Dynamics of Segregation, Mixing, and Coarsening of Granular Matter

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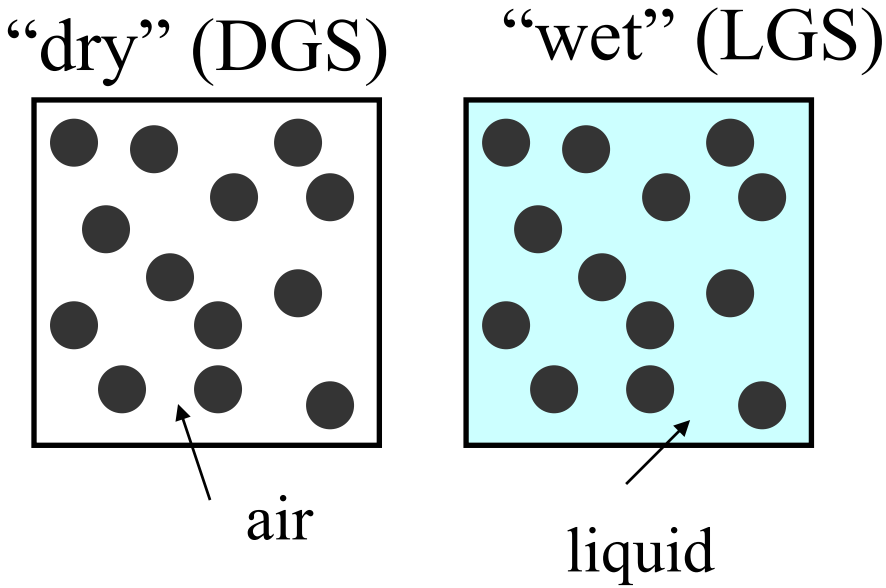
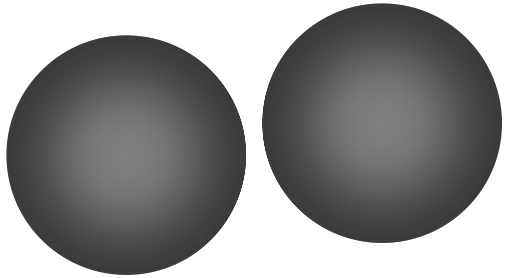
Granular Matter
dry, partially wet, and wet systems



2 phases only

Mechanisms, Interstitial Fluid (air, liquid)



diameter d, roughness ε

Collisional Forces $\rightarrow$ $\tau_C \sim \rho_{part} d^2 \dot{\gamma}^2$

Lubrication Forces $\rightarrow$ $\tau_L \sim \dfrac{\mu \dot{\gamma} d}{\varepsilon}$

$$\text{Bagnold Number (Ba)} = \frac{\tau_C}{\tau_L} = \frac{\rho_{part}\, d \dot{\gamma} \varepsilon}{\mu}$$

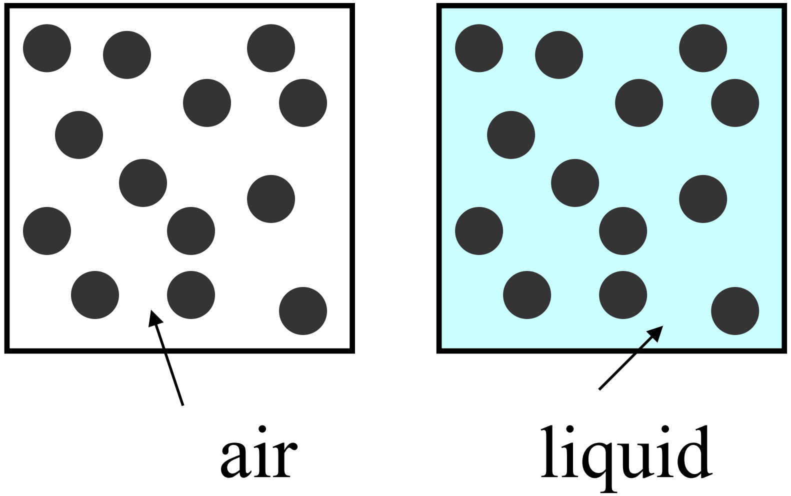
$Ba_{air} \sim 10^2,\ 10^3$

$Ba_{water} \sim 10^0,\ 10^1$

* Coussot and Ancey, PRE 1999

Granular Materials and Suspensions

“dry” (DGS) “wet” (LGS)



Stokes Number:

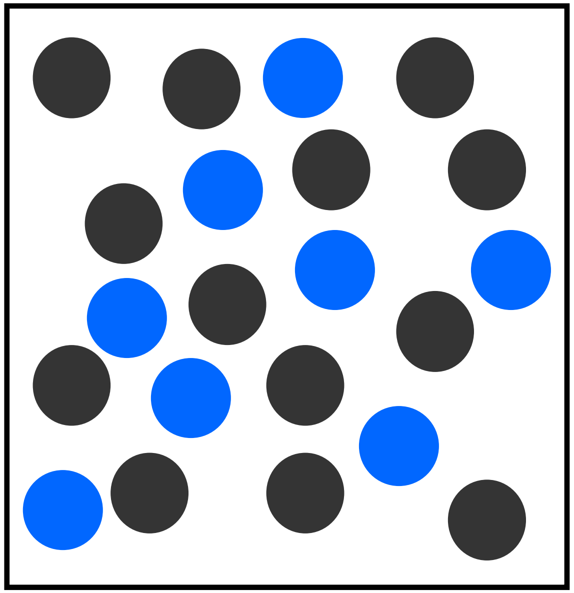
$$St = \frac{\text{"particle inertia"}}{\text{"viscosity"}}$$

DGS:
$St >> 1$

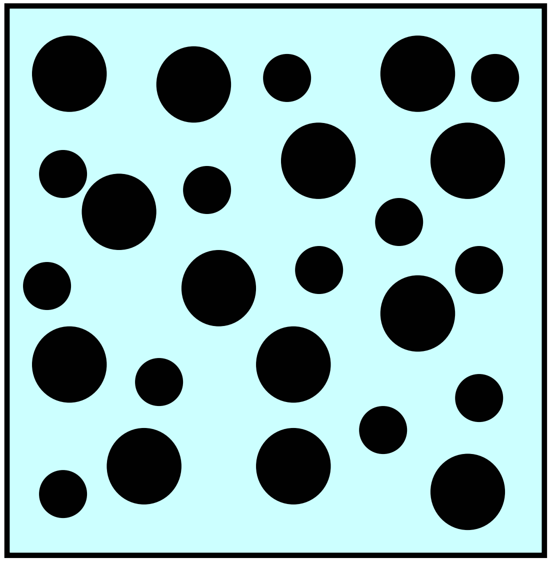
Distributions, bi-modal, etc.

DGS
(Dry Granular System)

LGS
(Liquid Granular System)



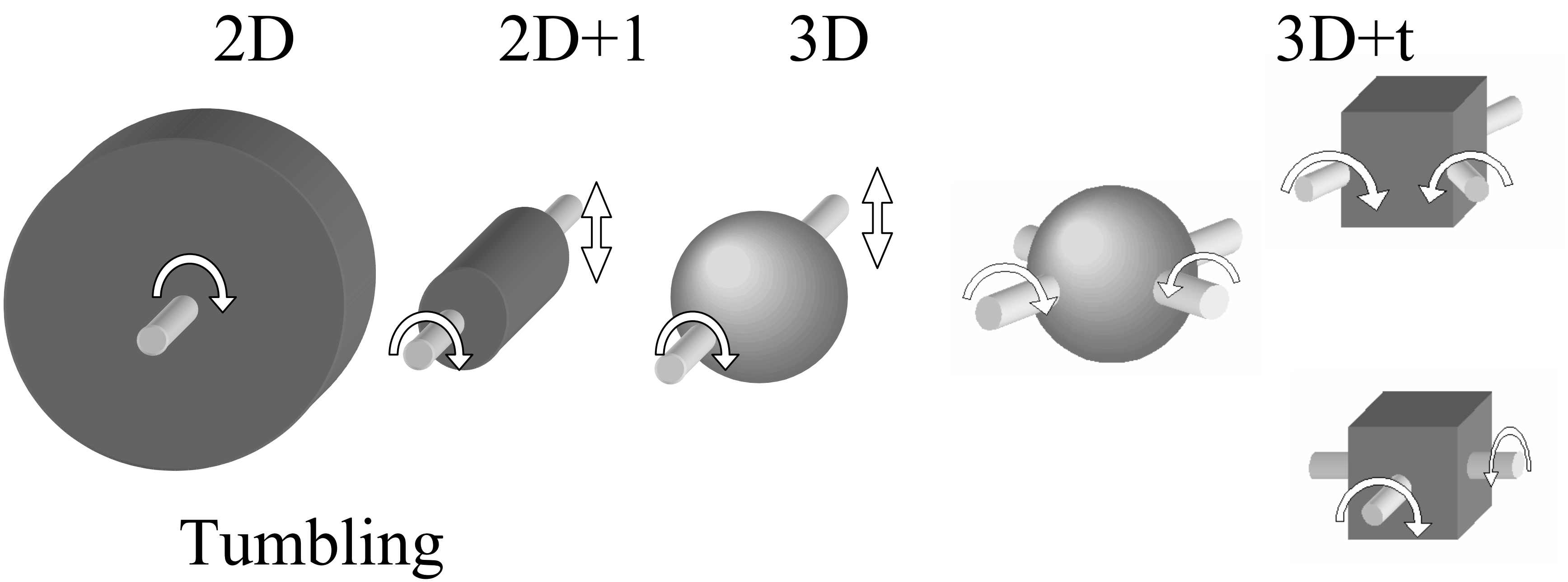
D-system (density)



S-system (size)

