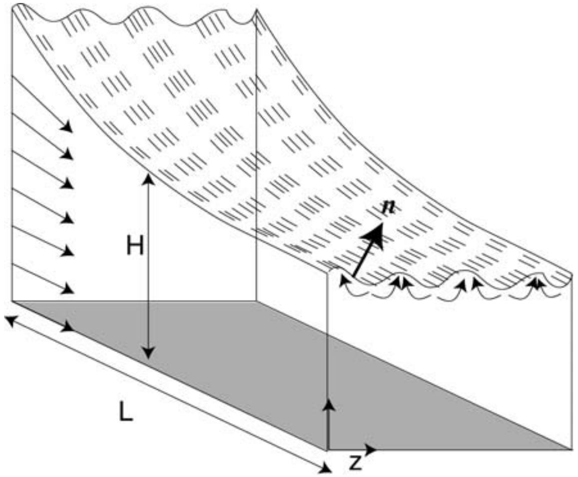
Interfacial hoop stress and viscoelastic free surface flow instability



Michael D. Graham
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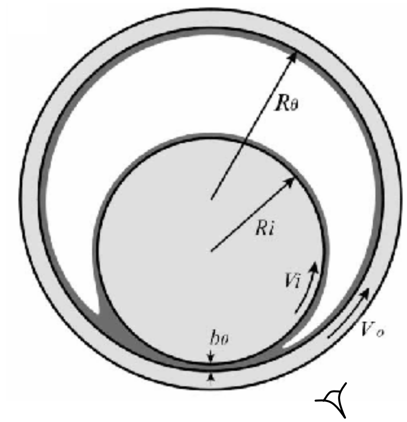
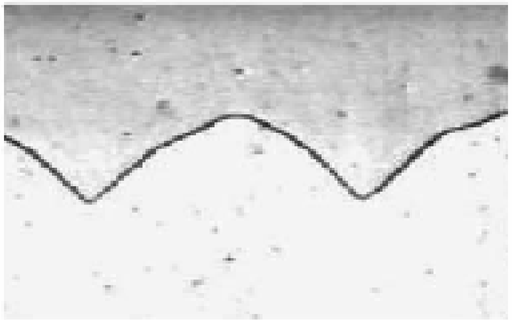
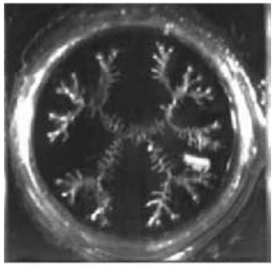
Free surface instabilities of viscoelastic flows

Eccentric cylinders (Varela-Lopez et al 2002)





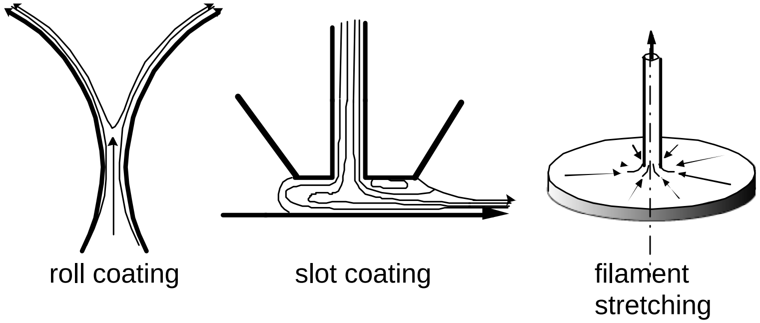
Filament stretching (Sridhar & McKinley 2002)



