McCormick
Northwestern Engineering

The Art of Mixing with an Admixture of Art: Fluids, Solids, and Visual Imagination

Julio M. Ottino
Northwestern University

What? How?

There is no scorn more profound, or on the whole more justifiable, than that of the men who make for the men who explain. Exposition, criticism, appreciation, is work for second-rate minds.

G.H. Hardy, *A Mathematician's Apology* (London 1941)

Creativity in Science and Art – Concept of Genius

Immanuel Kant (1724-1804), *Critique of Judgment*

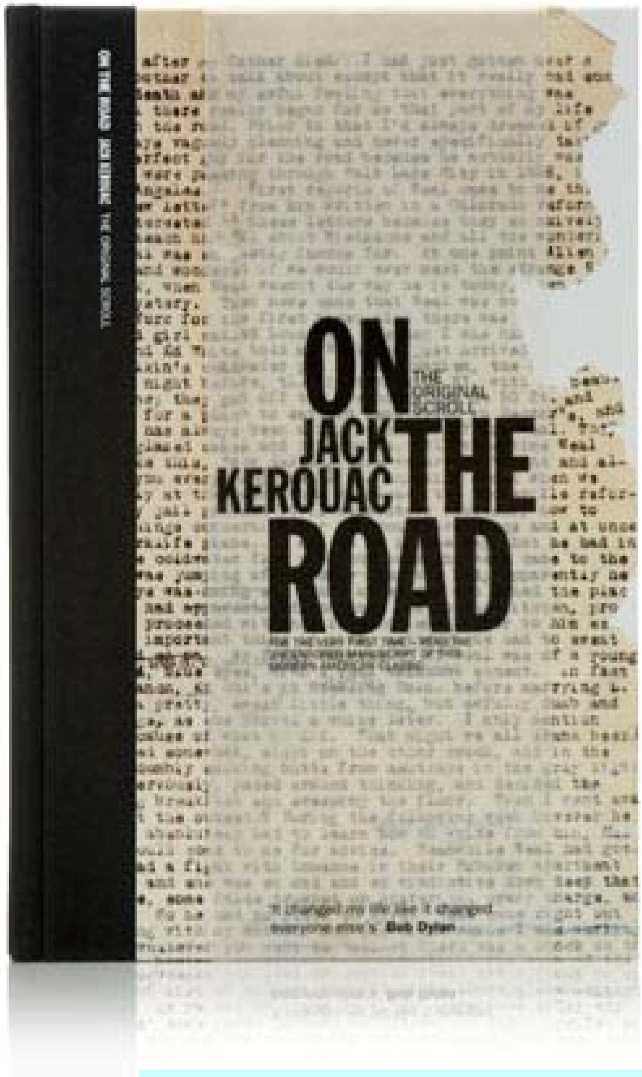
"Newton could show how he took every one of the steps he had to take to get from the first elements of geometry to his great and profound discoveries," Kant wrote, "not only to himself but to everyone else as well, in an intuitively clear way, allowing others to follow."

This is not the case with Homer and other great poets. "One cannot learn to write inspired poetry," continued Kant, "… however superb its models."

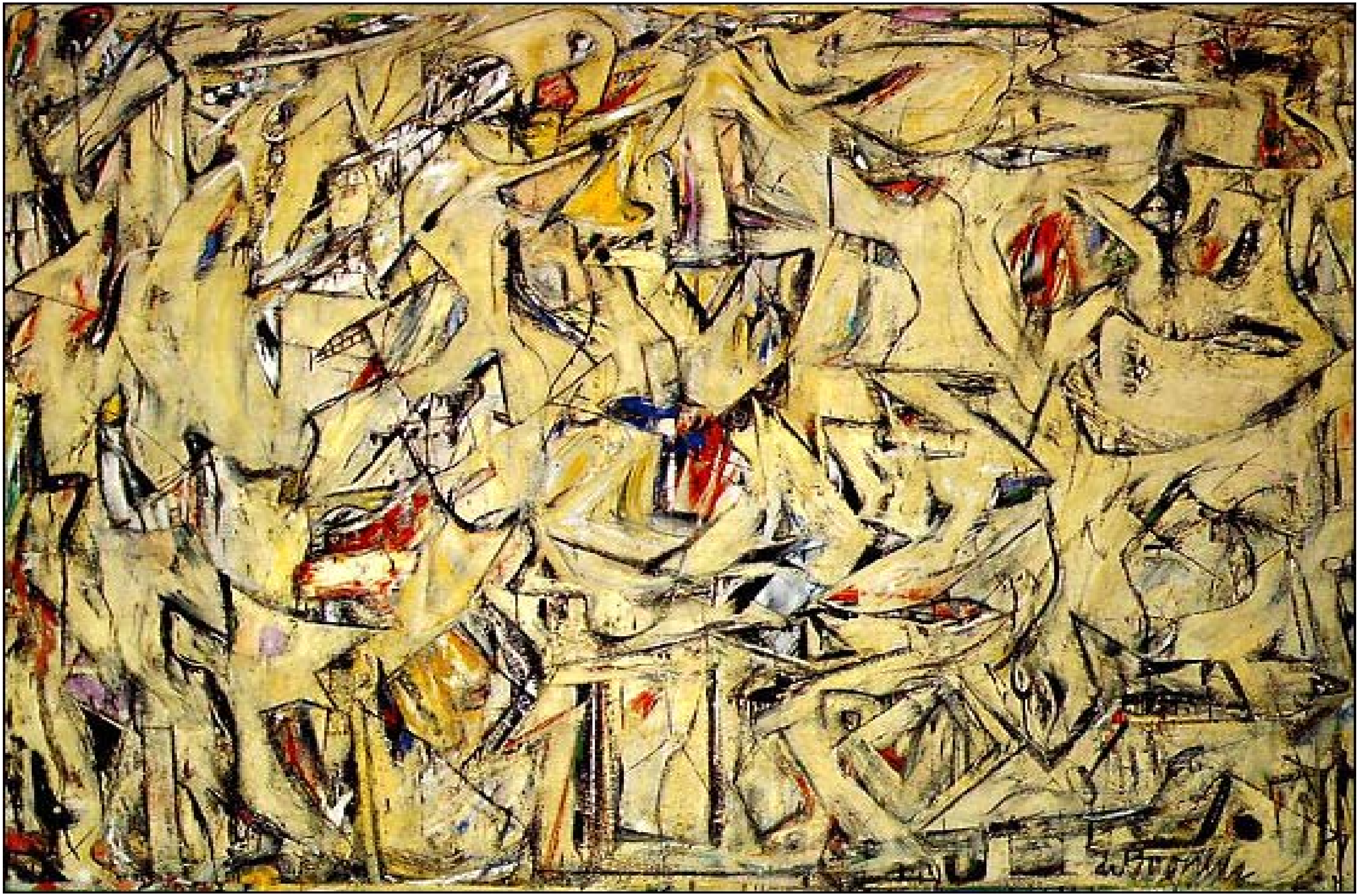
More…

"… had Newton or Leibnitz never lived, the world would have had the calculus, but that if Beethoven had not lived, we would never have had the C-Minor Symphony"

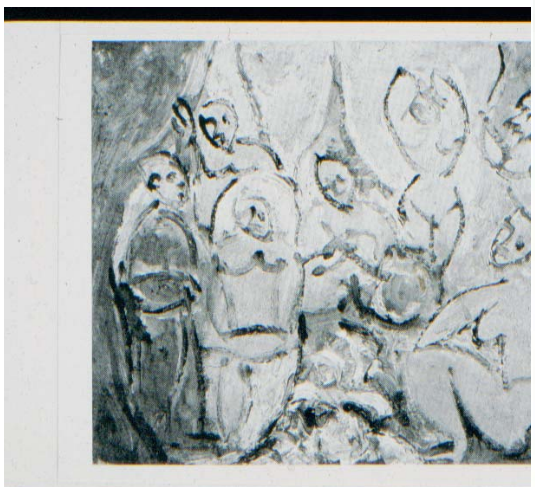
(Bernard Cohen, in *Franklin and Newton*, in remark attributed to Einstein)



Jack Kerouac (1922-1969)



Willem de Kooning (1904-1997)

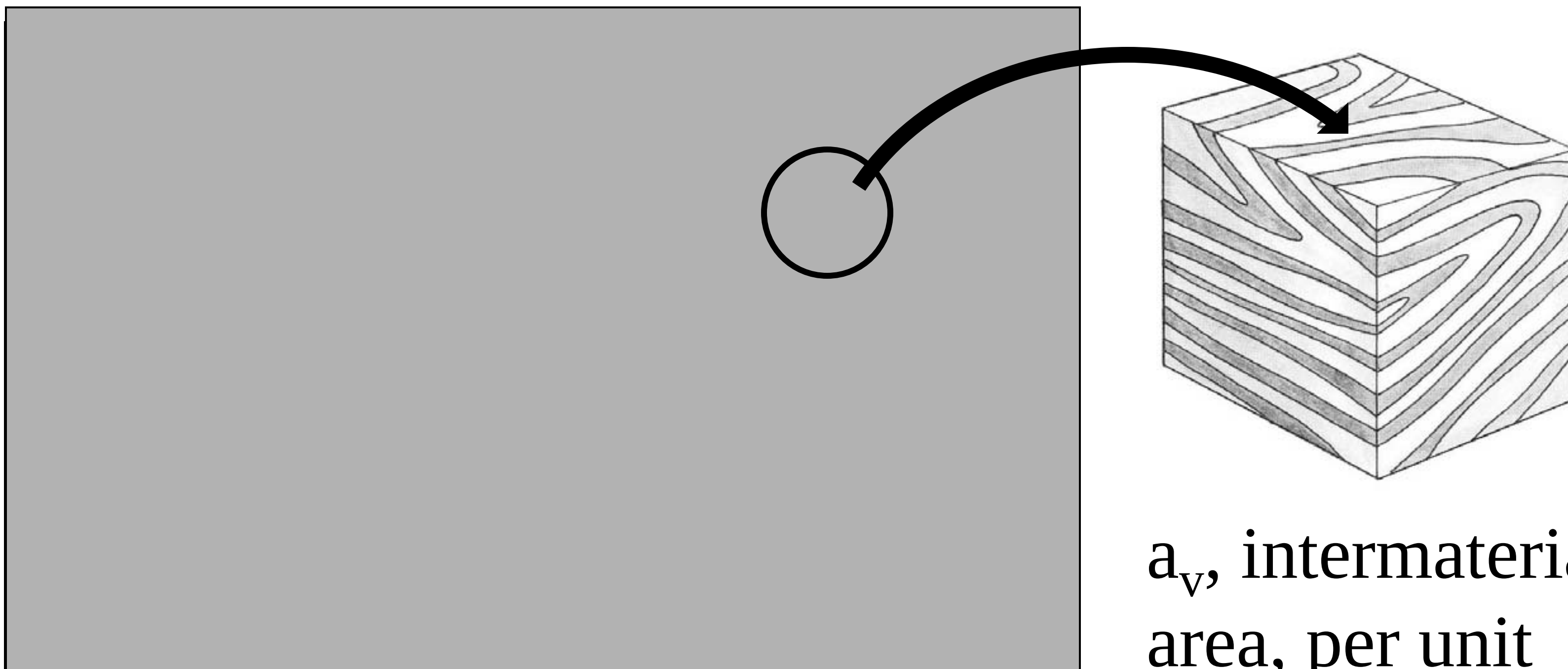


Pablo Picasso (1881-1973)

Pierre Matisse (1869-1954)

Van Rijn Rembrandt (1606-1669)

Conceptual Picture of Mixing – Lamellar Structures



Continuum theory in terms of a_v

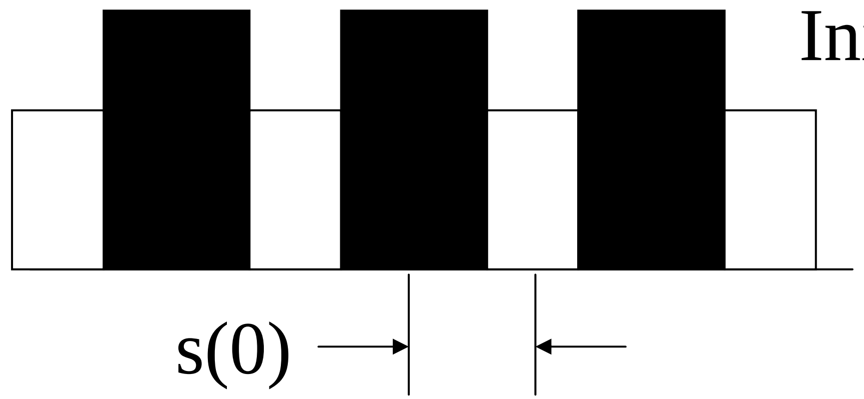
a_v, intermaterial area, per unit volume, $s \sim 1/a_v$ striation thickness

Ottino, 1979

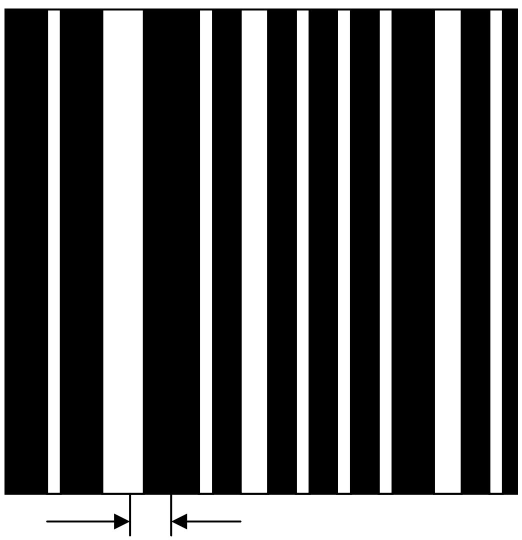
Fast reaction

A+B→P

Initial condition



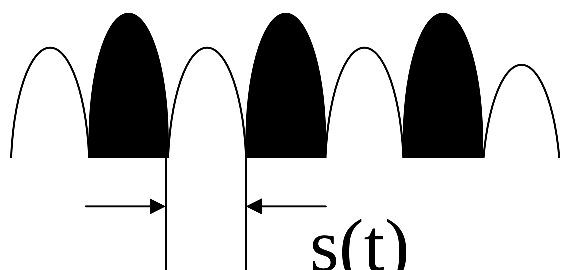
1D view



striation thickness

Infinite rate of reaction

After time t



striation thickness

fluid mechanical history contained in s=f(t)

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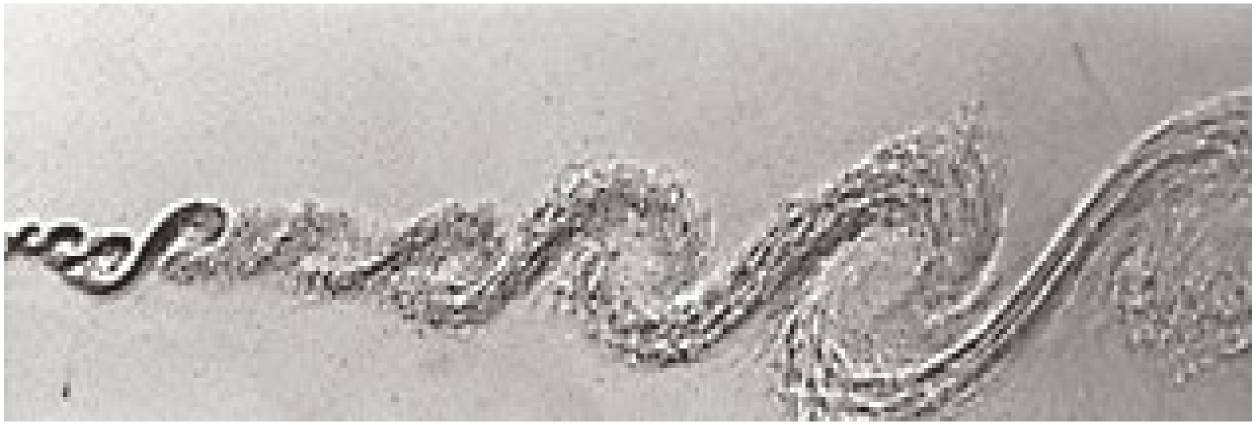
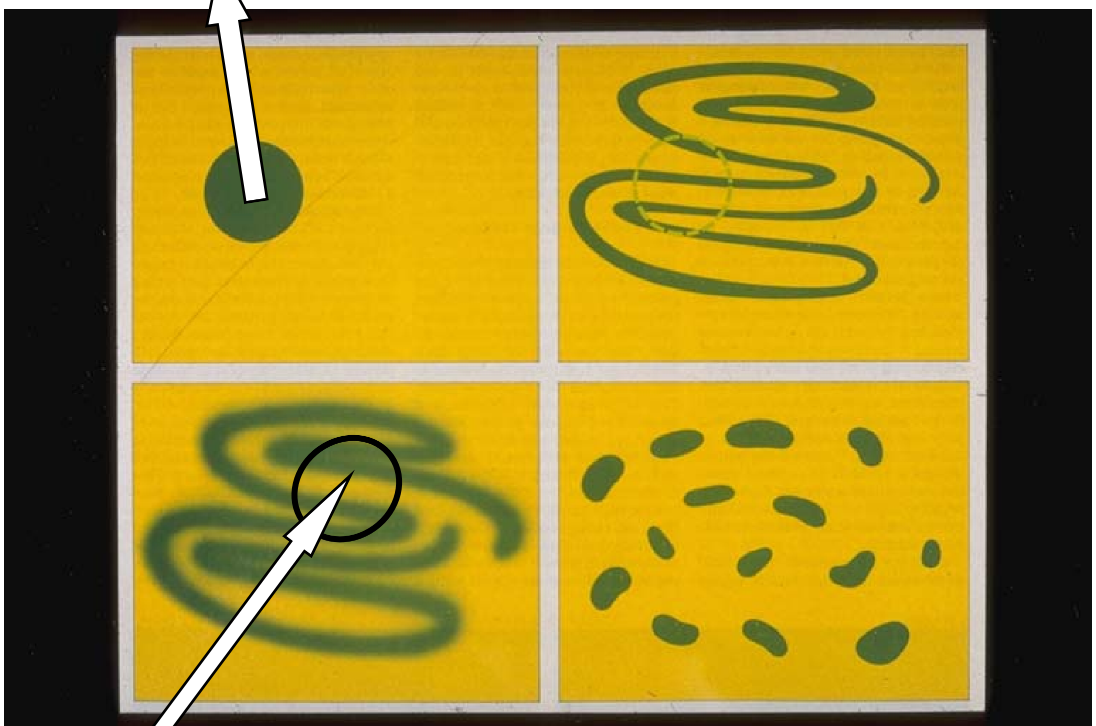
Roshko, 1976

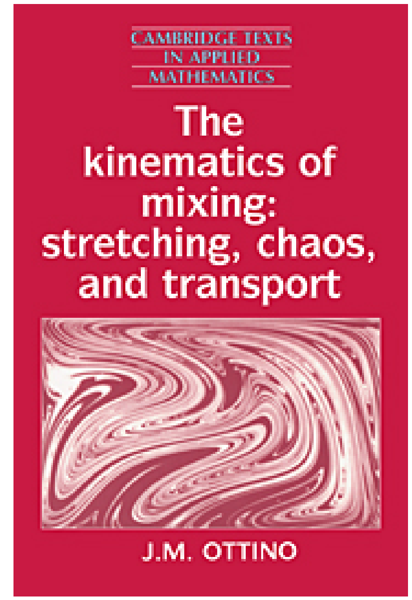


Conceptual Picture of Mixing – Stretching and Folding

Initial Condition



Striations



1989

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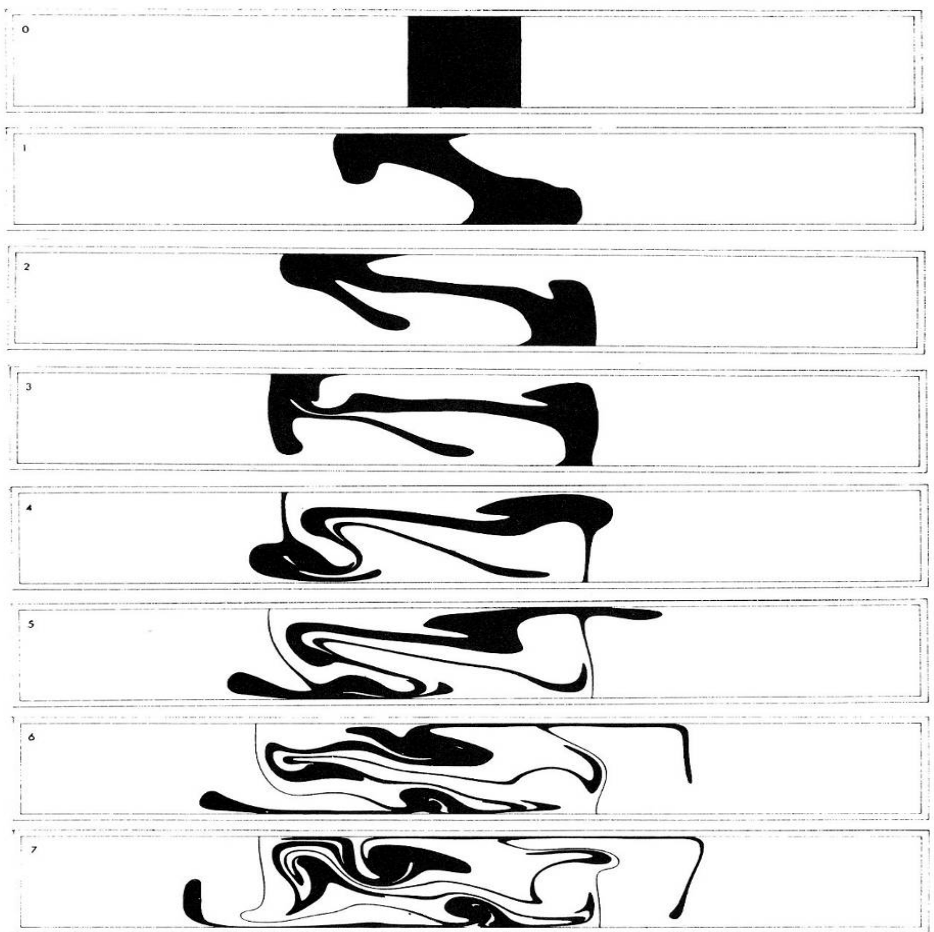
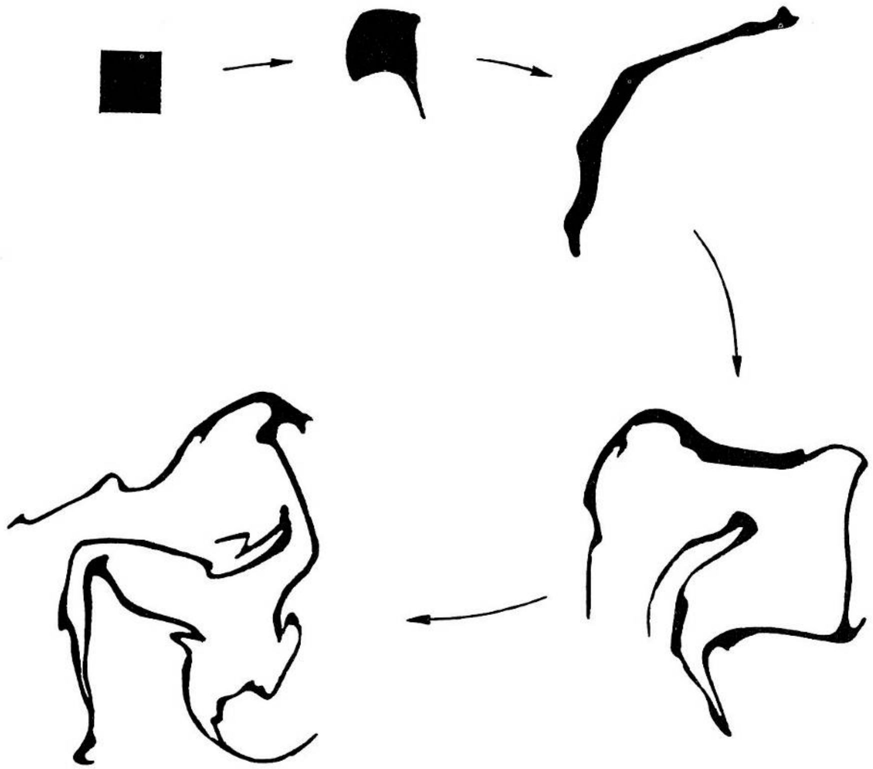
Figure 1.3.1. Deformation of a tracer in a numerical experiment of motion in the Earth mantle. The sides of the rectangle are insulating but the bottom is subjected to a constant heat flux while the temperature of the top surface is kept constant. The motion is produced by buoyancy and internal heating effects (the fluid is heated half from below and half from within). The Rayleigh number is 1.4×10^6, the time scale of the numerical simulation corresponds to 155 Myear, and the thickness of the layer is 700 km. An instantaneous picture of the streamlines reveals five cells. (Reproduced with permission from Hoffman and McKenzie (1985).)



Mixing in the Mantle of the Earth (Hoffman and McKenzie 1985)

Figure 1.2.1. Reproduction of one of the early mixing experiments of Welander (1955); evolution of an initial condition in a rotating flow. He used butanol floated on water and the initial condition (square) was made of methyl-red; unfortunately few additional details regarding the experiment were given in the original paper.



