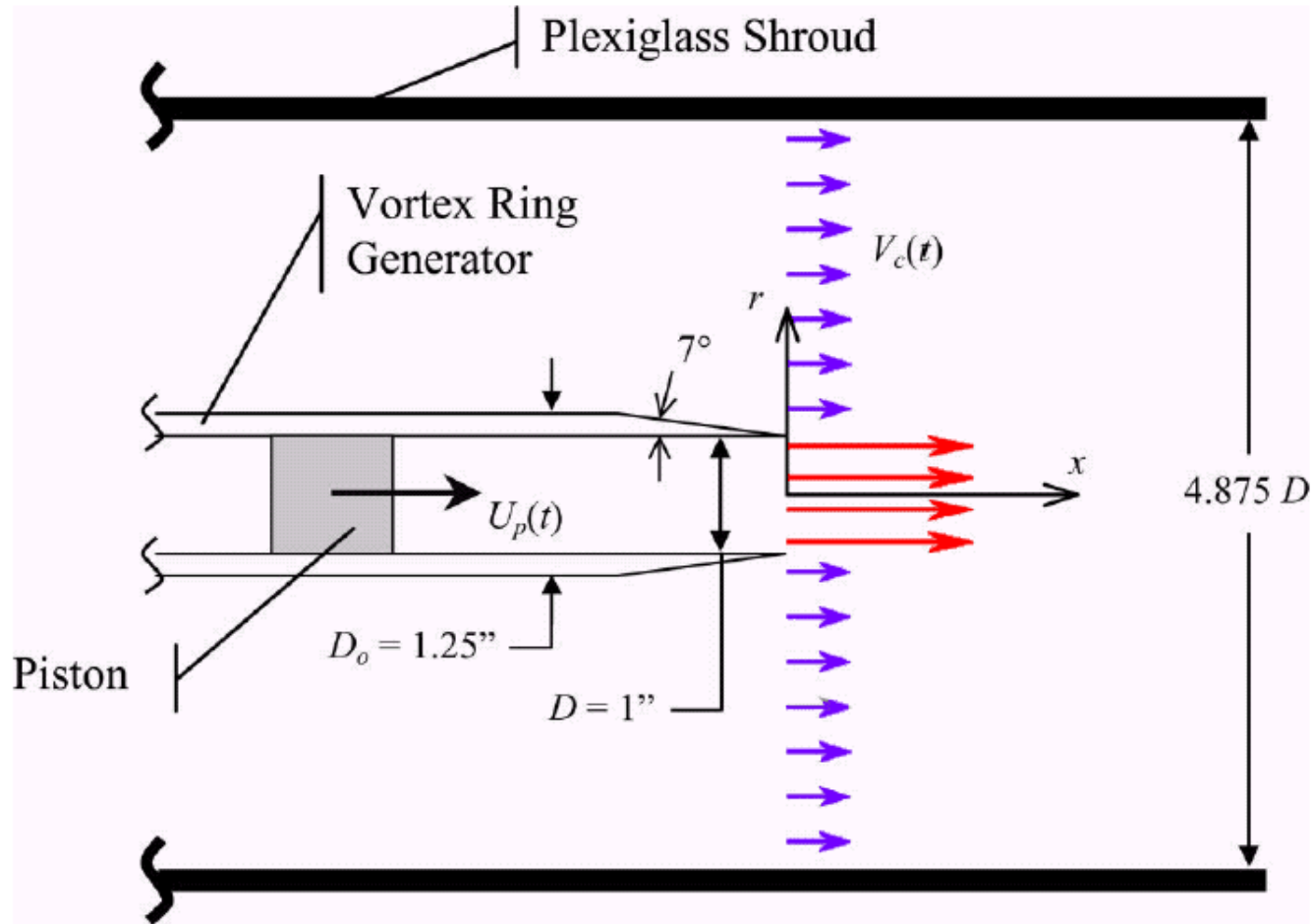
External interactions: **Flow past the vortex generator**



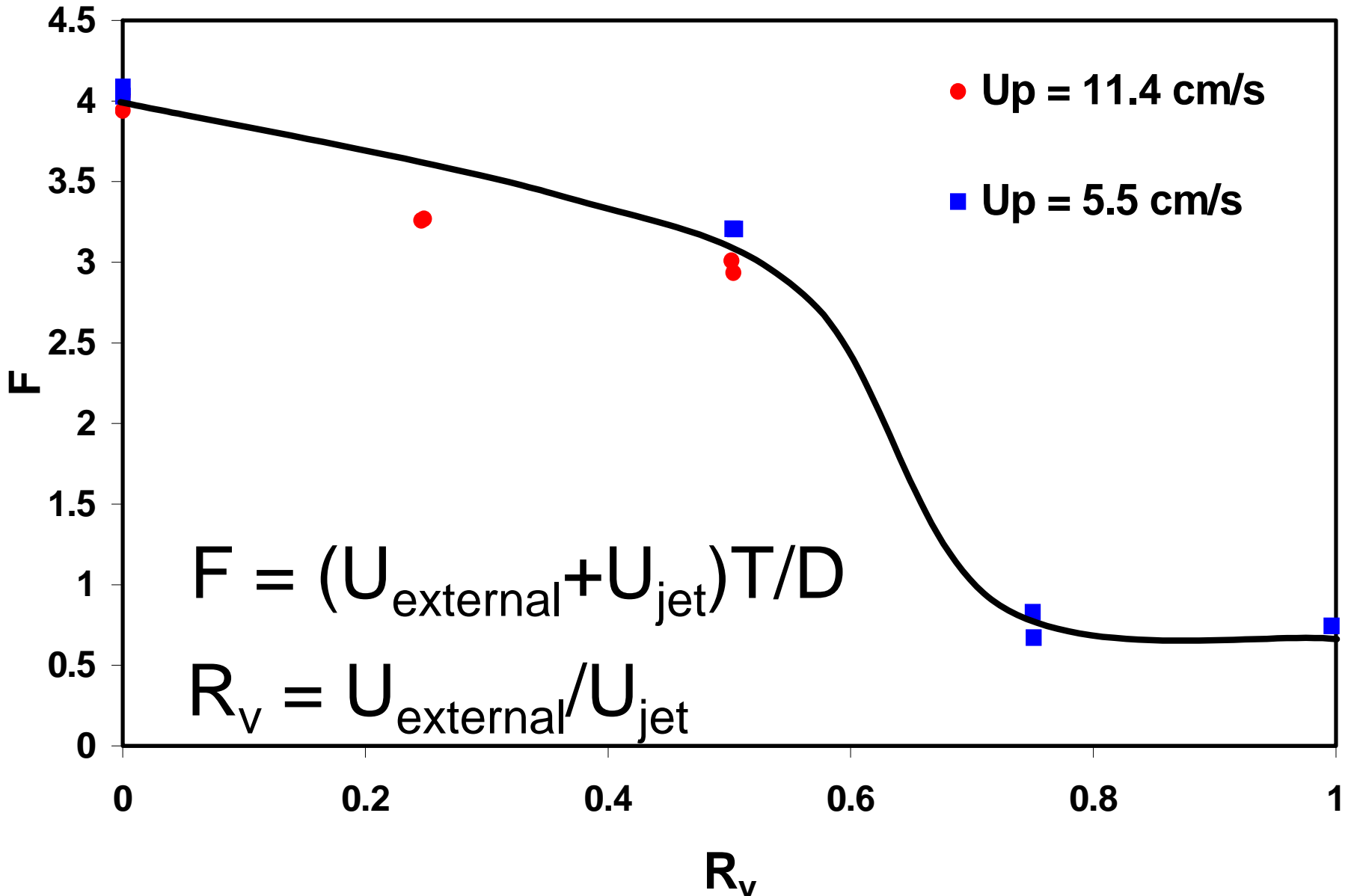
**Flow past the propulsor can affect
vortex ring formation**

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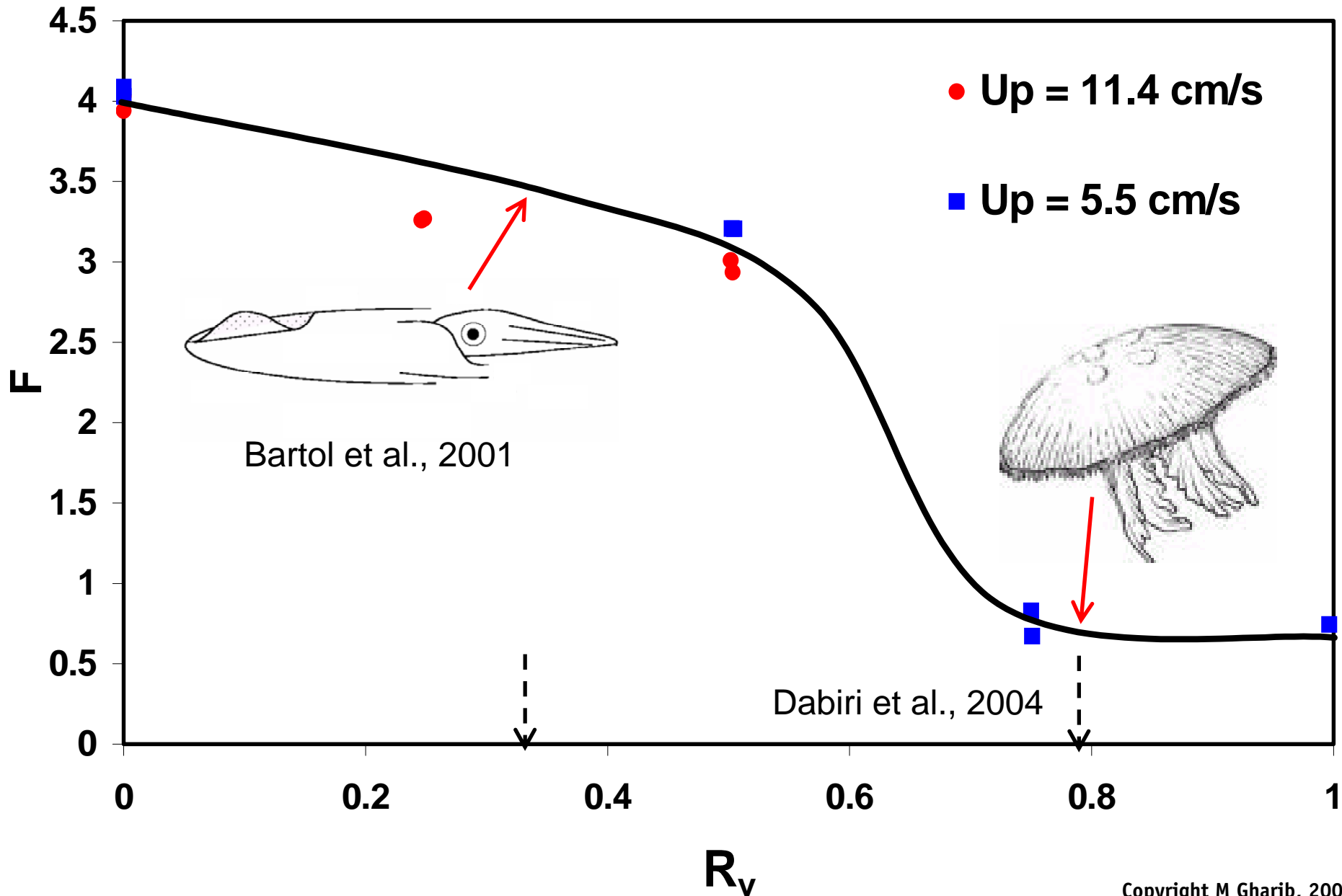
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Effects of Co-flow