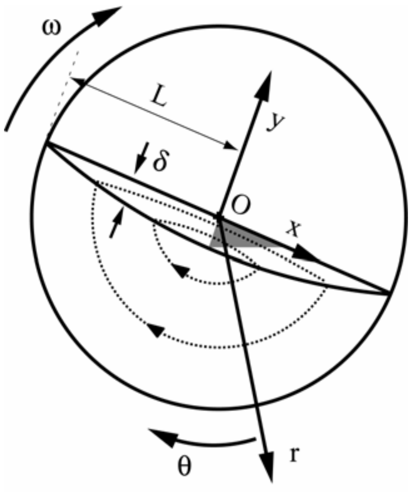
Combinations, e.g. S-DGS, D3-LGS, etc

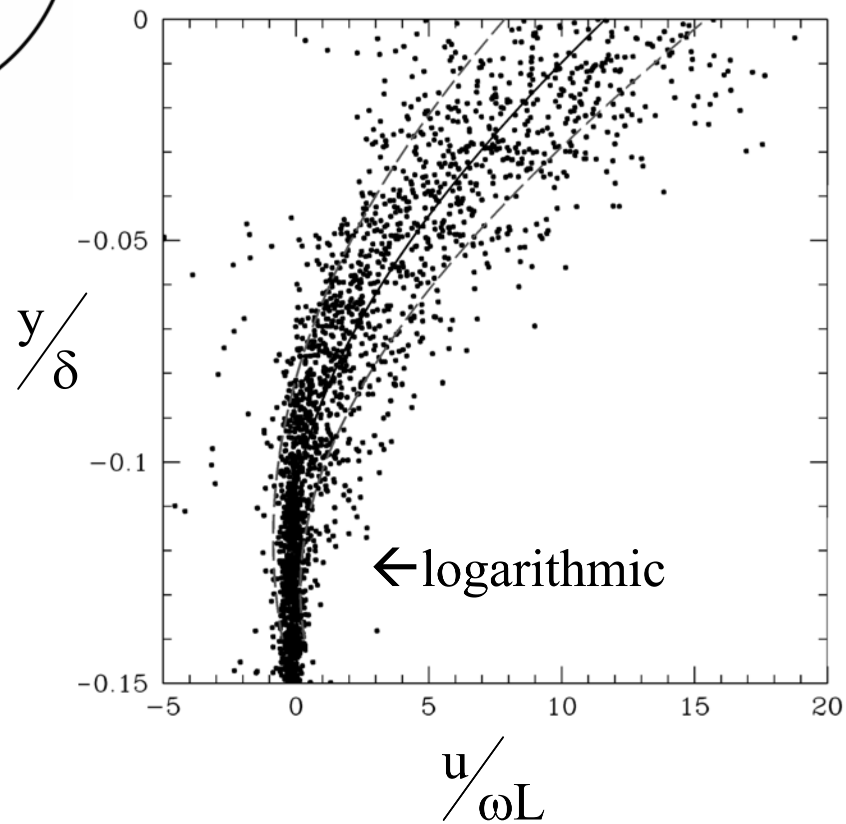
Granular matter/environment interaction

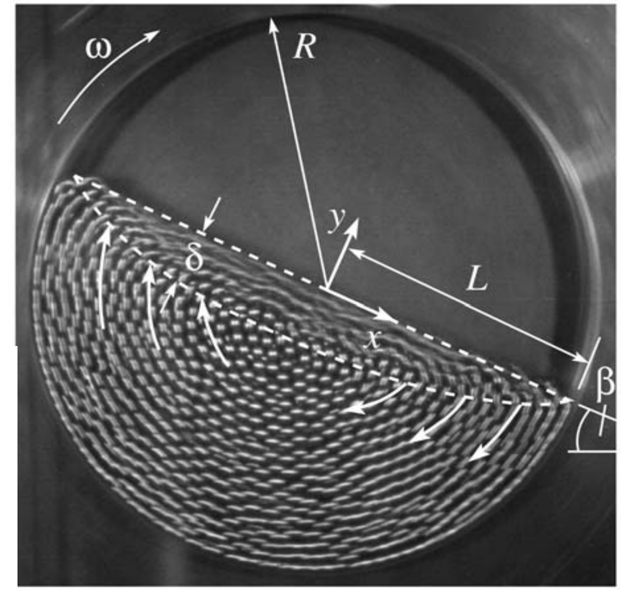


(S. Meier 2004)

Velocity field, fluid layer







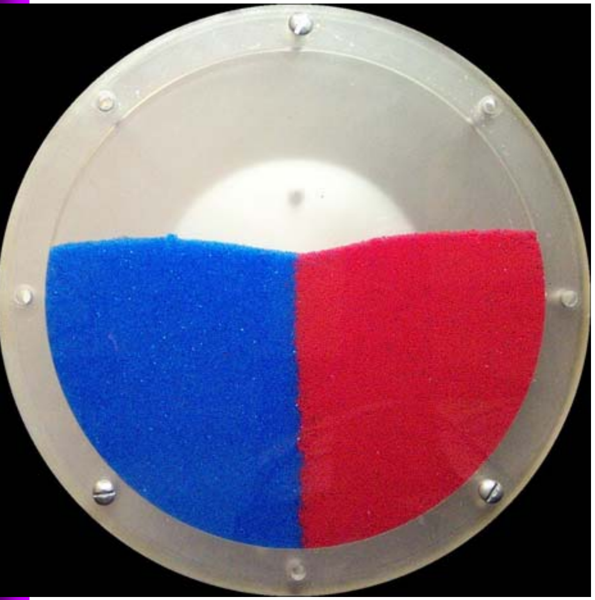
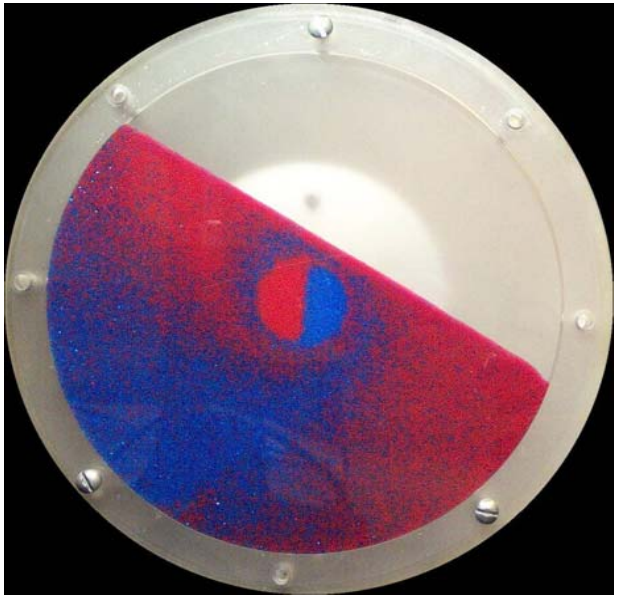
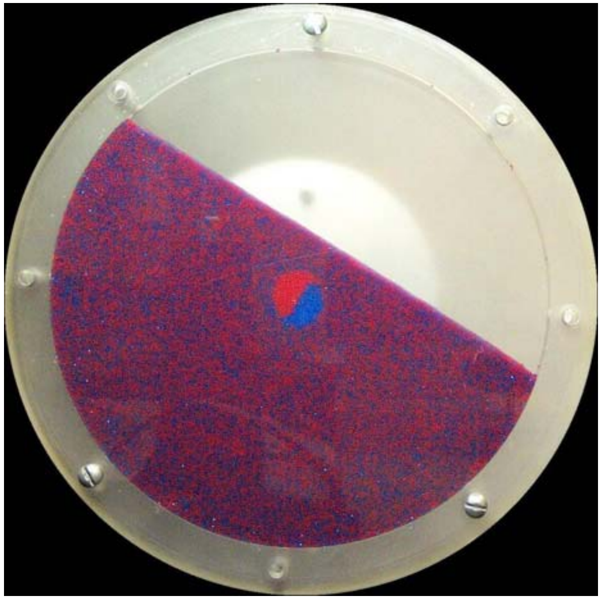
glass, steel,
d~1mm

"wall effects", 4 < *d/t* < 8

Experimental PIV

Jain *et al*. *Phys. Fluids* 2002 (DGS), Jain et al. *JFM*, 2004 (LGS)