P. Welander (1955) Studies on the general development of motion in a two dimensional, ideal fluid. *Tellus* 7:141–156

Continuum Mechanics, Lagrangian

Stretching

$$\lambda \equiv \lim_{|d\mathbf{X}|\to 0} \frac{|d\mathbf{x}|}{|d\mathbf{X}|} \qquad \eta \equiv \lim_{|d\mathbf{A}|\to 0} \frac{|d\mathbf{a}|}{|d\mathbf{A}|}$$

$$\frac{\mathrm{d}(\ln\lambda)}{\mathrm{dt}} = \mathbf{D}:\mathbf{mm}, \qquad \frac{\mathrm{d}(\ln\eta)}{\mathrm{dt}} = \nabla\cdot\mathbf{v} - \mathbf{D}:\mathbf{nn},$$

Efficiency

$$e_\lambda \equiv \frac{1}{(\mathbf{D}:\mathbf{D})^{1/2}}\frac{d(\ln\lambda)}{dt} < 1, \quad e_\eta \equiv \frac{1}{(\mathbf{D}:\mathbf{D})^{1/2}}\frac{d(\ln\eta)}{dt} < 1.$$

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Flow, 1-1 mapping

$$\mathbf{x} = \Phi_t(\mathbf{X}),\ \ \mathbf{X} = \Phi_{t=0}(\mathbf{X})$$

$$\left(\frac{\mathrm{d}\mathbf{x}}{\mathrm{dt}}\right)_{\mathbf{X}} = \mathbf{v}(\mathbf{x},\mathrm{t})$$

x …Eulerian
X…Lagrangian

If time-periodic **v**(**x**,t)=**v**(**x**,t+T), maps

$$x_{n+1} = \Phi(x_n) \qquad\qquad x_n = \Phi^n(x_0)$$

Map

n applications of map

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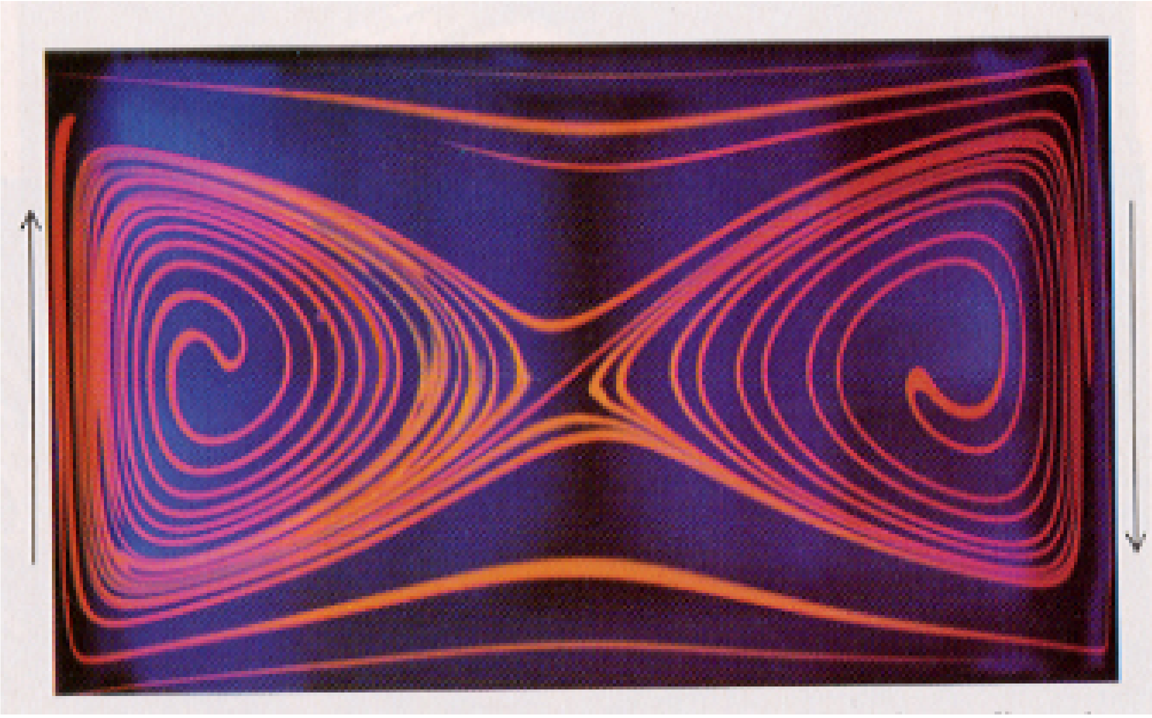
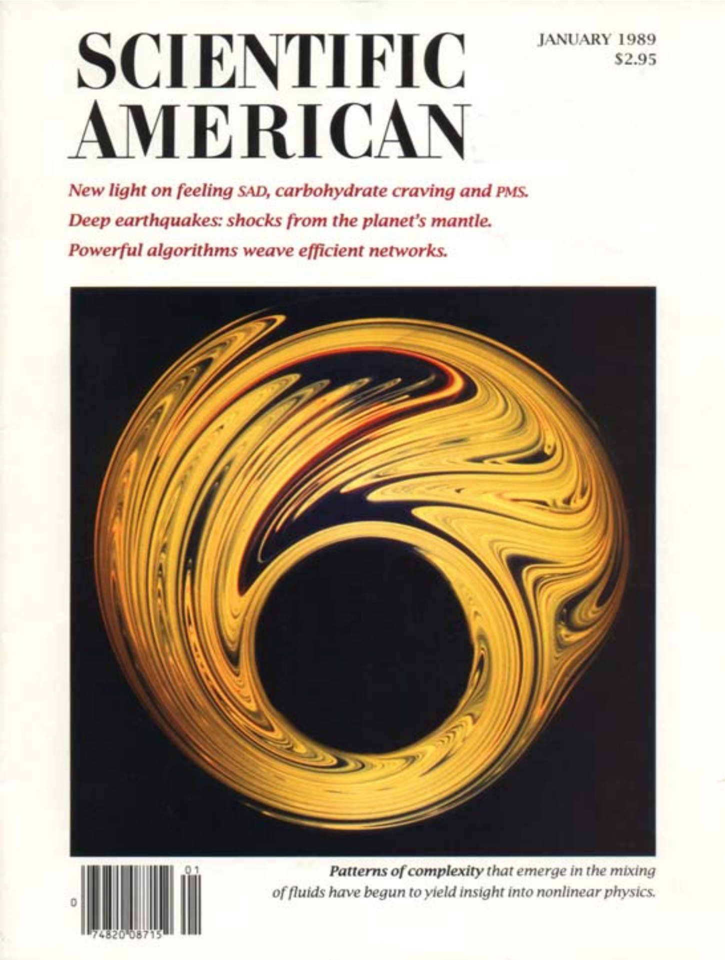
Candidate Flows





Cavity Flow

Ottino et al. 1986

Eccentric Cylinder Flow

Chevray et al. 1986,
Ottino et al. 1986

Start with streamline portrait…

2D flow, there exists Ψ(x,y) such that …

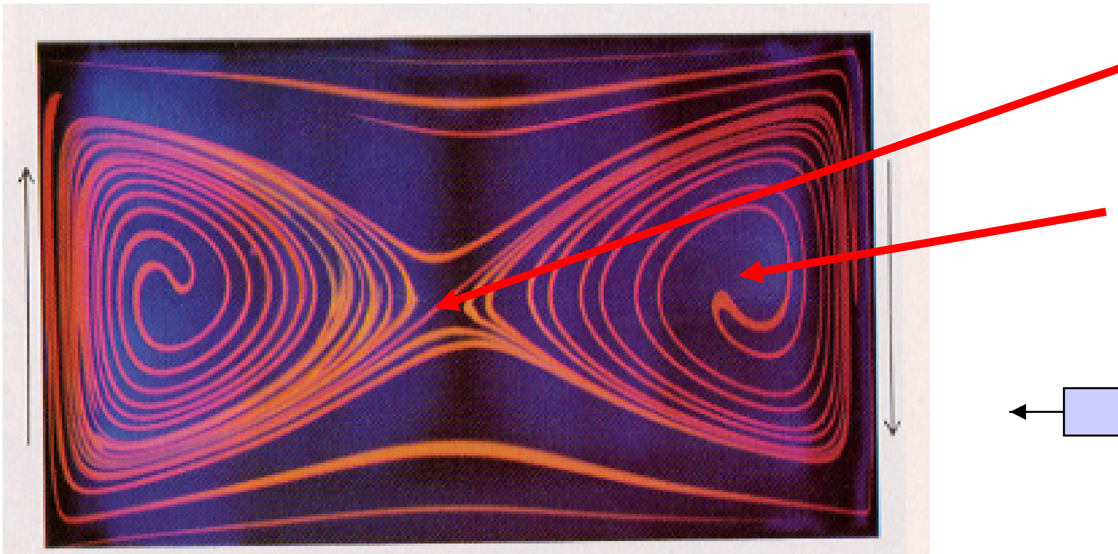
$$v_x = \frac{dx}{dt} = \frac{\partial \psi}{\partial y}, \qquad v_y = \frac{dy}{dt} = -\frac{\partial \psi}{\partial x}$$

System has a Hamiltonian structure (Aref)

Critical points **p** s.t. **v**(**p**)=0

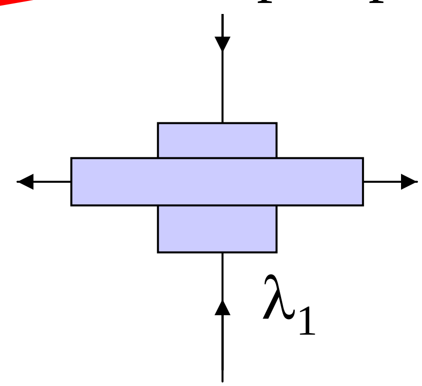
steady Ψ = Ψ(x,y), unsteady Ψ = Ψ(x,y,t) → chaos

Dynamical Systems Theory: invariant sets, manifolds



Hyperbolic point

Elliptic point

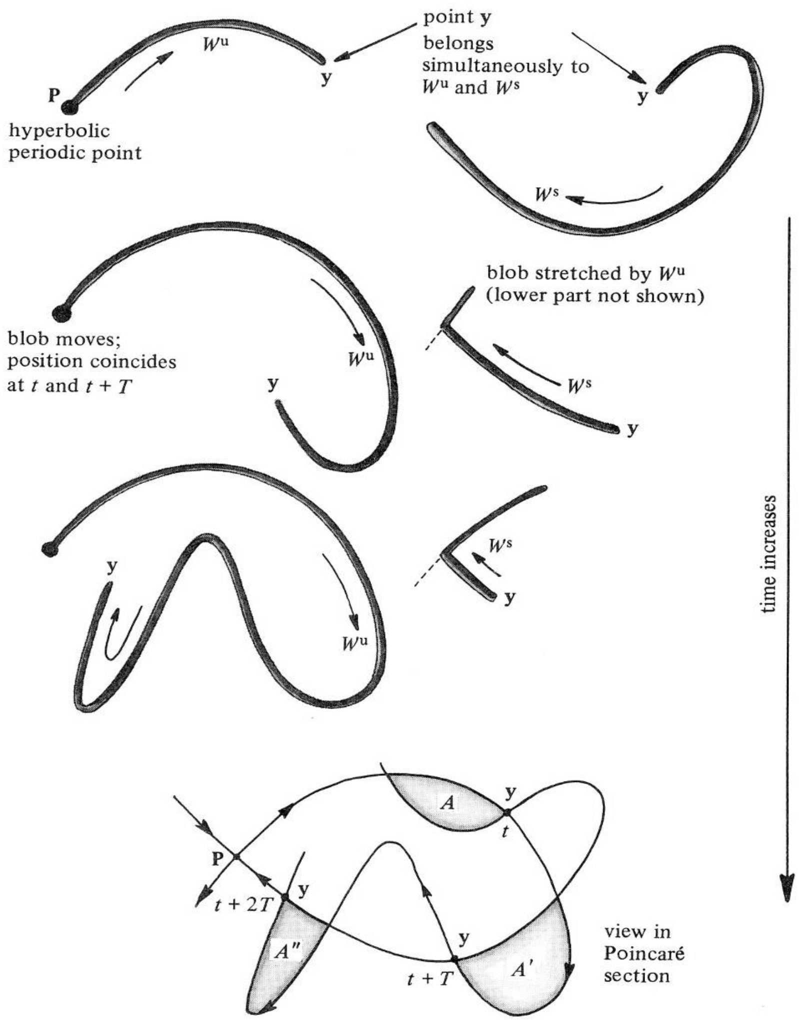


$$W^s(P) = \{ all \quad X \in R^n \quad s.t. \quad \Phi_t(X) \to P \quad as \quad t \to \infty \}$$

$$W^u(P) = \{ all \quad X \in R^n \quad s.t. \quad \Phi_t(X) \to P \quad as \quad t \to -\infty \}$$

Rising, Khakhar ~1986

Figure 5.8.3. Visualization of $W^u(\mathbf{P})$ in a time-periodic system by means of a fluid mechanical experiment. The manifolds $W^s(\mathbf{P})$ and $W^u(\mathbf{P})$ intersect transversally.



Picture near integrable system

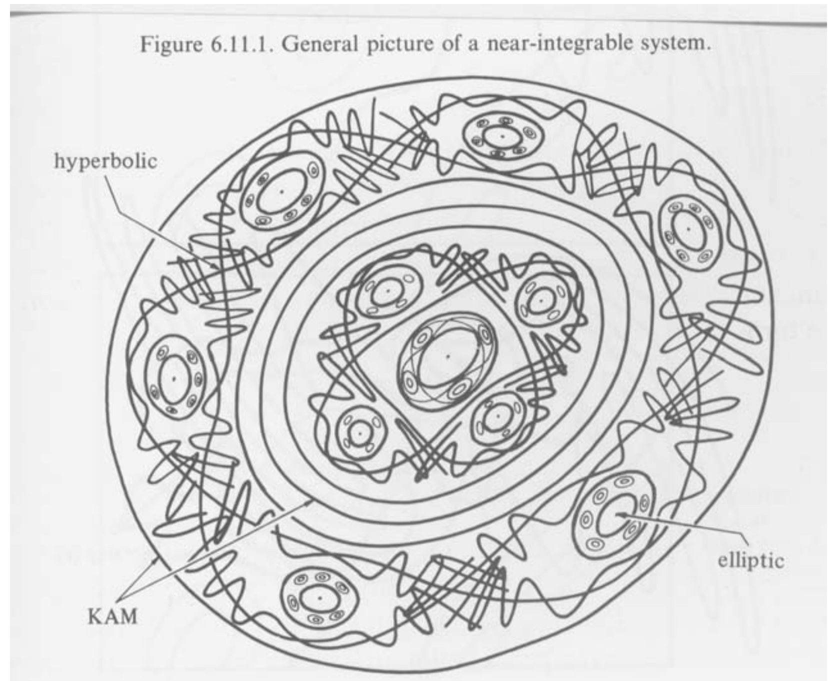


Figure 6.11.1. General picture of a near-integrable system.

Kolmogorov-Arnold-Moser theorem

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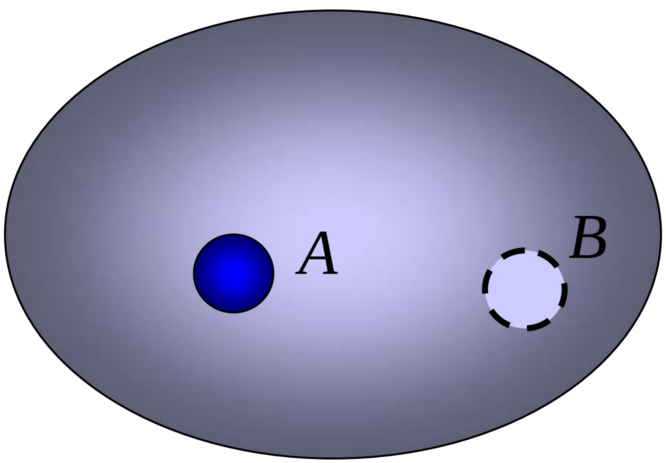
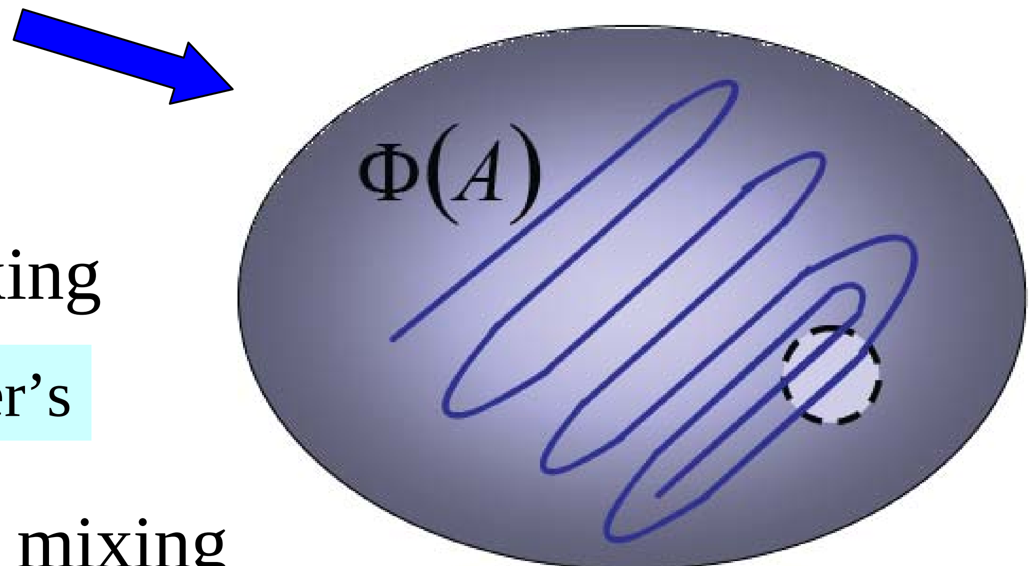
Definition of Mixing

$$\Phi(A) \cap B \neq 0$$

For “all of most” *A*’s and *B*’s





Measure-theoretic mixing

Baker’s

Strongly topologically mixing

Horseshoe

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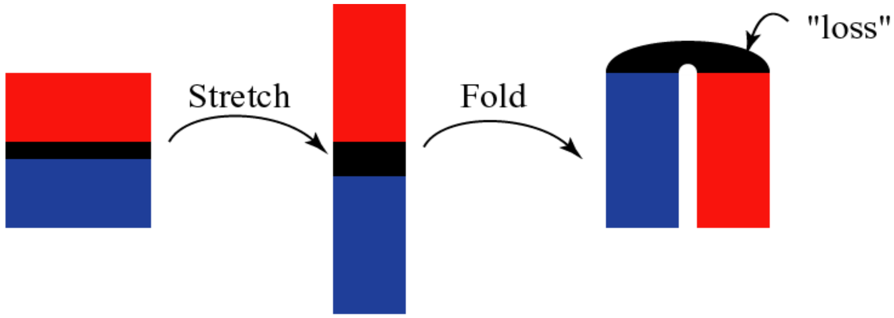
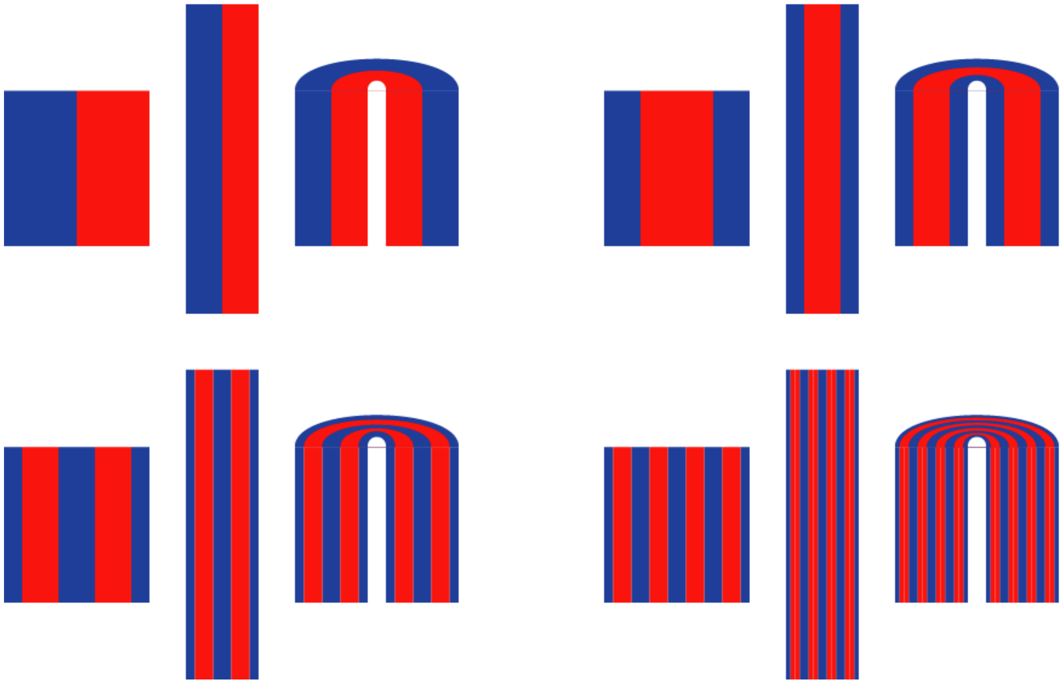
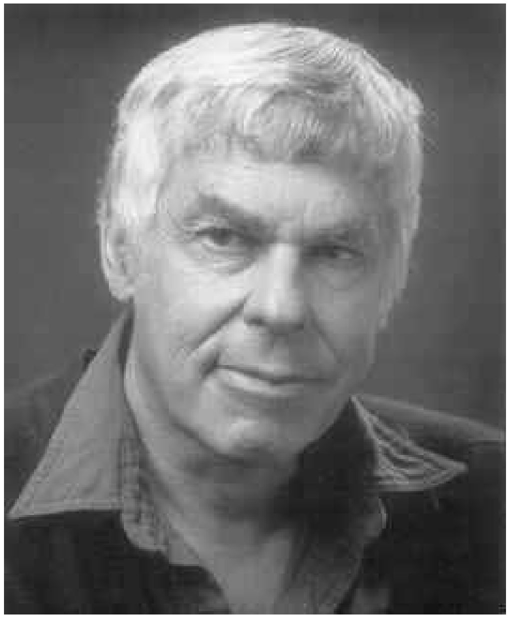
Smale Horseshoe map







Stephen Smale
b. 1930

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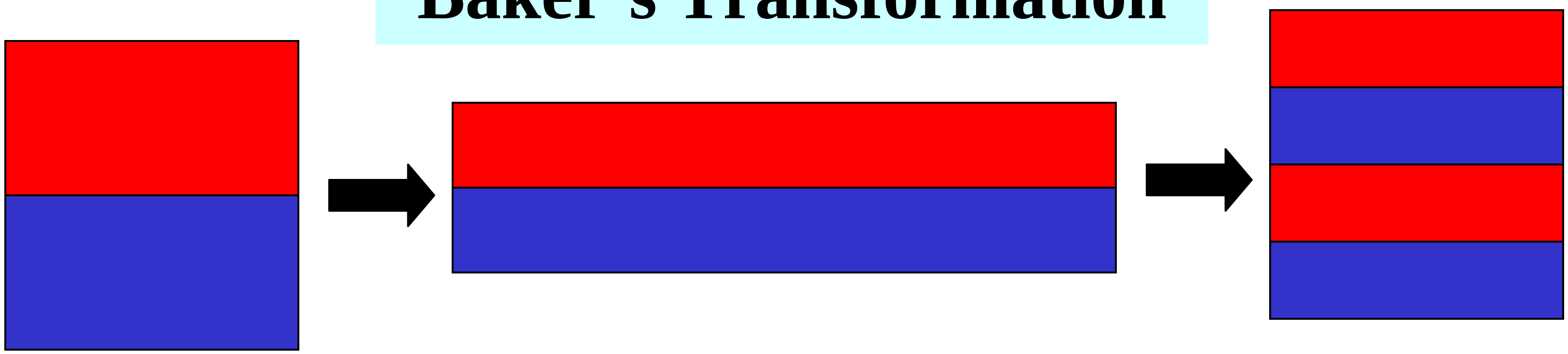
Baker's Transformation



$(x, y) \rightarrow (2x, y/2)$, cut and stack

x_1=0.10110001…→ x_2=0.0110001…→ x_3=0.110001

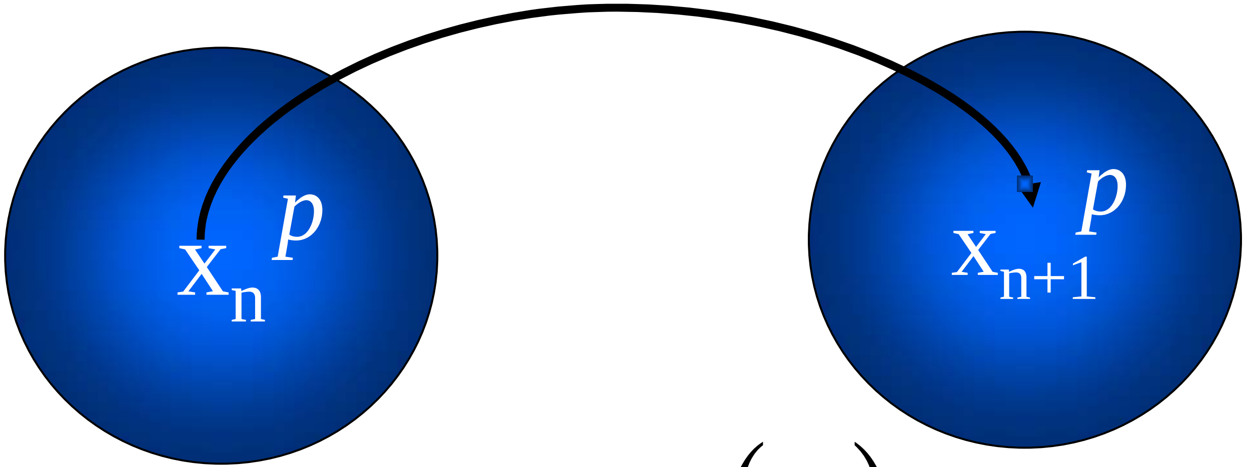
***x* left shift….and**

y_1=0.01110100…→ y_2 =0.101110100…→ y_3 = 0.01110100…

leftmost digit of *x* becomes leftmost digit of *y*

Bernoulli shift

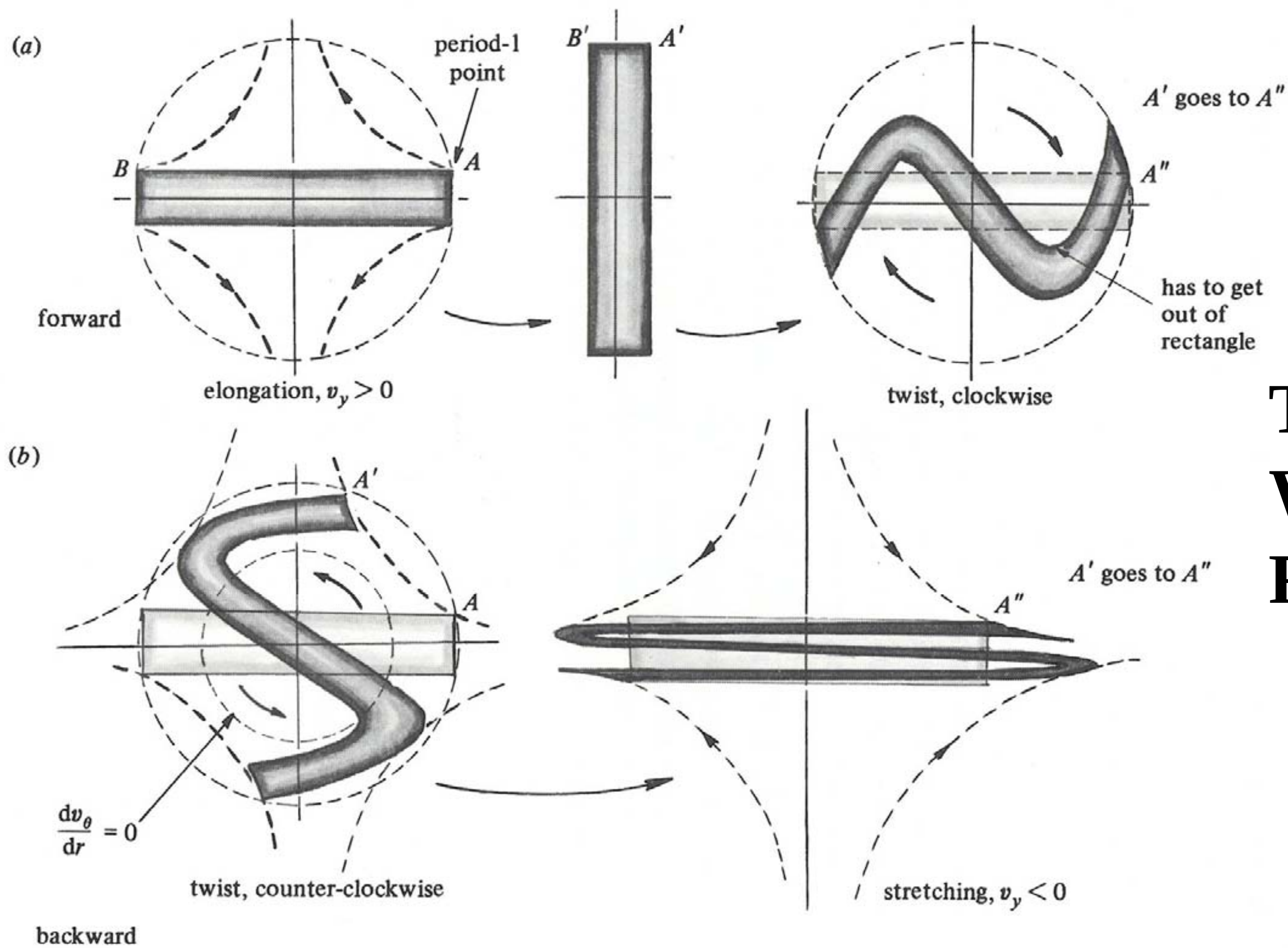
transformations or maps…
Brouwer's fixed point theorem (~1910)



$$p = \Phi(p)$$

Mapping of a singly connected domain onto itself
periodic points

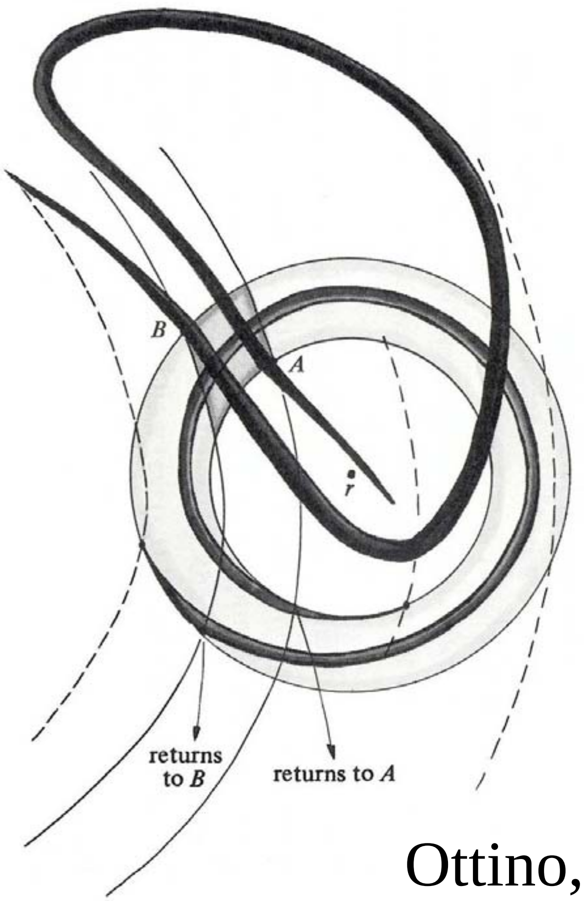
Figure 7.2.10. Horseshoe formation in the tendril–whorl flow. In (*a*) the elongational flow acts first, followed by a twist clockwise. In (*b*) the twist acts first, counter-clockwise, followed by the elongational flow. (*c*) Actual example corresponding to $\alpha = 10.000$ and $\beta = 2.180$.