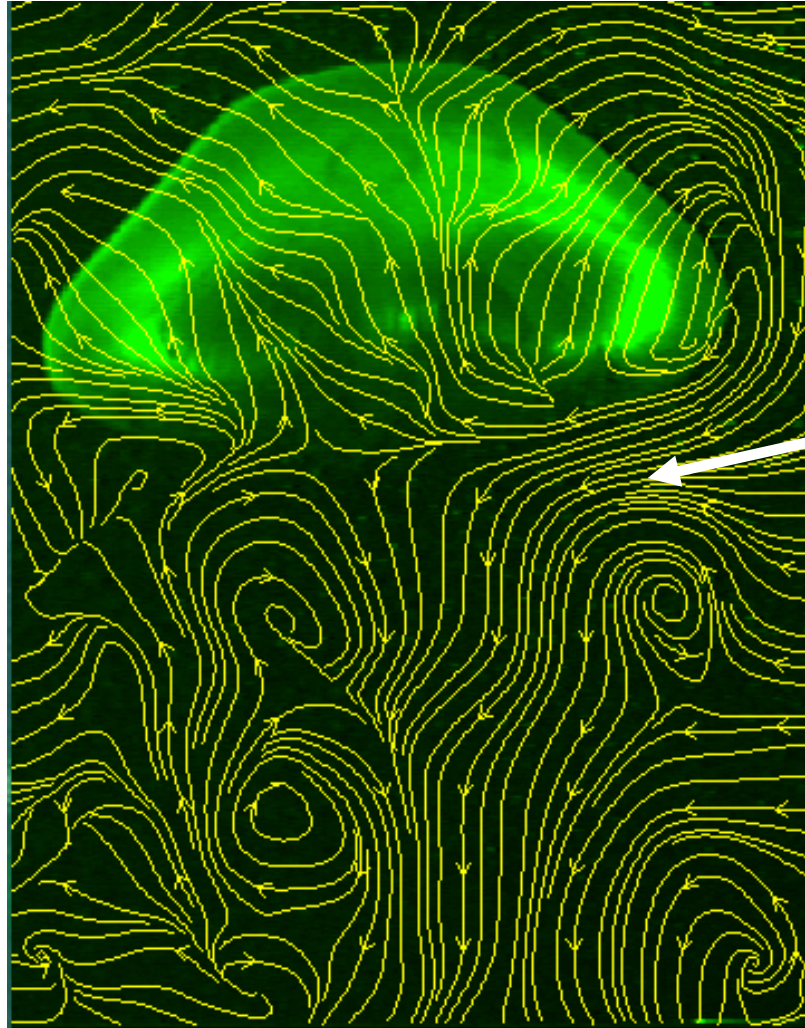
Effects of Co-flow



Effects of Co-flow

Animals can exploit external flow in other flow-related functions (i.e. feeding and maneuvering)

See Gallery of Fluid Motion Video #14 (Dabiri et al.)



feeding via
vortex ring fluid
entrainment

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Internal interactions: Dynamical effects of a variable exit $D(t)$

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Possible effects

- 1) As a **source** of additional vorticity

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- 1) As a source of additional vorticity
- 2) As a **manipulator** of existing vorticity

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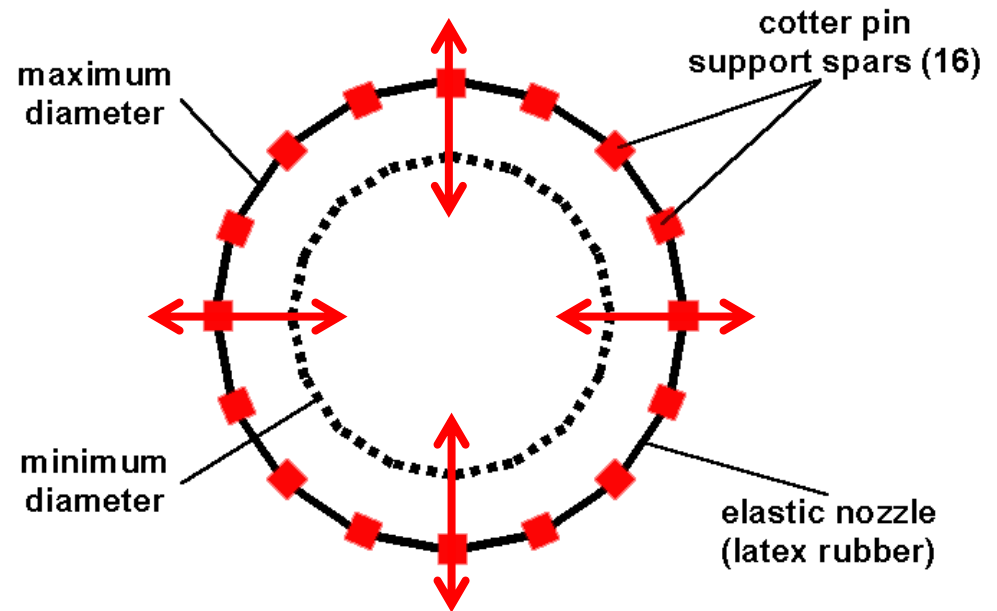
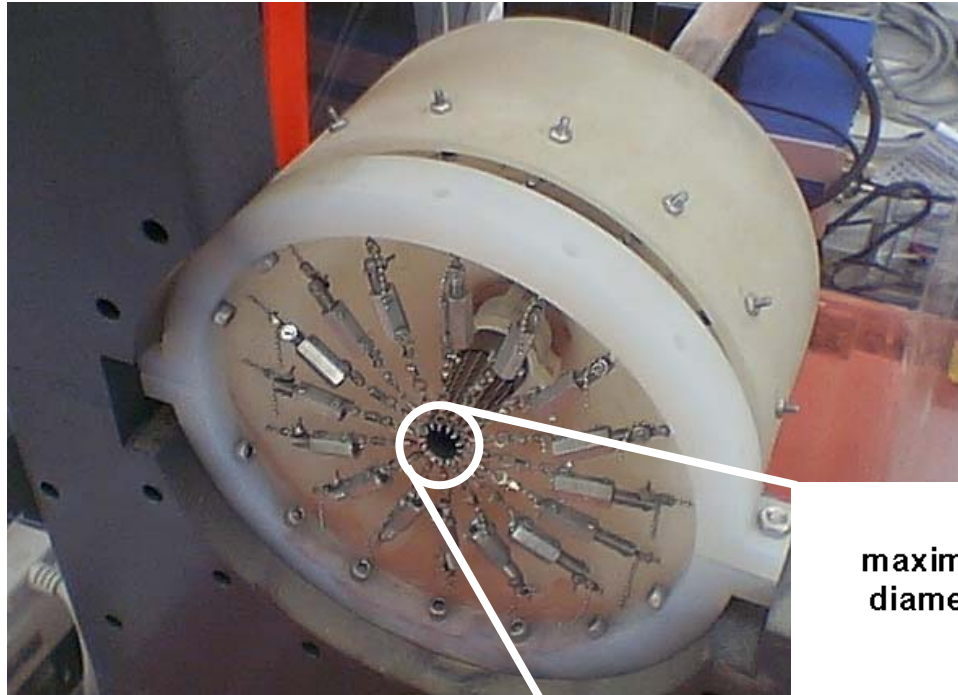
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**These effects are obscured when the time-dependent
 $D(t)$ is replaced by \bar{D}**

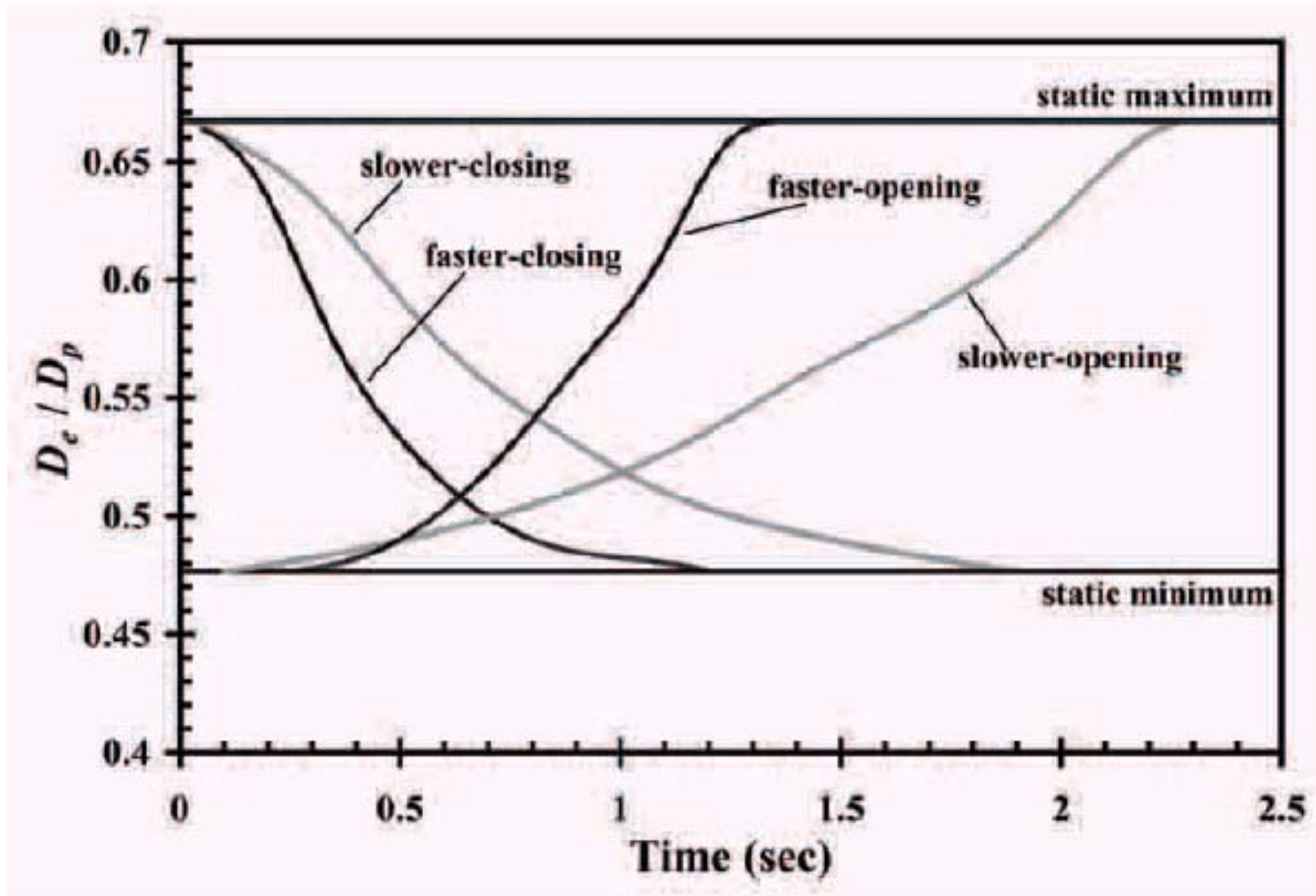
A *new technique* replicates fluid-structure interactions in variable-diameter jet flows