G .H. McKinley

- Viscoelasticity dramatically exacerbates many free surface instabilities

Viscoelastic free surface flows: theory and computation



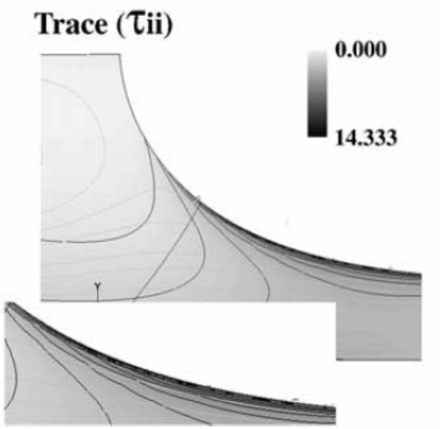
Weissenberg number

$$Wi = \lambda \dot{\gamma}$$

λ relaxation time

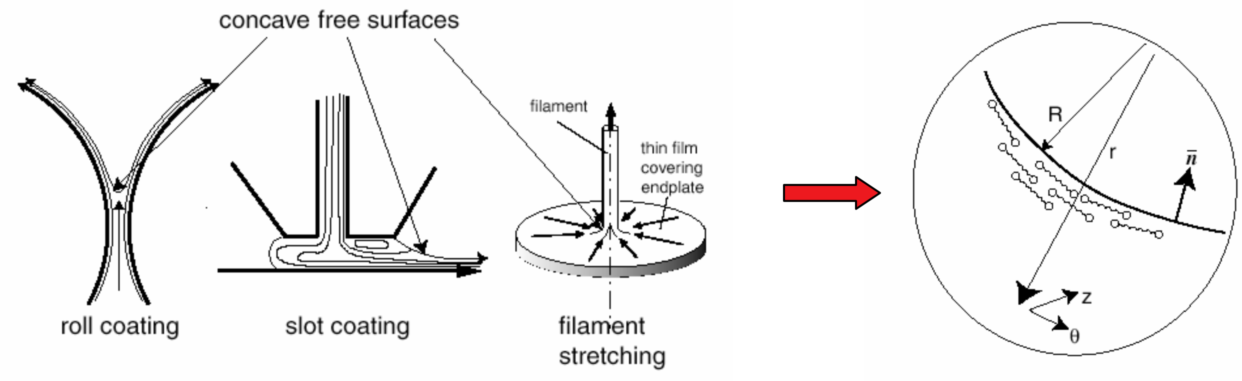
$\dot{\gamma}$ strain rate

- Lubrication approximation: tractable formulation at high *Wi* that keeps dominant viscoelastic effects has not been worked out
- Low *Wi* asymptotic analysis (Ro and Homsy 1995): effects of viscoelasticity are small
- CFD approaches (Scriven, Khomami, Pasquali)
  - can predict film thickening when *Wi*=O(1)
  - challenging: thin stress b.l.s arise at free surfaces
  - linear stability analysis at high *Wi* has not been performed
- Present work: simple models that incorporate key physical effects



Polymer stress in Hele-Shaw coating (Lee *et al.* 2002)

Stress at a concave free surface



Approximate local free surface shape as an arc, neglect gravity for clarity: normal stress balance for a 2D flow yields:

$$\frac{\partial \overline{\sigma}_{rr}}{\partial r} = \frac{\overline{\sigma}_{\theta\theta} - \overline{\sigma}_{rr}}{R}$$

$$\equiv \frac{\Delta \sigma}{R}$$

$$\equiv T$$

For Newtonian fluids, *T* can be estimated from the bulk pressure gradient

Perturbation normal stress balance

Let radial position of surface be

$$r = \bar{h} - \hat{h}(\theta,t)e^{ikz}$$

Normal stress balance becomes:

$$\hat{\sigma}_{rr}\Big|_{r=\bar{h}} = T\hat{h} + \gamma\left(-k^2 + \frac{1}{R^2}\frac{\partial^2}{\partial\theta^2} + \frac{1}{R^2}\right)\hat{h}$$

$$\approx \left(T - \gamma k^2 - \gamma S\right)\hat{h}$$

$$T - \gamma k^2 - \gamma S > 0 \rightarrow \quad \hat{h}\uparrow \Rightarrow \hat{\sigma}_{rr}\uparrow$$