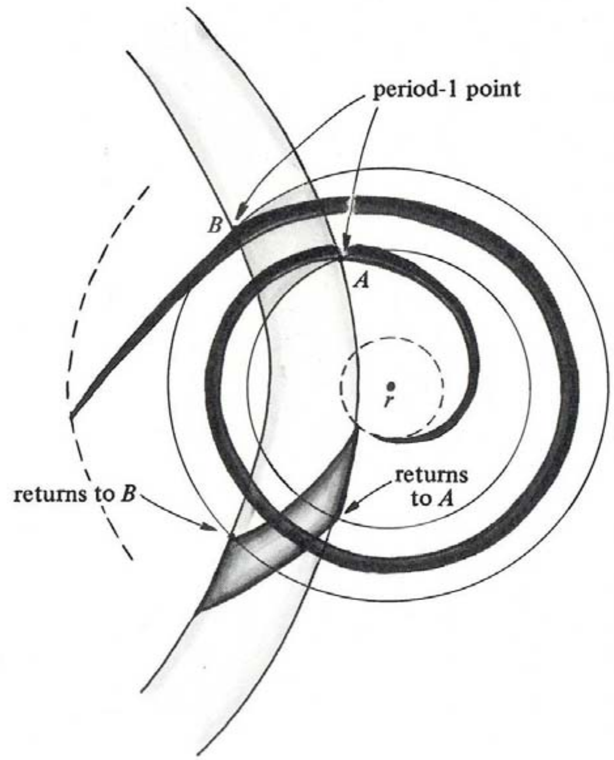
Tendril-Whorl Flow

Ottino 08

Construction of a Horseshoe



Ottino, 1989



Blinking
Vortex
Flow
(Aref 1984)

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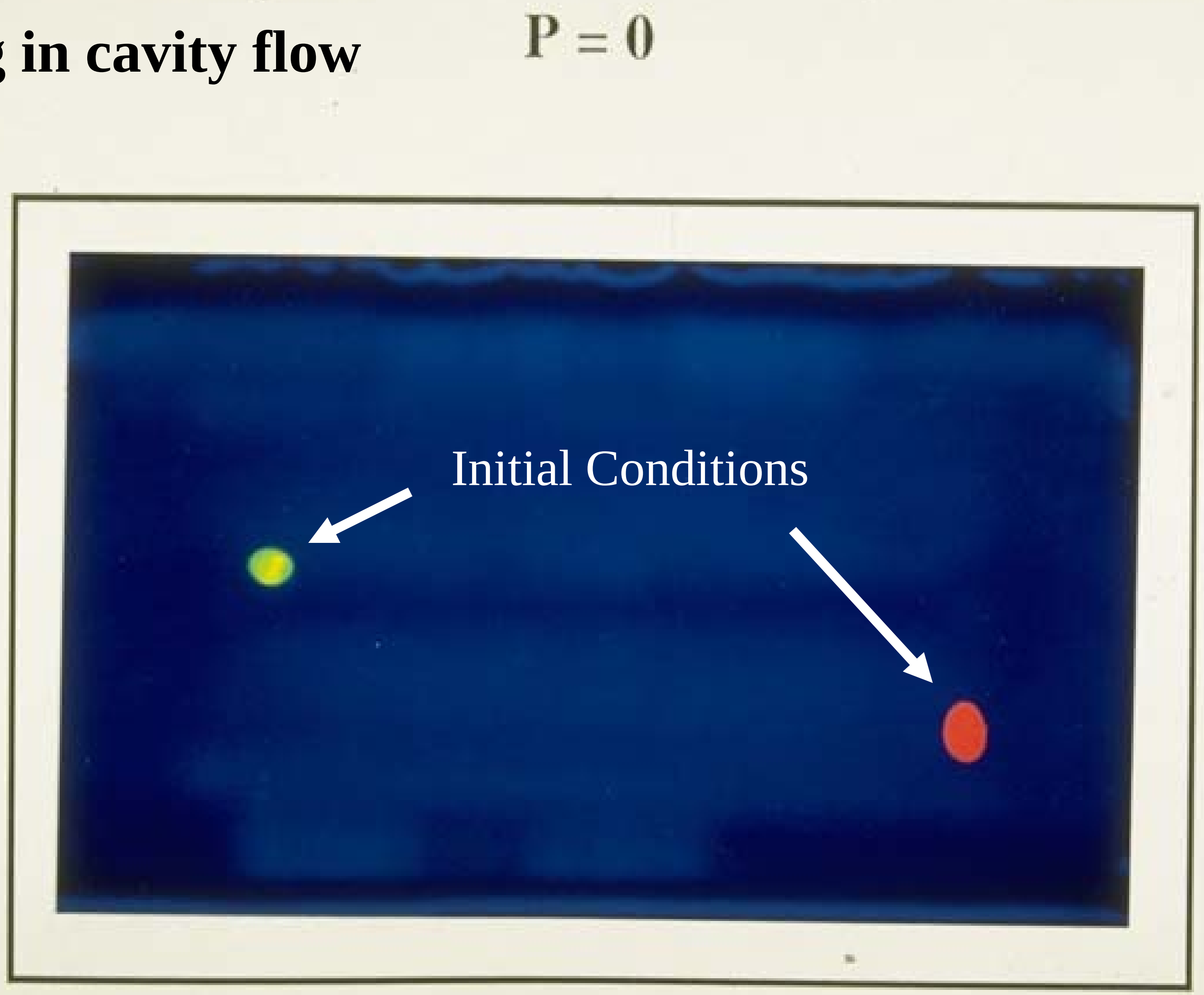
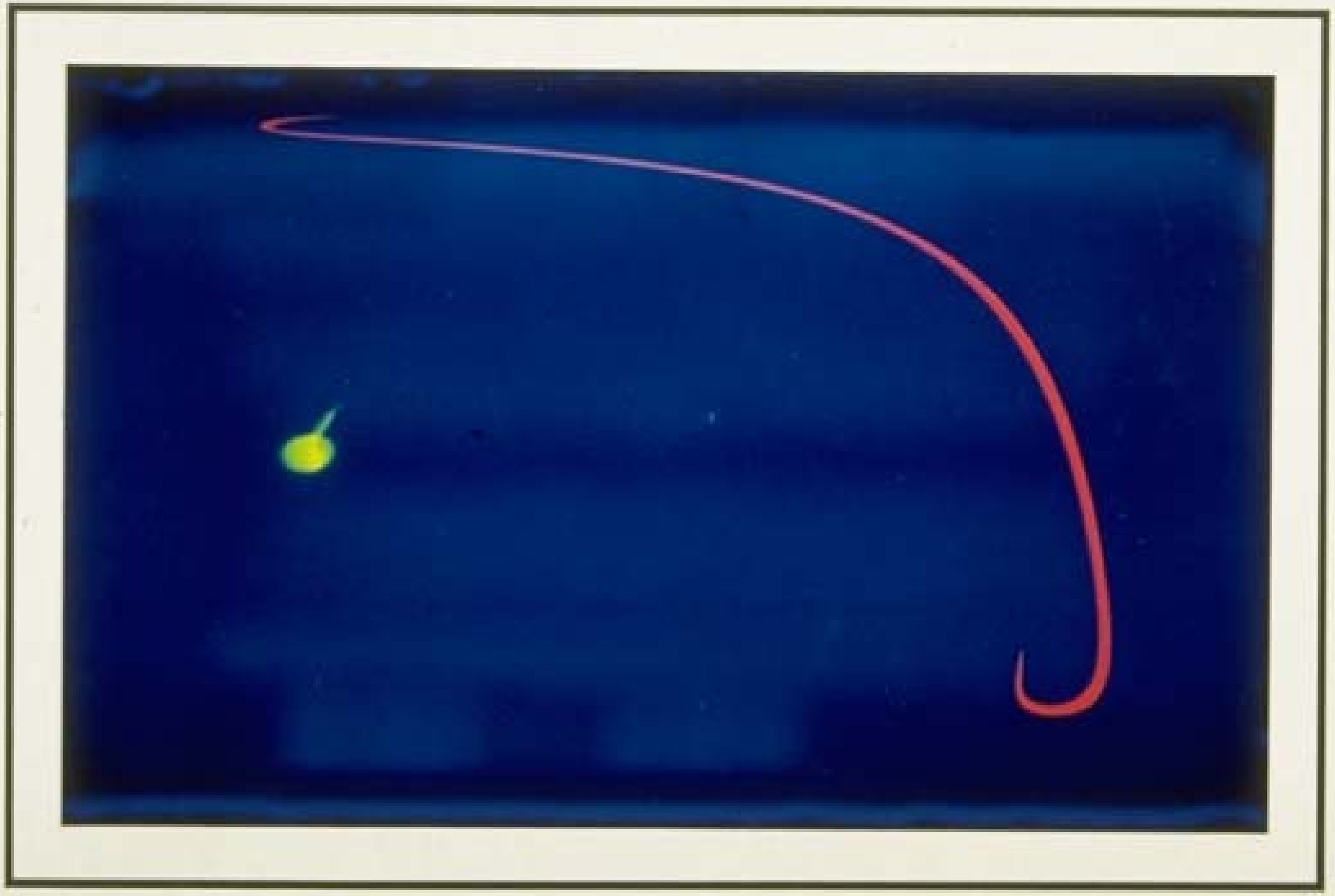
Mixing in cavity flow



P = 2



P = 3

P = 4

P = 5

P = 6

P = 8

P = 8

Motion of Islands P = 8.25

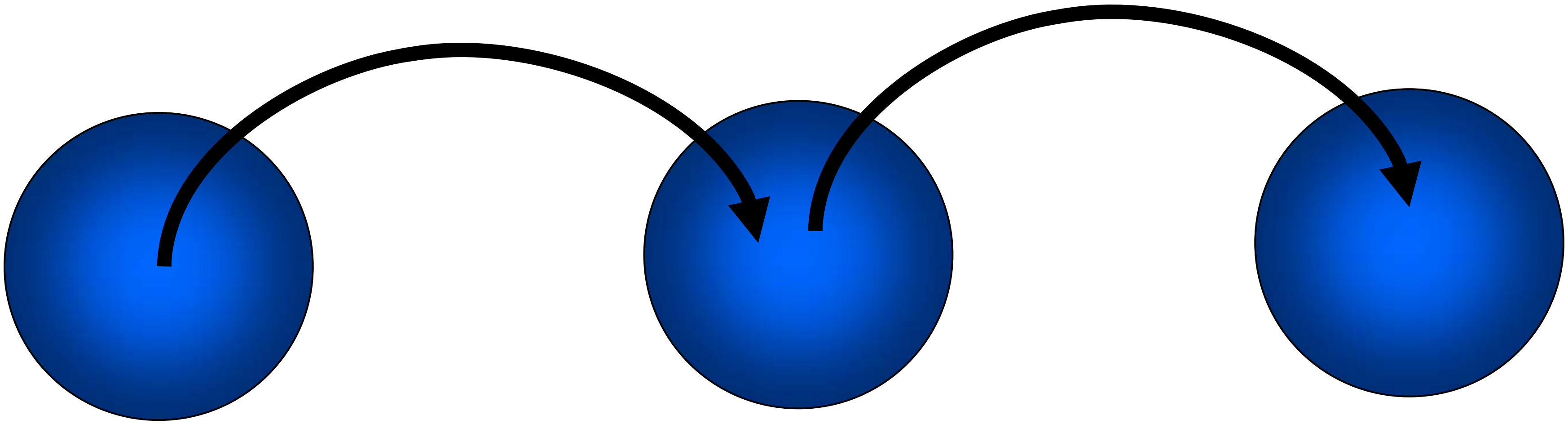
P = 8.50

P = 8.75

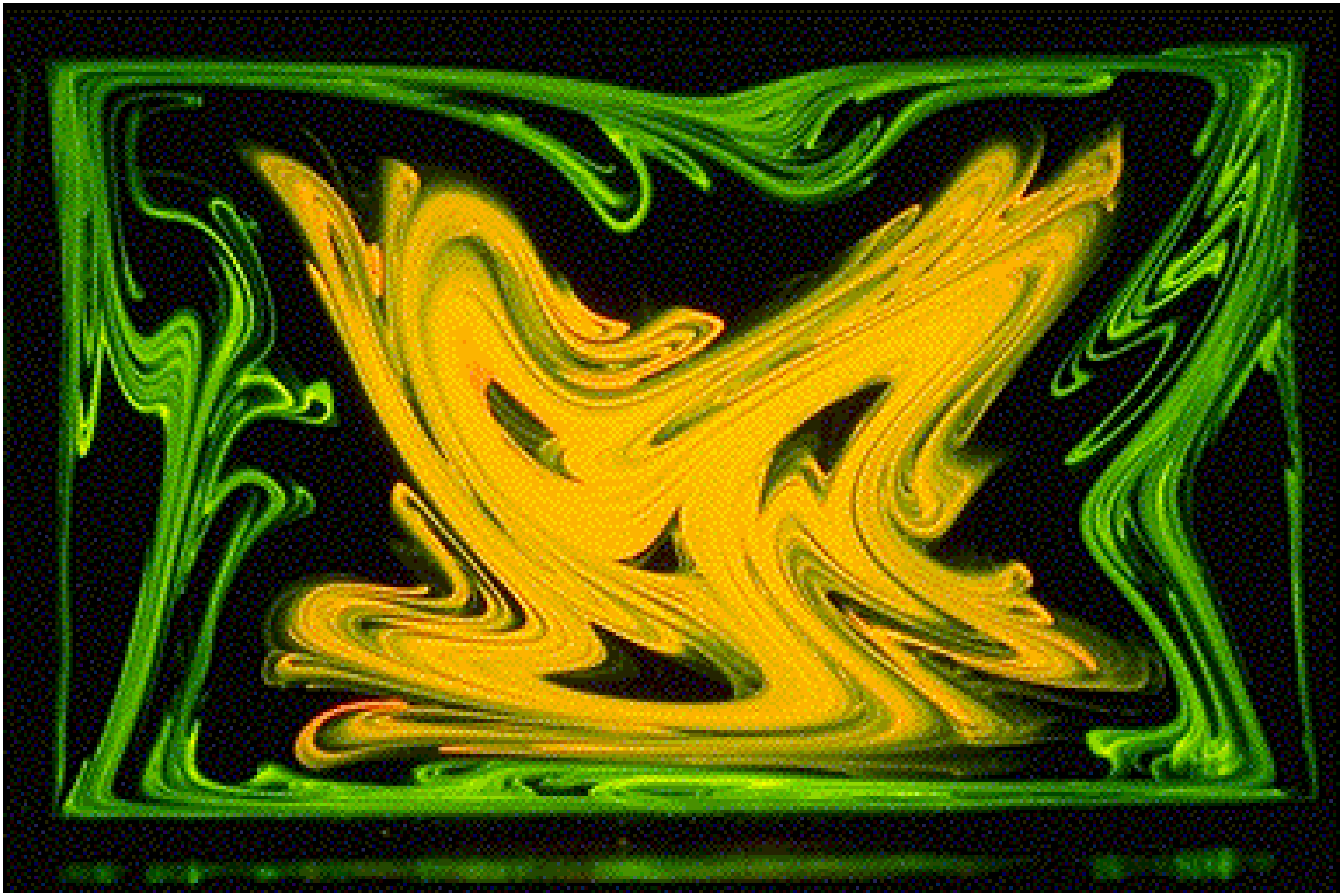
P = 9.00

Higher Order Periodic Points

Composition of fundamental maps



There exist *x* such that $x = \Phi^n(x)$ for any *n*



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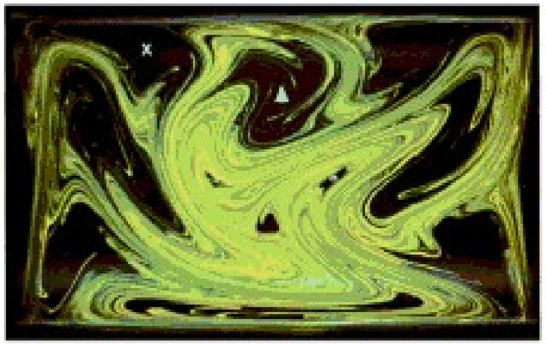
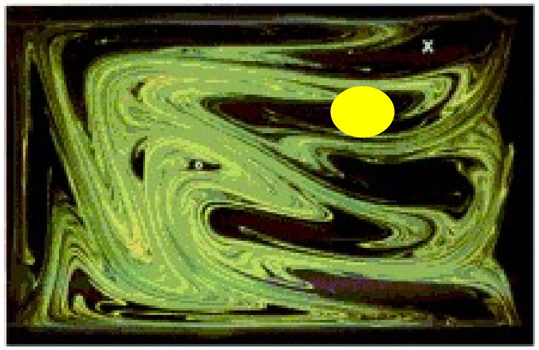
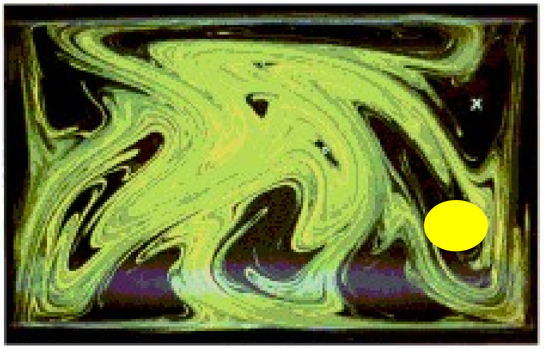
10 periods



20 periods

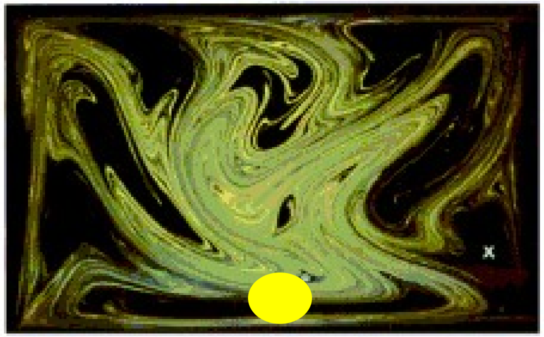


$20\frac{1}{4}$ periods



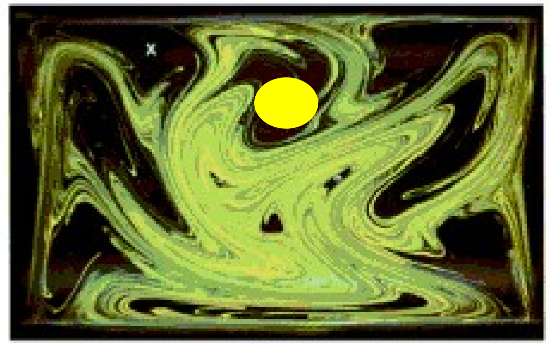
$20\frac{1}{2}$ periods

$20\frac{3}{4}$ periods

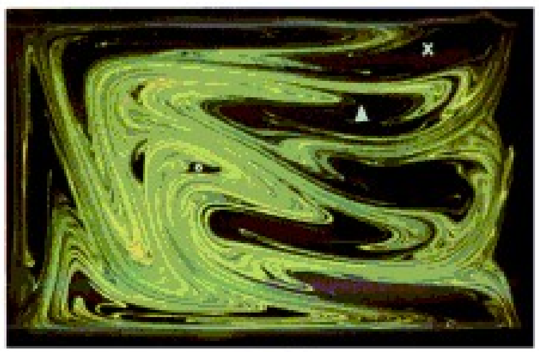


21 periods

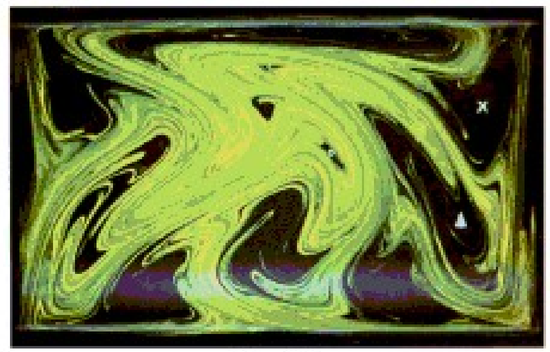
10 periods



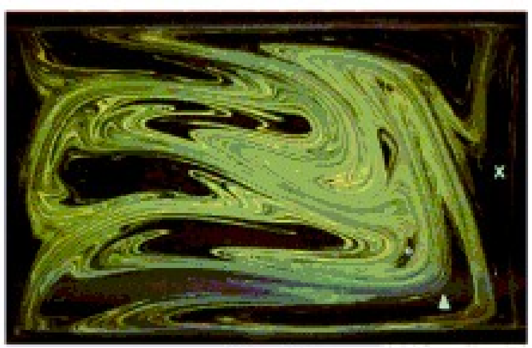
20 periods



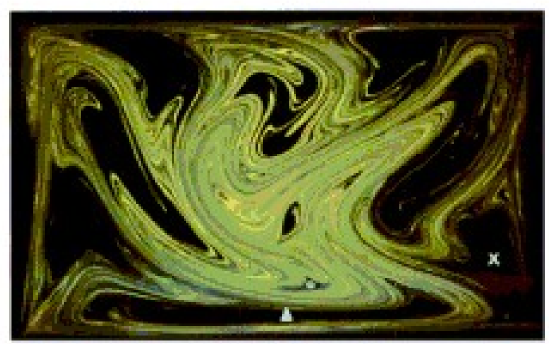
$20\frac{1}{4}$ periods



$20\frac{1}{2}$ periods

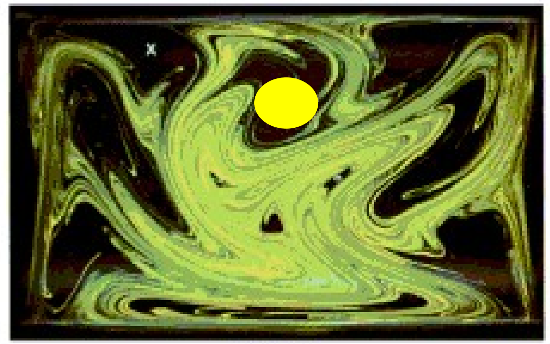


$20\frac{3}{4}$ periods

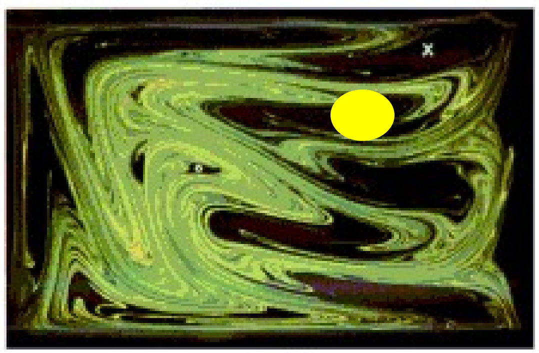


21 periods

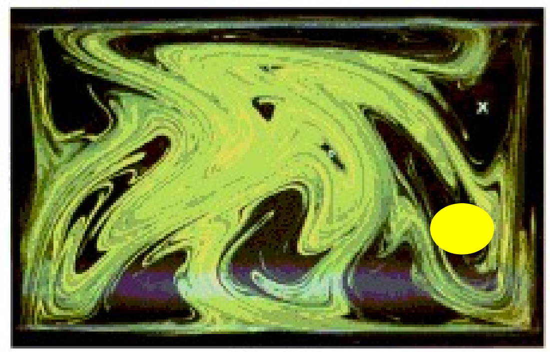
10 periods



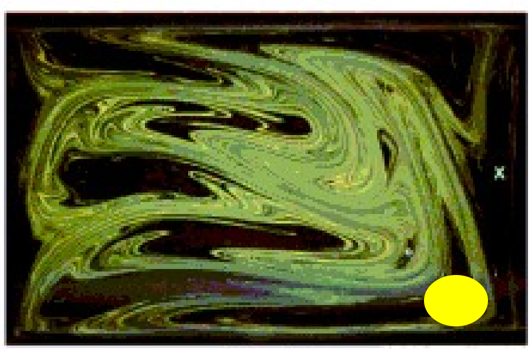
20 periods



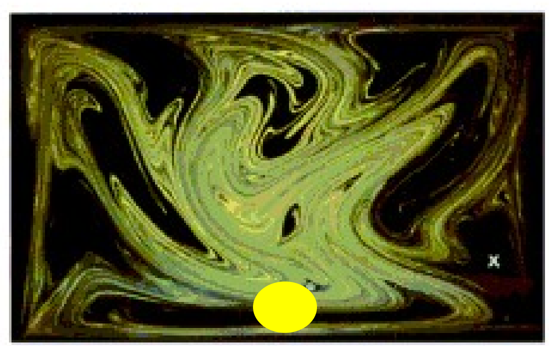
$20\frac{1}{4}$ periods



$20\frac{1}{2}$ periods



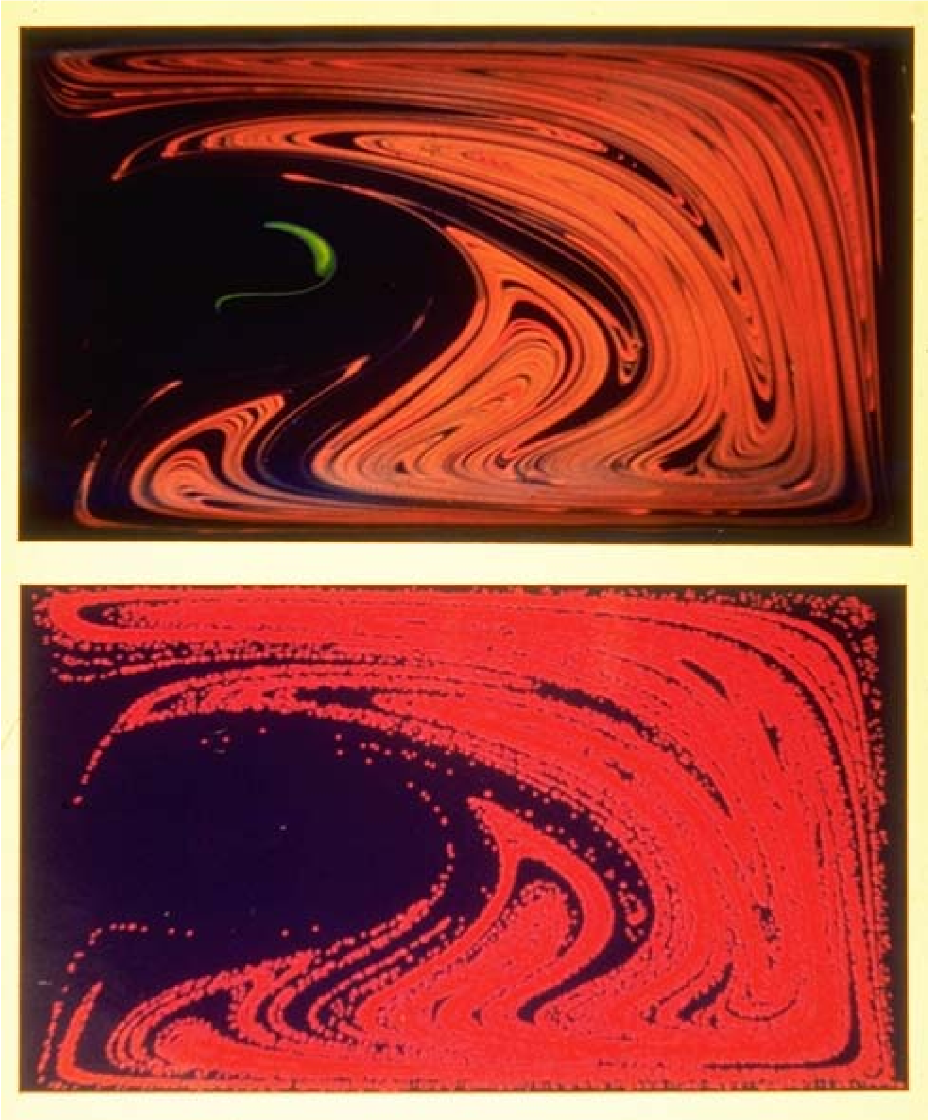
$20\frac{3}{4}$ periods



21 periods

Computations in Chaotic Systems

Experiment



Computer Simulation

$$dx \approx (dx_0)e^{\sigma t}$$

σ Liapunov exponent

Types of Errors

Round-off error
Integration
Discretization
Parameter mismatch

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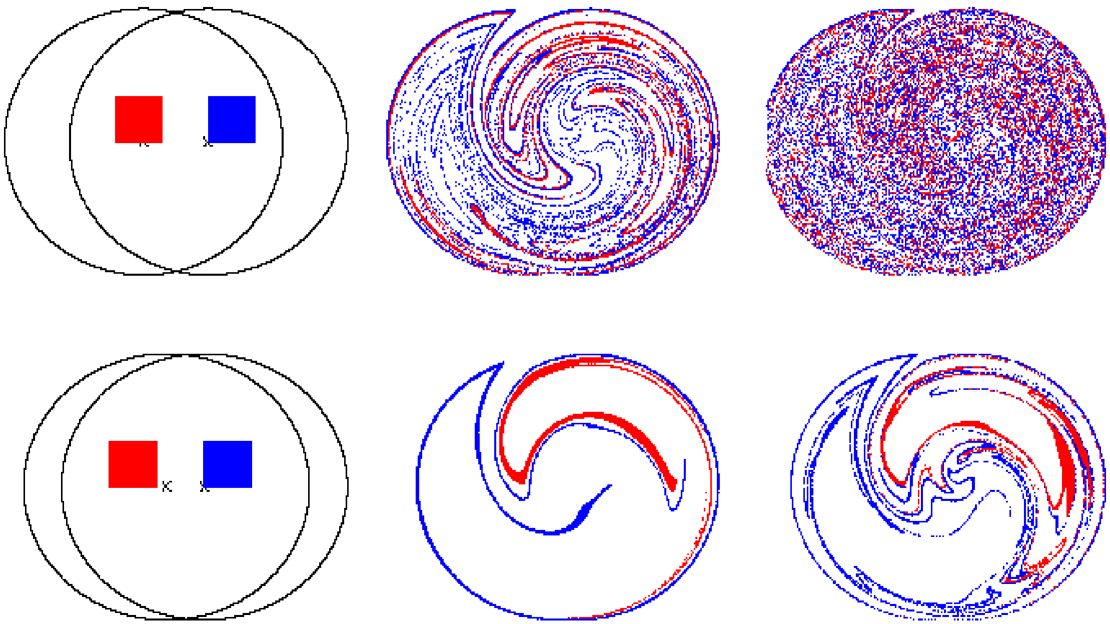
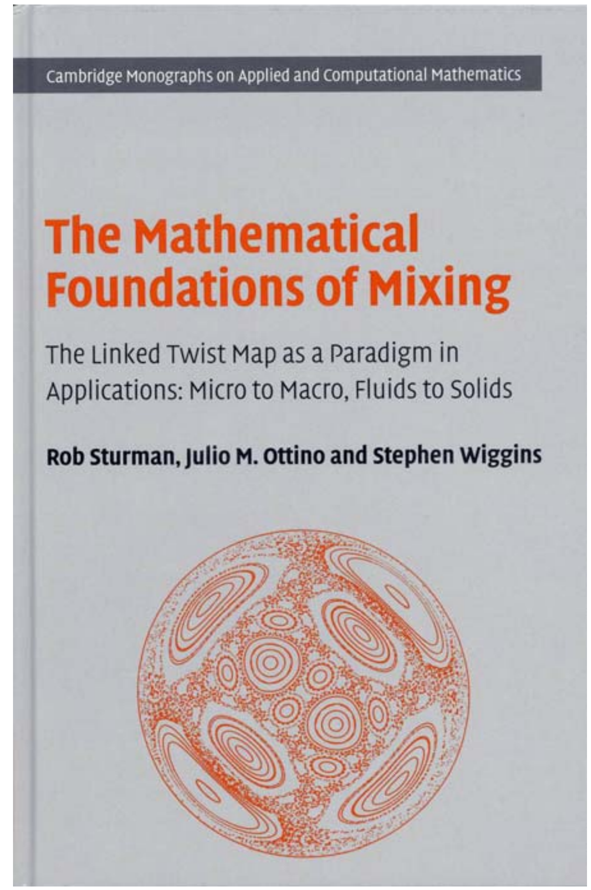
Linked Twist Maps and Mixing