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A *new index* for vortex formation properly accounts for time-dependent boundary conditions

“DYNAMIC FORMATION TIME”

Increase the vortex formation time incrementally:

$$\Delta(\mathbf{L}/\mathbf{D})^* \equiv (\mathbf{U}(\boldsymbol{\tau})/\mathbf{D}(\boldsymbol{\tau}))\Delta\boldsymbol{\tau}$$

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Integrating from flow initiation at $\tau = 0$ to flow termination at $\tau = t$:

$$(\mathbf{L}/\mathbf{D})^* = \int_0^t (\mathbf{U}(\boldsymbol{\tau})/\mathbf{D}(\boldsymbol{\tau}))d\boldsymbol{\tau}$$

Nondimensional Analysis

- Normalized Circulation

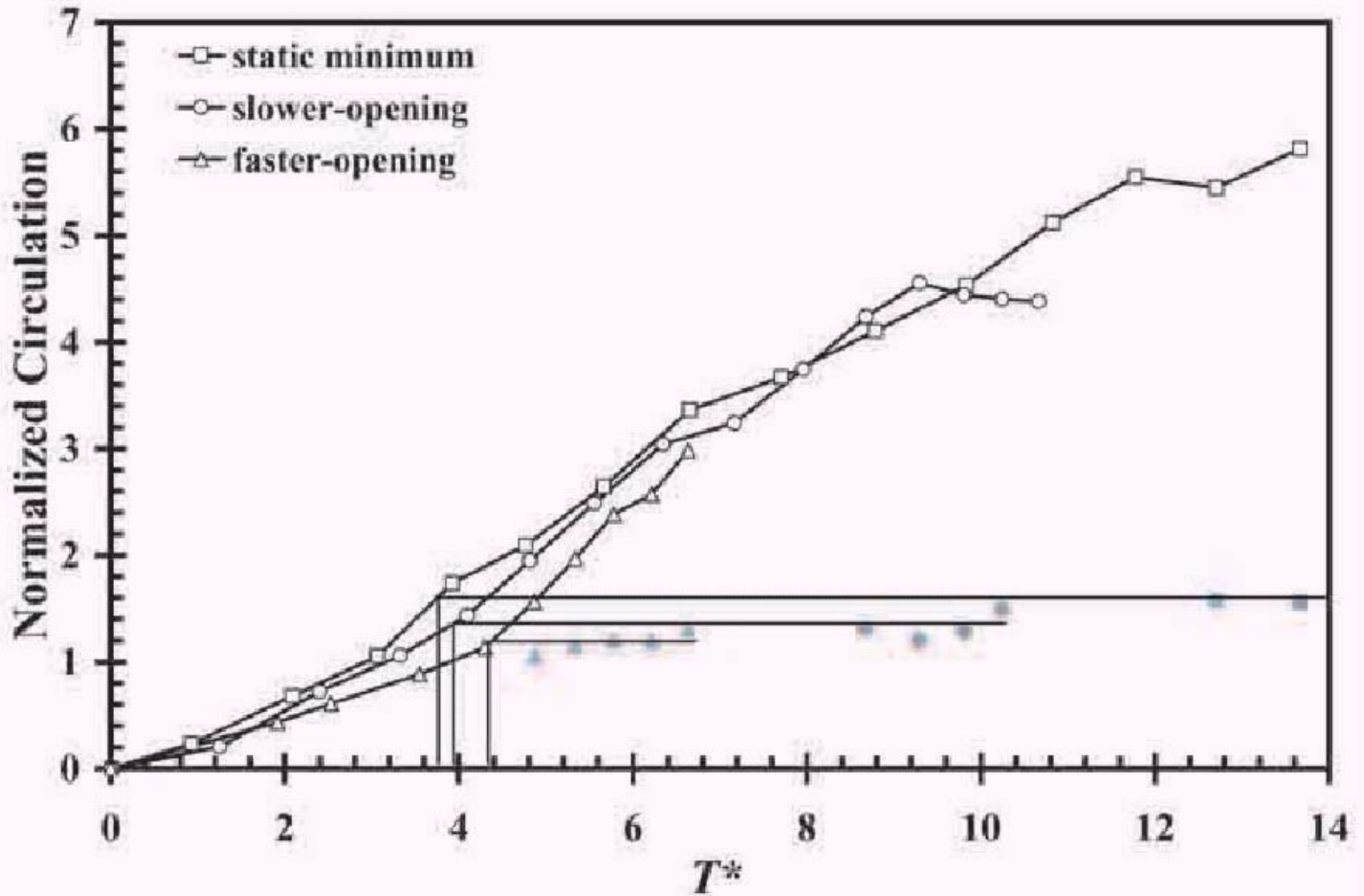
$$\Gamma^* = \frac{\overline{\Gamma(U_e/D_e)}}{U_e^2}$$

- Normalized Time

$$T^* = \int_0^t \frac{U_e(\tau)}{D_e(\tau)} d\tau = \overline{\left(\frac{U_e}{D_e} \right)} t$$

Vortex formation time is unaffected by temporal increases in jet diameter

- $(L/D)^* = 4.0 \pm 0.5$ for all $dD(t)/dt > 0$ tested



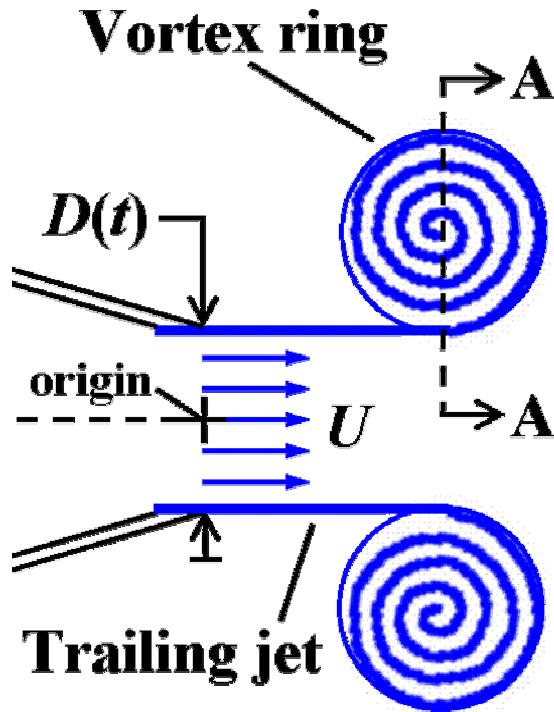
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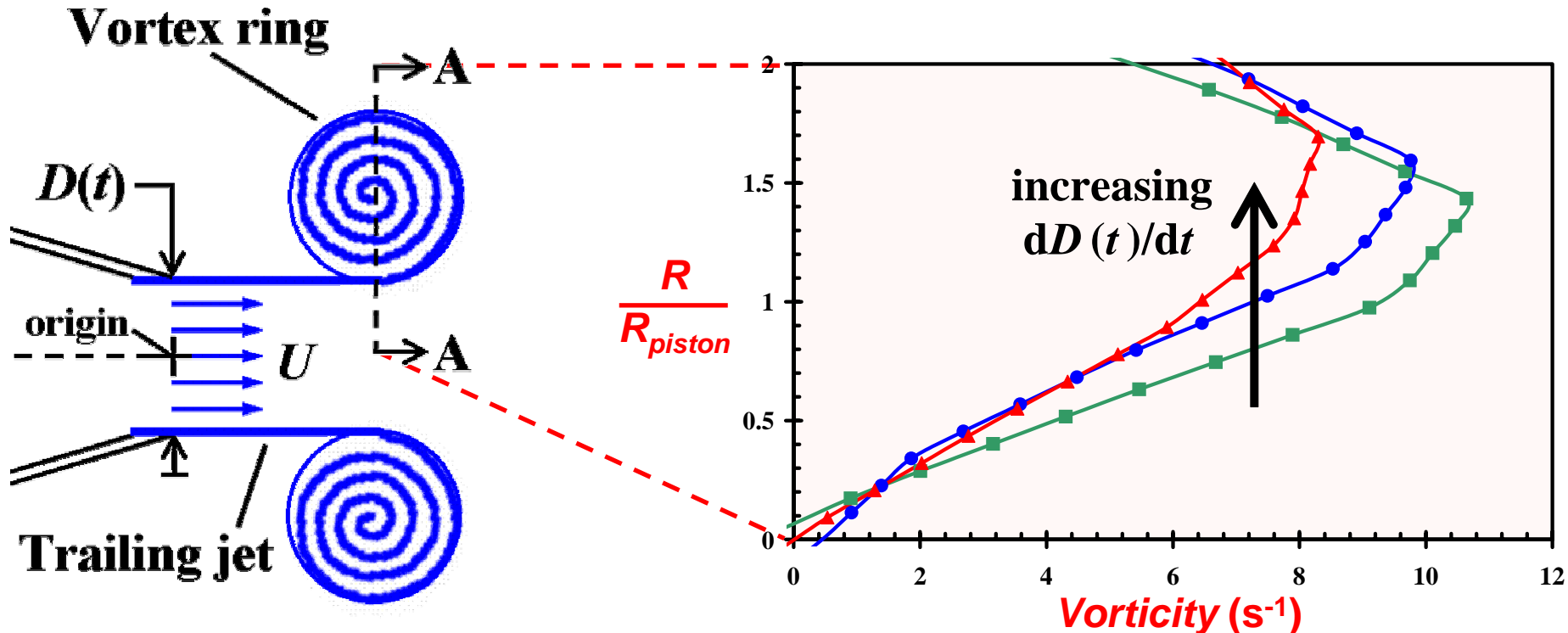
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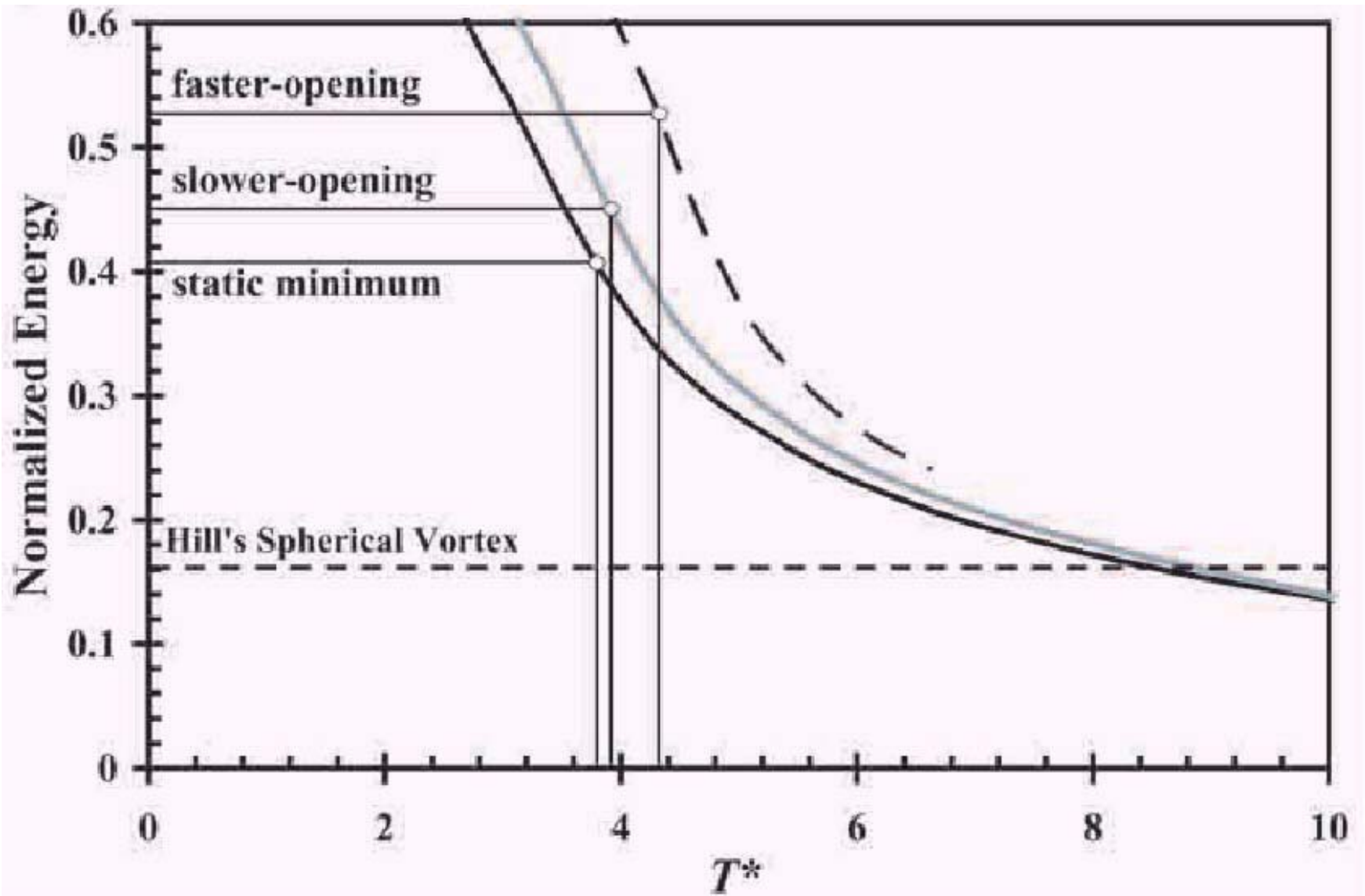
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*These results suggest the first set of
engineering strategies for optimal fluid transport*