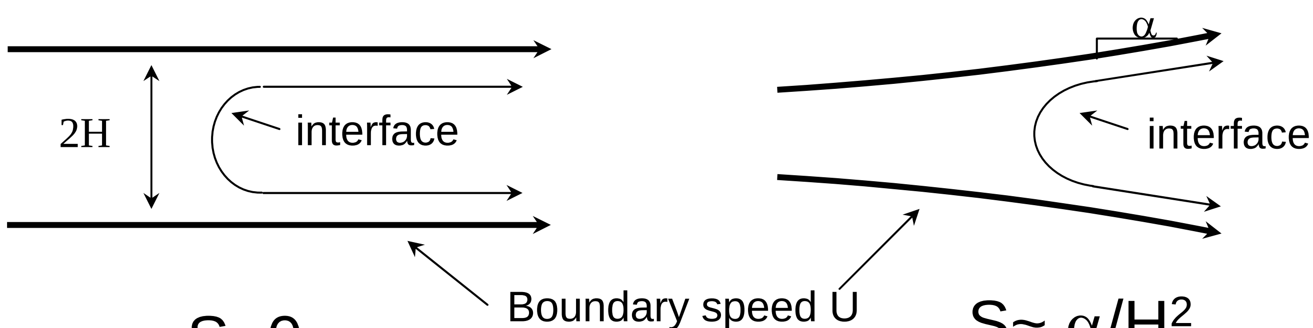
Inward pull on interface increases

increase in inward radial velocity

Increase in inward interface displacement

T = Δσ/*R*=hoop stress * curvature = general driving force for free surface flow instability

Newtonian Hele-Shaw and roll-coating flows



S=0

$S \sim \alpha/H^2$

(Pearson 1960)

Newtonian flow:

- $\Delta\sigma \sim \eta U/H$ (bulk stress ~ interfacial stress ~ viscous stress)
- $R \sim H$
- $T = \Delta\sigma / R \sim \eta U/H^2$

Newtonian instability criterion becomes

$$Ca = \eta U/\gamma \sim \alpha + (kH)^2$$

← Incorrect exponent comes from local approx.

(Saffman & Taylor, Pearson, Pitts & Greiller…)

Effect of viscoelasticity

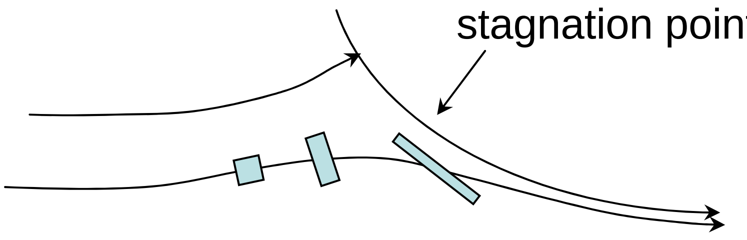
Is the bulk stress a good estimate of the interfacial stress?

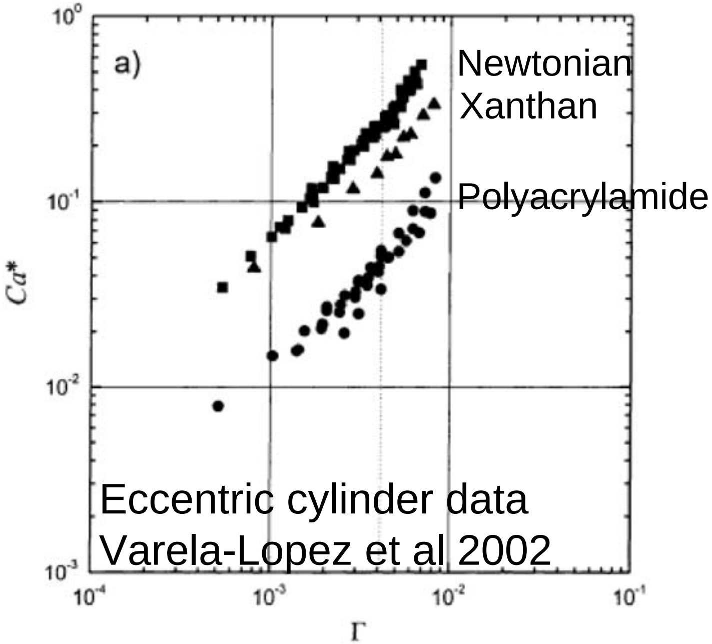
No.