Core precession and erosion

0

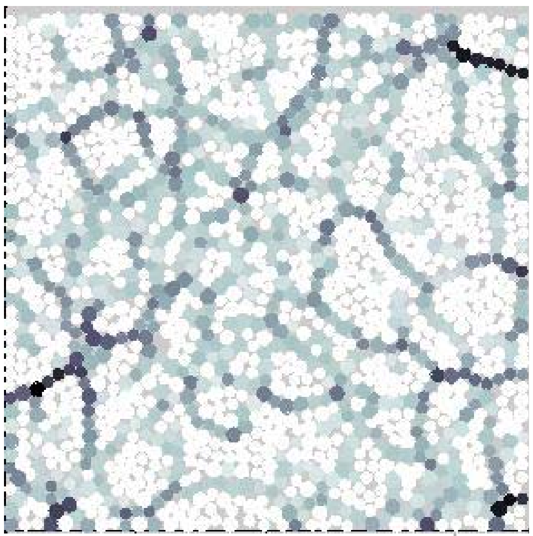
Initial Condition

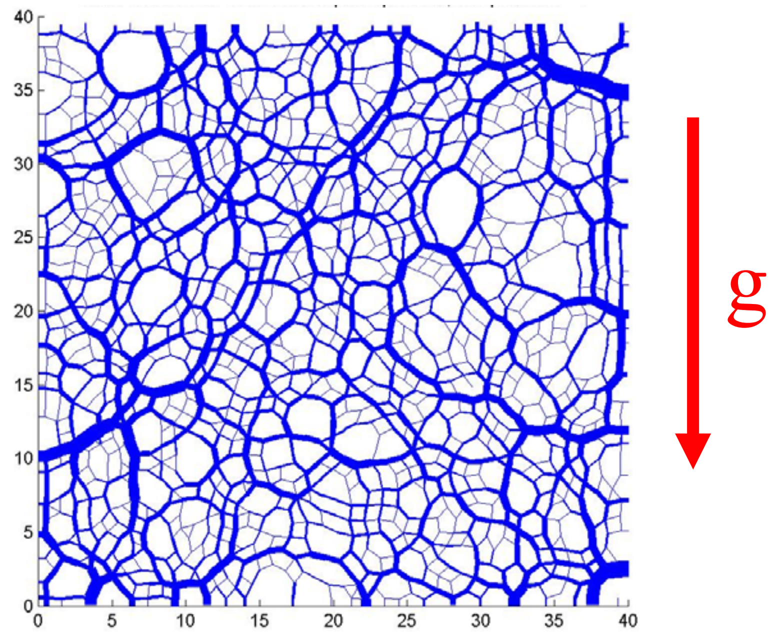
4 revolutions

1000

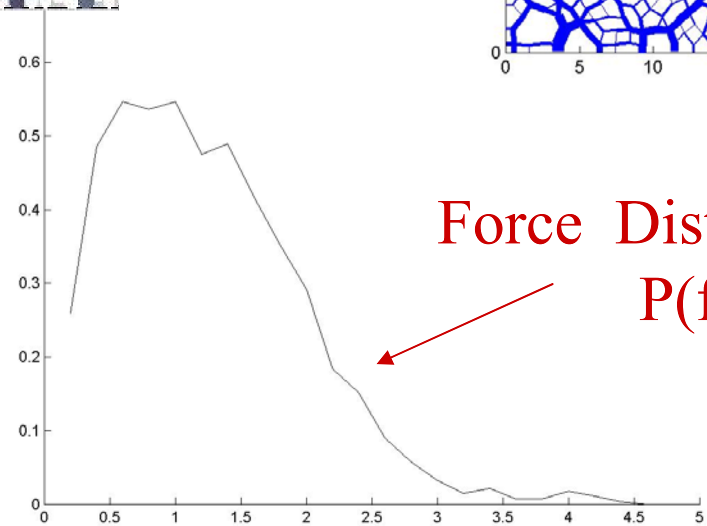
DH8 Socie et al. 2004

granular force network, re-arrangements





P(f)



Force Distribution, P(f)

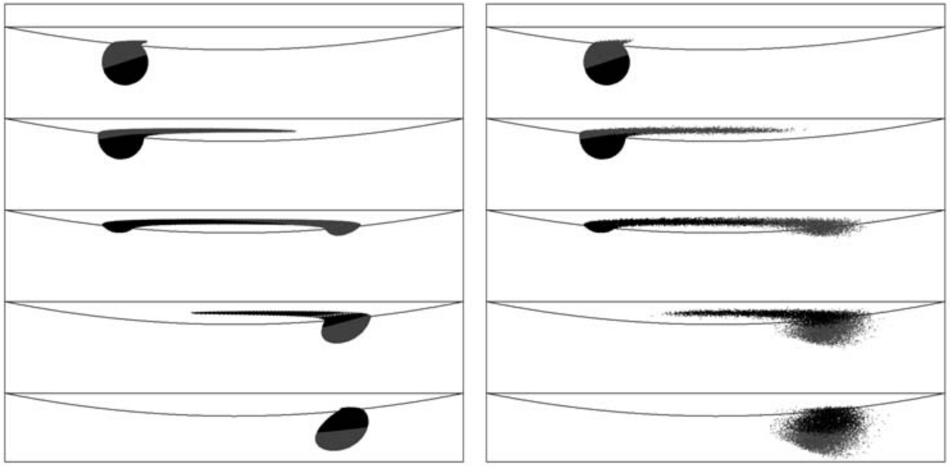
f /<f>

Ashley Smart 2004 unpublished

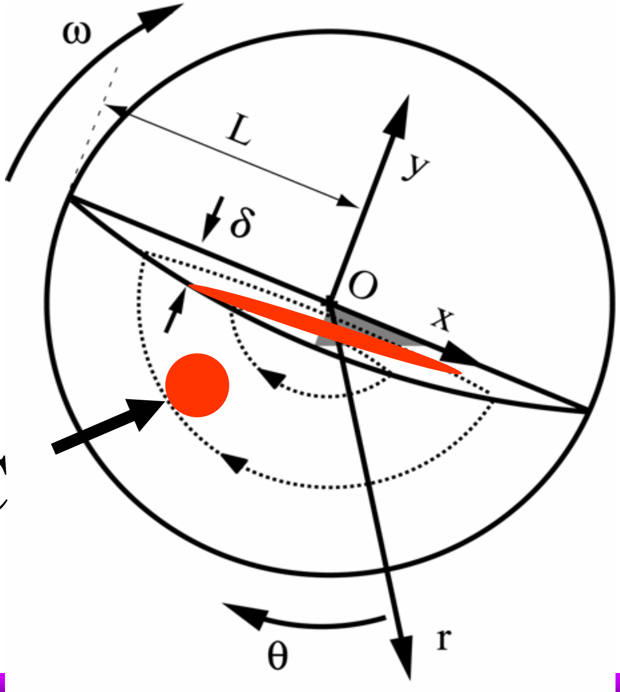
How Mixing Occurs (basis of continuum model)

$$D_{coll} = f(\nu)d^2 \frac{dv_x}{dy}$$

Collisional Diffusion



no diffusion with diffusion



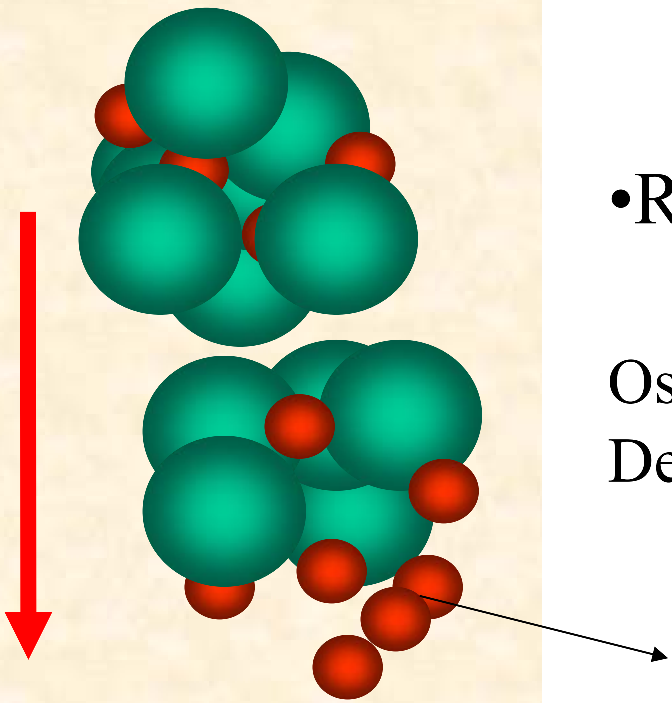
IC

$\delta = \delta(x)$ layer thickness

Khakhar et al. 2004

Segregation due to flow

(size) S-systems, (density) D-systems



Flux, Field

•Reynolds's dilatancy (percolation)

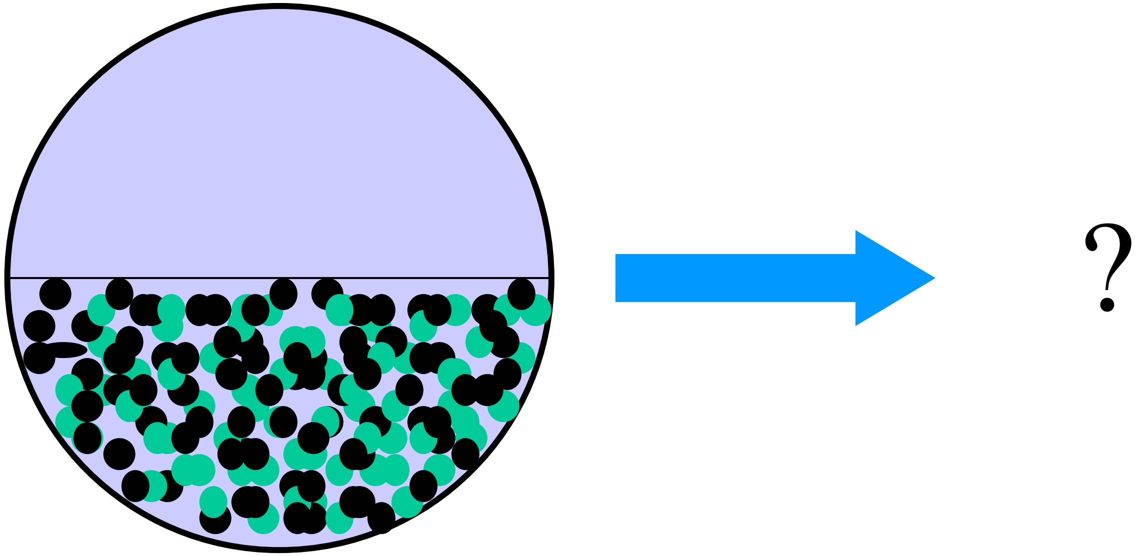
Osborne Reynolds, *Philosophical Magazine*, December, 1885.

Size segregation

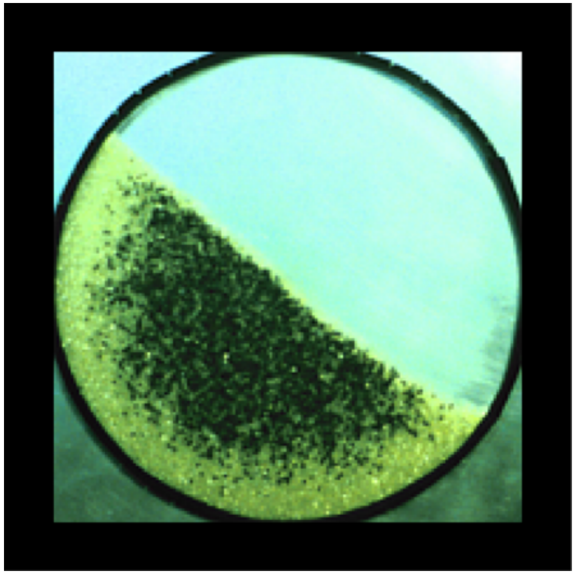
"Flux Model"…heuristic, PD-based…measurement?