During fluid ejection at $(L/D)^* < 4$:

$$I_{(L/D)^* < 4} \sim D^2$$

(J.O. Dabiri & M. Gharib, *J Fluid Mech*)

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Transport fluid
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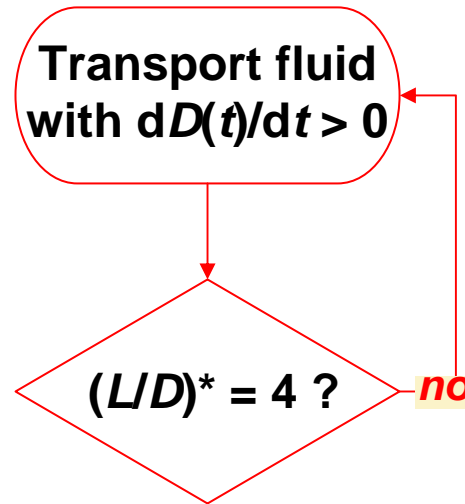
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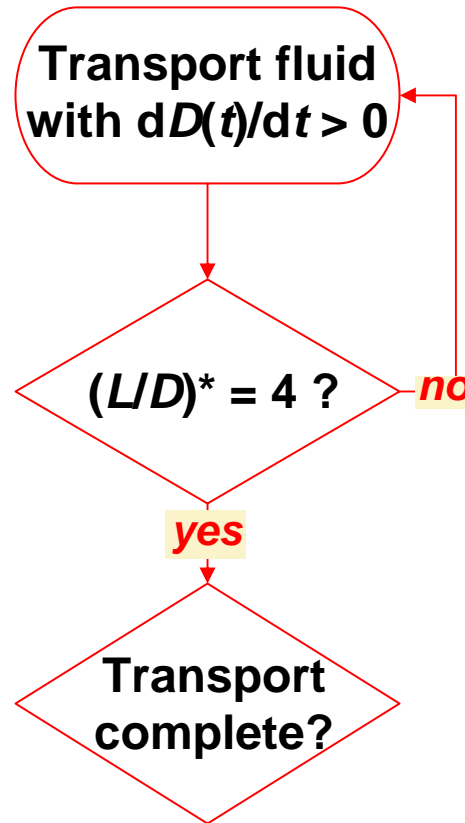
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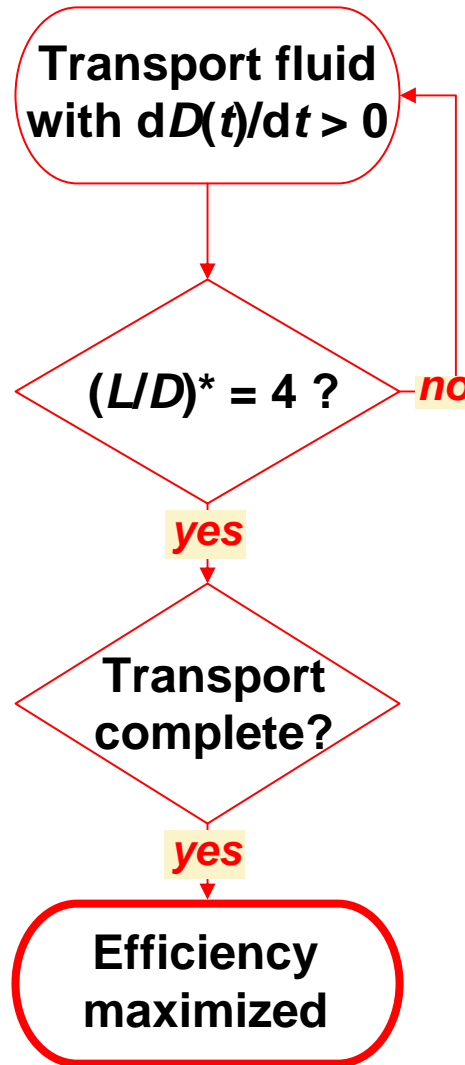
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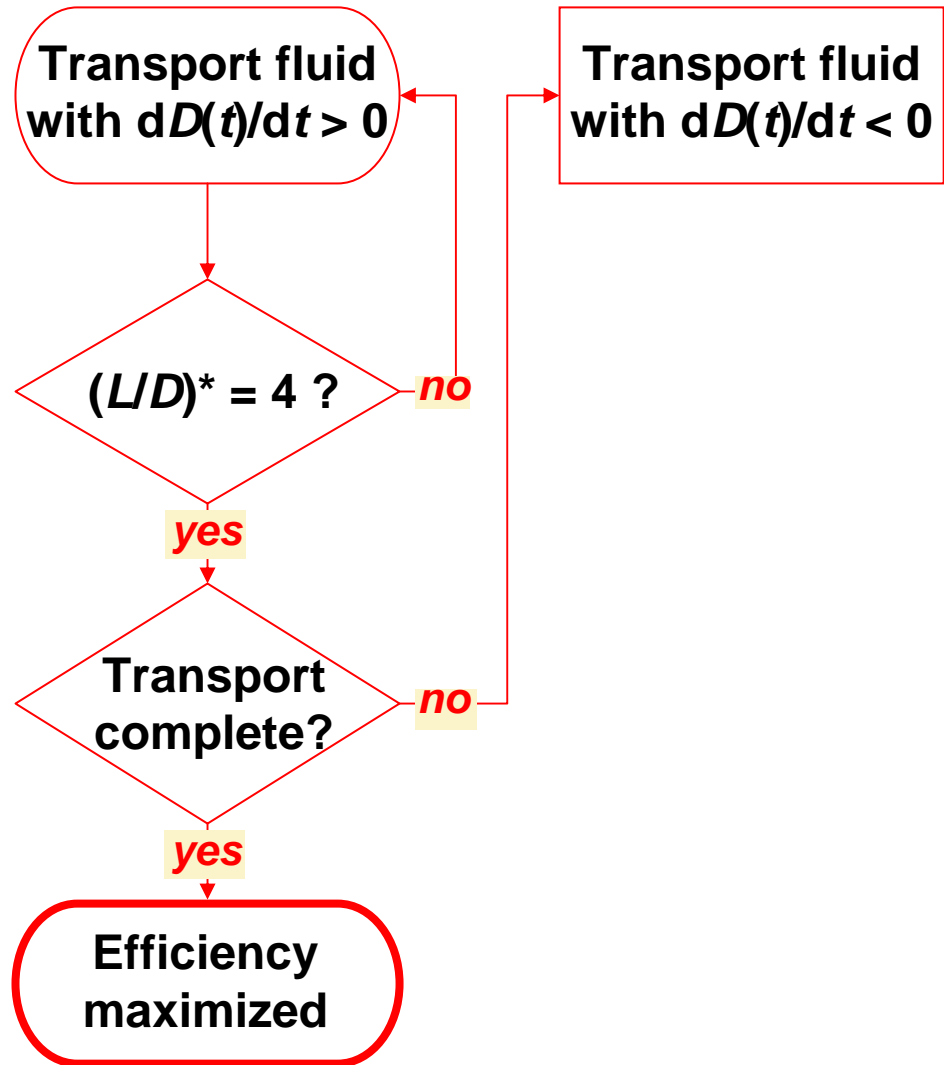
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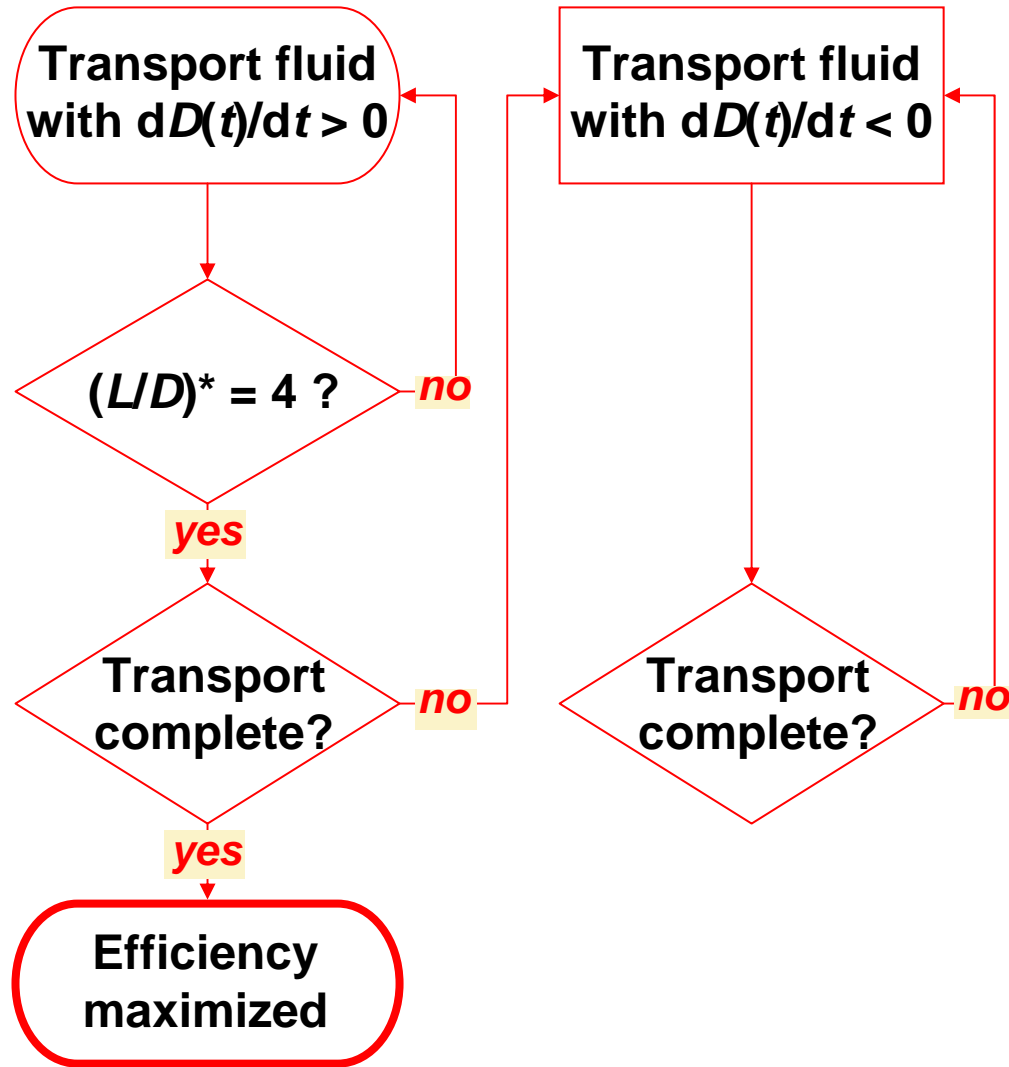
If total vortex formation time $(L/D)^* \rightarrow 4$, fluid is transported with maximum efficiency