- Flow near a free surface is always extensional
- strain ~ (distance from free surface)$^{-1}$

$\Rightarrow$ stress boundary layers

$\Rightarrow$ We should estimate $\Delta\sigma$ with an extensional viscosity η_e: $\eta_e >> \eta$



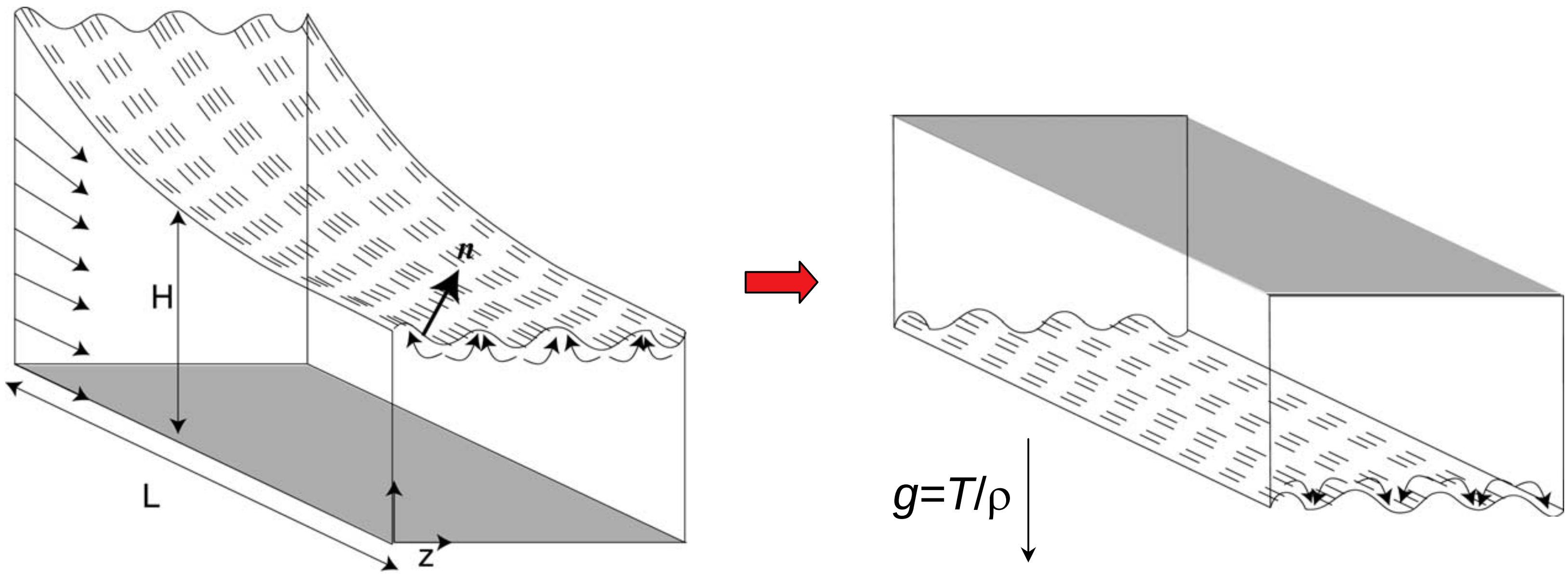


=> Modified instability criterion

$$Ca\ \eta_e/\eta \sim \alpha+(kH)^2$$

Viscoelastic free surface instability

a solvable special case



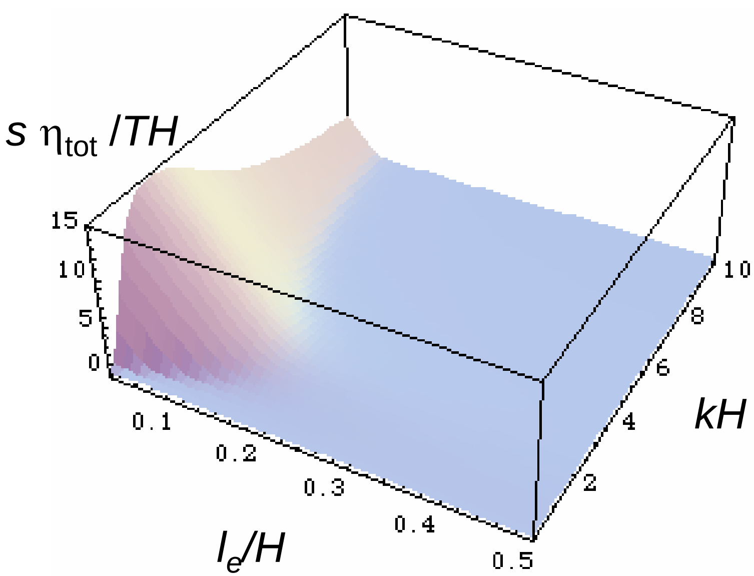
- $H \ll L$, $R = L$
- Let curvature $K=H/R$ --> 0 with $K\Delta\sigma$ finite.