Ottino & Khakhar *ARFM* 2000

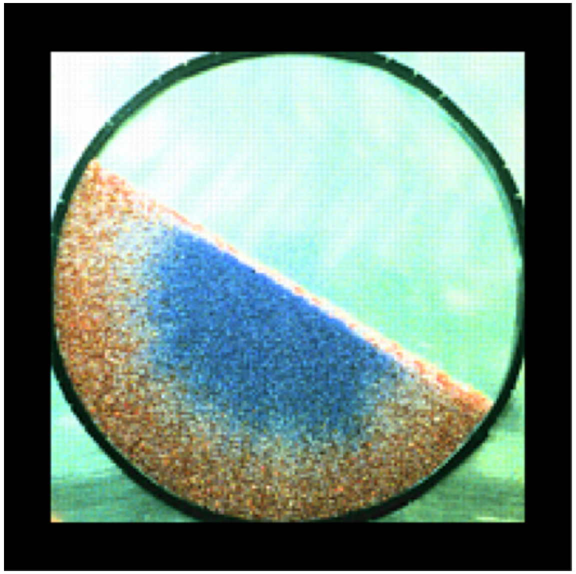
- Mixing → Unmixed



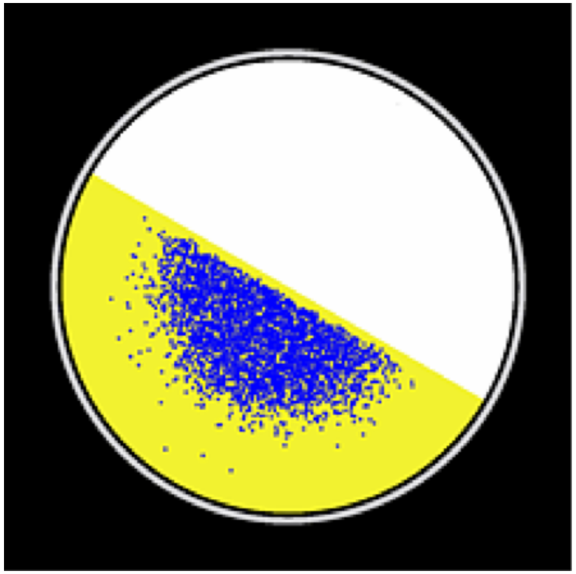
“Radial Segregation”, *O*(1) rotations



D-system



S-system

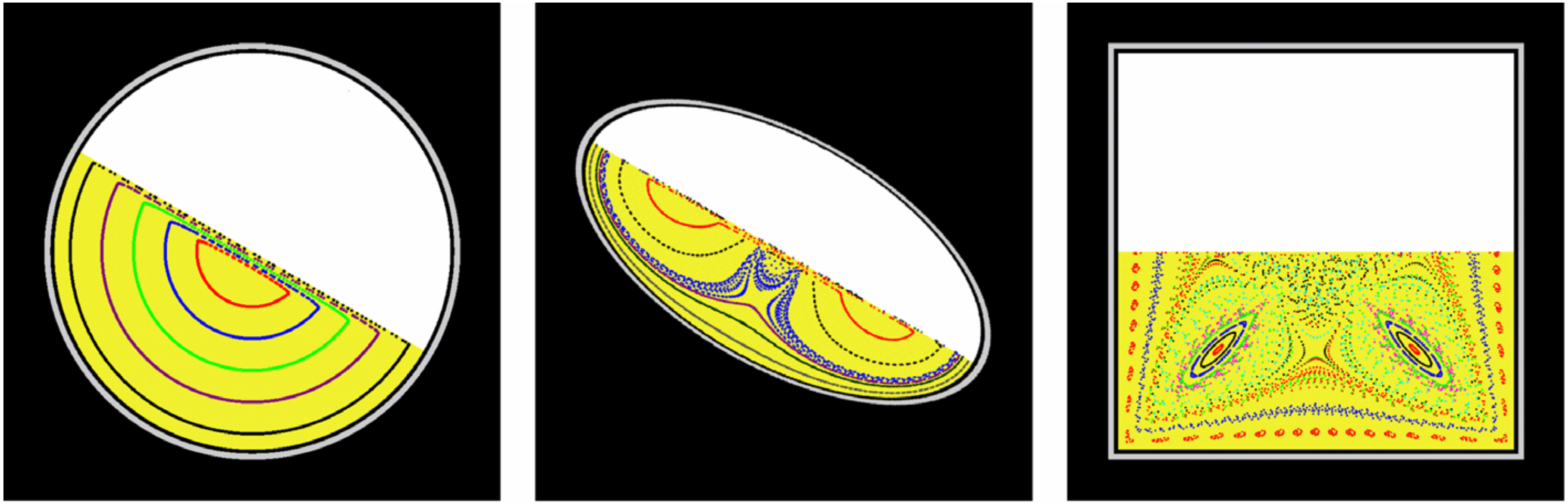


Computation

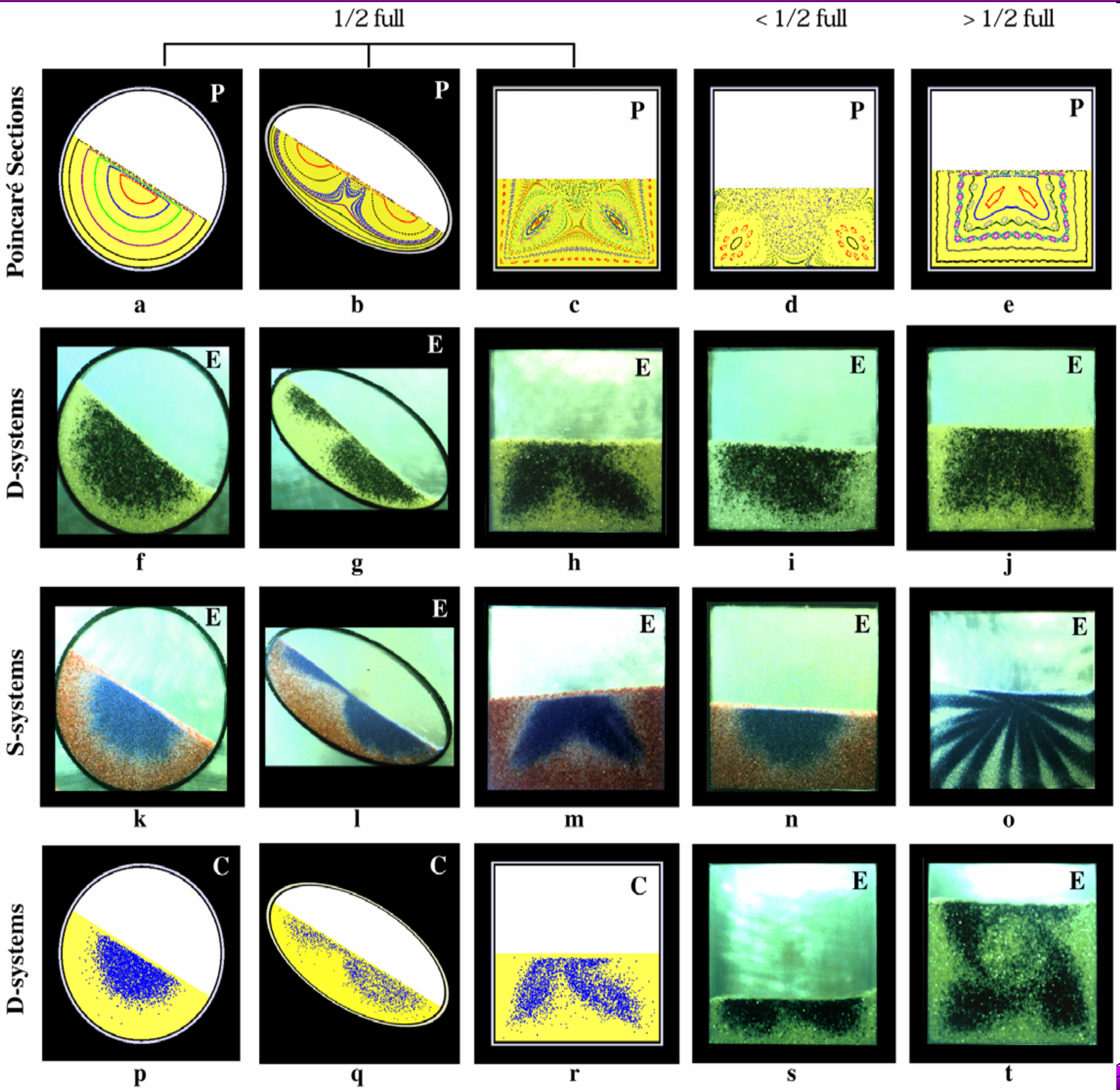
Non-circular geometries

continuum model, Poincaré plot

IC's $\xrightarrow{\text{Flow}}$ plot $\xrightarrow{\text{Flow}}$ plot…



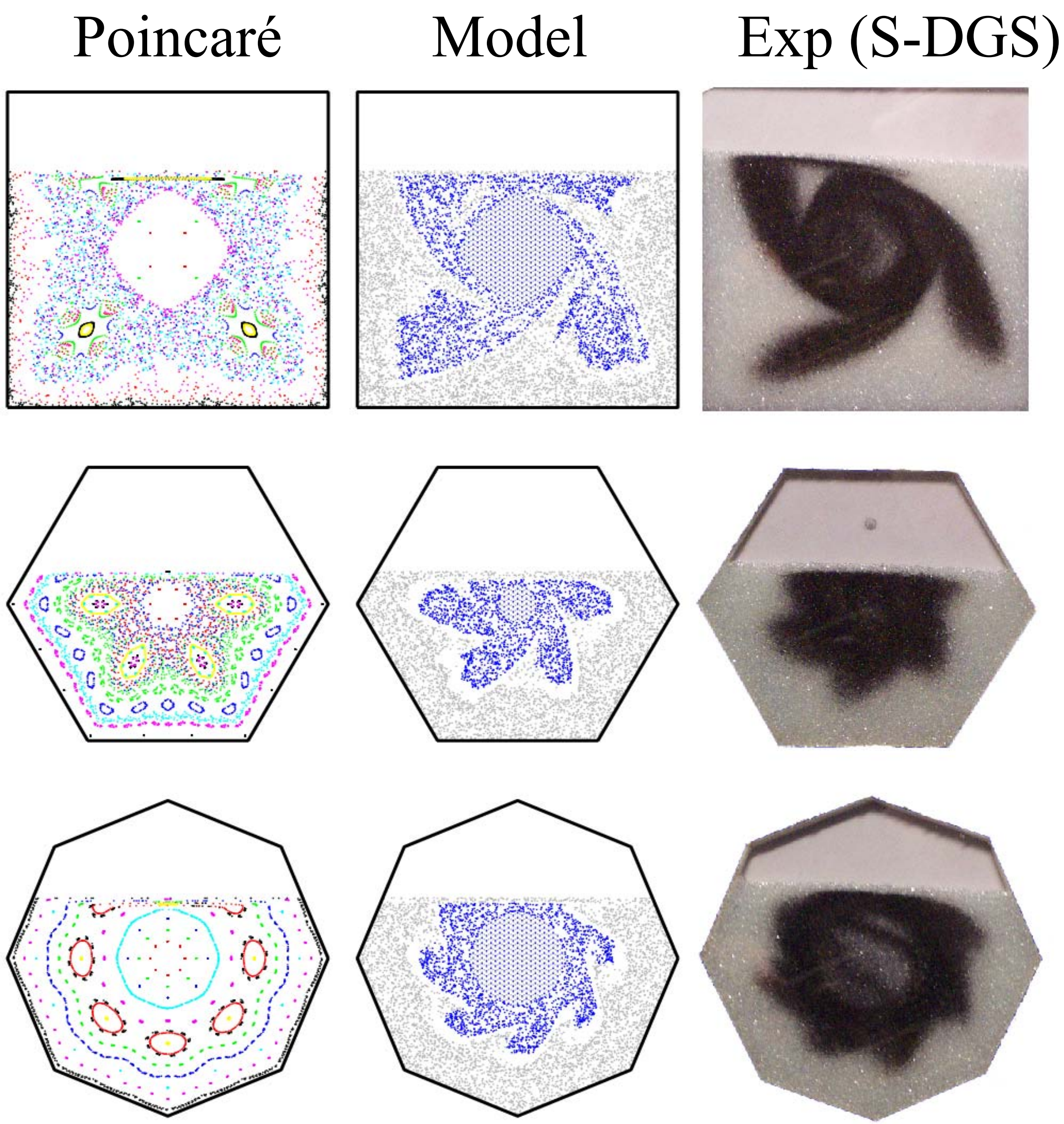
Khakhar *et al*., Chaos, 1999



S2,3-DGS

Hill et al. PNAS 1999

Interpenetrating Continua Model



Steve Cisar 2004
unpublished

Locally in layer… Species 1 and 2

$$\frac{dx}{dt} = v_x = 2u\left(1 + \frac{y}{\delta}\right)$$