(P.S. Krueger & M. Gharib, *Phys Fluids*)

During fluid ejection at $(L/D)^* > 4$:

$$I_{(L/D)^* > 4} \sim D^{-4}$$

(J.O. Dabiri & M. Gharib, *J Fluid Mech*)



These results suggest the first set of engineering strategies for optimal fluid transport

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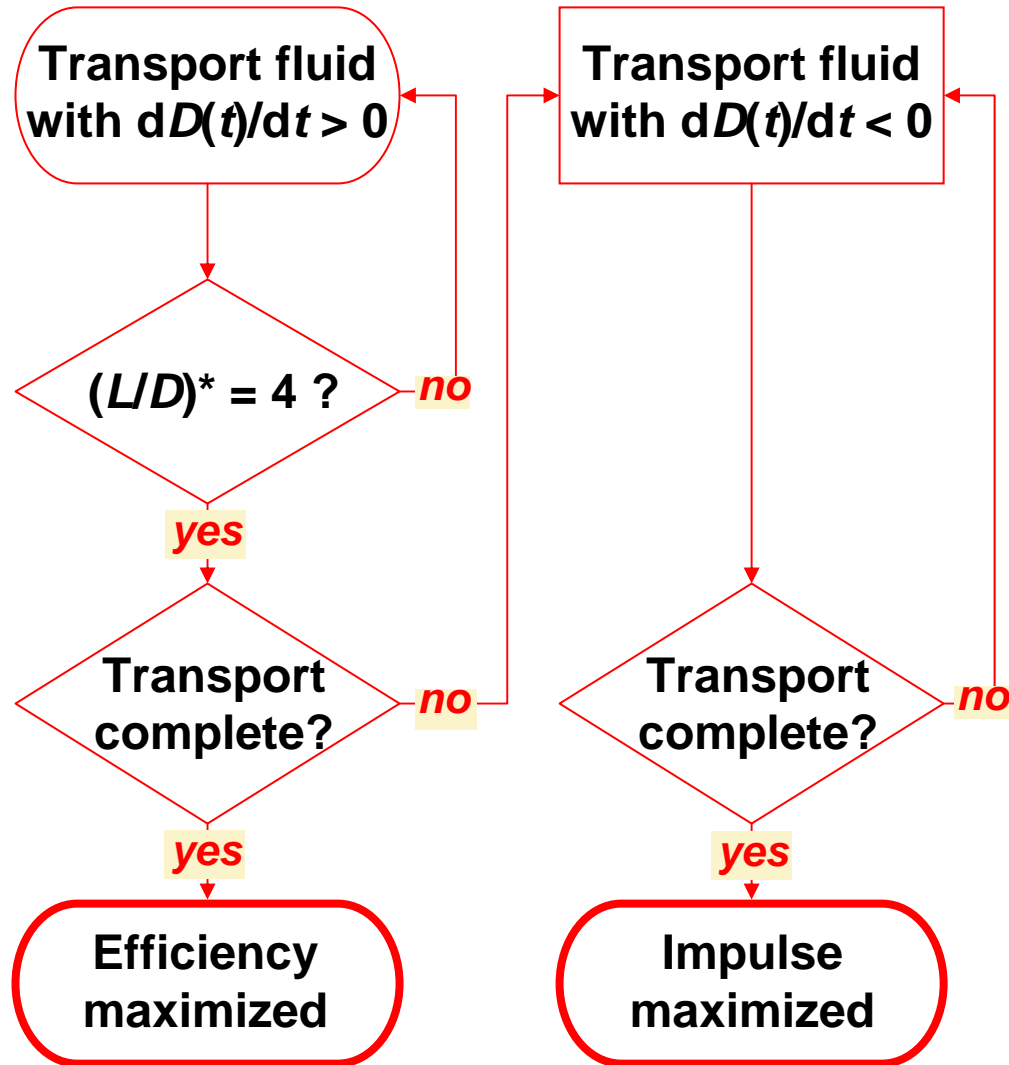
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Are these strategies observed in biological systems?

Animal swimming revisited

- Squid rely on jet flow for **high-speed swimming** and **escaping predation**

Are these strategies observed in biological systems?