$\Rightarrow$ viscoelastic Rayleigh-Taylor problem (*cf*. Aitken & Wilson 1993), but with ρg replaced by T

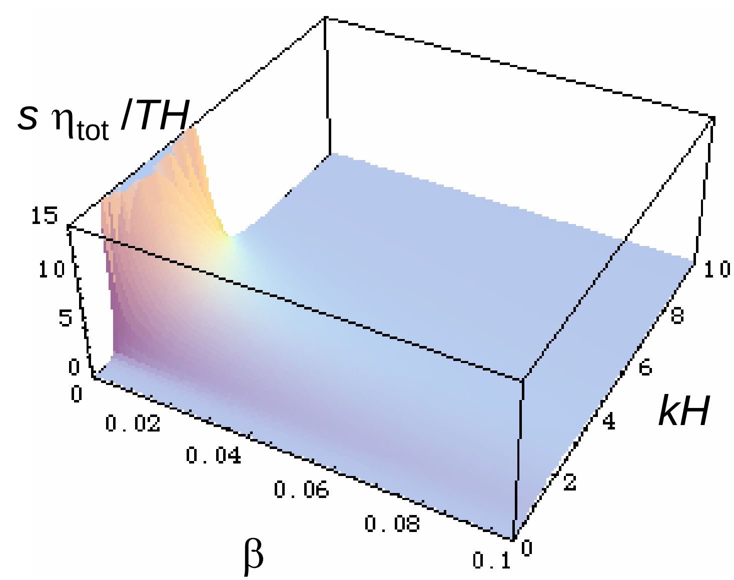
New intrinsic elastic length scale $l_e = G/T$, where G is shear modulus

Results for inertialess Oldroyd-B model, $\gamma=0$

$\beta = \eta_s/\eta_{tot}=0.01$



$l_e/H = 0.1$



- Maximum growth rate s is at $kH = 2.2$
- Newtonian: $s\,\eta_{tot}\,/TH < 0.16$ for all k
- Viscoelastic: $s\,\eta_{tot}\,/TH \sim \beta^{-1}$ for $l_e<0.16H$; why the blowup?