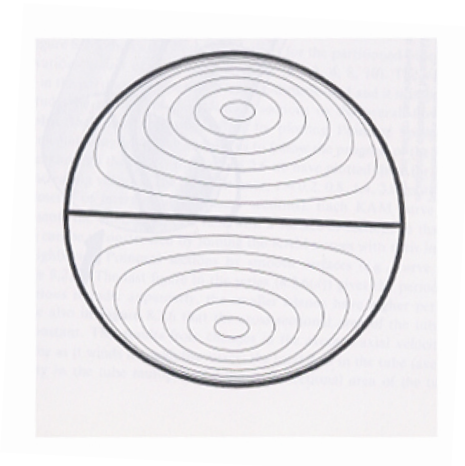


$n = 0$ $n = 5$ $n = 10$

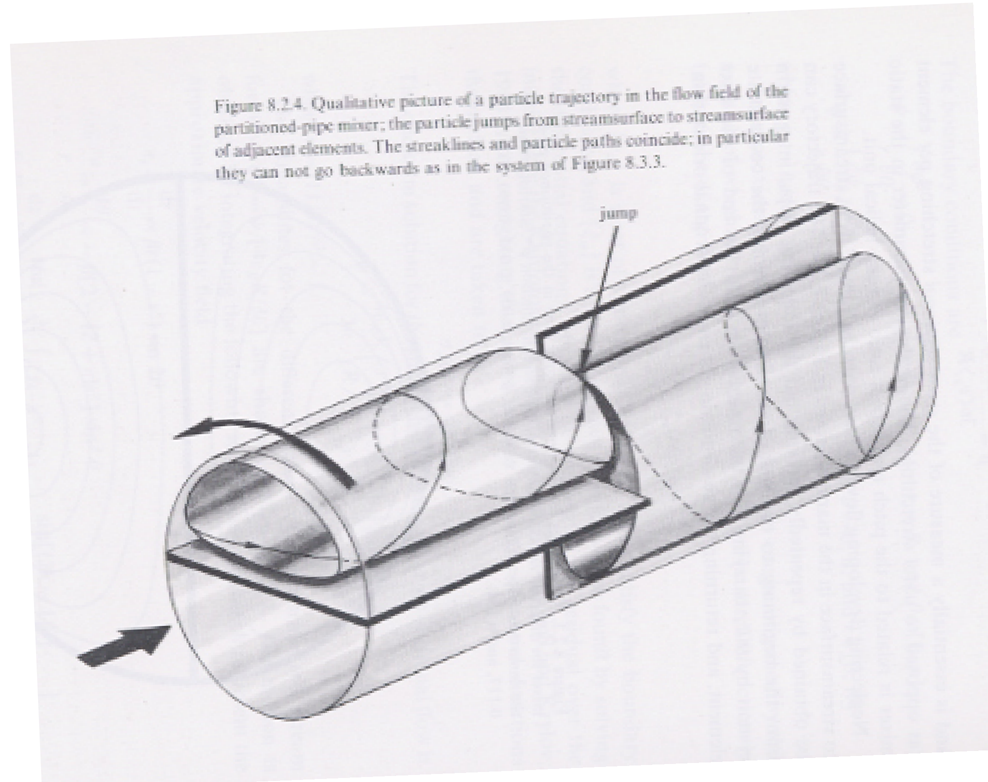


Rigorous mathematical foundation –can show that “mathematical mixing” exists on sets of full measure (with Sturman and Wiggins)

Realization: Partitioned Pipe Mixer



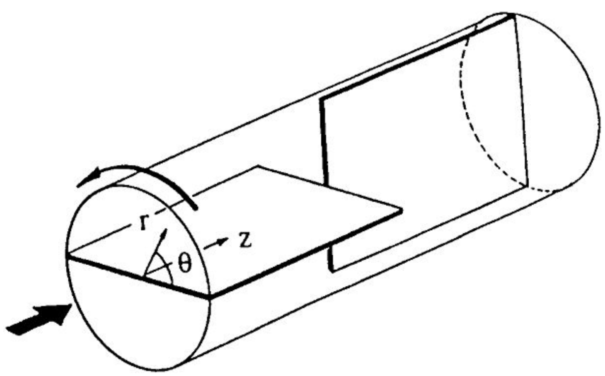
Cross section



Pictorial view of mechanism

Franjione, Khakhar, Ottino,

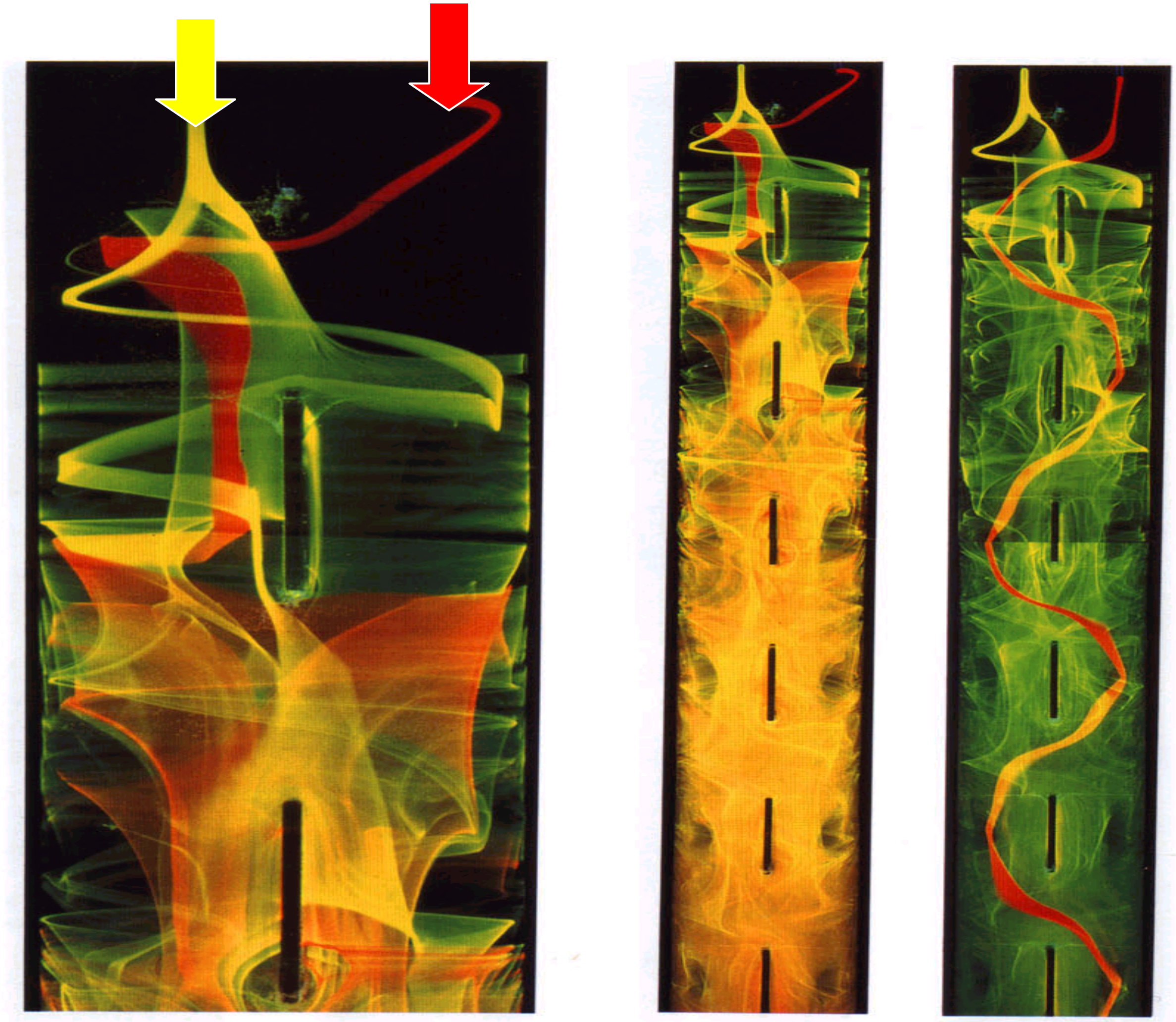
Chem. Eng. Sci. 1987



Parameter=cross twist/axial flow

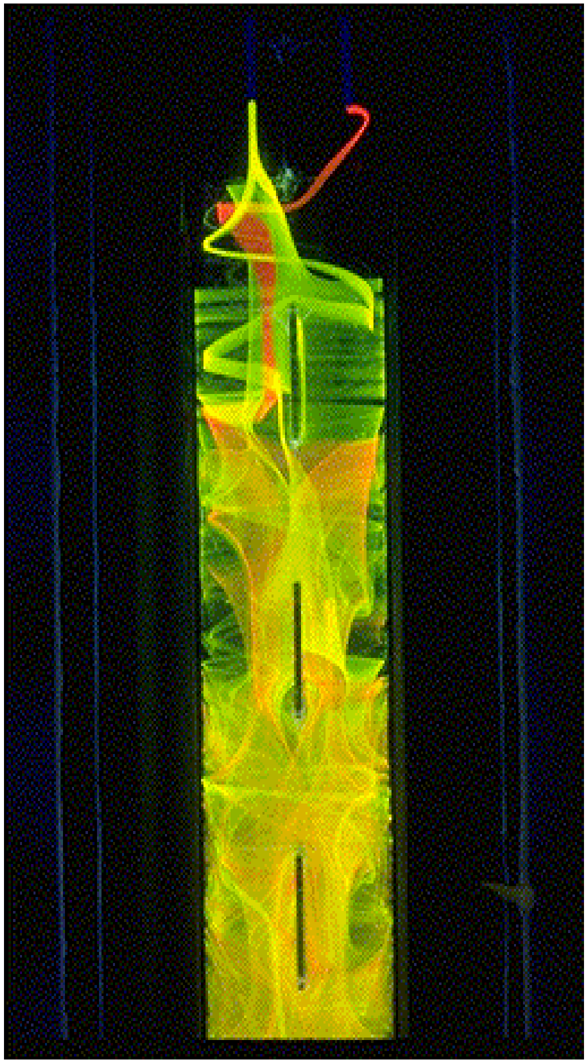
Difference: Injection point

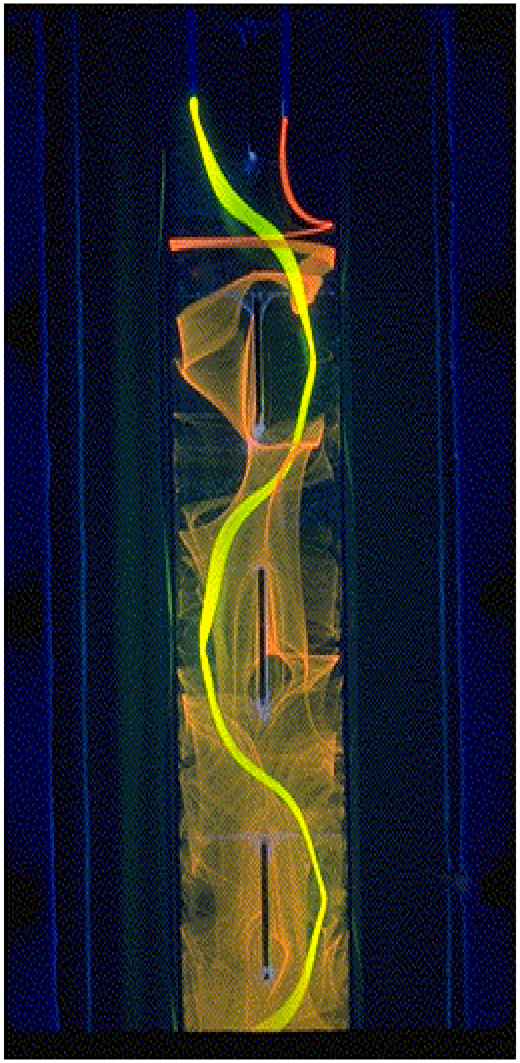
Kusch & Ottino, *JFM*

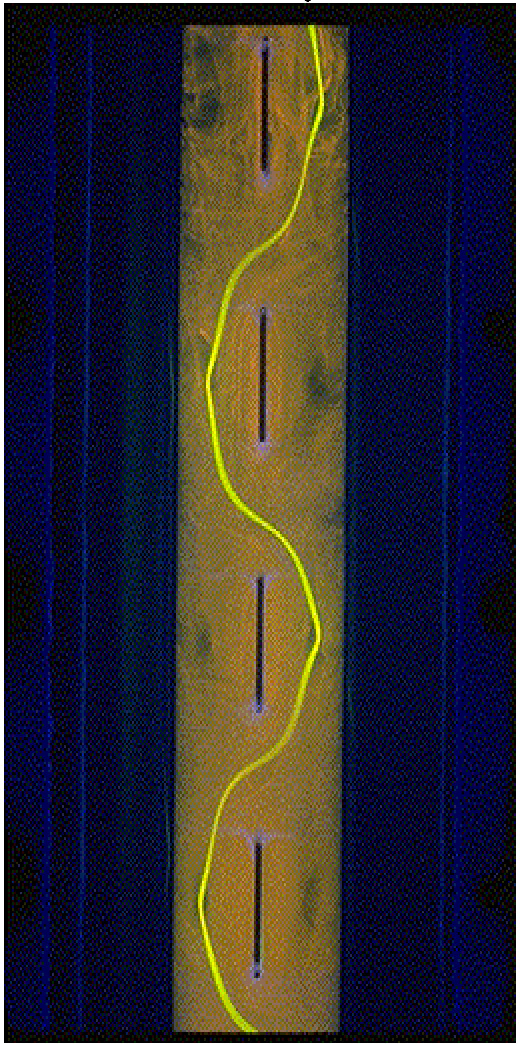


(a) (b) (c)

FIGURE 7. Order within chaos, existence of KAM-tubes. The mixing strength parameter for the flows is $\beta=10\pm0.3$, and the Reynolds numbers are $Re_{PPM:axial}=0.3$ and $Re_{PPM:cs}=1.8$. The differences between the flows (*b*) and (*c*) is the position of the orange dye streak feed point; (*a*) is a close-up of (*b*).





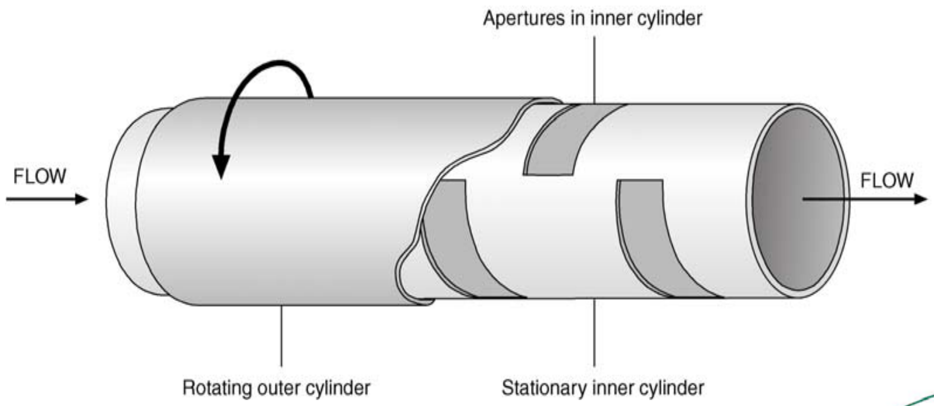


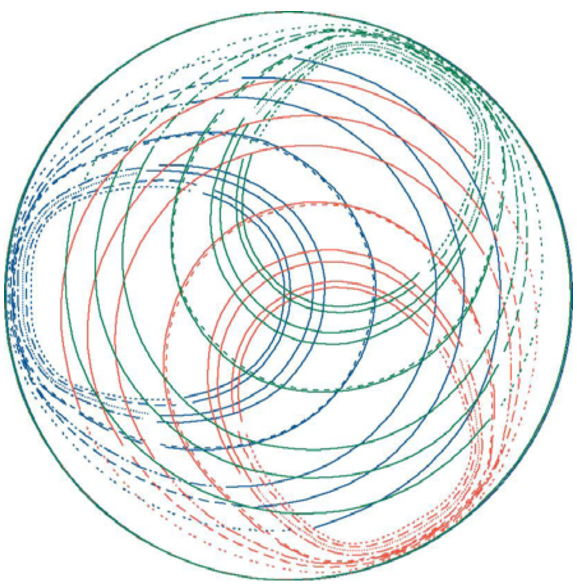
Continued on right…

JMOttino 08

Rotated Arc Mixer

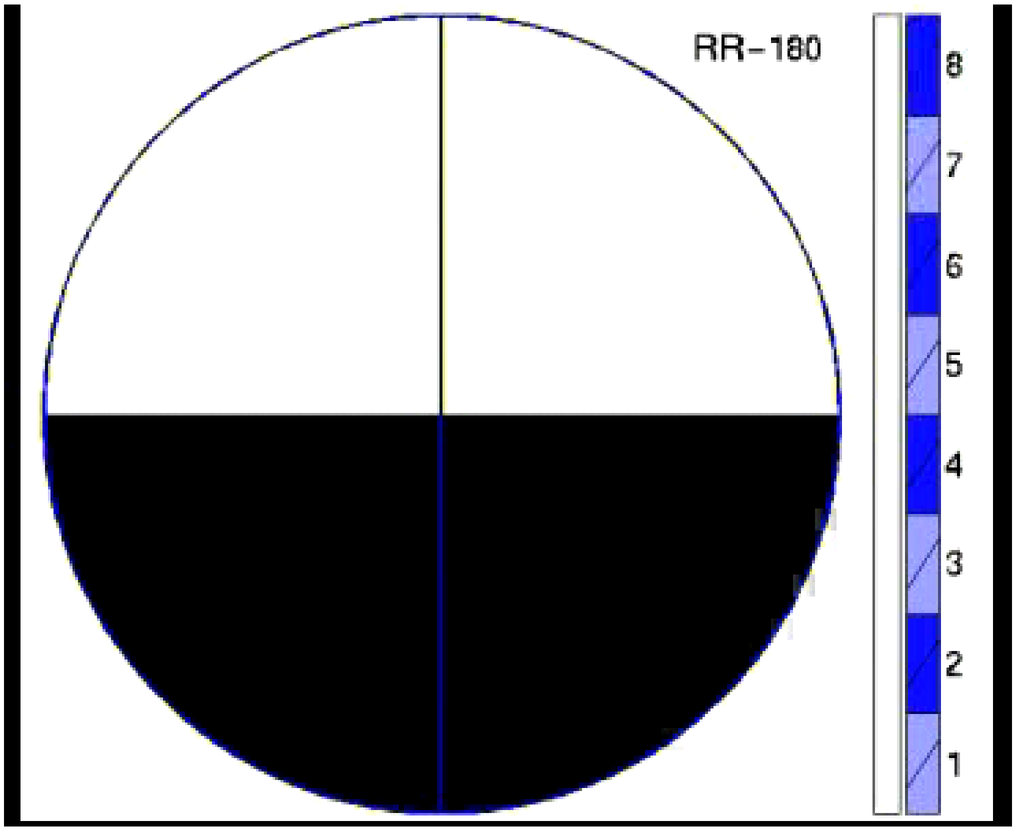
Motion of boundaries



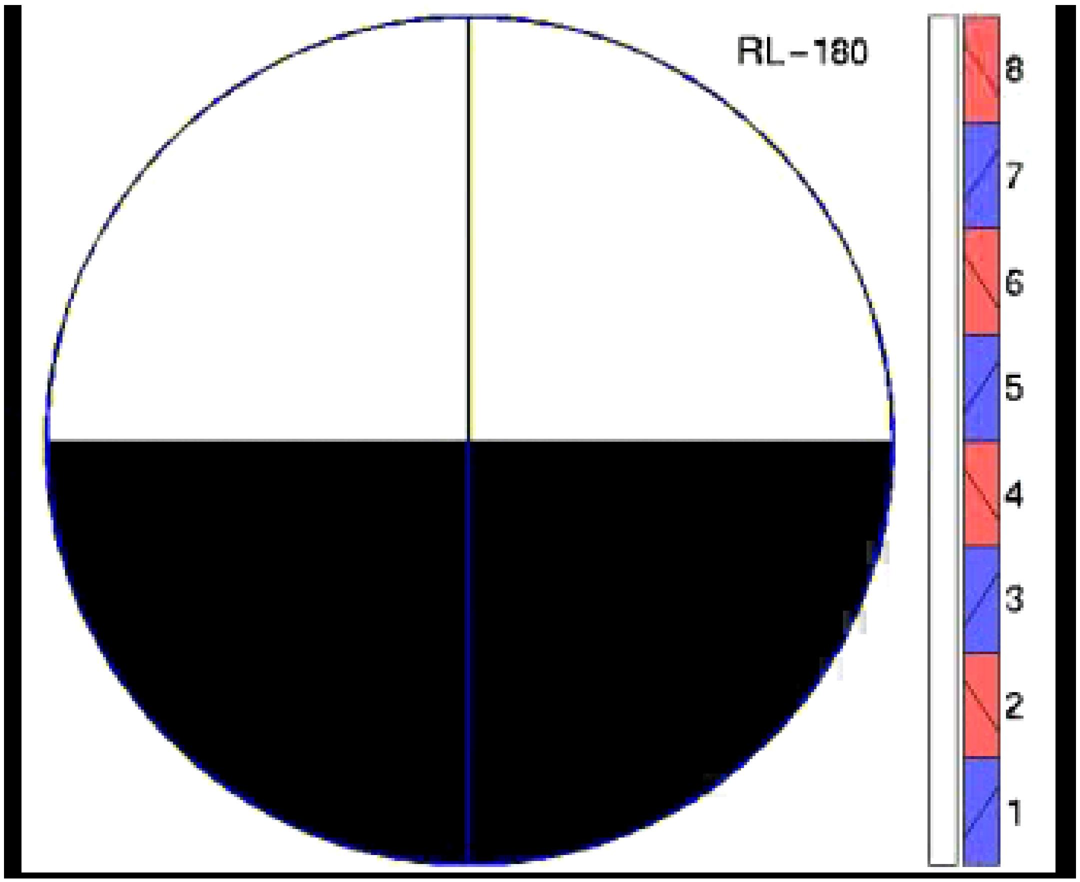


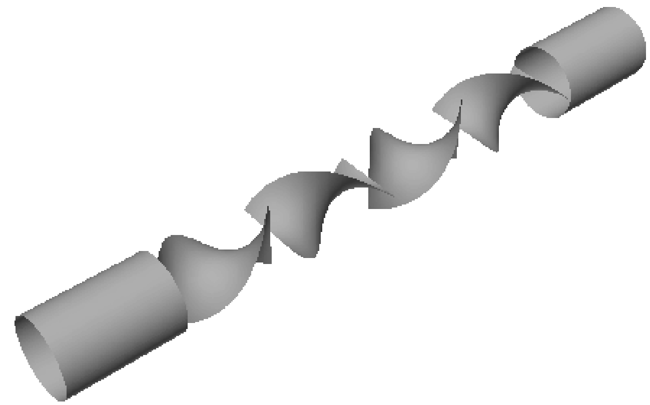
Metcalfe et al.
AIChE J, 2001

Parameter:
cross twist/axial flow

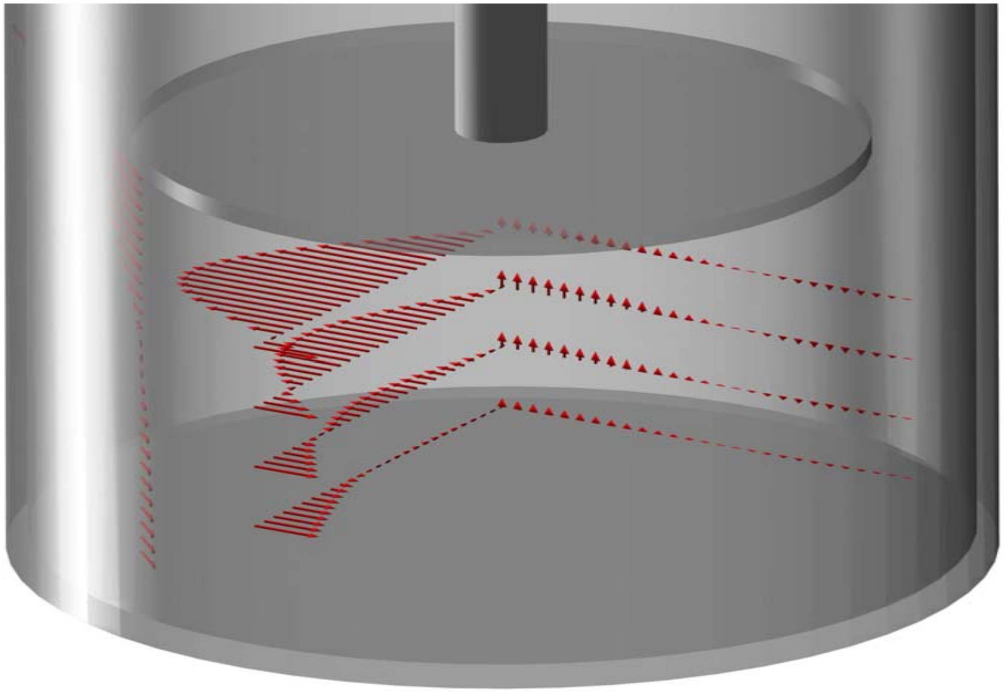


by
P.D. Anderson
O.S.Galaktionov,
G.W.M. Peters
H.E.H. Meijer
Eindhoven, Netherlands
2002-3