$$\frac{dy}{dt} = v_y = -\omega x\left(\frac{y}{\delta}\right)^2$$

$$Pe = uL / D_{coll} >> 1$$

$$\frac{dx}{dt} = v_x$$

$$\frac{dy}{dt} = v_y + S$$

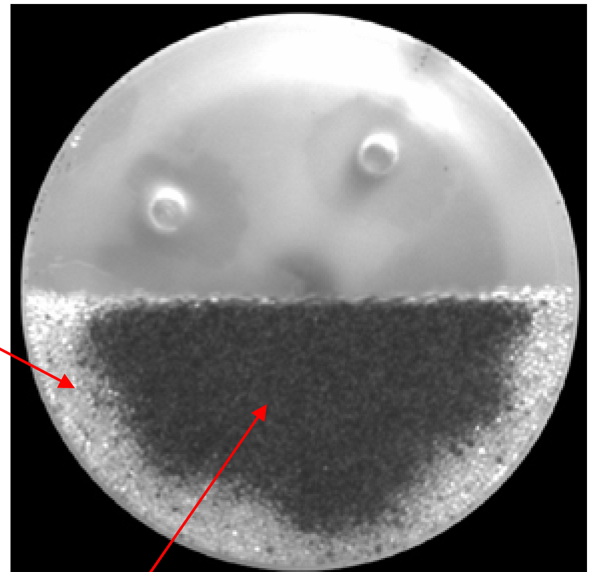
$$\frac{dy_1}{dt} = -\omega x_1\left(\frac{y_1}{\delta}\right)^2 + S - \frac{2\beta(1-\bar{\rho})D_{coll}(1-f)}{d}$$

$$\frac{dy_2}{dt} = -\omega x_2\left(\frac{y_2}{\delta}\right)^2 + S + \frac{2\beta(1-\bar{\rho})D_{coll}\, f}{d}$$

Segregation model

Segregation in Tumblers

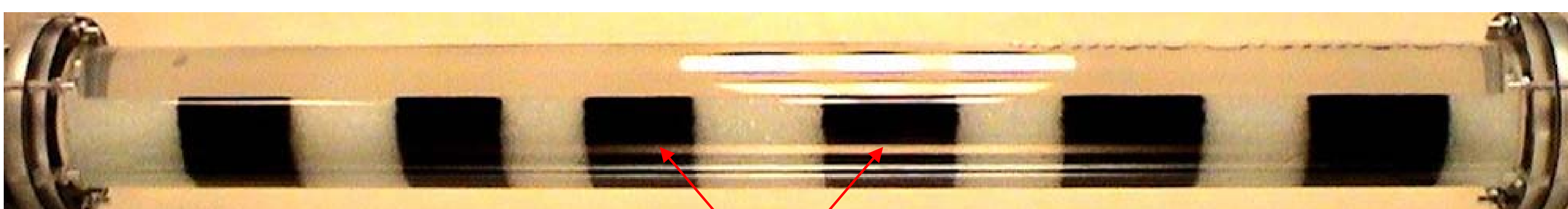
Large particles



2D

Smaller particles

2D+1



smaller particles

Difficulties, Questions

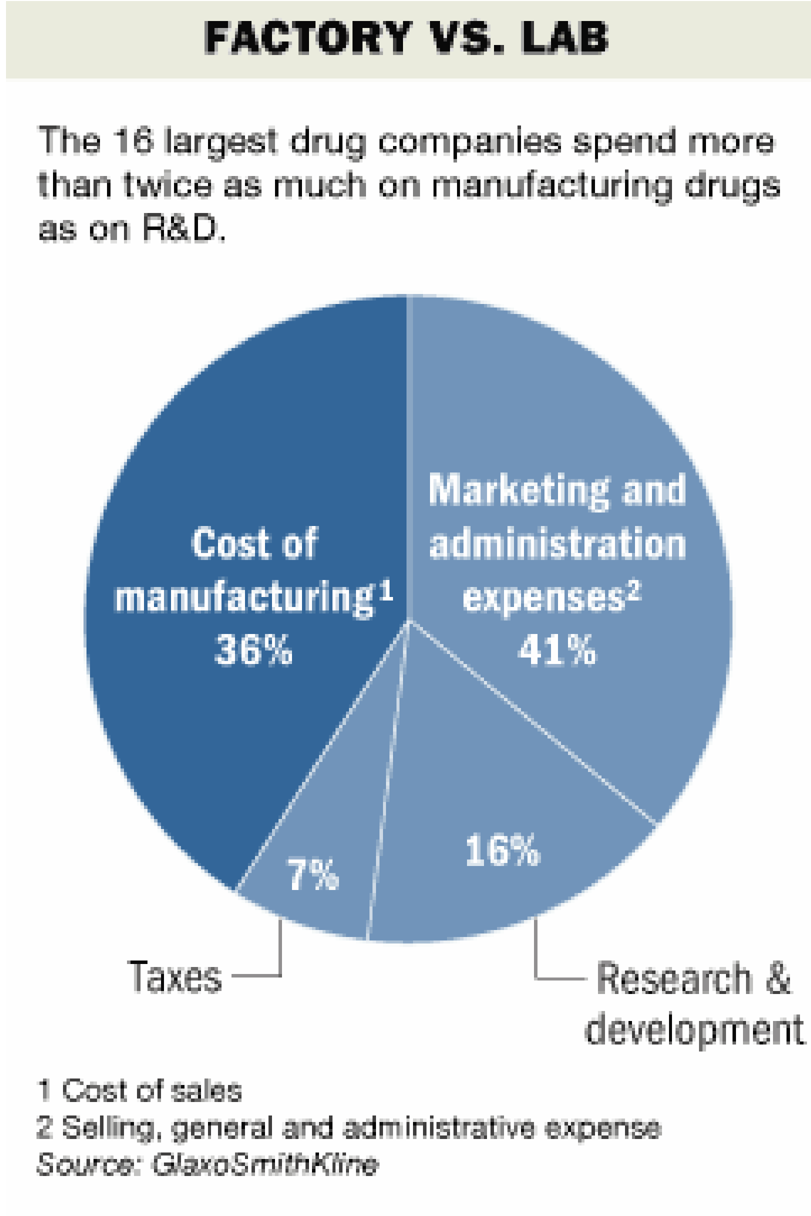
- No fit-all approach.