Energy integral scalings

$$kH \gtrsim 1$$

K = kinetic energy: $\dfrac{\rho}{k^2} s^3$

E = strain energy: Gs

W = work done by the surface perturbation: $\dfrac{T}{k} s$

D = dissipation: $\eta_s s^2$

$$K+E+D=W$$

$W \gg E$ for $T \gg Gk$ ($kl_e \ll 1$)

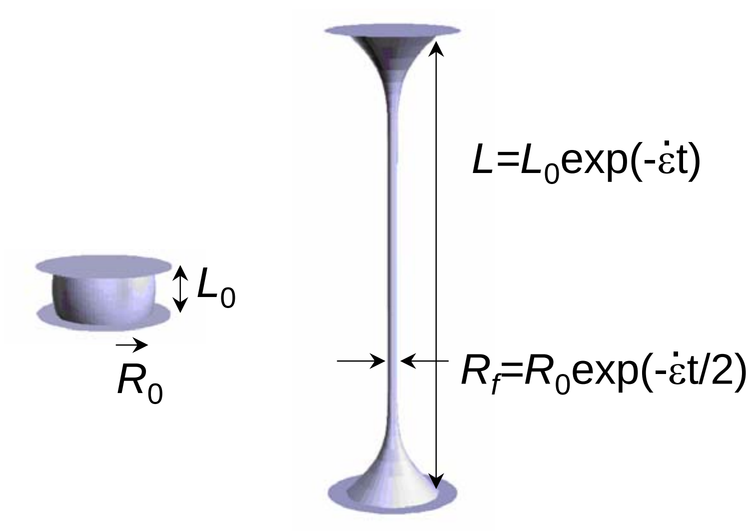
l_e: length scale where surface work and strain energy balance

$\Rightarrow$ For small l_e, surface work must be balanced by inertia or solvent viscosity
-- strain energy can't keep up

$\Rightarrow$ s blows up for small ρ and η_s

Filament stretching rheometer

Schematic



Instability near endplates
(McKinley & coworkers)

side view

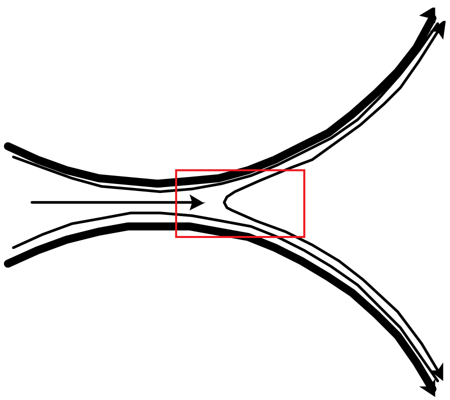
bottom view

- Extensional rheology of polymer solutions
- Simulations show stress boundary layers near free surface

Roll coating: computational results

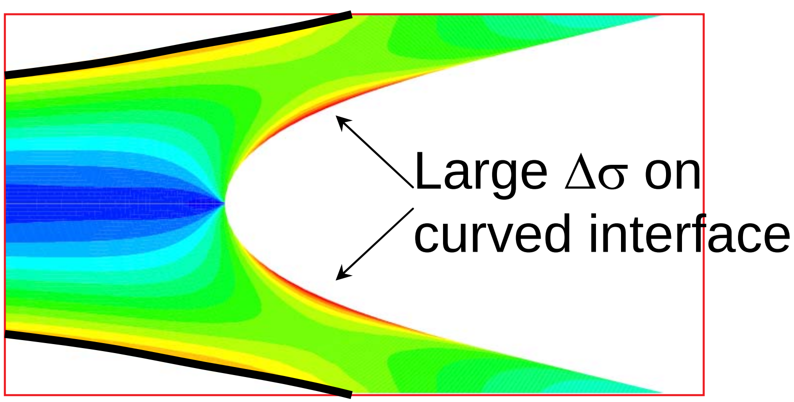
(courtesy of M. Pasquali)