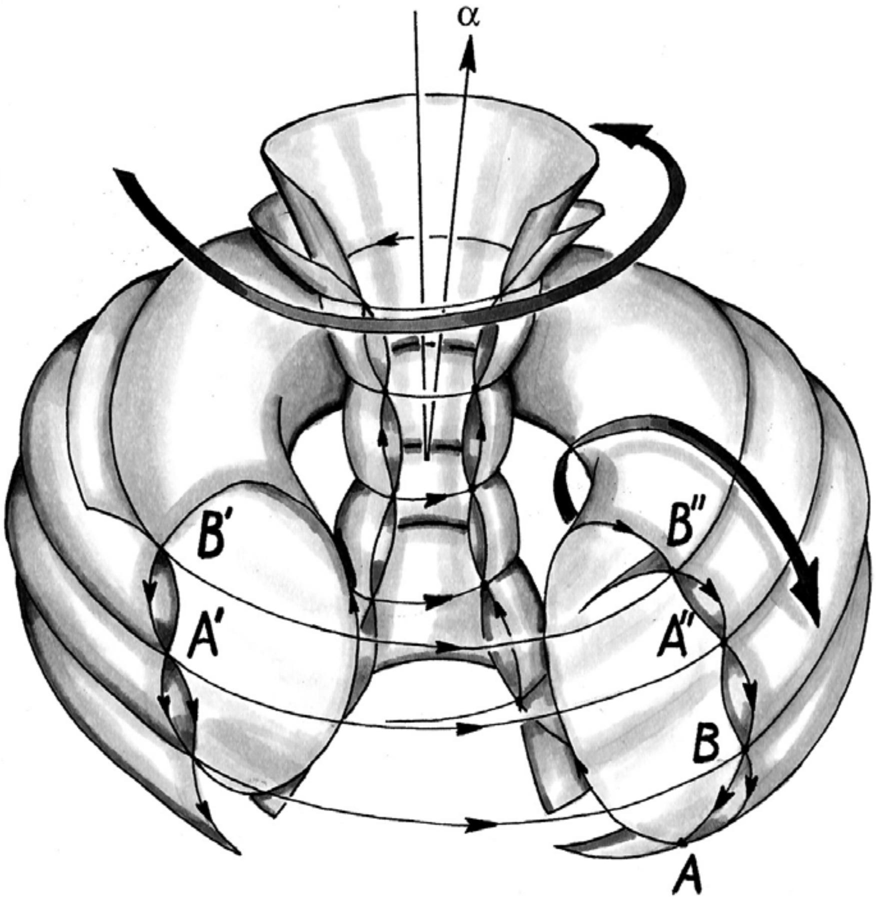




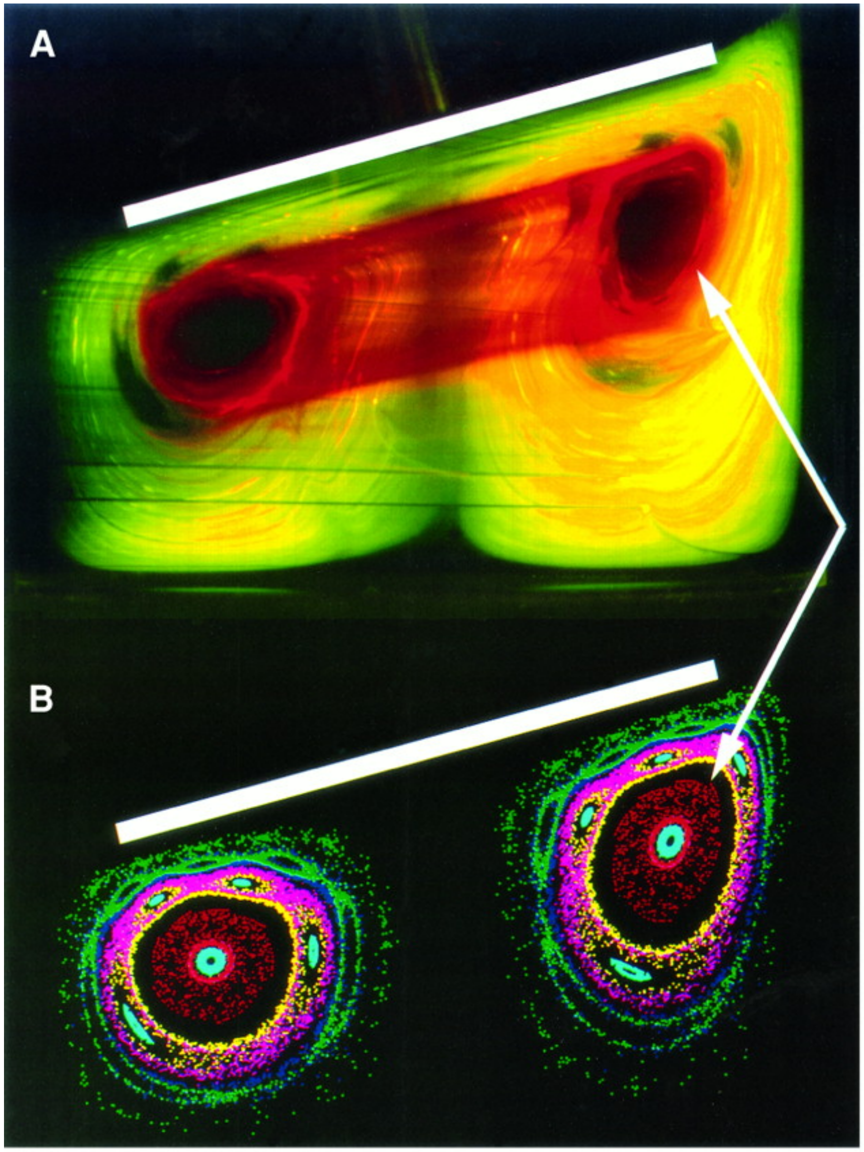
by
P.D. Anderson
O.S.Galaktionov,
G.W.M. Peters
H.E.H. Meijer
Eindhoven, Netherlands
2002-3

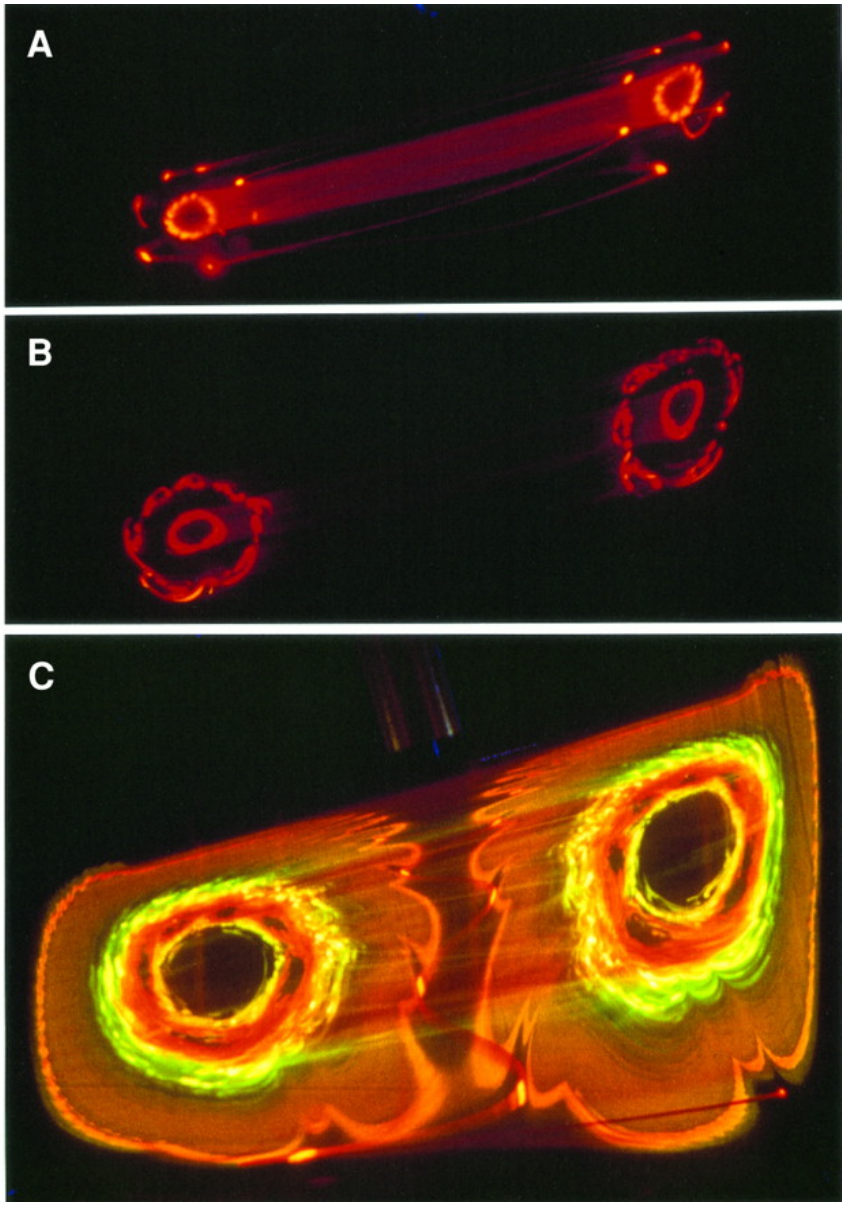


Break symmetry

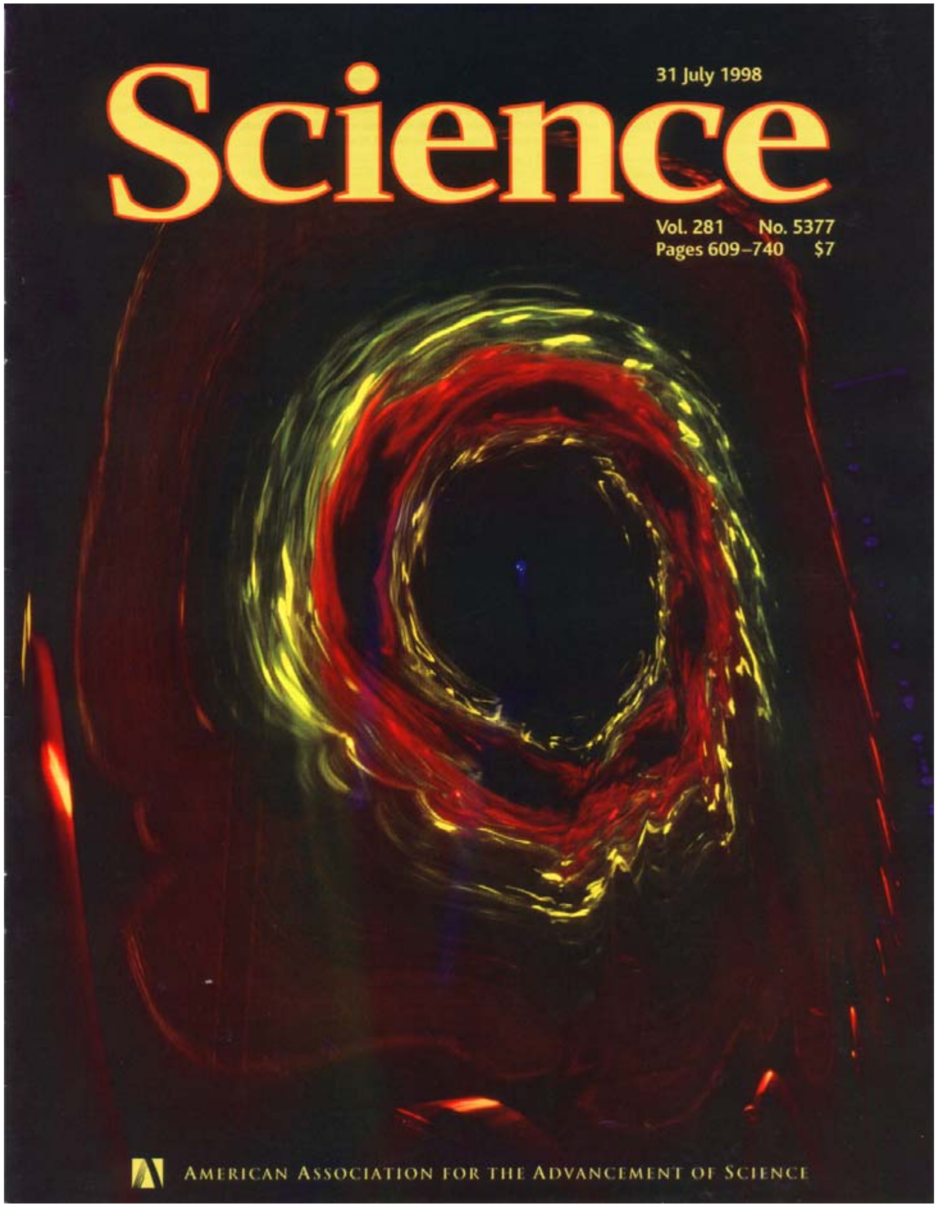


Fountain et al. *JFM* 2000



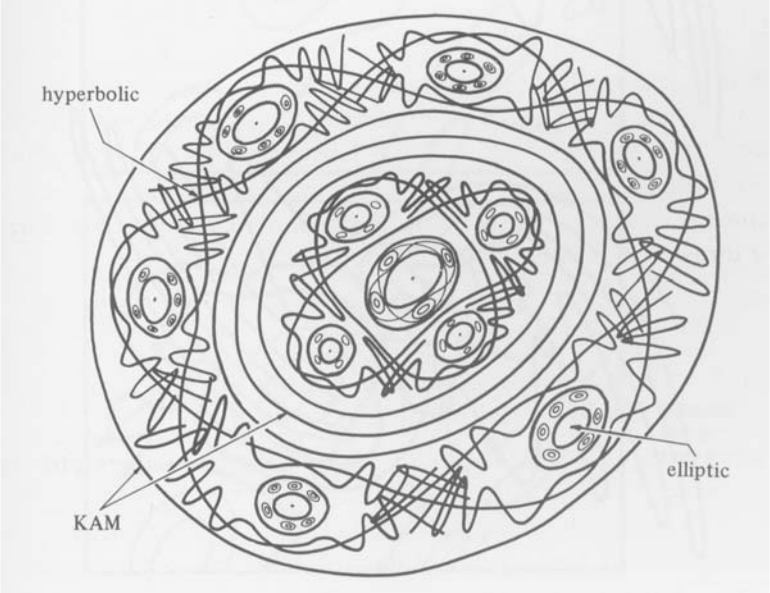


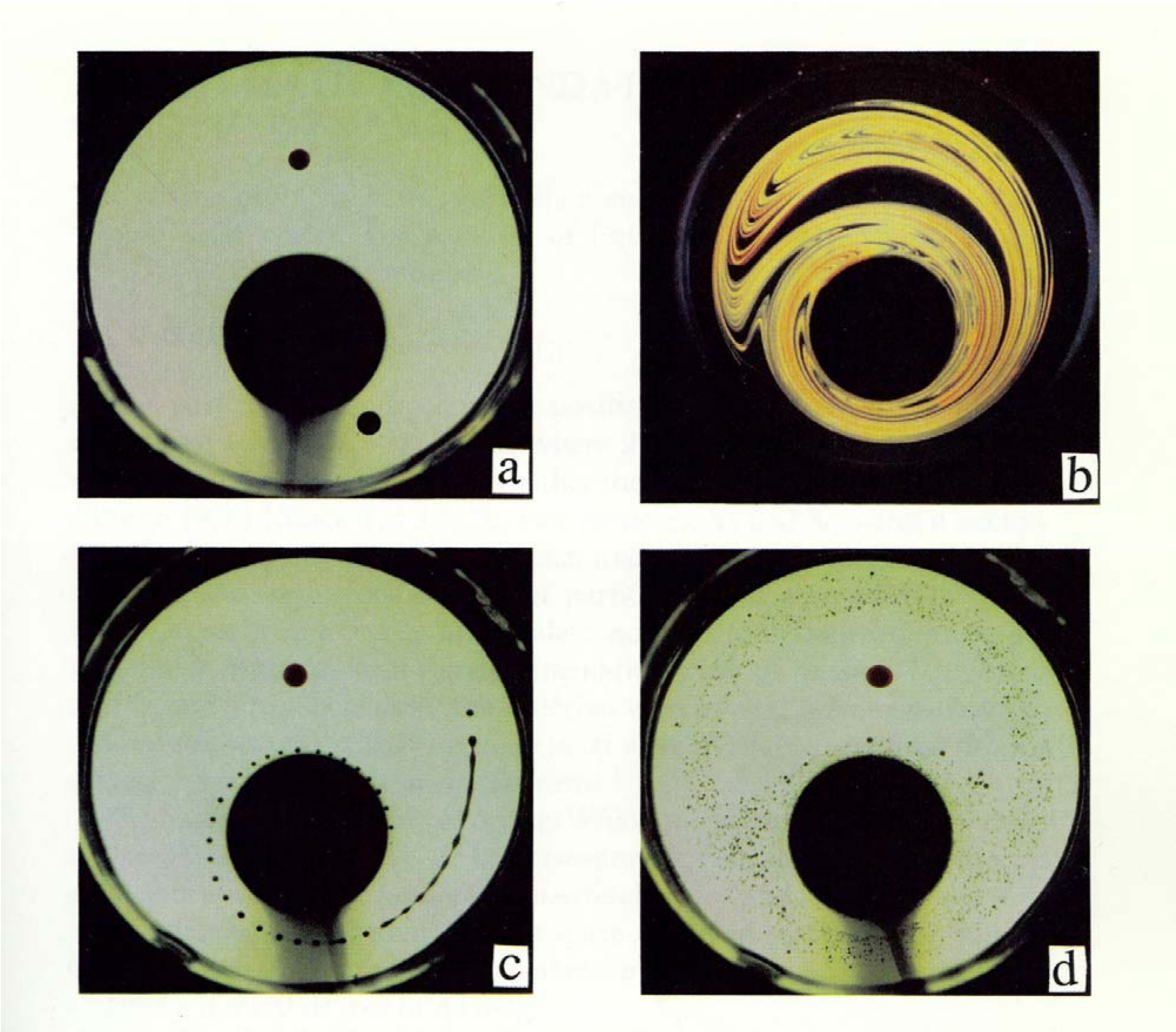
JMOttino 08



Kolmogorov-Arnold-Moser theorem

Figure 6.11.1. General picture of a near-integrable system.





Chaos as a Fabric

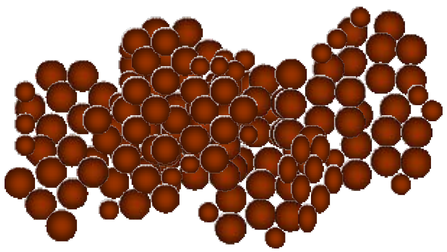
Drop breakup
in chaotic flows

Tjahjadi & Ottino
J. Fluid Mech. (1991).

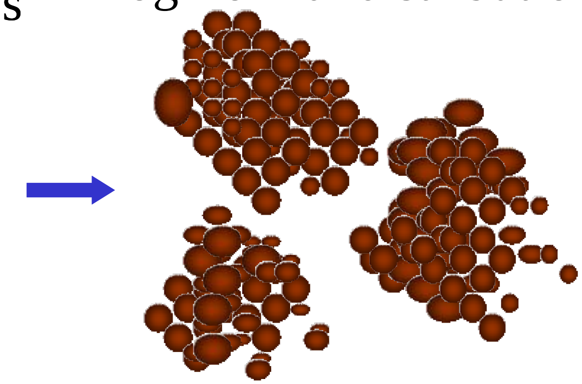
Chaos as a Fabric

Fragmentation

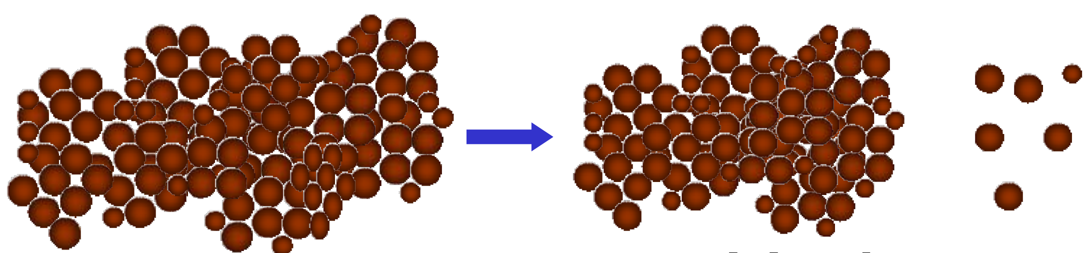
Rupture – critical stress



Log-normal distribution



Erosion – low stress

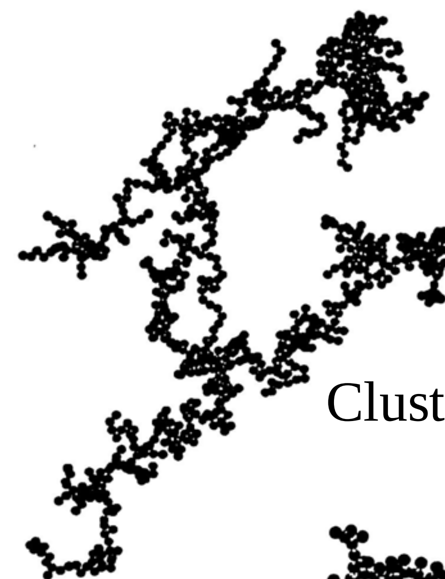


Normal distribution

Hansen & Ottino *Phys. Rev. E* (1996)

J. Colloid and Int. Sci. (1996).

Aggregation



Cluster-cluster

Particle-cluster

Evolution and Applications

- Mathematical proof of chaos
- Artificiality of the examples, add more realism?
- Shape of the forcing, do discontinuities matter?
- Realism of the flow. Due corners matter?
- Are there 3D examples?
- Agreement with experiments. Nature of errors, round off, computation of the velocity field
- Role of inertia
- But mixing is a means to an end.. Mixing as a fabric.
- Reactions, how they are affected by mixing
- Coagulation…structure of clusters formed
- Breakup
- Droplets and other microstructures
- Character of the fluid, elasticity, etc.

- Chemical Engineering
- ………..
- Oceanography
- Microfluidics
- Geophysics

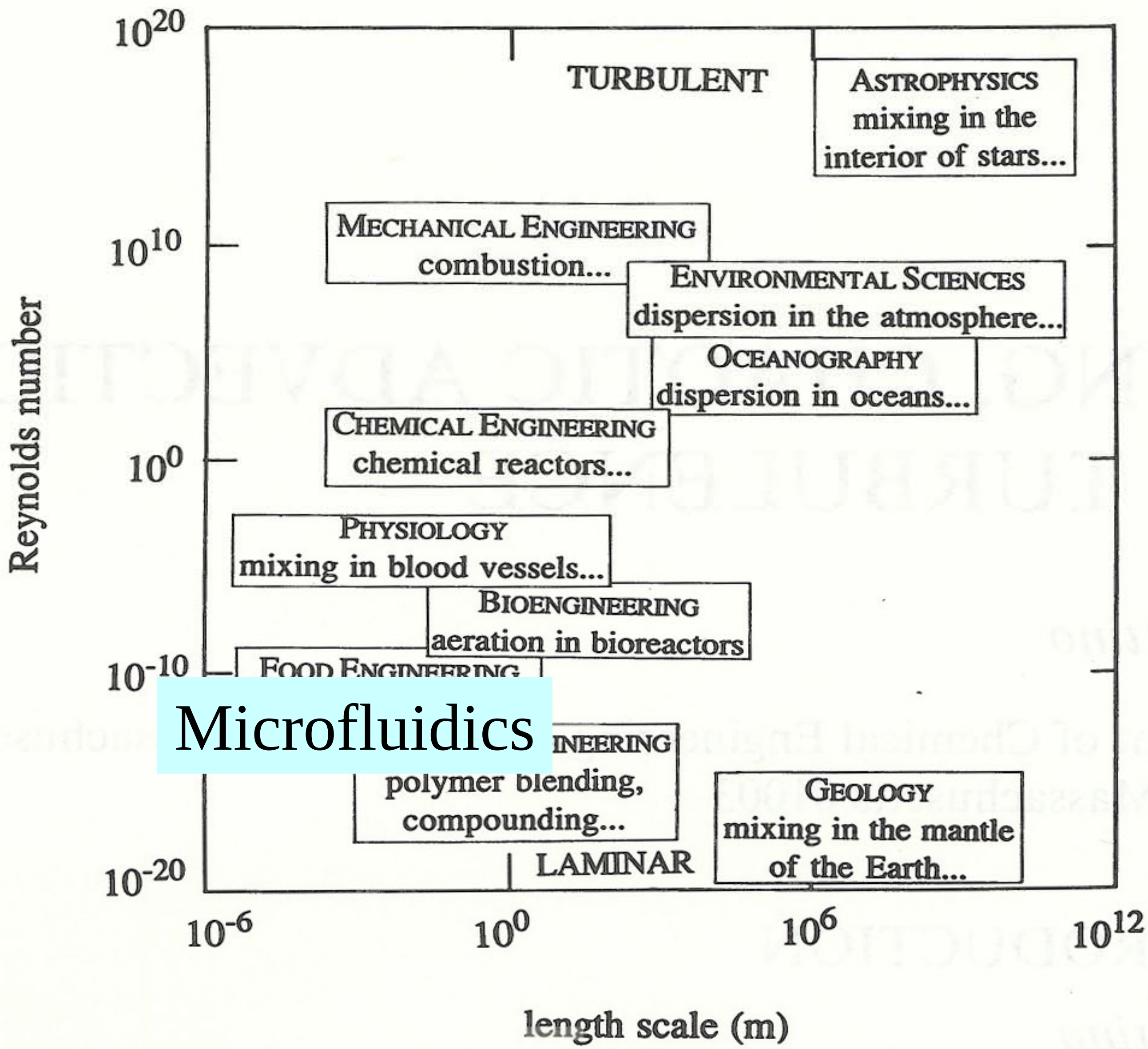


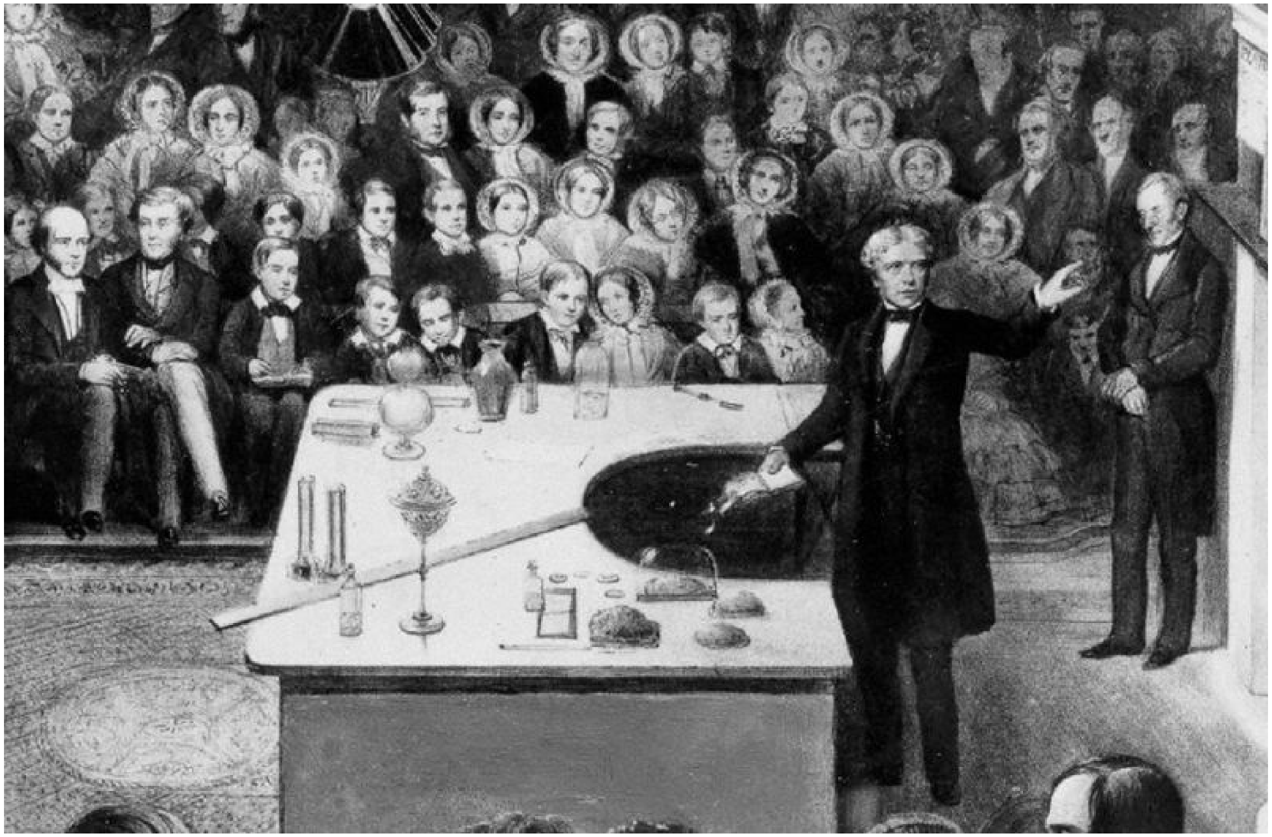
Figure 1 Spectrum of problems studied in various disciplines in which mixing is important.

ARFM
1990

Timelines

- Brouwer Theorem ~ 1910
- Kolmogorov-Arnold-Moser theorem ~1954
- Baker's map ~1960s
- Smale Horseshoe ~1967
- Linked Twist Maps (Burton & Easton, 1980; Wojtkowski 1980; Przytycki 1983)

- U Minn (many ~1979)
- Fluid mechanics (Aref ~1982; Moin ~1991)
- Dynamical Systems (Rising ~1982; Wiggins ~1986 Guckenheimer, Holmes, math; Gollub, Swinney; physics)
- Multifractals, microscales (Meneveau & Sreenivasan ~1991; Dahm ~1988)



Michael Faraday's Christmas Lectures

Osborne Reynolds
1842 - 1912

NOTICES

OF THE

PROCEEDINGS

AT THE

MEETINGS OF THE MEMBERS

OF THE

Royal Institution of Great Brit

WITH

ABSTRACTS OF THE DISCOURSES

DELIVERED AT

THE EVENING MEETINGS.

VOLUME XIV.
1893—1895.

LONDON:
PRINTED BY WILLIAM CLOWES AND SONS, LI
STAMFORD STREET AND CHARING CROSS.
1896.

WEEKLY EVENING MEETING,

Friday, June 2, 1893.

Sir Douglas Galton, K.C.B. D.C.L. LL.D. F.R.S. Vice-President, in the Chair.

Professor Osborne Reynolds, M.A. LL.D. F.R.S.

Study of Fluid Motion by means of Coloured Bands.