  Discrete and continuum (*Gollub*)

  Particle dynamics (PD), Lattice Boltzman, Monte Carlo (MC), Cellular Automata

- Role of thermodynamics (κT)
  - *Behringer*, *Edwards*, *Makse*, *others*

Applications

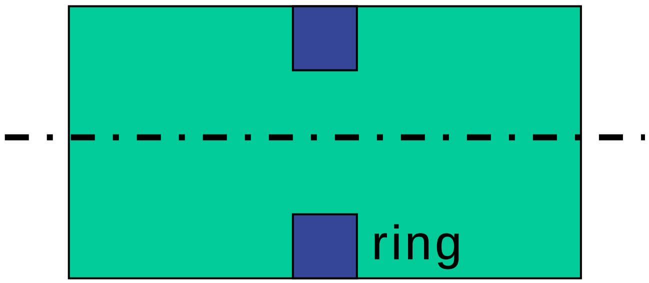


WSJ page 1, Sept. 4, 2003

Consequences of Axial Segregation

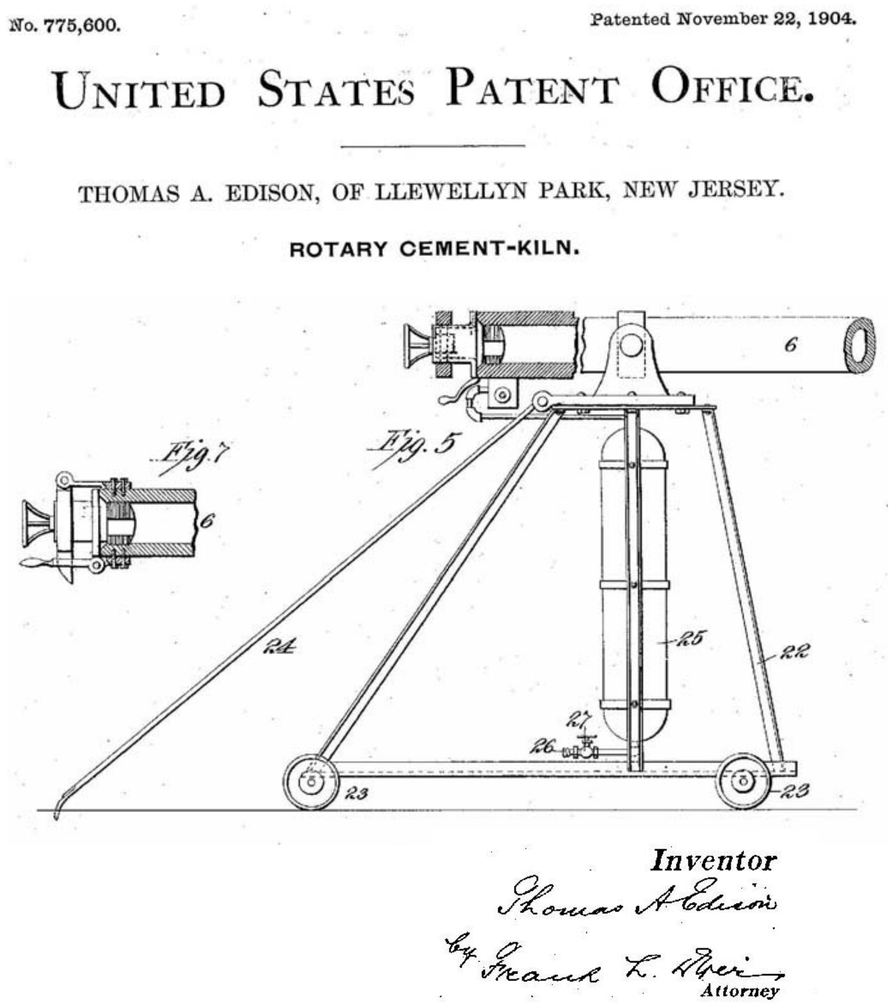
Ring formation

Molten clinker may solidify to form annular rings:



Has been a significant problem for rotary cement kilns for many years

Device for breaking rings (1904)



No. 775,600. Patented November 22, 1904.

UNITED STATES PATENT OFFICE.

THOMAS A. EDISON, OF LLEWELLYN PARK, NEW JERSEY.

ROTARY CEMENT-KILN.

SPECIFICATION forming part of Letters Patent No. 775,600, dated November 22, 1904.

Application filed July 22, 1903. Serial No. 166,519. (No model.)

To all whom it may concern:

Be it known that I, THOMAS A. EDISON, of Llewellyn Park, Orange, in the county of Essex, State of New Jersey, have invented certain Improvements in Rotary Cement-Kilns, (Case No. 1,104,) of which the following is a description.

Prior to my work in connection with the manufacture of Portland cement the Portland-cement mixture has been, and in most cases still is, calcined in rotary kilns about sixty feet in length and heated by means of pulverized coal. As the material progresses slowly through the kiln toward the zone of highest heat it first becomes very viscous; but as the chemical reactions progress it is converted into the usual hard clinker-balls of varying sizes. Although the melting-point of the material in this latter condition is much higher than when the material is introduced into the kiln, yet in the hottest portions of be softened and rendered worthless in about fifteen seconds, more or less.

With my new cement-kiln I make use of a structure approximately two and one-half times the length of the kilns now used, and in consequence the zone in which the mass forms into aggregates is proportionately removed from the lower end, so as to make it in many cases practically impossible to break up any of such coherent masses by any of the expedients as now employed.

The object of my invention, therefore, is to provide a kiln, or rather attachments thereto, by which any aggregates or masses of material can be effectively broken up, no matter how far their formation may take place from the lower end, and at the same time this operation will be performed much more expeditiously than heretofore and with resulting economies and improvements in uniformity of burning.

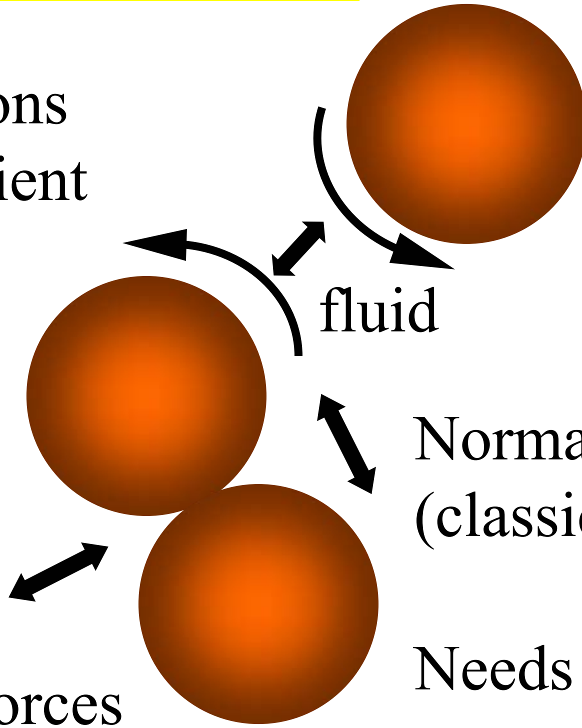
Particle Level

Building understanding up…

O(1 mm)

Non elastic collisions
Restitution coefficient

Exchange linear and
angular momentum



Normal forces, Hertzian
(classical elasticity)

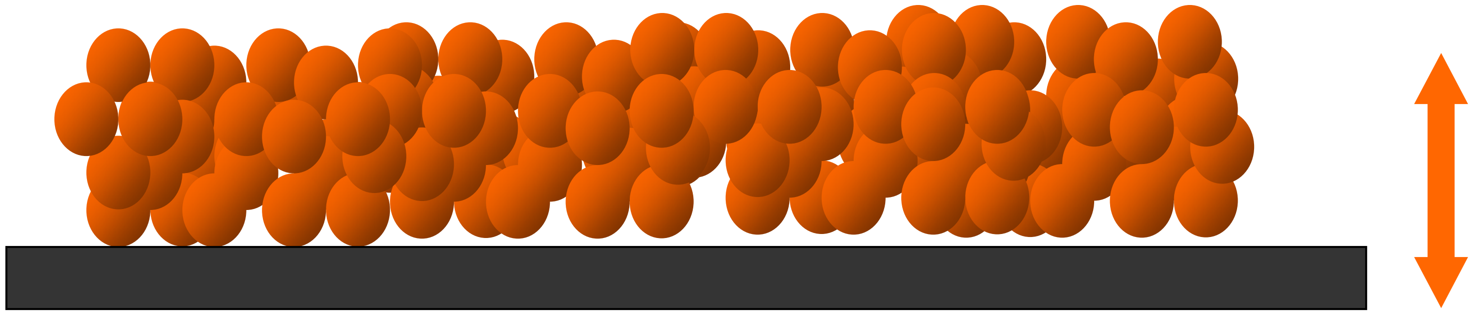
Needs to add dissipation

Tangential forces
Coulombic

"Regimes"…rolling regime, fast flow regime, etc.