geometry



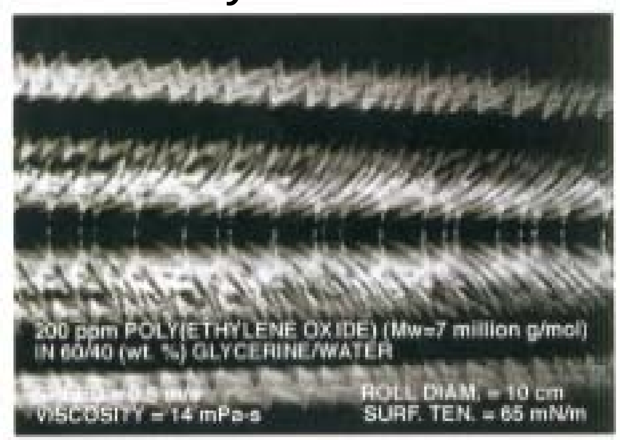
- Steady state viscoelastic computations, FENE-P model
- Stress boundary layers form
- $\Delta\sigma$ greatly exceeds Newtonian value

σ_{tt} in meniscus region



Ca=2.0, We=2.0, FENE-P, b=50, β=0.59

Instability in VE roll coating



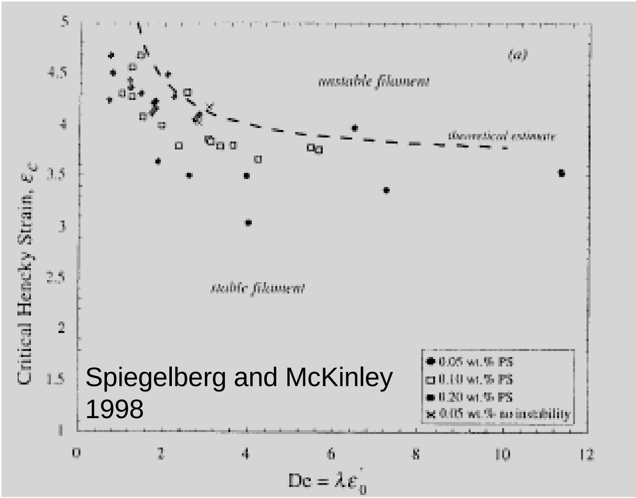
Carvalho et al. 1994

Instability prediction for filament stretching

$$\frac{\Delta\sigma}{R} > k^2\gamma$$

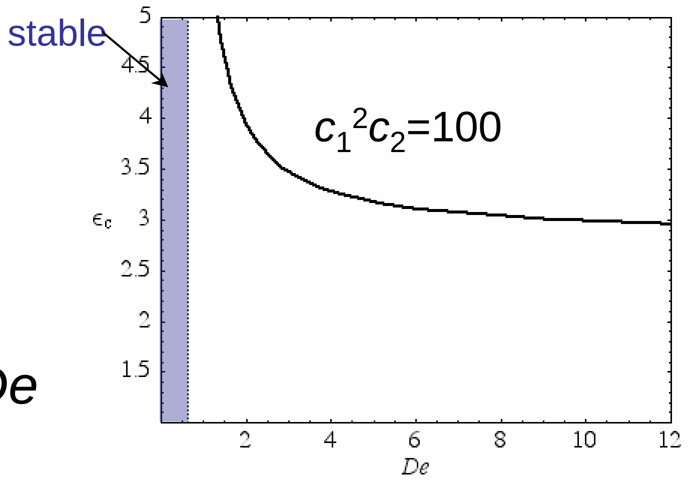
Estimates:

- $\Delta\sigma$: Oldroyd-B, uniaxial extension
- wavenumber $k = c_1/R_f$, $c_1 = O(1)$
- radius of curvature $R = c_2 R_f$, $c_2 = O(1)$
- $\gamma/GR_0 = 0.69$