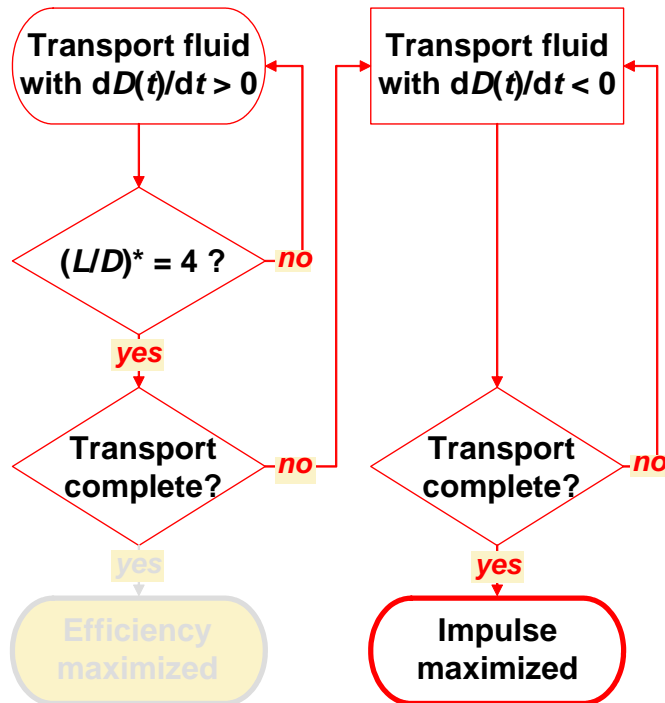
Animal swimming revisited

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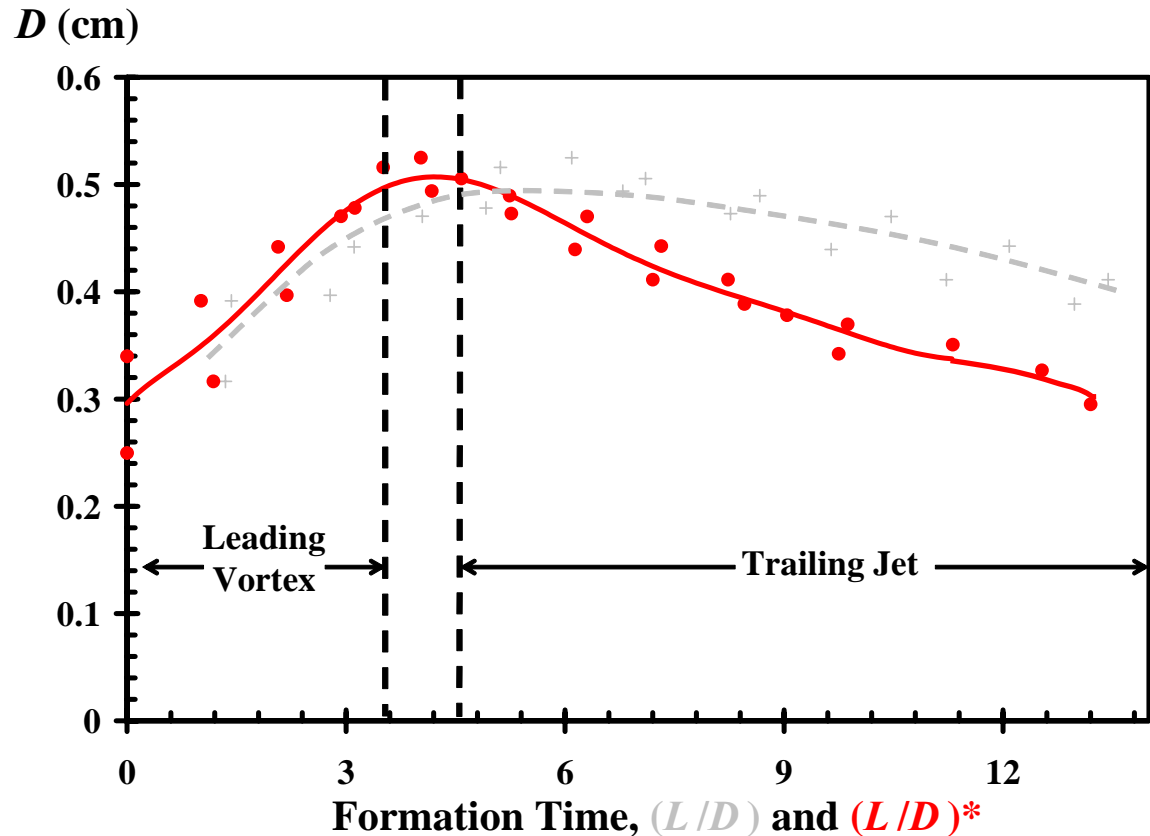
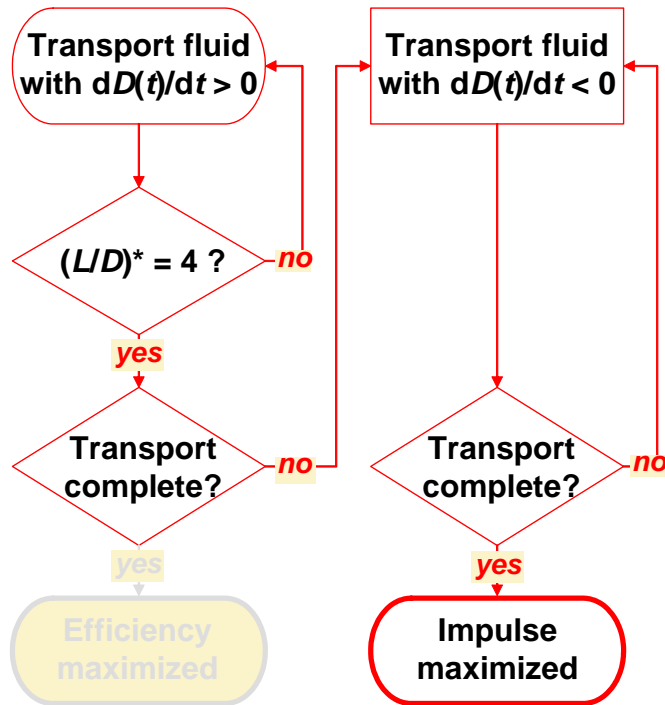
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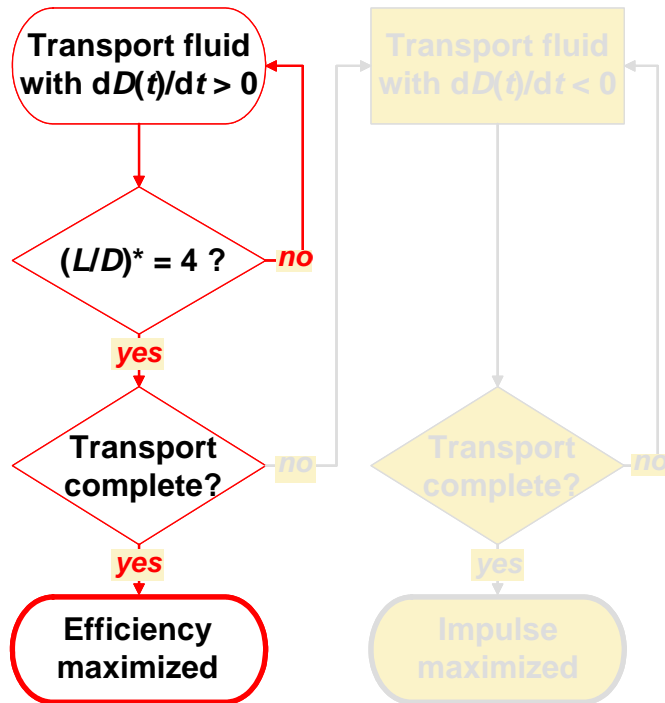
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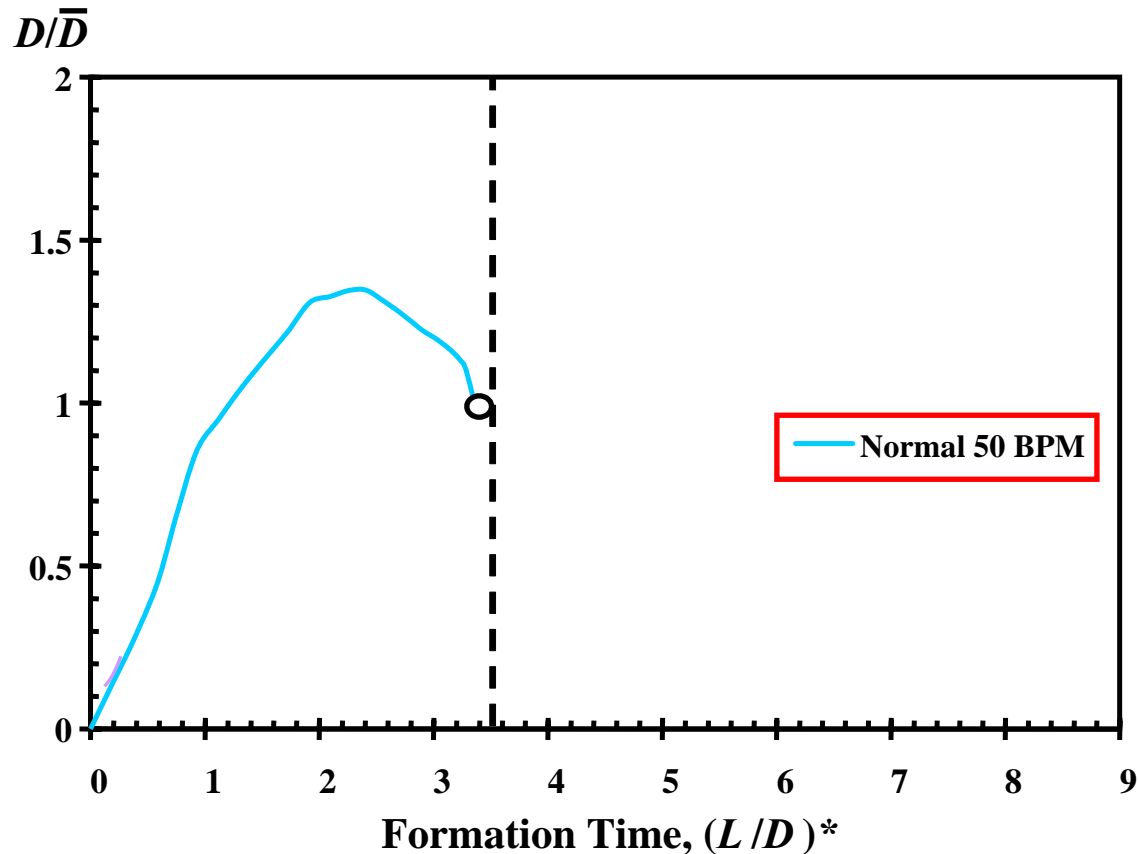
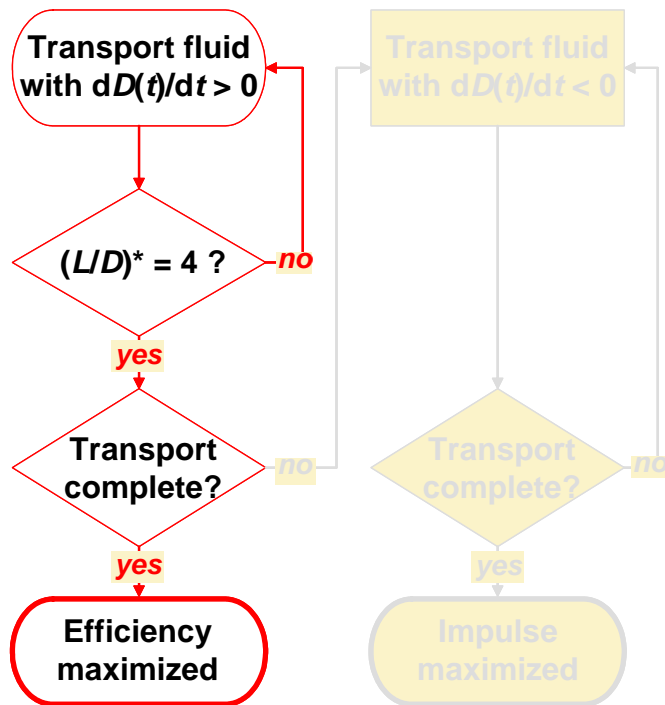
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- The optimal ejection strategy depends on **cardiac health**
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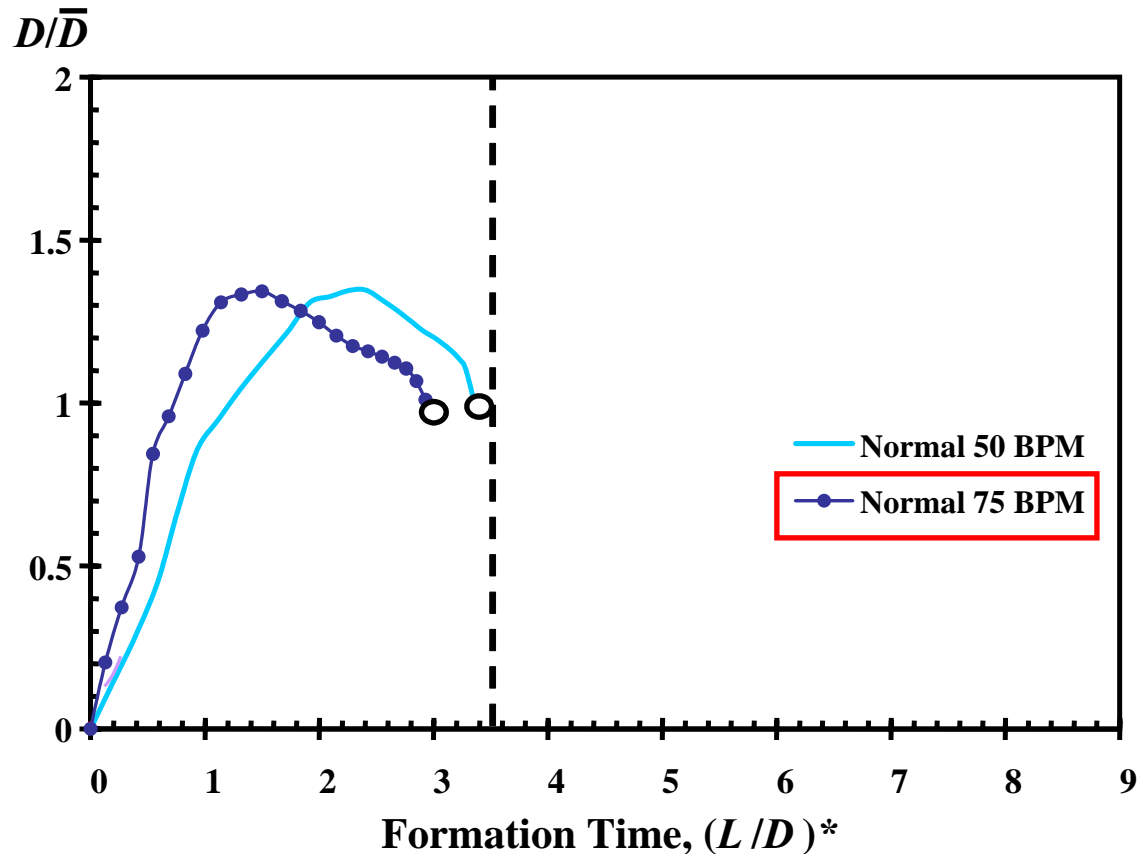