Vibrated layer

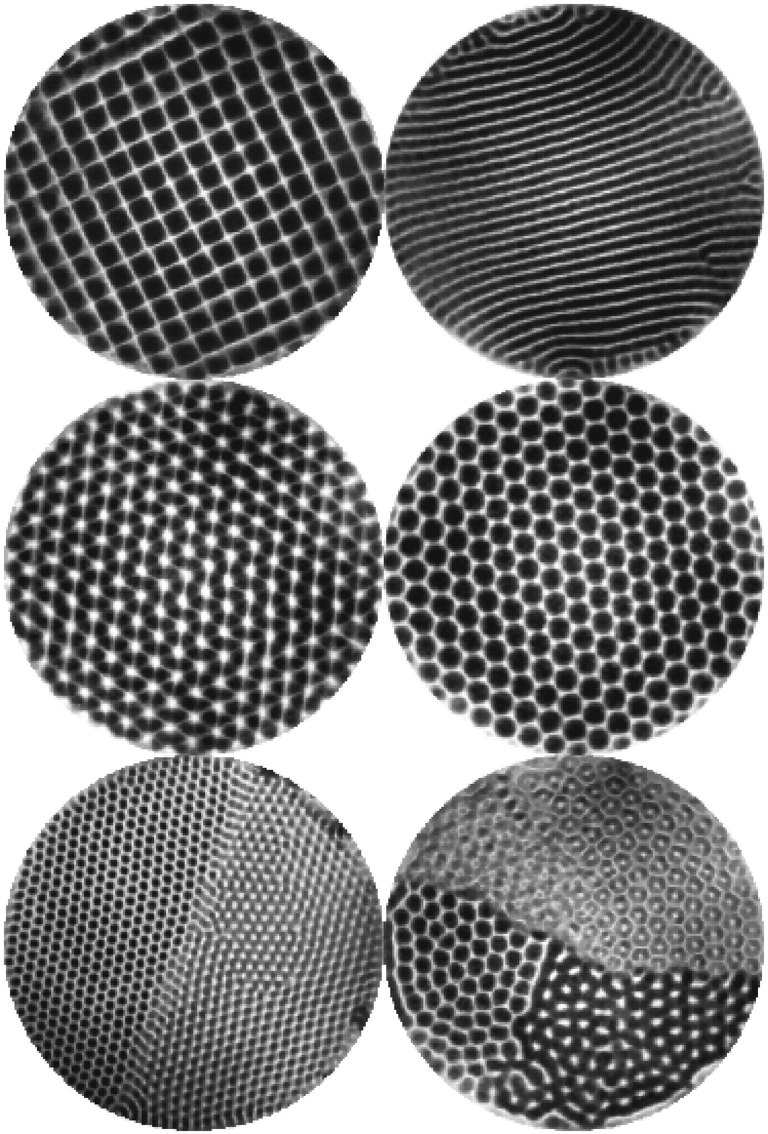
Small brass spheres



Energy in….Vibrate amplitude *A* frequency *f*

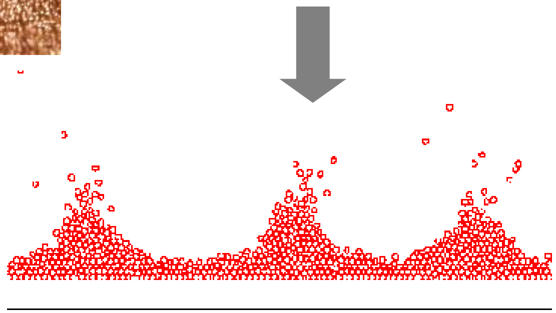
Umbanhowar, Swinney

top view



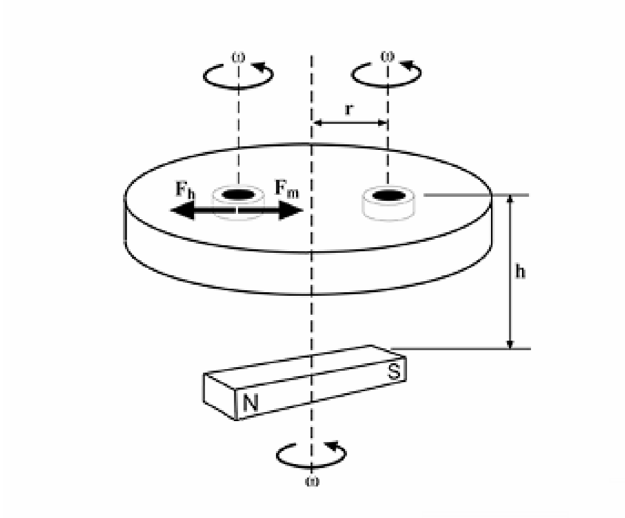
perspective view

side view

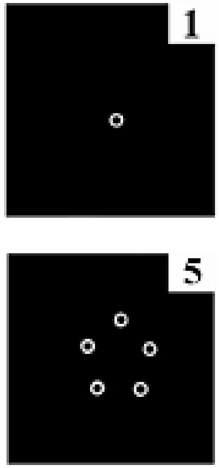


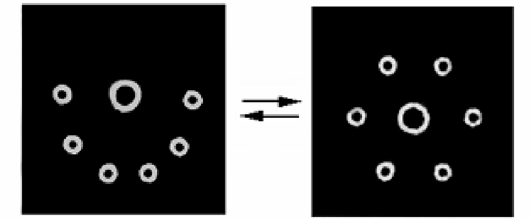
Vibrated granular matter
Umbanhowar, Swinney *et al*.

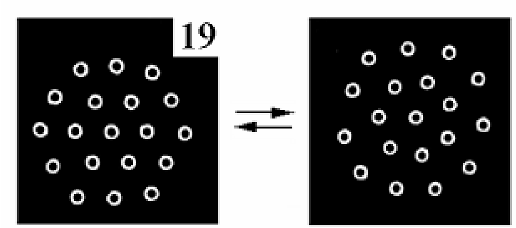
Dynamic Self-Assembly of Rotating Disks



Monomorphic Pattern **Symmetry-Breaking** **Polymorphic Pattern**







Grzybowski and Whitesides, *Science* 2003

…Beyond Granular Matter…

- GM serves as a prototype of collective systems far from equilibrium
- Concepts apply across a wide range of scales – from fine particles to ice floes to asteroid belts…
- Example of Complex Systems

Complex system… recognize by…

(1) **What is does**: *Display organization without any organizing principle being applied, i.e. behavior emerges*

(2) **How can be analyzed**: *Decomposing the system and analyzing a part does not give a clue as to the behavior of the whole.*

→ Rich behavior with simple parts

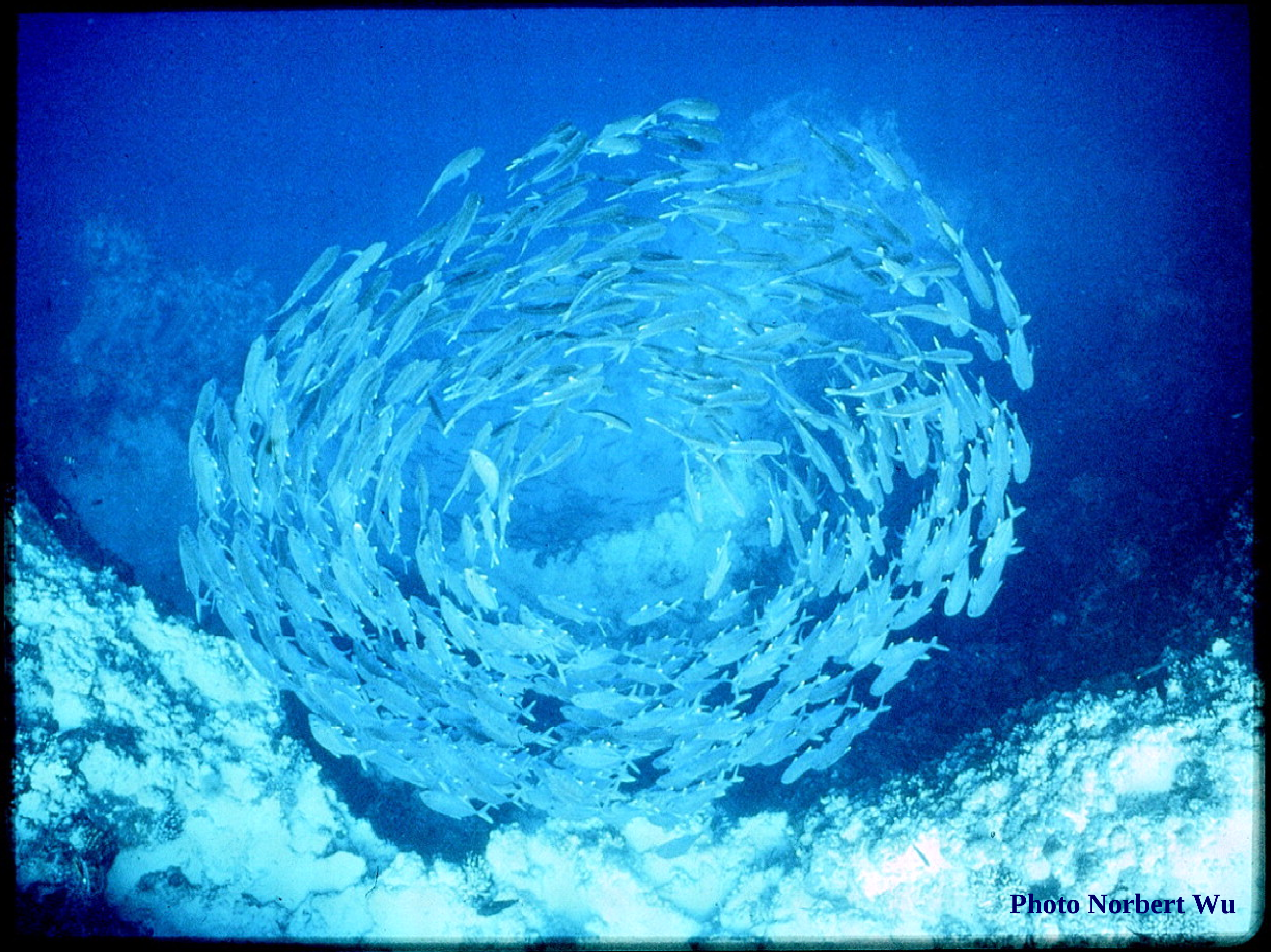
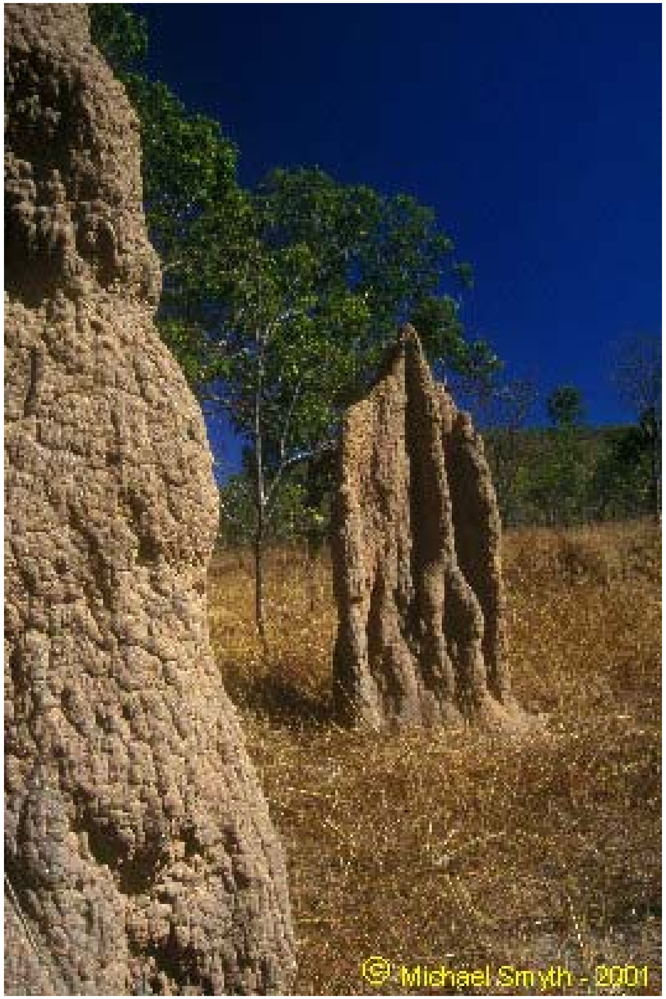