Predicted critical strain:

$$\varepsilon_c = \left(\frac{3}{2} - \frac{1}{De}\right)^{-1} \log\left(\frac{c_1^2 c_2 \gamma}{GR_0}\left(1 - \frac{1}{2De}\right)\right)$$



$\Rightarrow$ Stable region captured well

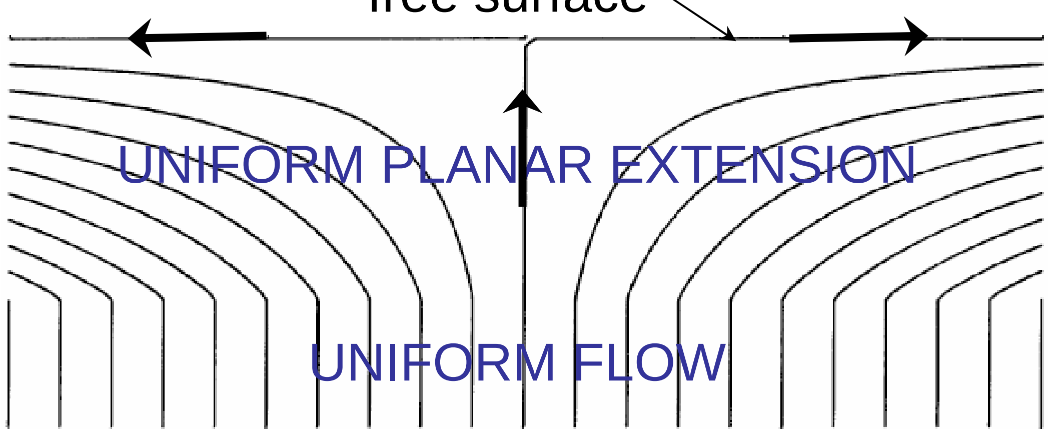
$\Rightarrow$ Overall trend reasonable until high *De*

Conclusions

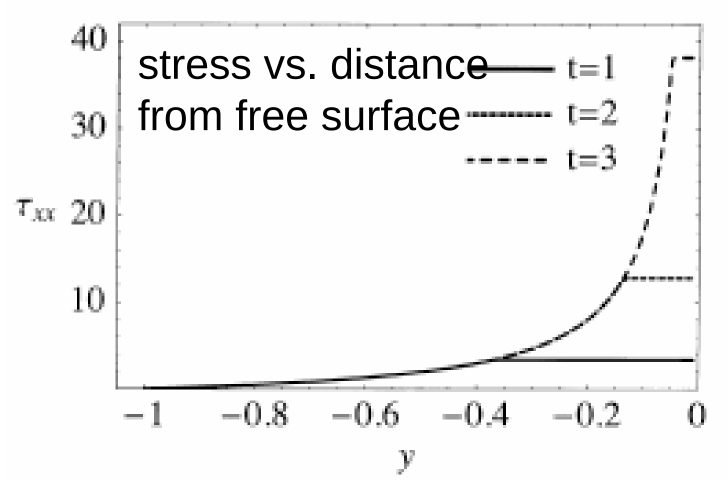
- Interface tension * curvature drives free surface flow instabilities:
  - $\Rightarrow$ generalized Saffman-Taylor result
  - $\Rightarrow$ connection to bulk viscoelastic instabilities
- A special case can be explored in detail by reduction to viscoelastic Rayleigh-Taylor instability
- Elasticity introduces a new length scale
- Growth rates can be large: surface work overcomes strain energy
- Application of simple theory to filament stretching gives good agreement with O(1) free parameters

Thanks: G. M. Homsy, KITP (UC-Santa Barbara), NSF, ACS/PRF

Stress boundary layers in viscoelastic free surface flows: a model flow



- Stress is independent of x
- Stress increases exponentially toward free surface for Wi>1/2.



Kumar and Graham 2000