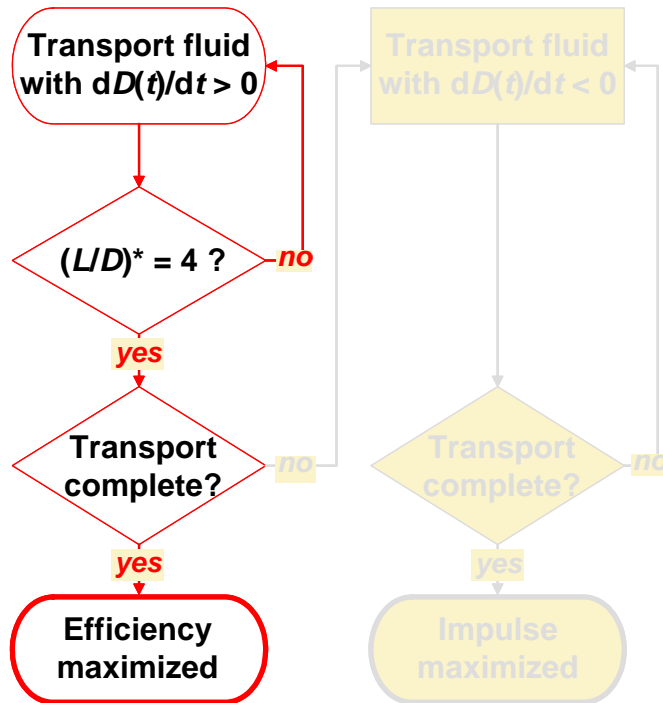
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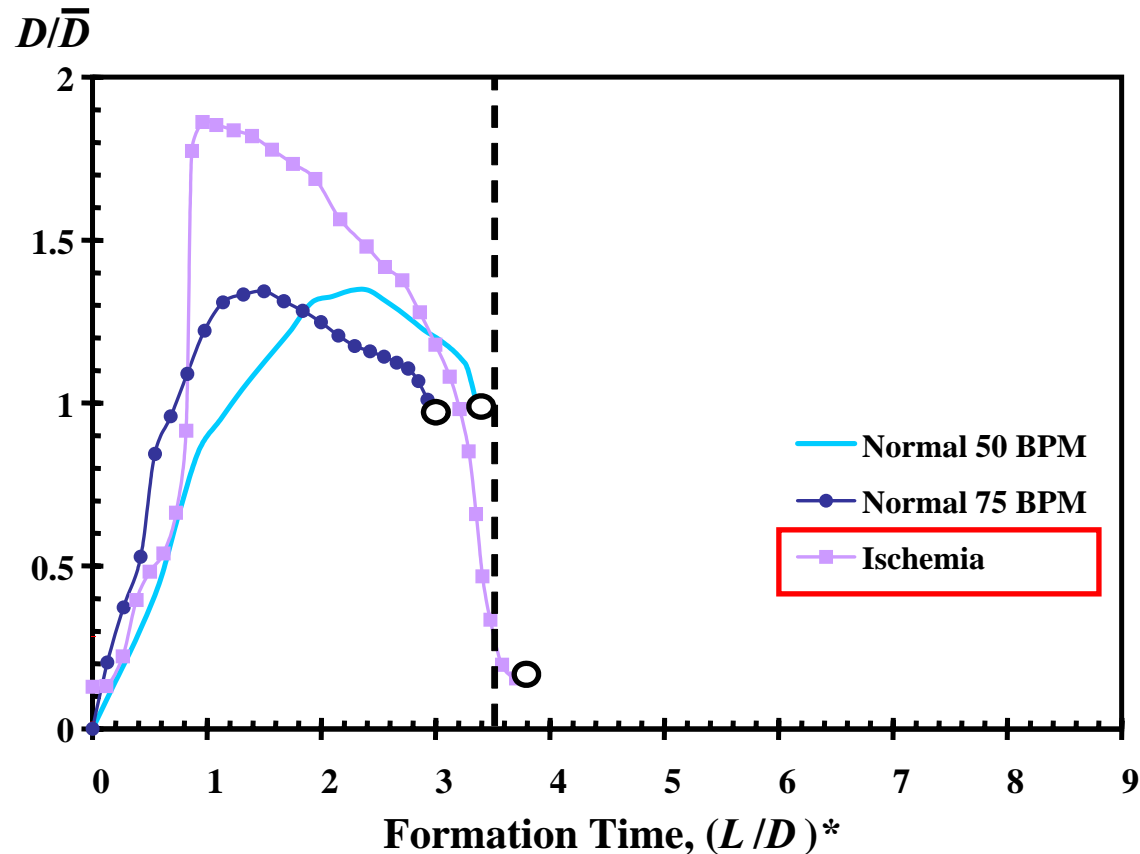
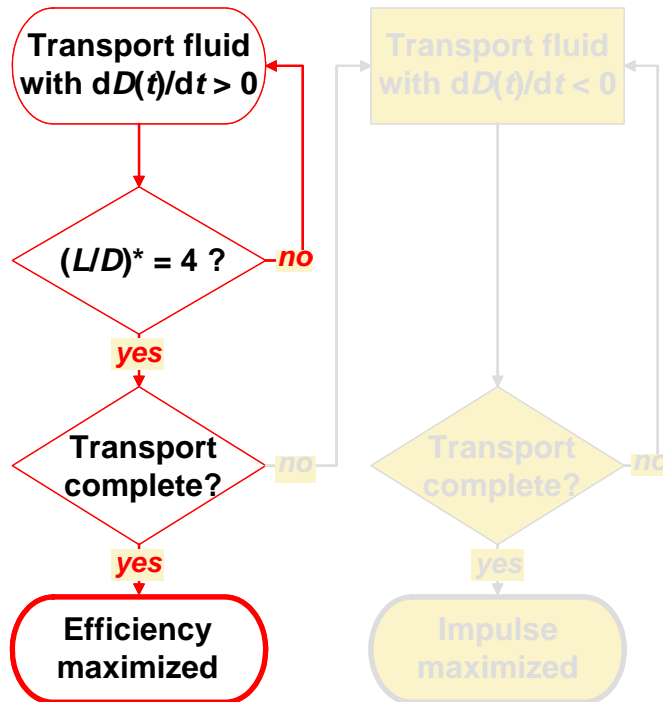
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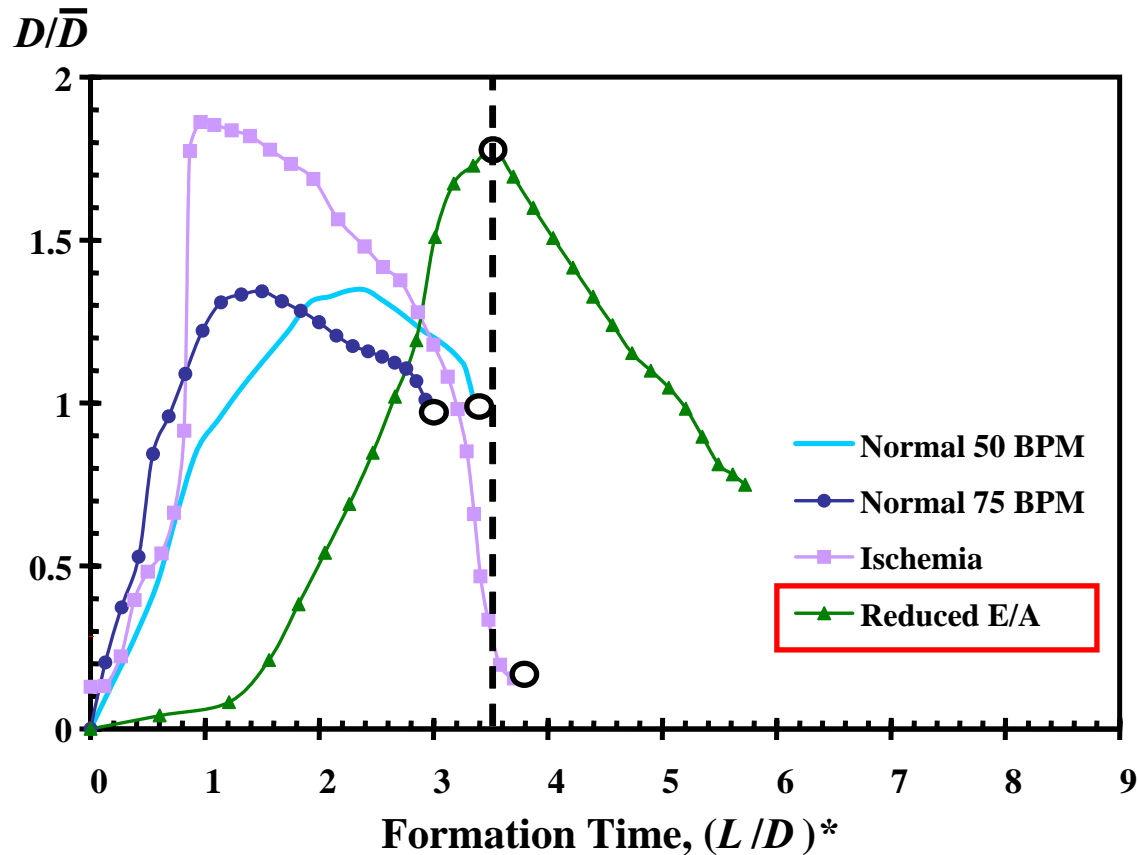
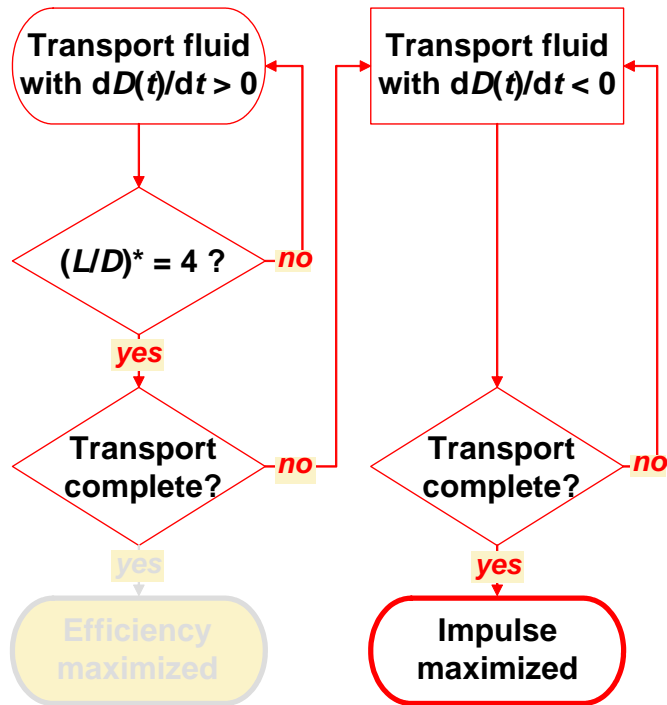
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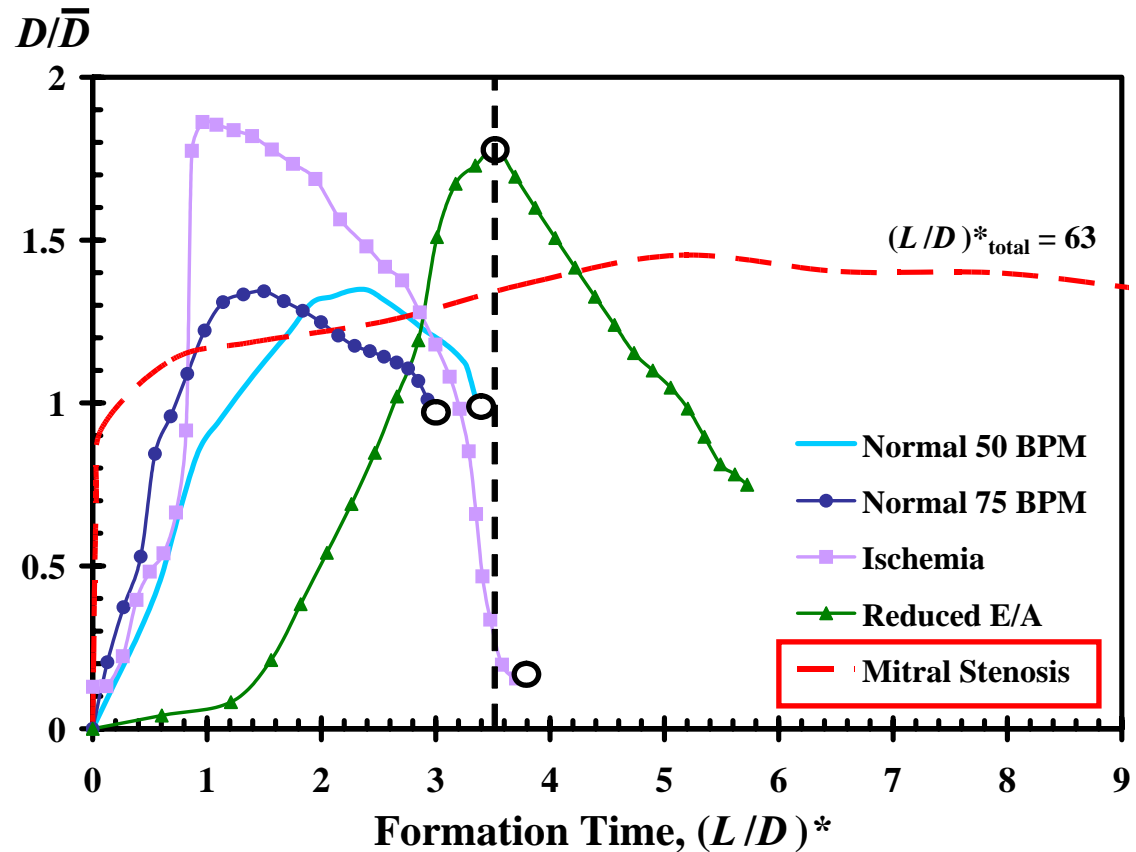
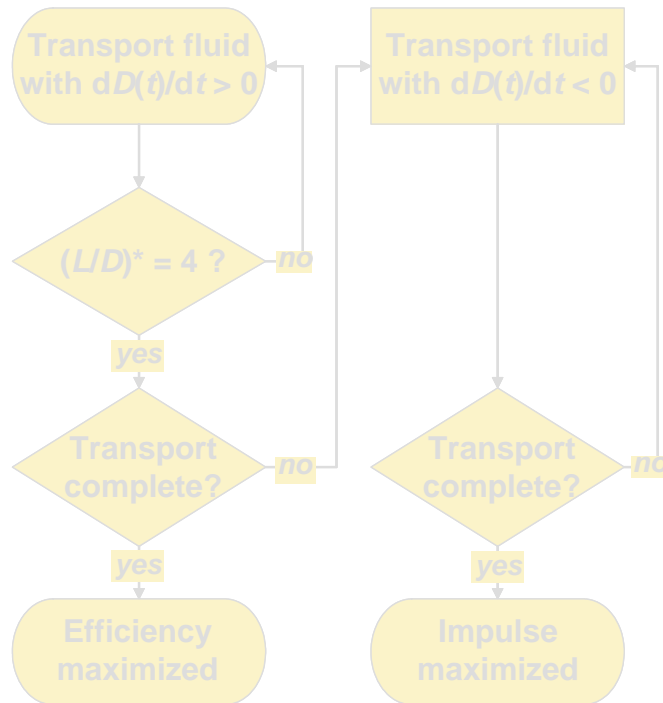
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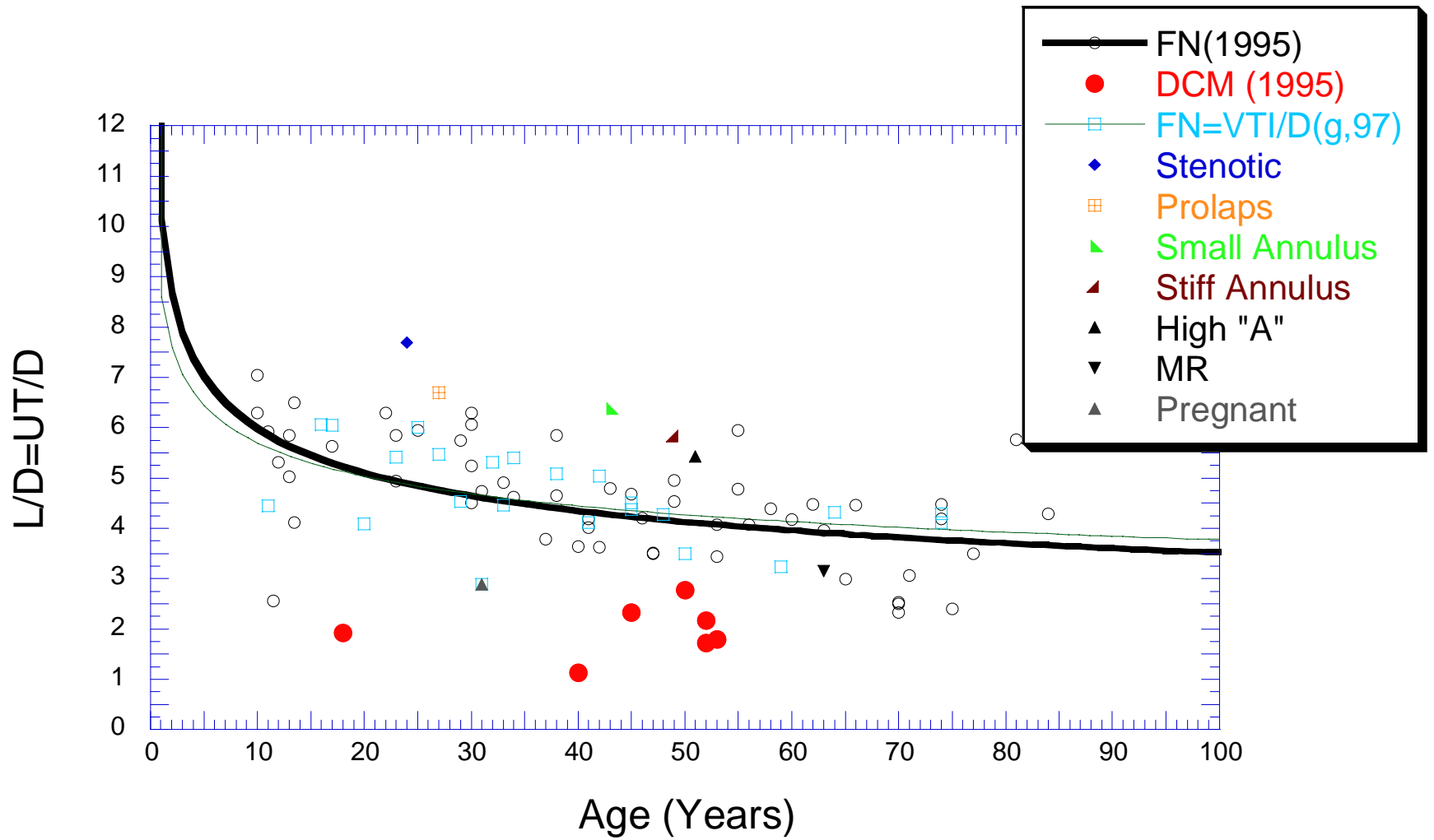
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Conclusions