In his charming story of 'The Purloined Letter,' Edgar Allan Poe tells how all the efforts and artifices of the Paris police to obtain possession of a certain letter, known to be in a particular room, were completely baffled for months by the simple plan of leaving the letter in an unsealed envelope in a letter-rack, and so destroying all *curiosity* as to its contents; and how the letter was at last found there by a young man who was not a professional member of the force. Closely analogous to this is the story I have to set before you to-night—how certain mysteries of fluid motion, which have resisted all attempts to penetrate them are at last explained by the simplest means and in the most obvious manner.

Osborne Reynolds, 1893

In fluids [..] this attenuation is only the first step in the process of mixing – all involve the second process, that of folding, piling, or wrapping, by which the attenuated layers are brought together.

Identification of stretching and folding as basic mechanism for mixing

Conceptual Frameworks

$$u(x, y, z, t) = \bar{u} + u'$$

- O. Reynolds, "On the dynamical theory of incompressible viscous fluids and the determination of the criterion," *Philos. Trans. R. Soc. London, Ser*. A 186, 132 (1895).

- **"a perfectly random motion of particles [where] no basic pattern should or could exist."** T. Theodorsen, "The structure of turbulence," 50 Jahre Genszchichtsforschung: Ludwig Prandtl, Friedr (Viegeg and Sohn, Braunschweig, 1955), pp. 55-62

Osborne Reynolds, 1893

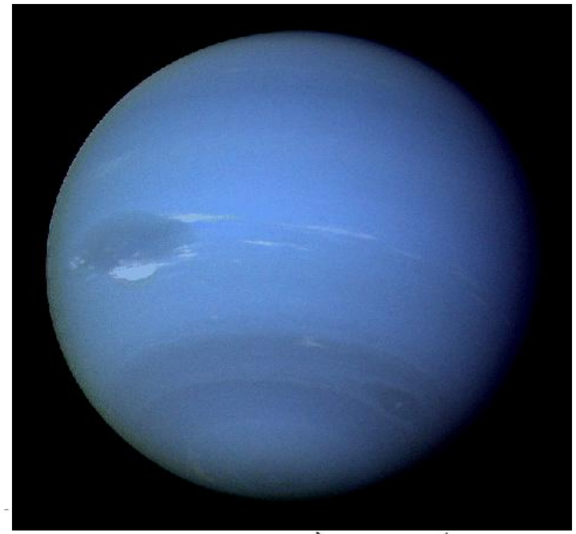
In fluids [..] this attenuation is only the first step in the Process of mixing – all involve the second process, that of folding, piling, or wrapping, by which the attenuated layers are brought together.

Speaking of (the work focusing on motion of fluids)…
"in respect of the mental effort involved, or the scientific importance of the results goes beyond that which resulted in the discovery of Neptune."

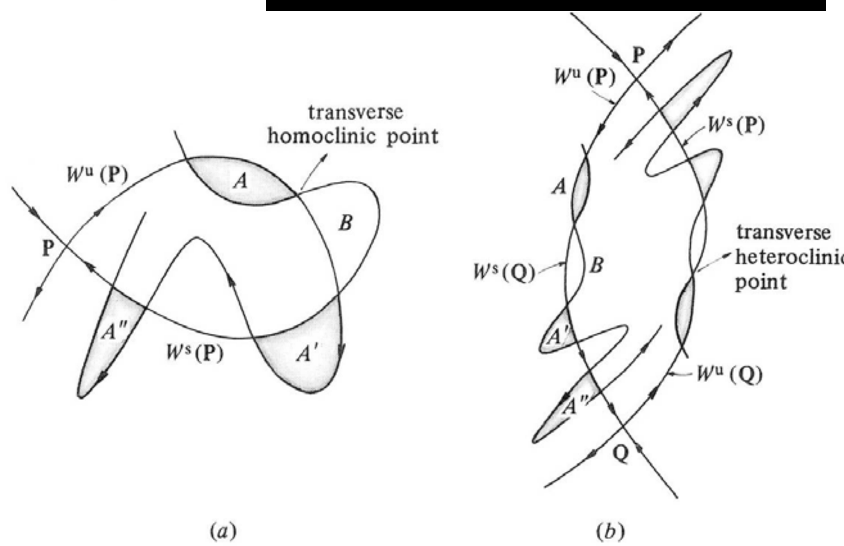
Jules Henri Poincaré
1854-1912

Osborne Reynolds
1842 - 1912

- Neptune’s discovery – 30 s deviation from Newtonian predictions; trans-Uranian.
- Jean Leverrier (France); John Couch Adams (England).
- Prize closed-form analytical solution for the three-body problem (Oscar II, King of Sweden)



Winner: Poincaré, 1889
(…. homoclinic orbits;
three bodies produce chaos.)



JMOttino 08

Jules Henri Poincaré
1854-1912

Osborne Reynolds
1842 - 1912

JMOttino 08

Jacques Hadamard
(1865-1963)

Psychology of Invention
In the Mathematical Field
(1945, 1954)

Visual Imagination

Non-Visual

Joseph Louis Lagrange

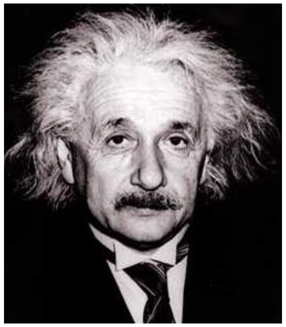
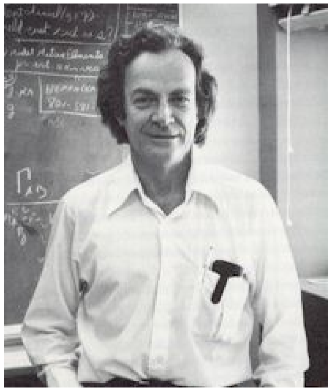
One of Lagrange's more famous books, *Analytical Mechanics*, which, he boasted proudly, contains no pictures.

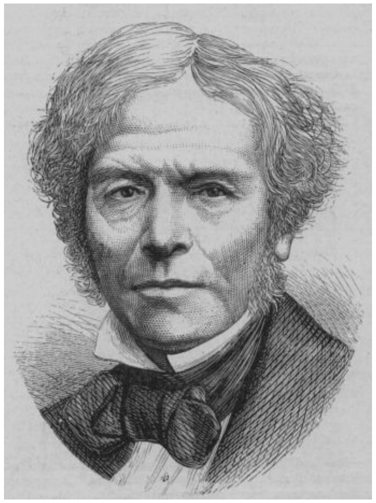
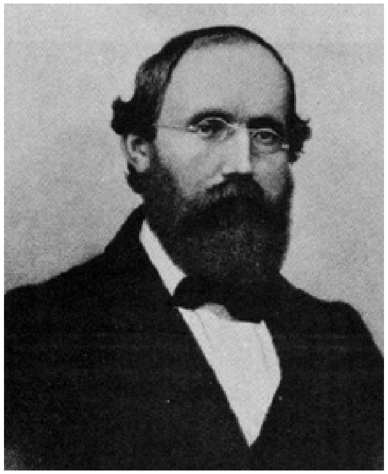
Visual

David Hilbert

Commenting on one of David Hilbert's work Jacques Hadamard said: "diagrams appear in every other page."

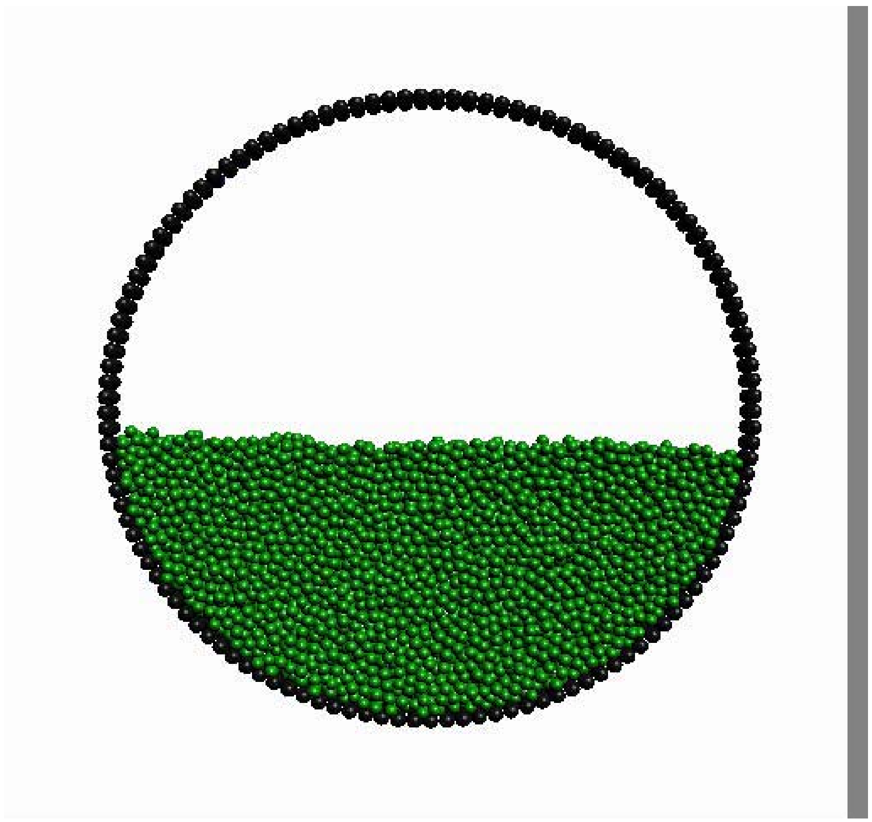


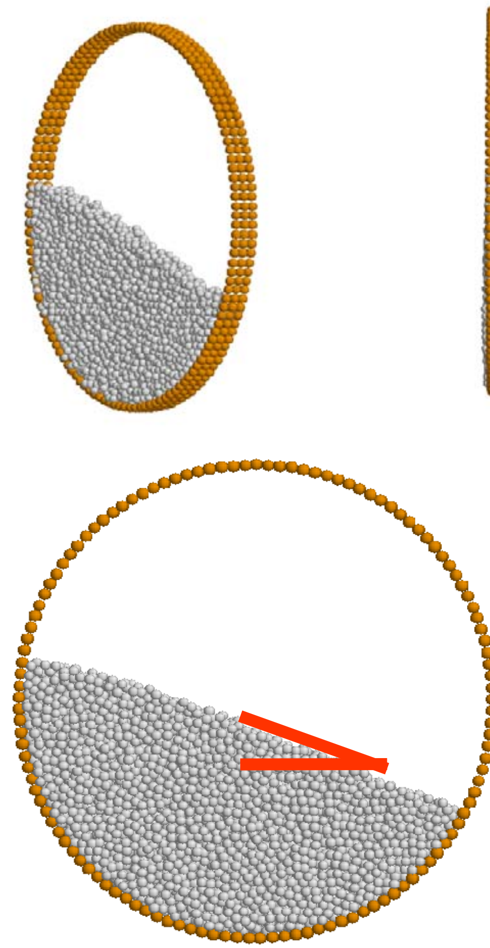




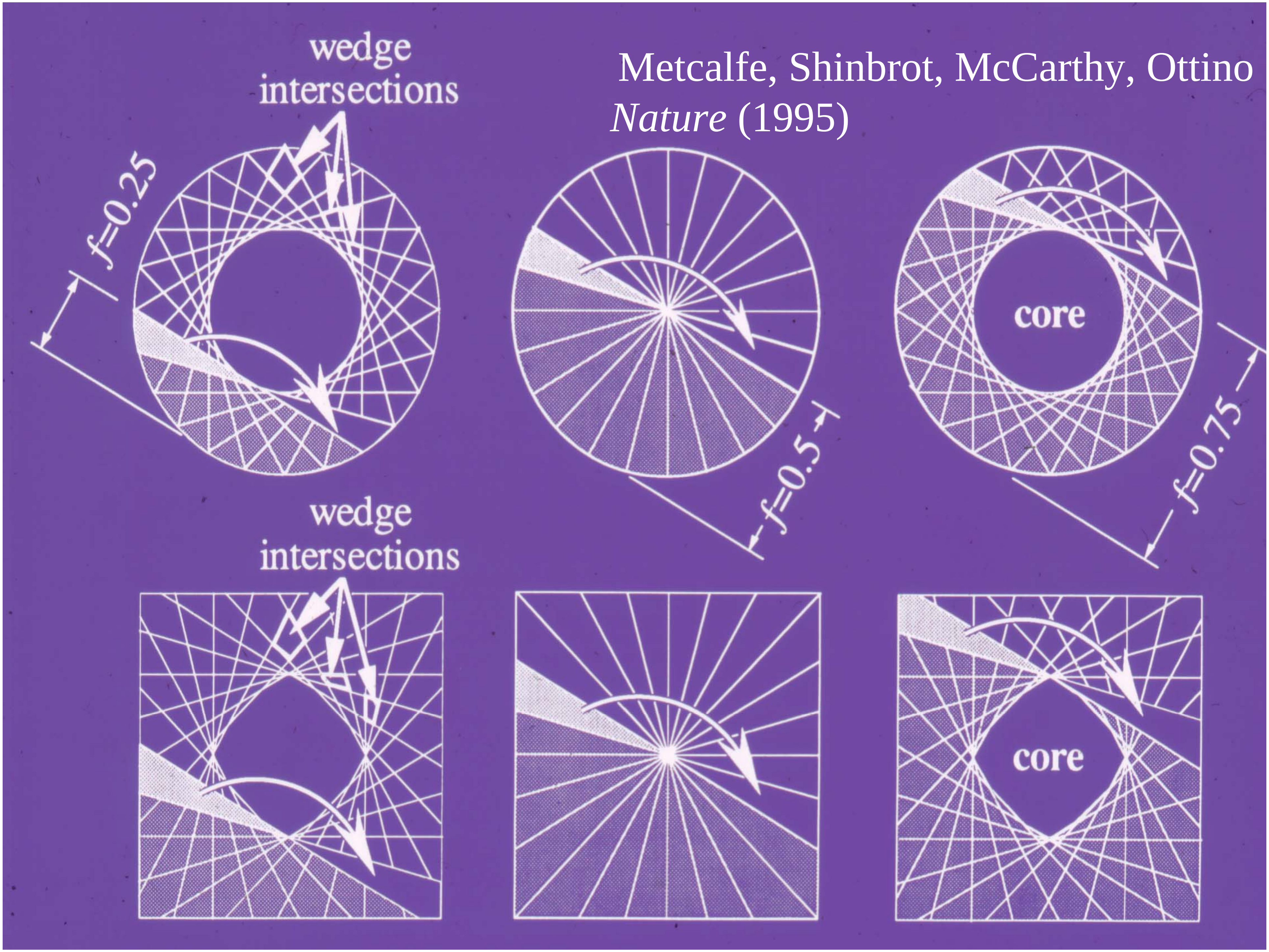




JMOttino 08

Mixing of Solids

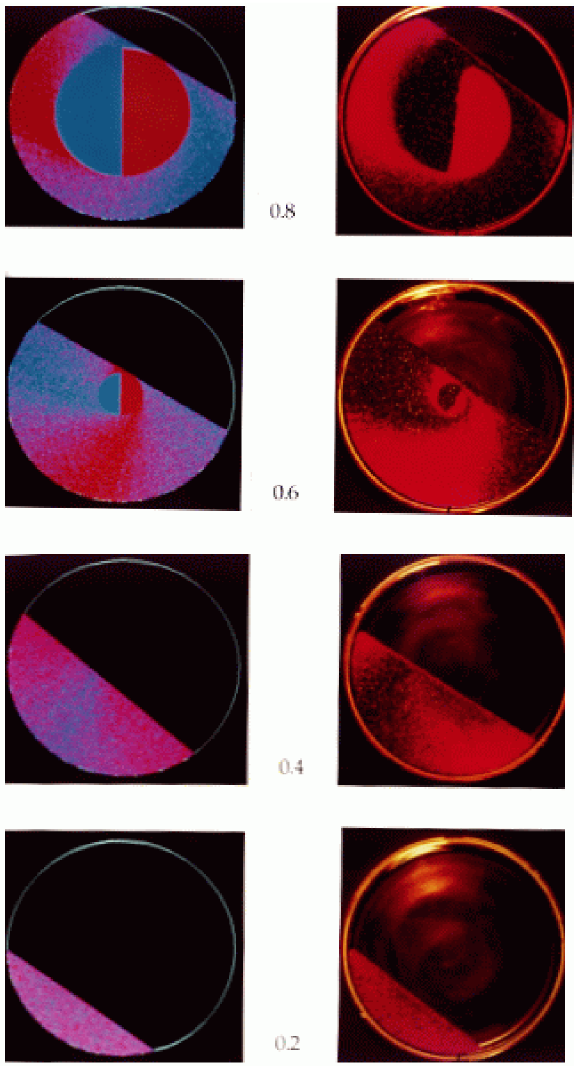






wedge
intersections
Metcalfe, Shinbrot, McCarthy, Ottino
Nature (1995)
f=0.25
f=0.5
f=0.75
core
wedge
intersections
core

Mixing by avalanching



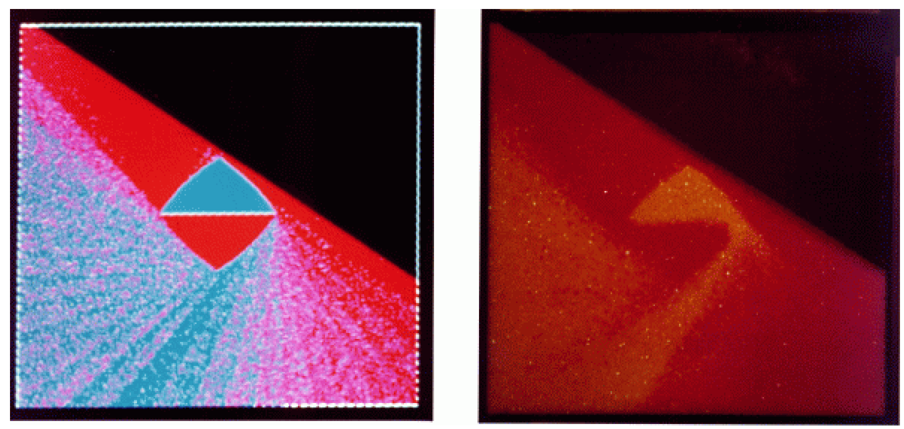
Model Experiment

(1) Geometry

wedges ⇒ wedges

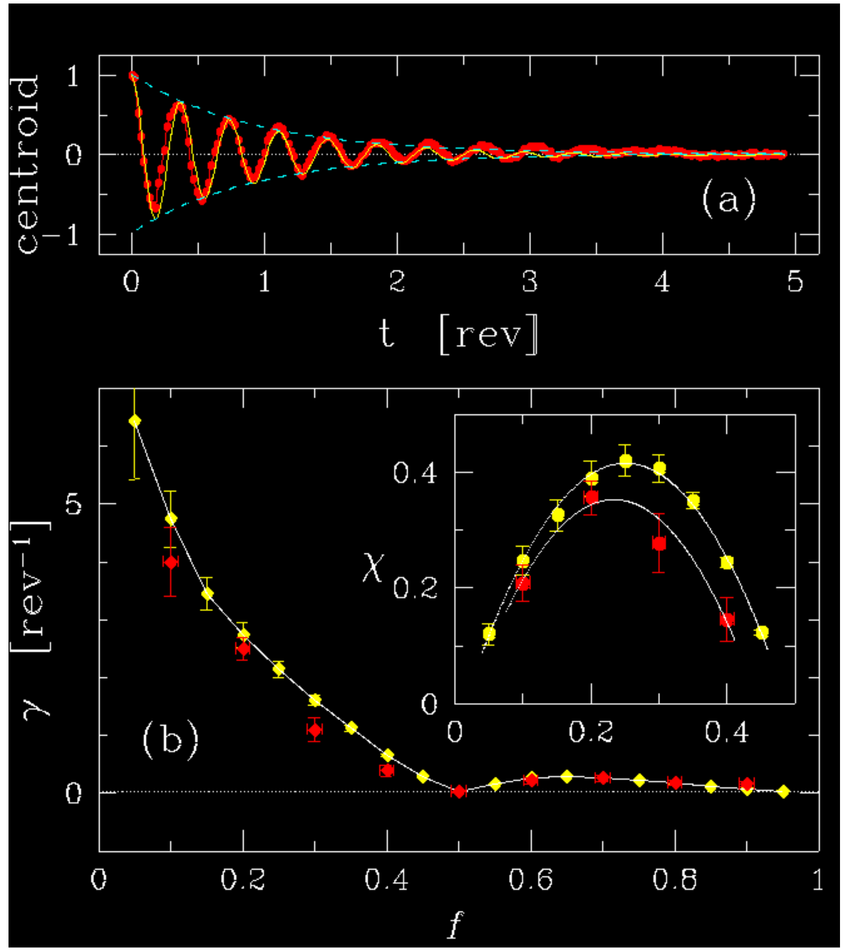
(2) Dynamics

Random mixing within wedges



Model Experiment

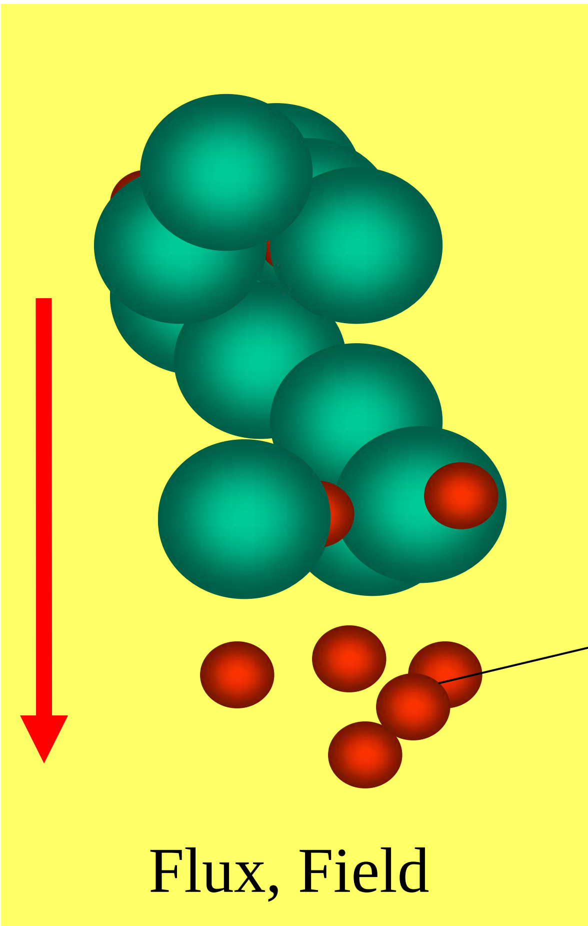
Metcalfe, Shinbrot, McCarthy, Ottino
Nature (1995)



γ = rate of mixing, $\chi = \gamma$ x total area being mixed

JMOttino 08

Segregation due to flow



(size) S-systems, (density) D-systems

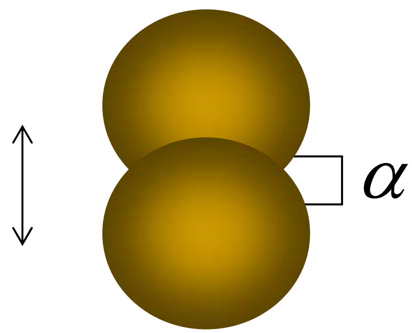
Reynolds's dilatancy (percolation)

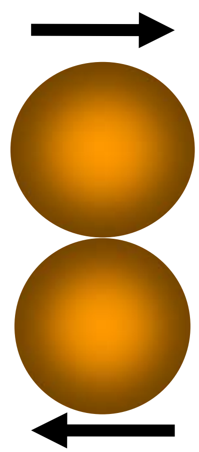
Osborne Reynolds, *Philosophical Magazine*, December, 1885

Size segregation

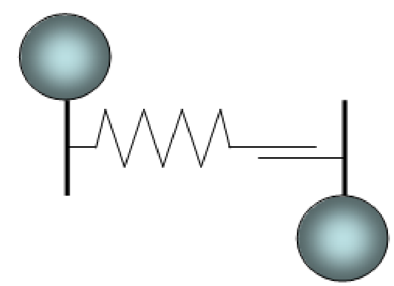
How Two Particles Interact: Contact Forces

Normal Forces





Tangential Forces



Spring and Slider

$$F_n = k_n \alpha^{3/2} - k_d v_n \sqrt{\alpha}$$

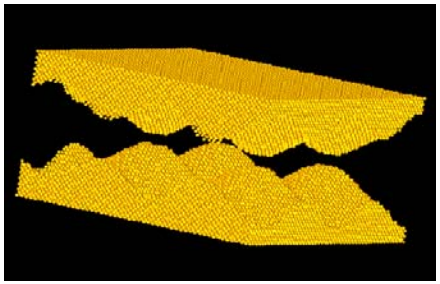
Hertz repulsion with dissipation

$$F_t = \min\left(\left|k_t s\right|, \left|\mu F_n\right|\right)$$

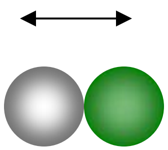
Linear tangential spring with maximum

JMOttino 08

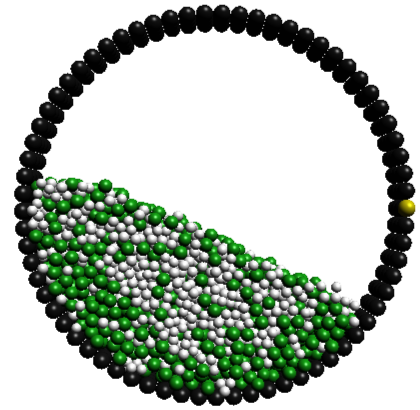
A Nano to Macro Problem



Origins of
Friction/Adhesion



Particle-Particle
Interacti



Collective Flow
ior

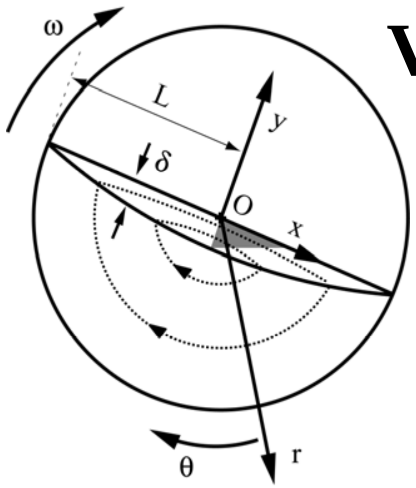
$10^{-9}\,m$ $10^{-7}\,m$ $10^{-5}\,m$

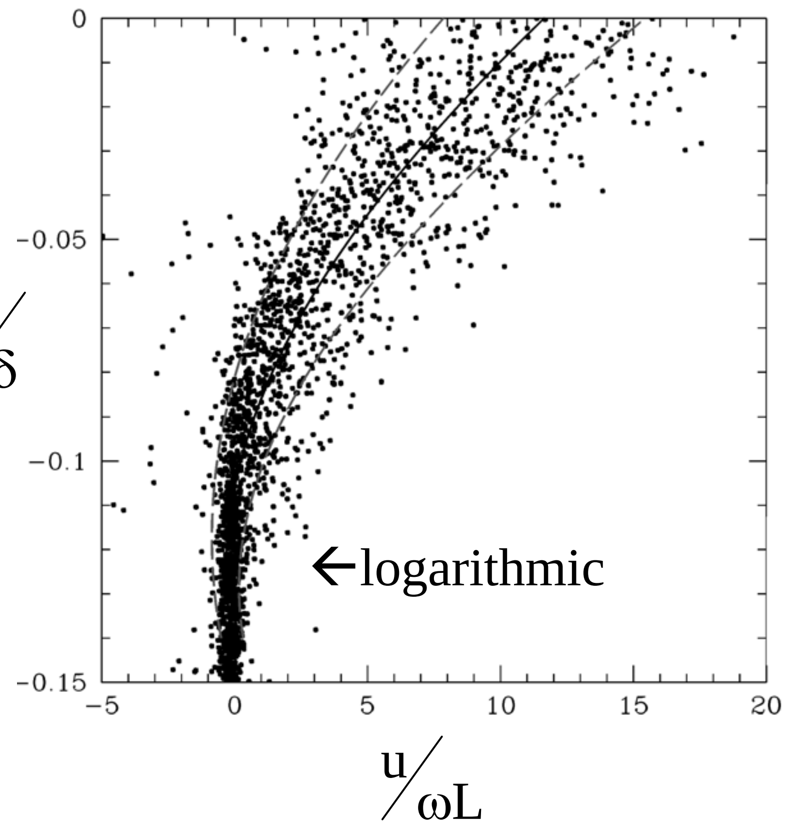
Specifics
chemistry

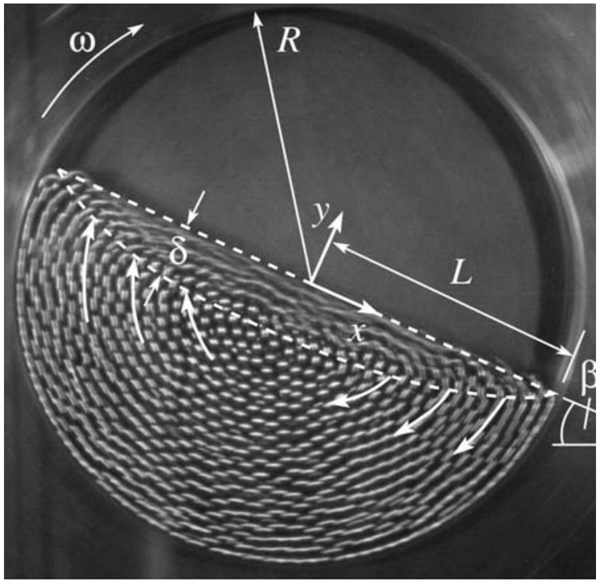
Generic
physics

JMOttino 08

Velocity field, fluid layer







glass, steel,
d~1mm

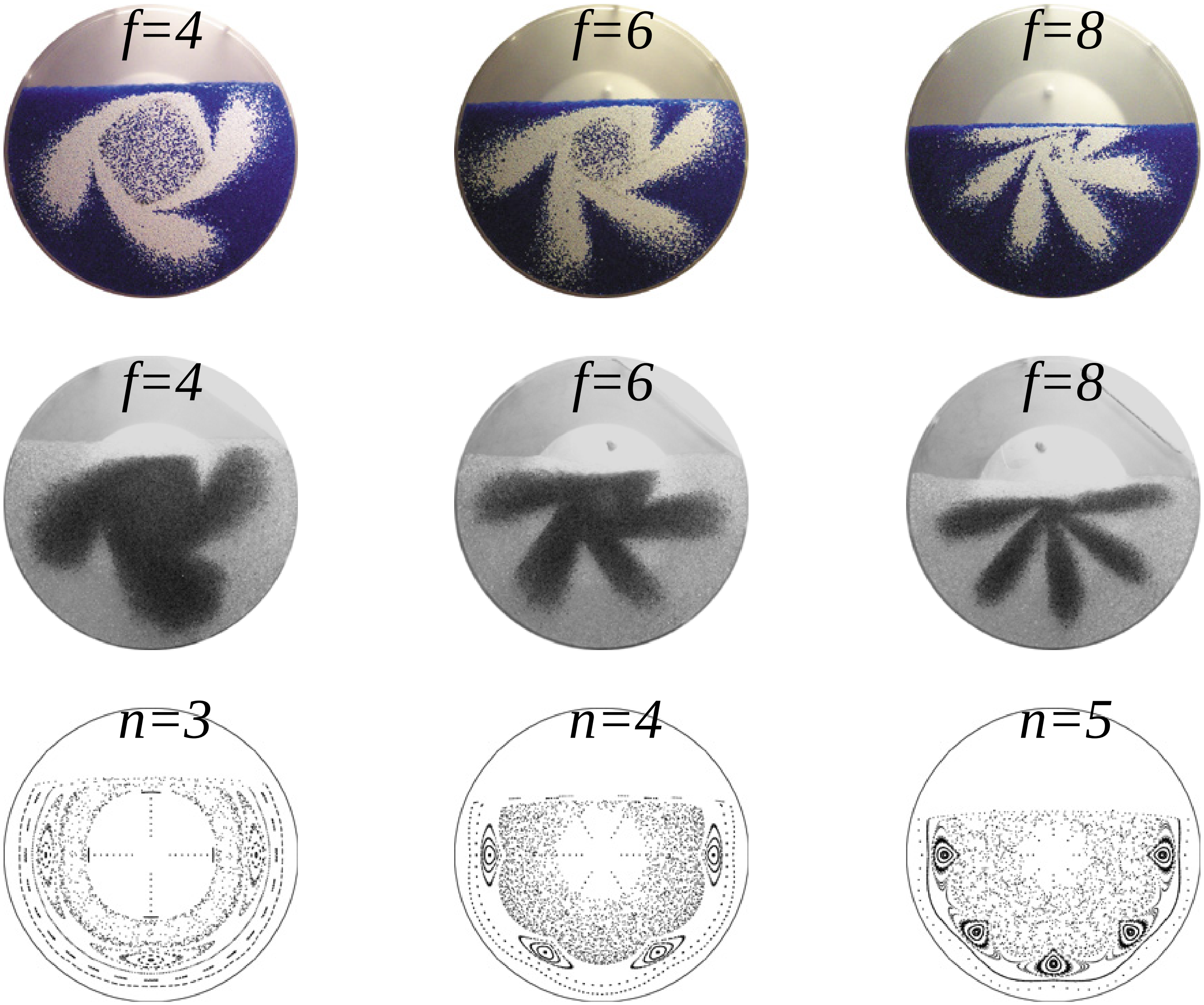
"wall effects", 4 < *d*/*t* < 8

Experimental PIV

Jain *et al*. *Phys. Fluids* 2002 (DGS), Jain et al. *JFM*, 2004 (LGS)