*Do biological systems exploit
vortex ring formation for optimal
fluid transport?*

Conclusions

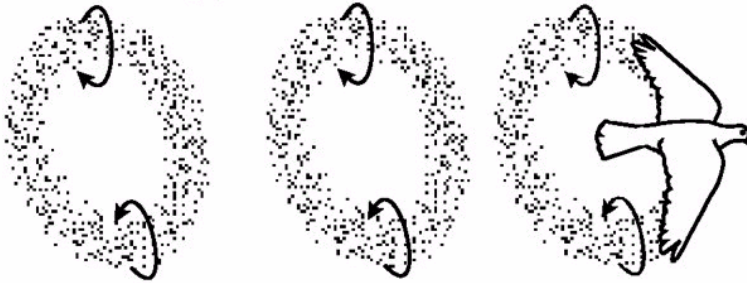
Do biological systems exploit

Yes, both mobile and

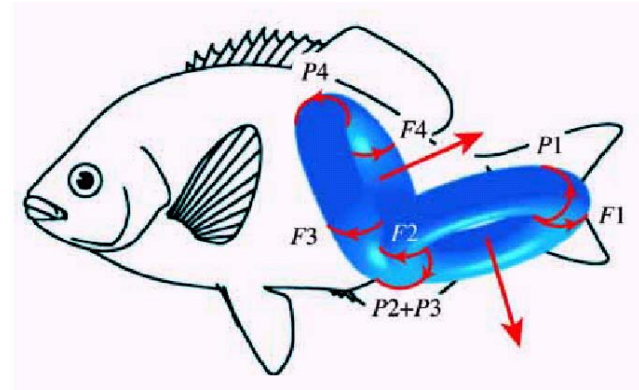
Comments

The *vortex ring motif* studied here is not limited to jet-based fluid transport...

Flapping

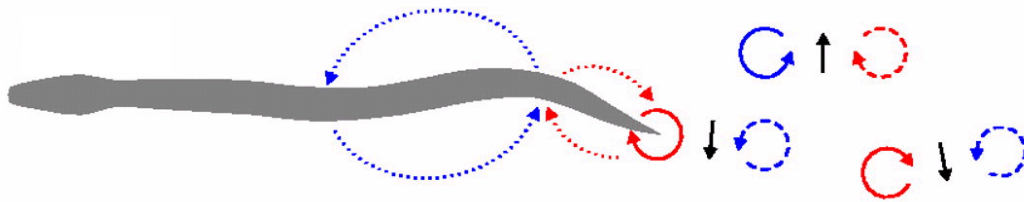


birds (Rayner, 1988)



fish (Drucker, 2000)

Undulating



eels (Tytell, 2004)

Paddling



frogs (Johansson, 2004)

Comments

...therefore, in order to better understand the physics and evolutionary incentives behind **other vortex-based mechanisms**, we need

To include **realistic boundary and flow conditions** such as compliance and co-flow

To investigate physics of **individual events of vortex formation** in the context of “**Dynamic Formation Time**” rather than Strouhal frequency

Acknowledgements

Special Thanks to

John O. Dabiri

And

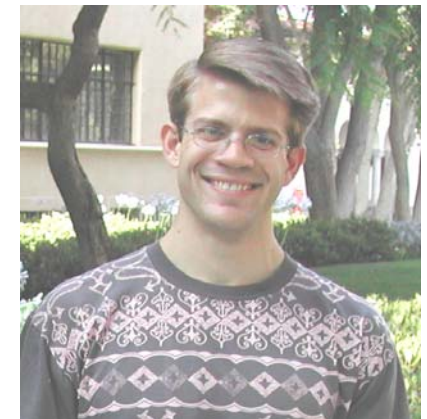
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**K. Mohseni (Colorado-Boulder), I. Bartol (Old Dominion),
Michael Shusser (Technion), Moshe Rosenfeld (Tel Aviv),
S. Colin (Roger Williams), J. Costello (Providence)**

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