Unmixing: *t*-periodic forcing

Fiedor & Ottino
J. Fluid Mech. (2005)



DGS

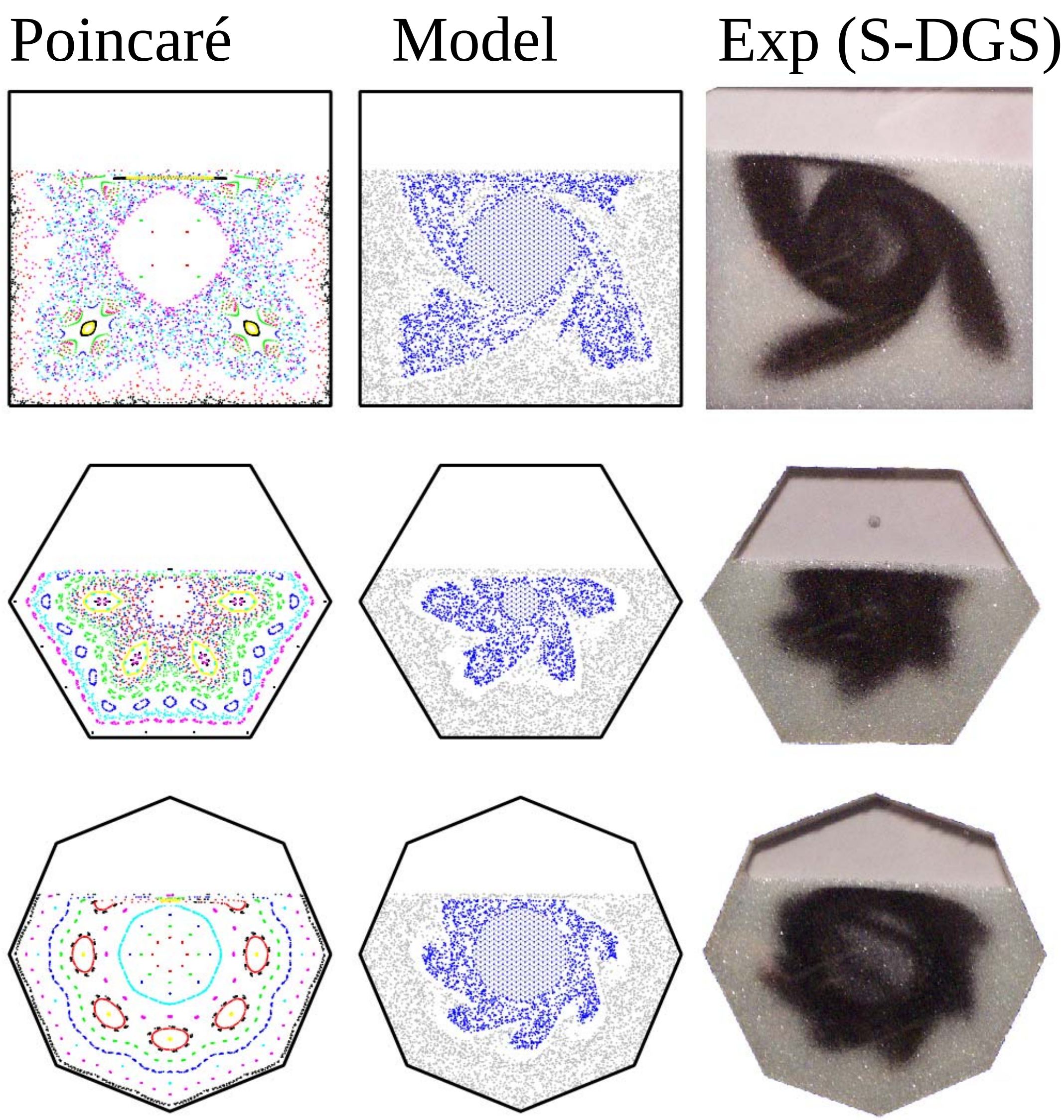
LGS

Poincaré Sections

Unmixing

Interpenetrating
Continua Model
2D systems

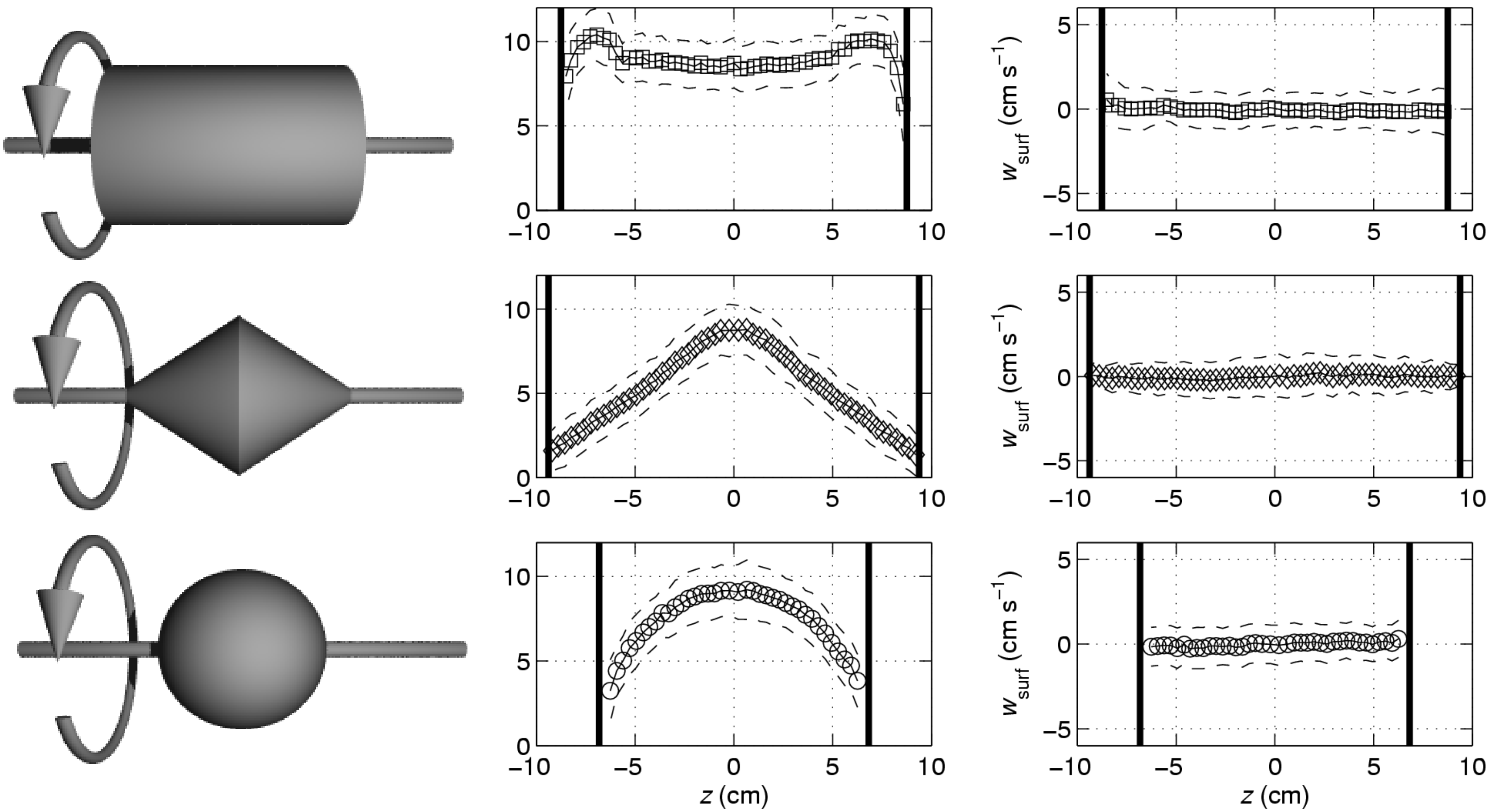
Cisar, Umbanhowar
Ottino, PRE, *2006*



JMOttino 08

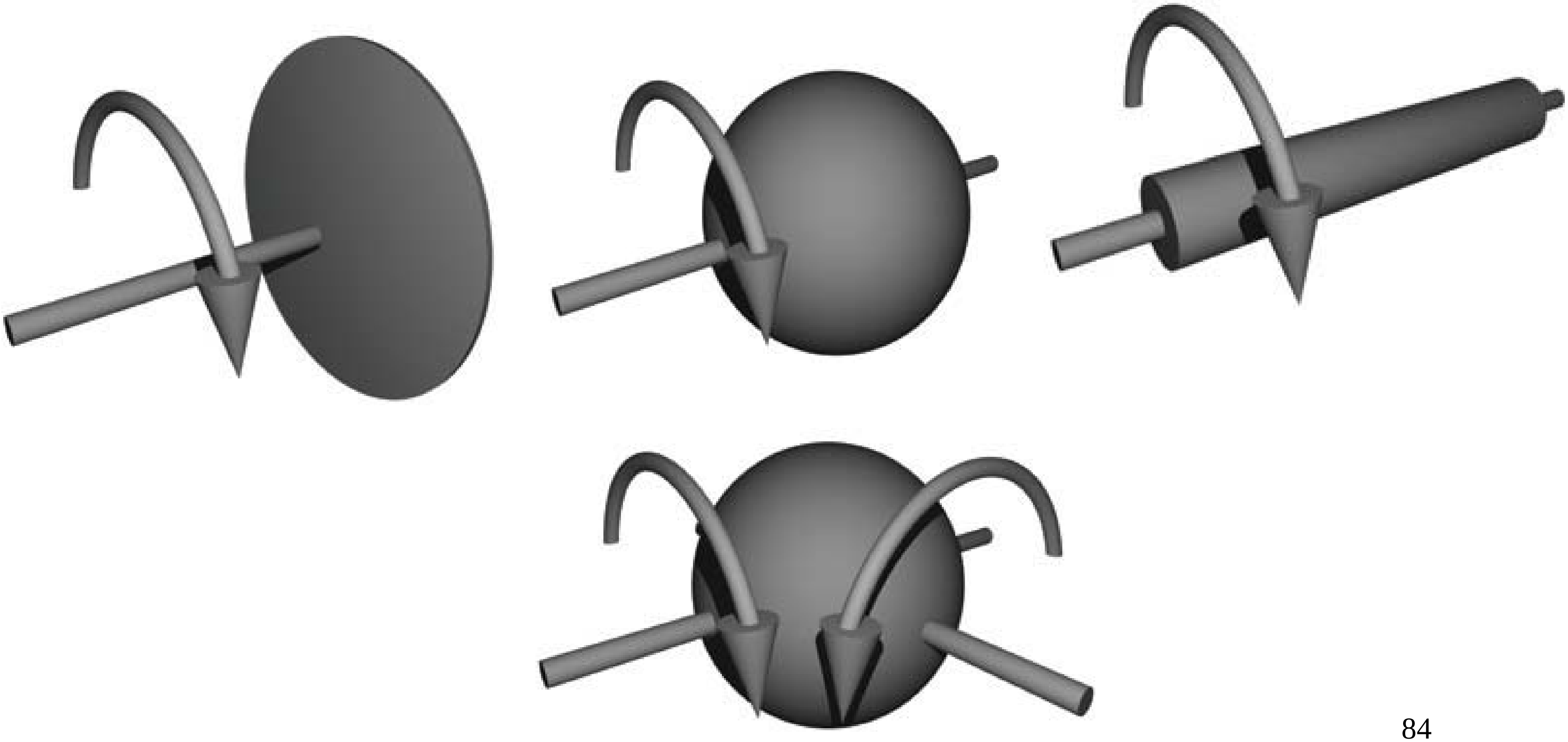
McCormick
Northwestern Engineering

Velocity Profiles: 1 mm Particles, ω= 2 RPM

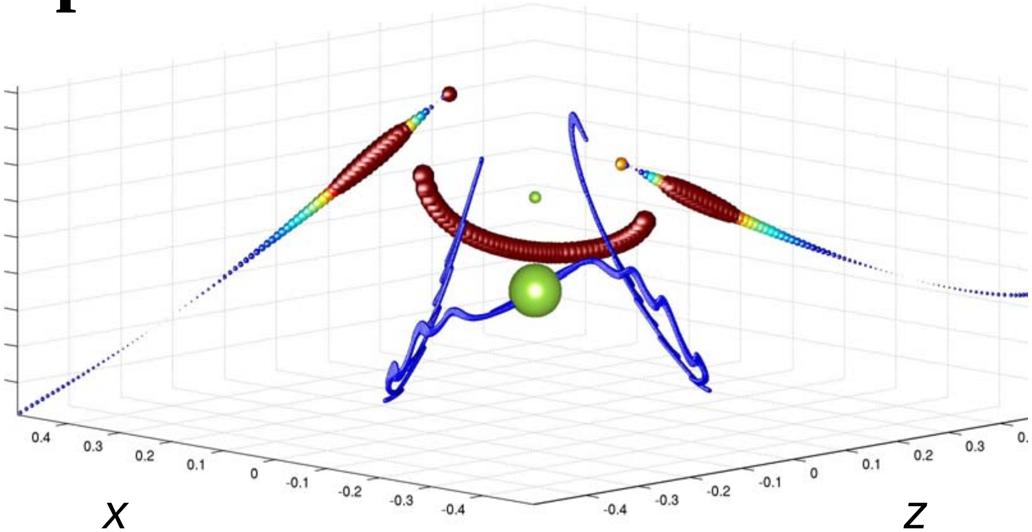


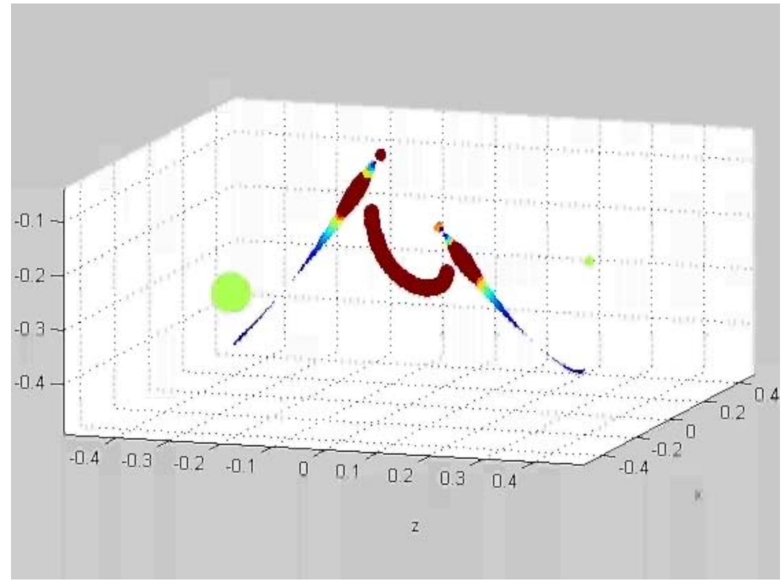
JMOttino 08

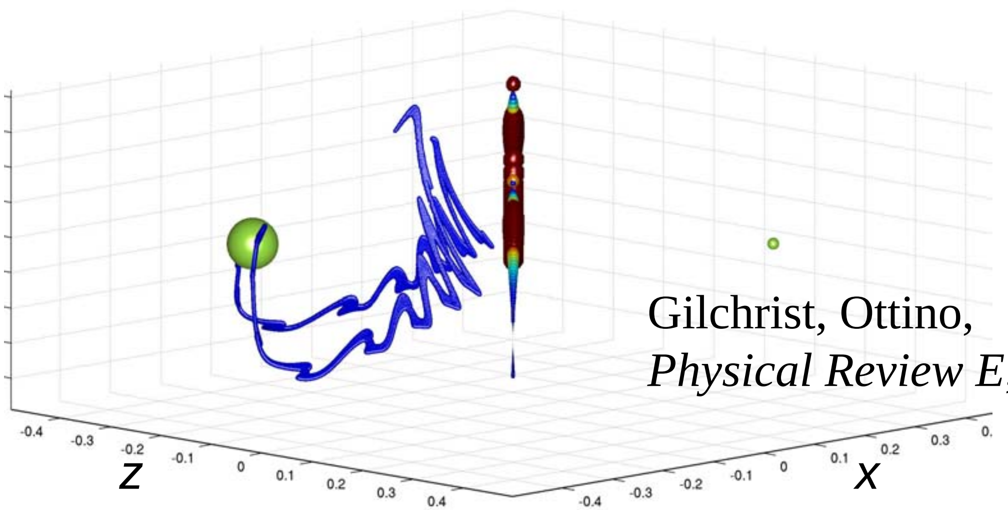
Tumbling Geometries and Forcings



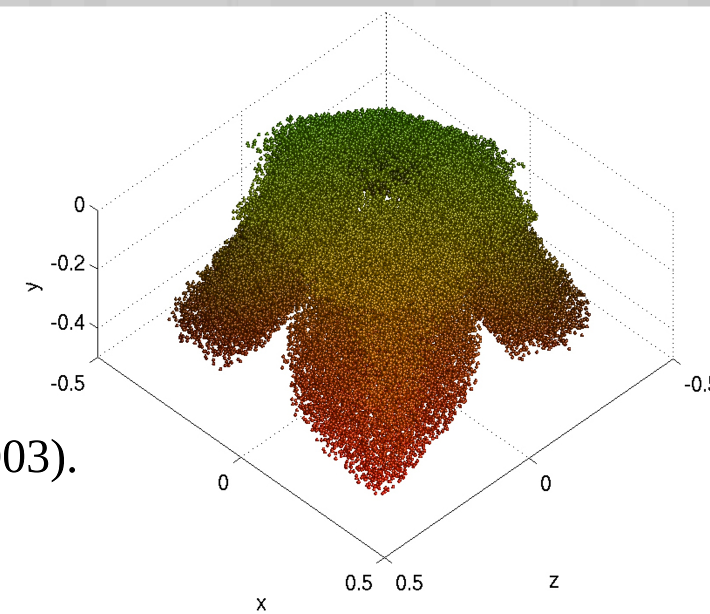
Cube: Skeleton, periodic points and unstable manifolds



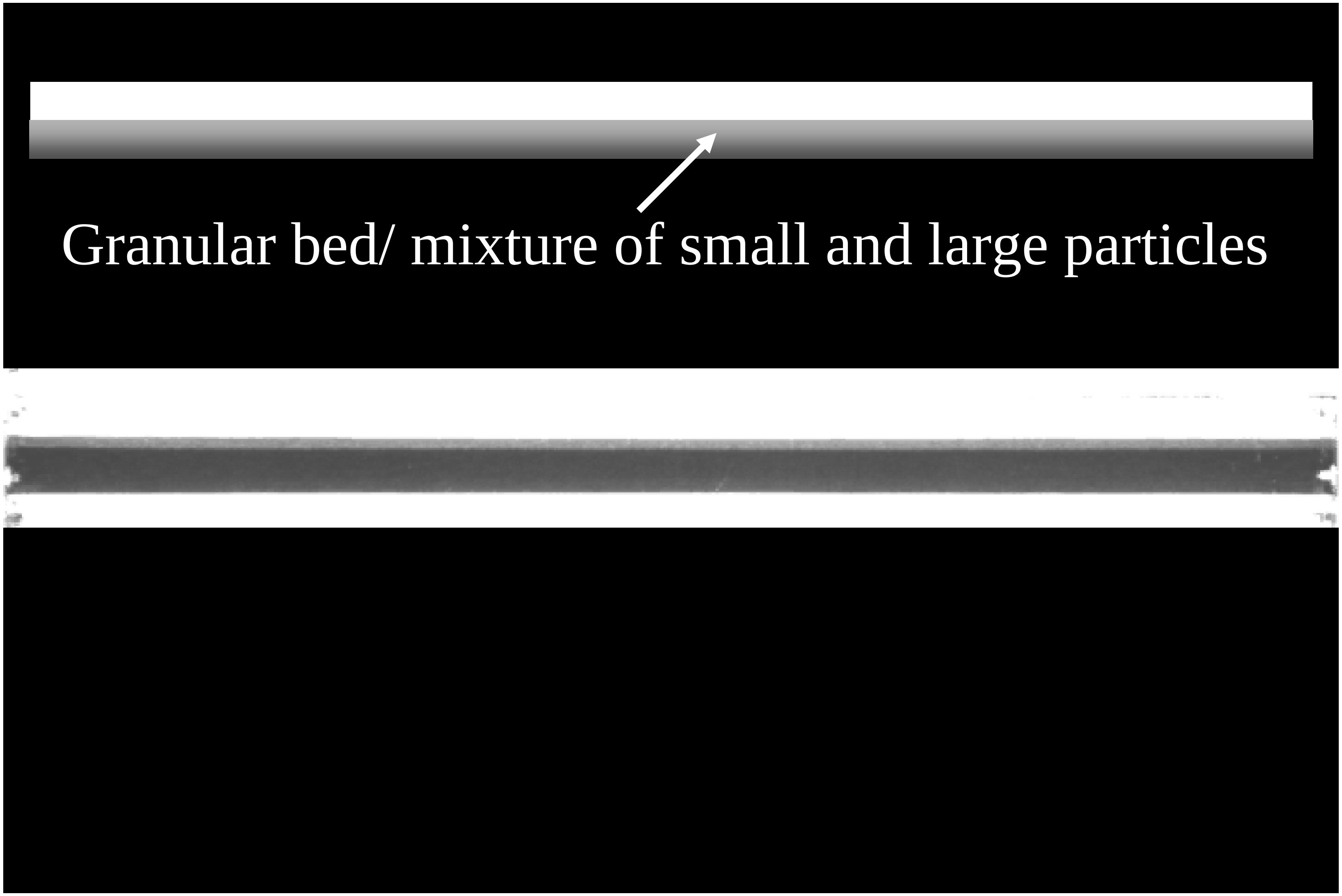




Gilchrist, Ottino,
Physical Review E, **68** (2003).



Segregation in long rotating cylinder (LGS)



JMOttino 08

Effect of rotation rate - DGS

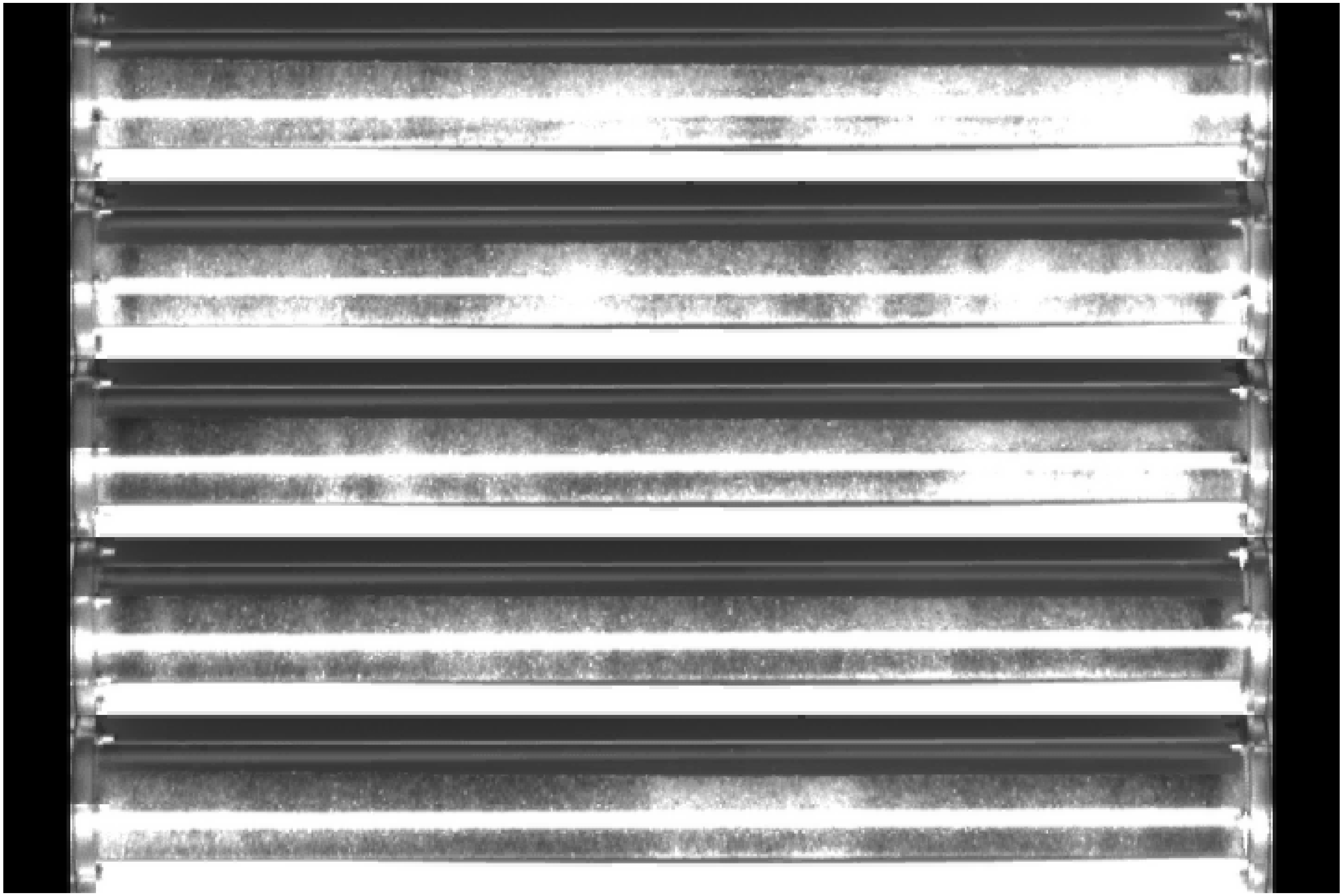
5 RPM

10 RPM

15 RPM

20 RPM

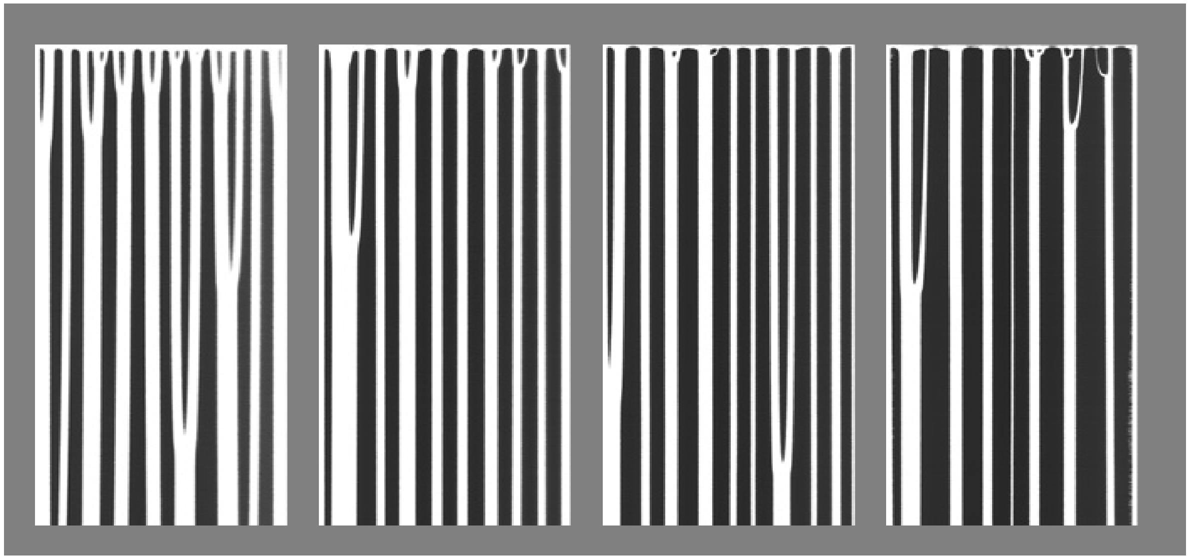
25 RPM

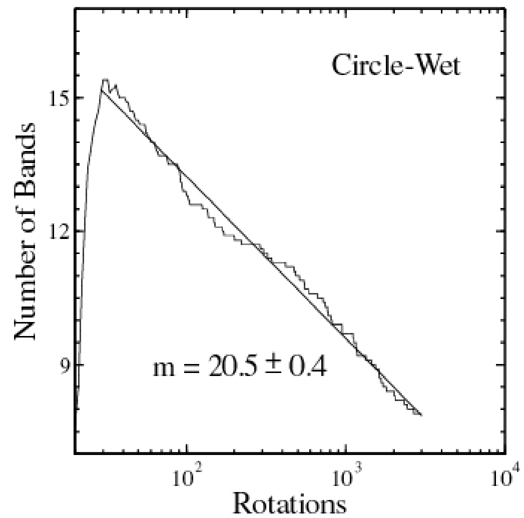


JMOttino 08

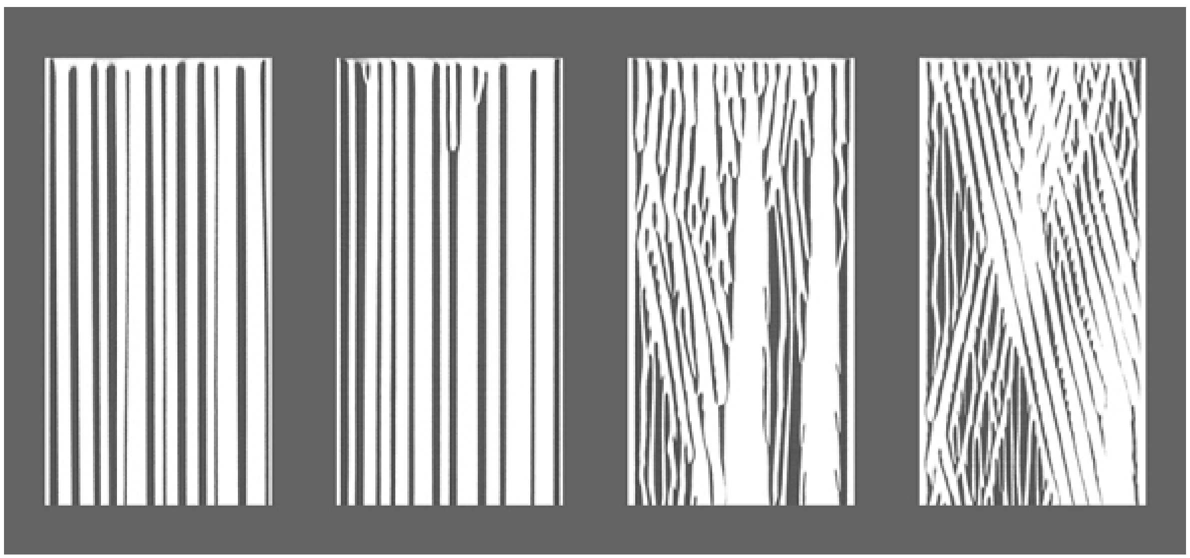
Coarsening, Quasi 1D, Logarithmic decay

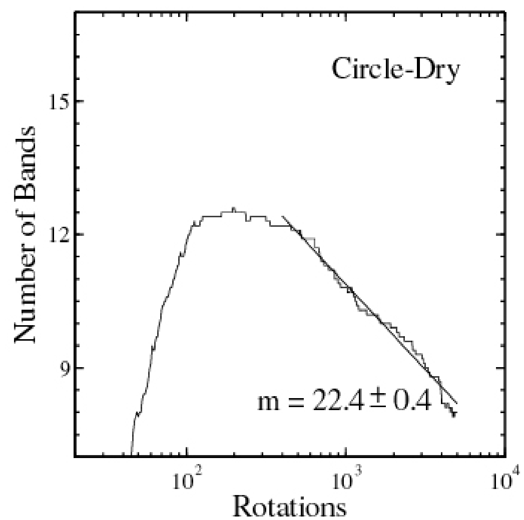
LGS





DGS





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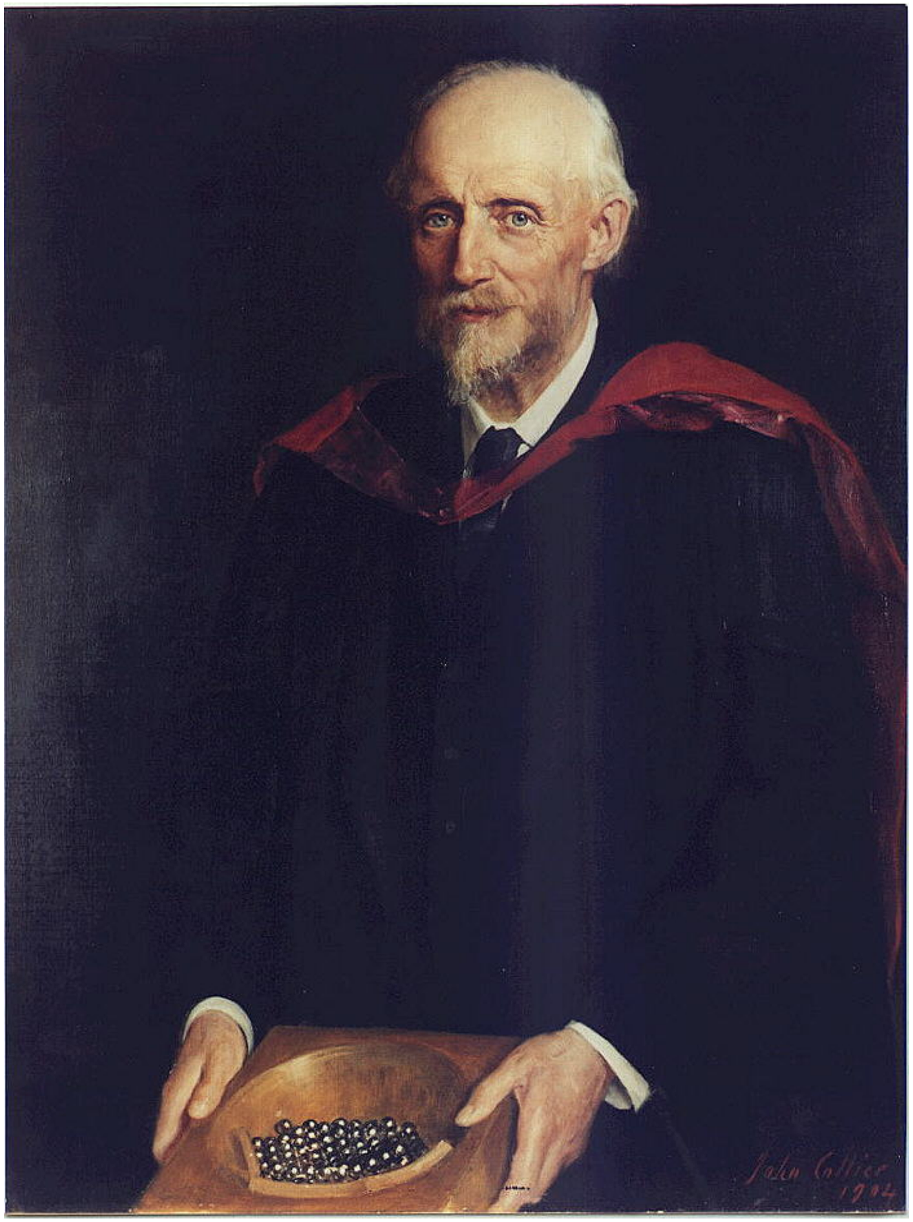
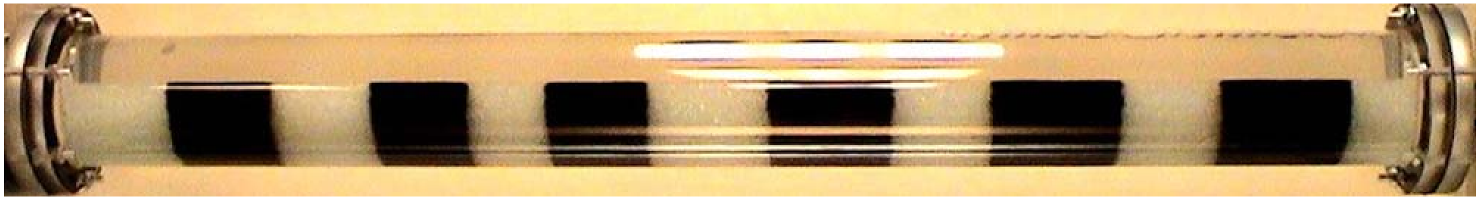


Osborne Reynolds
(1842-1912)

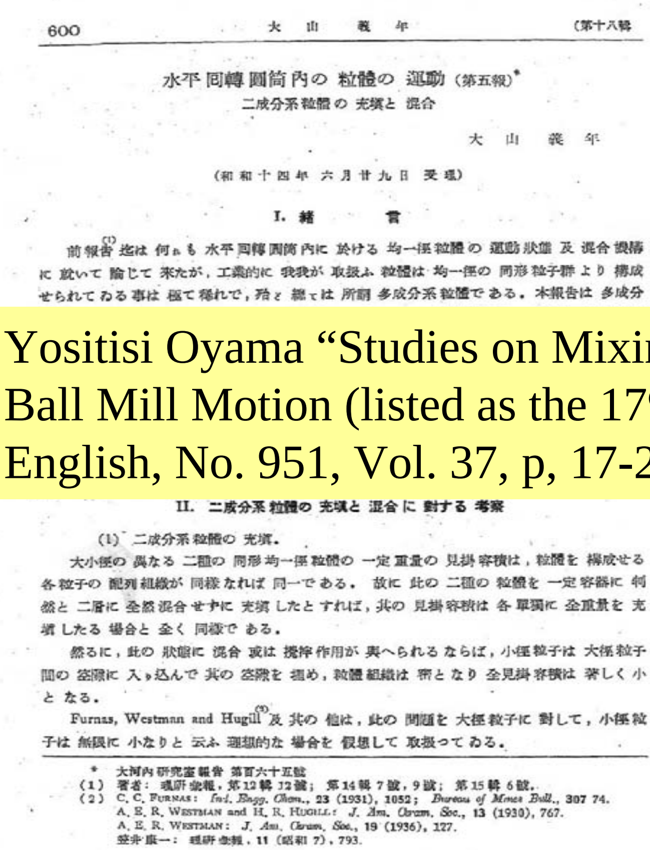
- Osborne Reynolds 'On the dilatancy of media composed of rigid particles in contact. With experimental illustrations'. *Philosophical Magazine*, December 1885).

- Rode Lecture in 1902 ("On an inversion of ideas as to the structure of the universe")

  "I have in my hand the first experimental model universe, a soft India rubber bag filled with small shot".



Oyama 1939



600 大山義年 (第十八輯

水平回轉圓筒内の粒體の運動（第五報）*

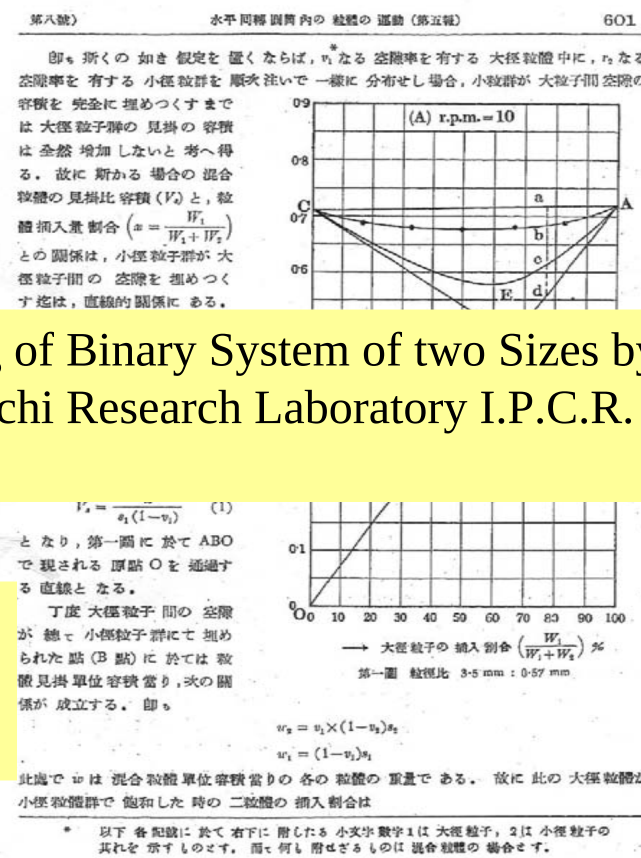
二成分系粒體の充塡と混合

大山義年

(昭和十四年六月廿九日受理)

I. 緒言

II. 二成分系粒體の充塡と混合に對する考察



第八號) 水平回轉圓筒内の粒體の運動（第五報） 601

Yositisi Oyama “Studies on Mixing of Solids. Mixing of Binary System of two Sizes by Ball Mill Motion (listed as the 179th report from Okochi Research Laboratory I.P.C.R. In English, No. 951, Vol. 37, p, 17-29 year 1939).

Prematurity Origins

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Prematurity

Fitting with the canonical knowledge of the times

"Sleeping Beauty" Papers

- "…a publication that goes unnoticed ('sleeps'), gathering less than one citation a year for many years, and then, almost suddenly, attracts a lot of attention (the paper is awakened by a "prince").

van Raan (*2004*)

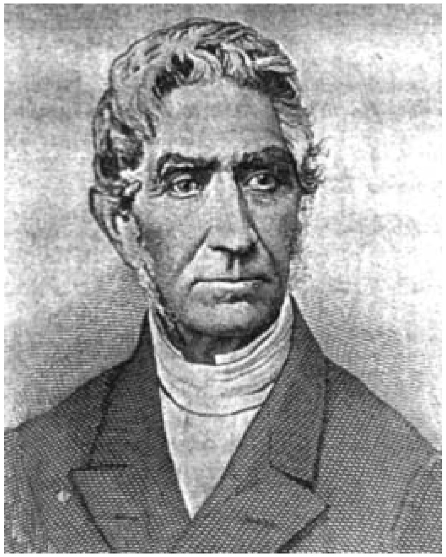
The work of the Innovator

"Never forget what I believe was observed to you by Coleridge, that every great and original writer, in proportion as he is great and original, must himself create the taste by which he is relished"

William Wordsworth (English poet, 1770–1850) in Letter to Lady Beaumont, 21 May 1807; in E. de Selincourt (ed.) *Letters of William and Dorothy Wordsworth* vol. 2; revised by M. Moorman, 1969.

Origins

James Clerk Maxwell
1831-1879



Adolphe Quetelet
1796-1874
Mécanique Sociale

Maxwell's arguments about constancy of averages (foundation of Statistical Mechanics)

P. Ball, *Physica A* (2002)
Critical Mass, (2004)

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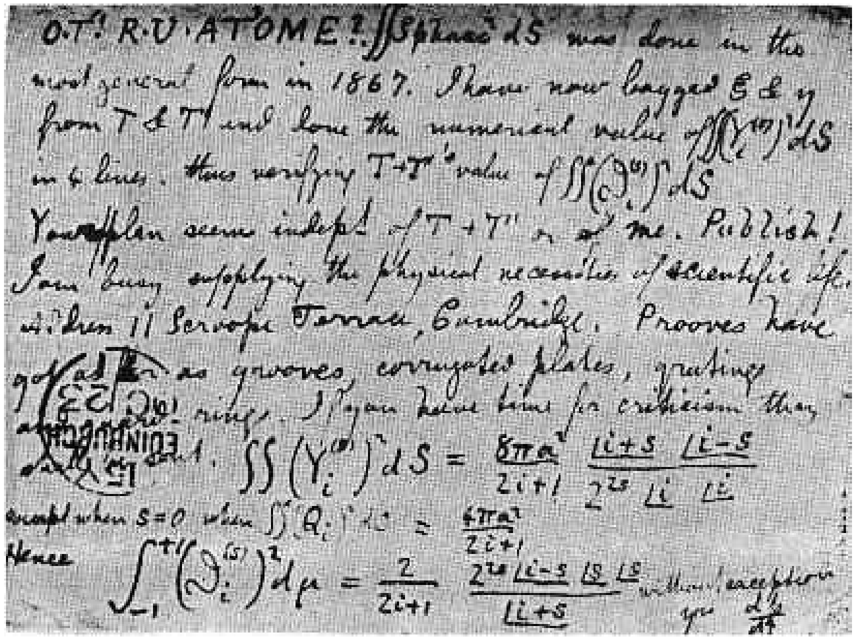
Peter Guthrie Tait, "On the foundations of the kinetic theory of gases', *Scientific Papers*, vol.2, p. 126, CUP 1898-1900.

"…the extraordinary steadiness with which numbers of such totally unpredictable, though not uncommon phenomena as suicides, twin or triple births, dead letters*, &c., in any populous country, are maintained year after year."

*Laplace had also commented on this issue.



O.T.! R.U. ATOME? ∬Sphaer dS was done in the most general form in 1867. I have now lugged ξ & η from T & T' and done the numerical value of $\iint (Y_i^{(\sigma)})^2 dS$ in 4 lines. thus verifying T+T' value of $\iint (Q_i^{(\sigma)})^2 dS$. Your plan seems indept of T+T' or of me. Publish! I am busy supplying the physical necessities of scientific life. Address 11 Scroope Terrace, Cambridge. Proofs have … as grooves, corrugated plates, grating rings. If you have time for criticism than …

Ref. *Critical Mass*, Philip Ball, FSG, 2004

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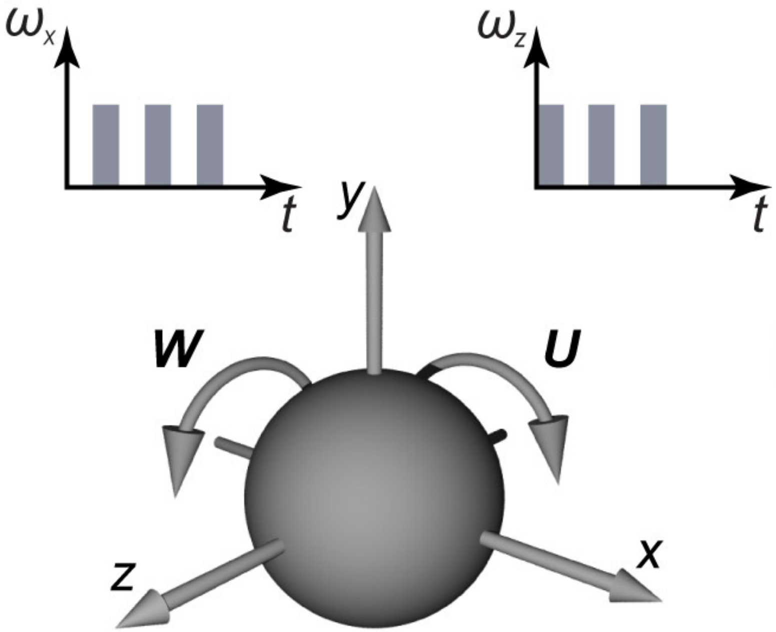
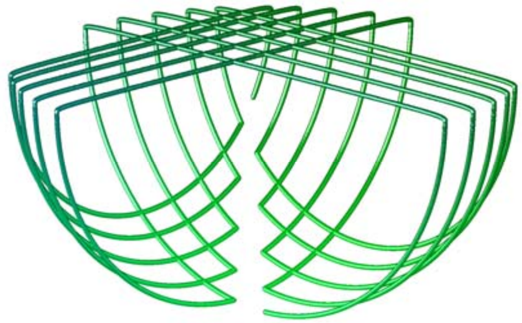
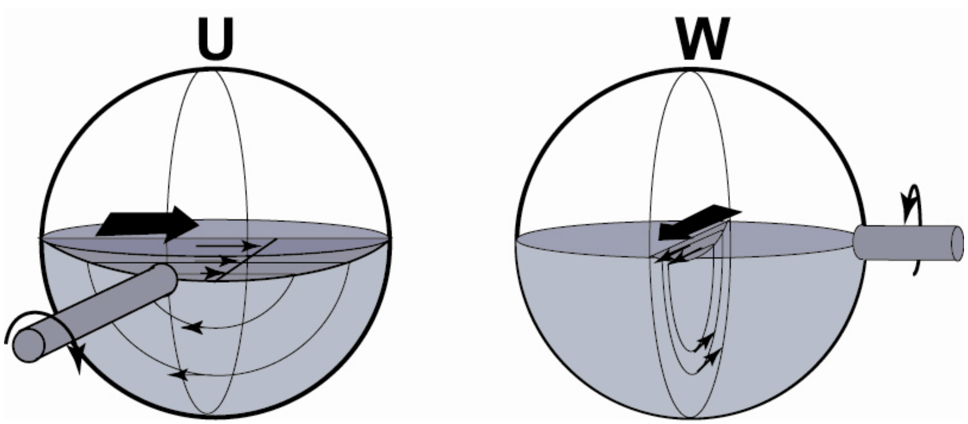
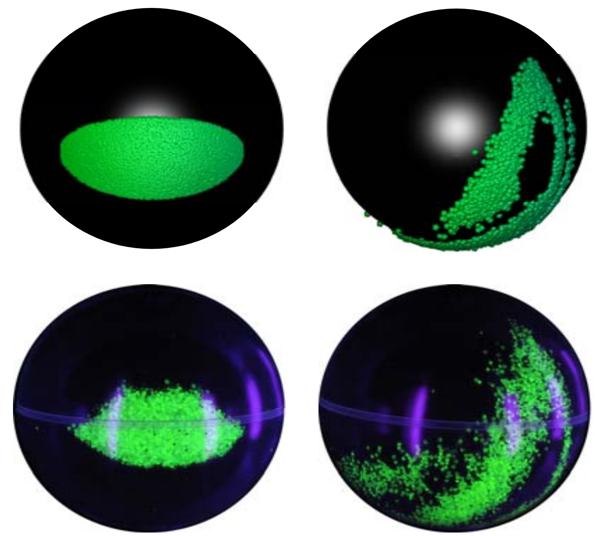
3D Granular Mixing





Meier, Lueptow, Ottino, *Advances in Physics* (2007).





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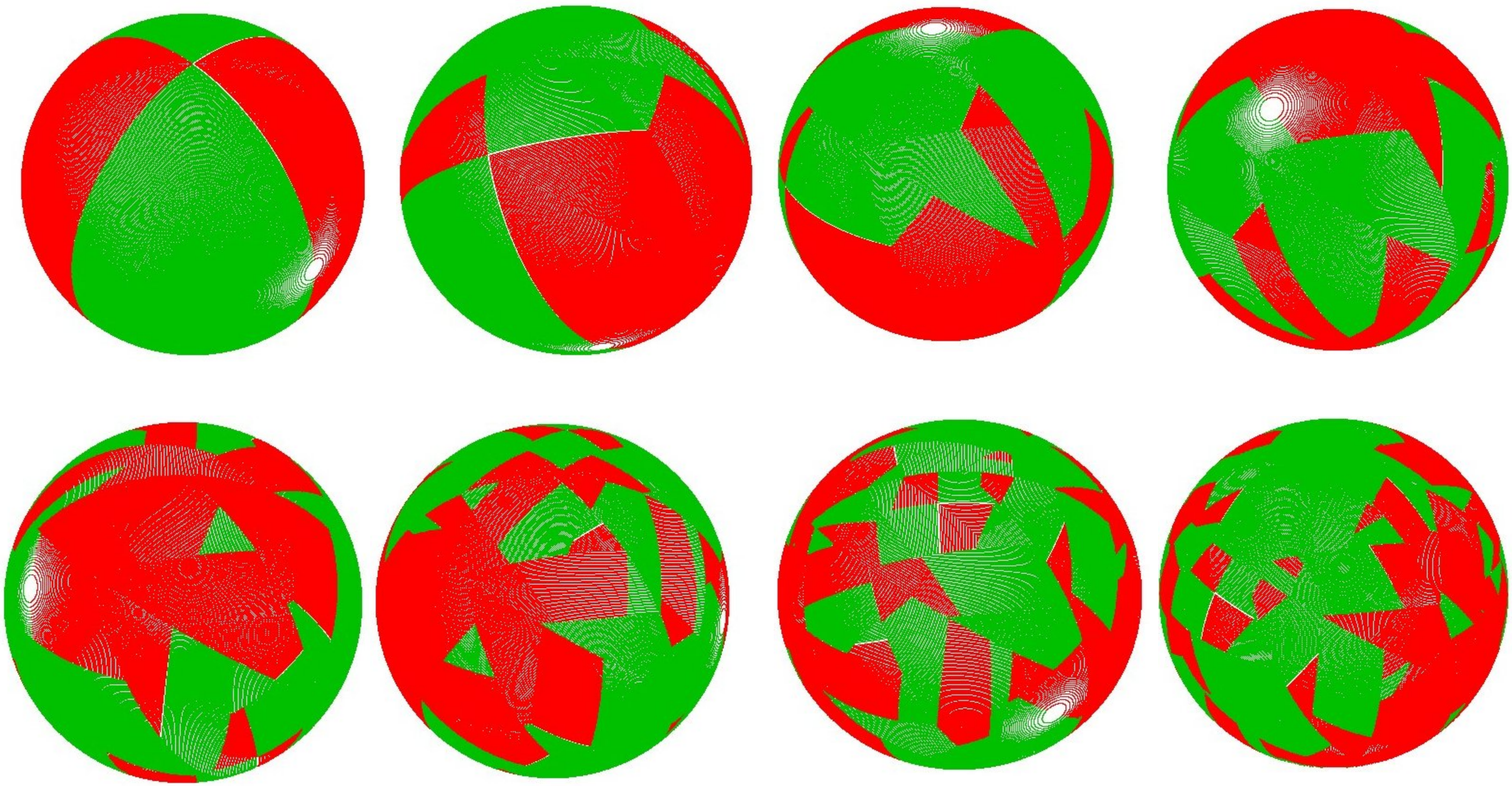
McCormick
Northwestern Engineering

Piecewise isometries: Mixing without stretching



with Sturman, Wiggins, in progress

JMOttino 08

Looking Back

- The Deception of Final Pictures (…Gauss and Riemann leaving no trace of the scaffold that lead to their final results; Hermann von Helmholtz and the "smooth royal path"; Ludwig Wittgenstein's metaphor of kicking the ladder after climbing up…Can anyone retrace Riemann?
- Bifurcation Points

  Reynolds vs. Reynolds, mixing without stretching
- The Role of the Environment

  Visual vs. non-Visual
- A historical perspective fosters creativity