Photo Norbert Wu

Termites, mounds

Segregation, cities



Granular Matter, Metaphors

- Sandpile Avalanches

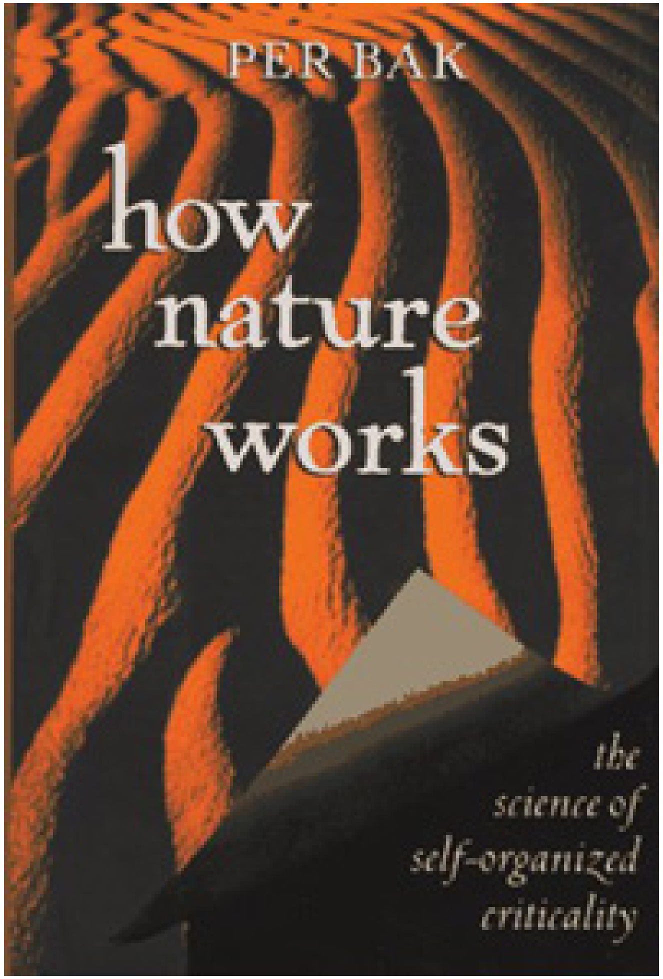
  …motion of flux lines in a type-II superconductor (*de Gennes*).
- Self-Organized Criticality, microscopic to astrophysical scales
- Excited granular matter, *slow relaxation* found in glasses, spin glasses, and the like
- *Fluid-like behavior*, convection

Beyond Granular Matter…

Per Bak

"How Nature Works:

The Science of Self Organized Criticality"

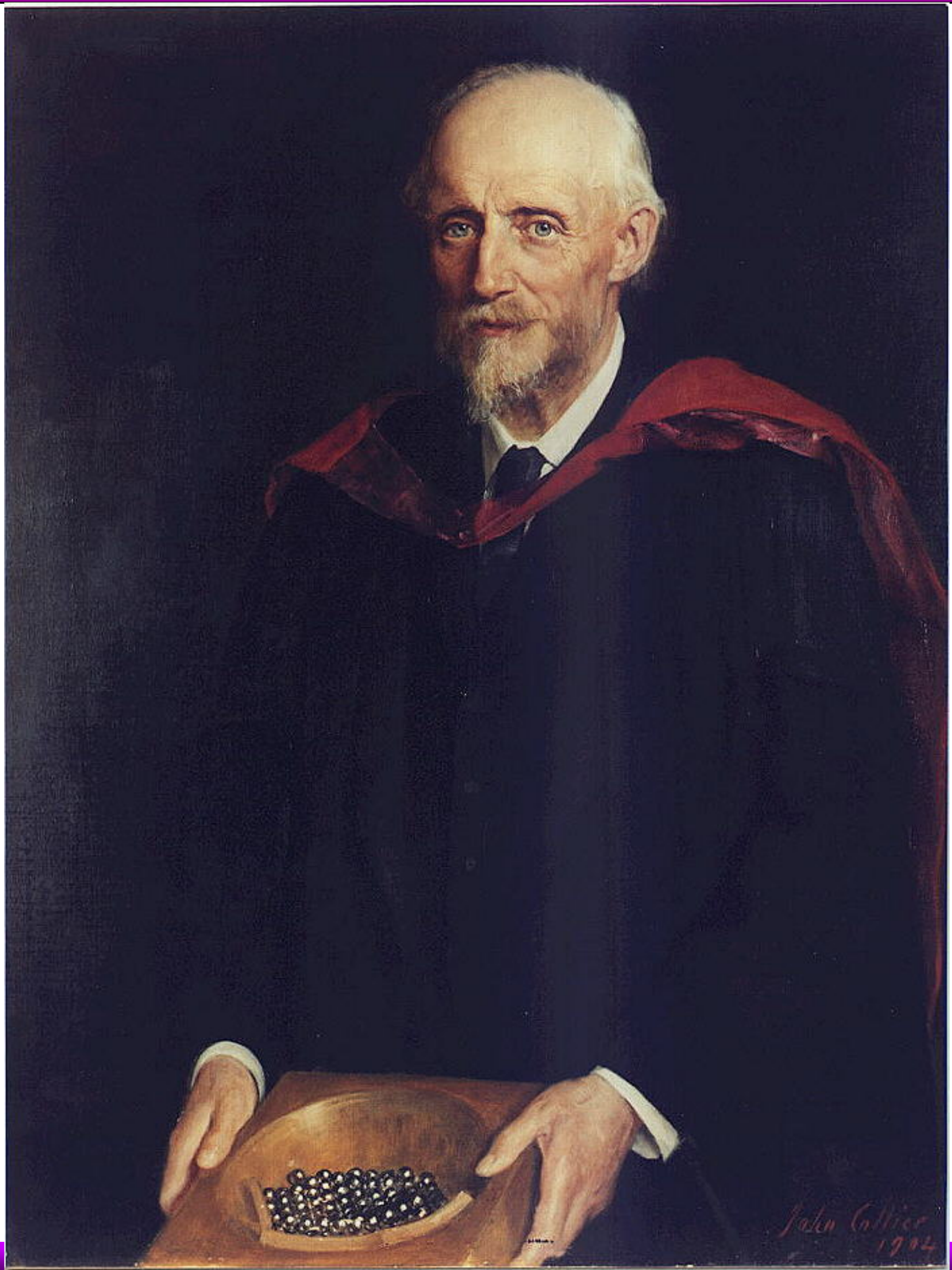


Reasons for "Success" of Granular Matter

- New physics, open theoretical questions, continuum, discrete.
- Experimentation still accessible and creativity still plays an important role.
- Intuition – often built on fluids – often does not work.
- Interplay

    Science (understanding and explaining)

    Technology (making and building).
- The "appeal" of metaphors.



Osborne Reynolds
(1842-1912)

- Osborne Reynolds "On the dilatancy of media composed of rigid particles in contact. With experimental illustrations". *Philosophical Magazine*, December 1885.

- Rode Lecture in 1902 ("On an inversion of ideas as to the structure of the universe")

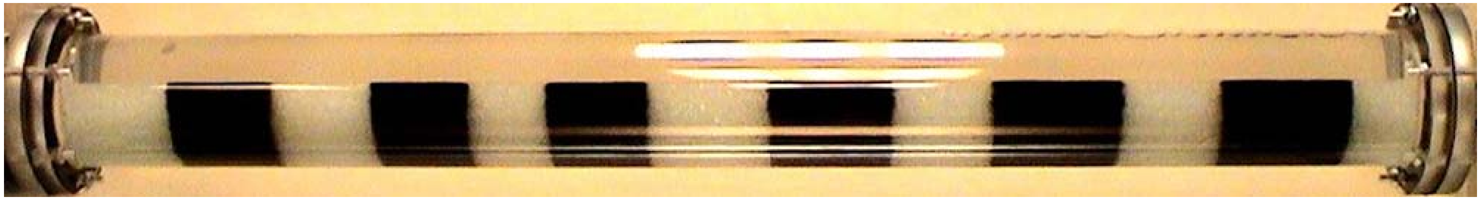
  "*I have in my hand the first experimental model universe, a soft India rubber bag filled with small shot*".

- "**The Sub-Mechanics of the Universe**" (Reynolds 1903). "*By this research it is shown that there is one, and only one, conceivable purely mechanical system capable of accounting for all the physical evidence, as we know it, in the Universe.*"

- Mechanical theory of the ether. Universe filled with rigid grains size: 5.534×10^{-18} cm, mean free path 8.612×10^{-28} cm.
- Planck length ('quantum of length') smallest measurement of length with any meaning:

  1.6×10^{-37} cm or about 10^{-20} times the size of a proton.
- ….but Reynolds was a teacher of J.J. Thomson (discoverer of the electron).

  …so much for the modernity of far-reaching analogies…



Oyama 1939

水平回轉圓筒內の 粒體の 運動 (第五報)*

二成分系粒體の 充塡と 混合

大 山 義 年

(昭和十四年 六月廿九日 受理)

I. 緒 言

前報告[1]迄は 何れも 水平回轉圓筒內に 於ける 均一徑粒體の 運動狀態 及 混合攪拌に 就いて 論じて 來たが，工業的に 我我が 取扱ふ 粒體は 均一徑の 同形粒子群より 構成せられてゐる事は 極て稀れで，殆と 總ては 所謂 多成分系粒體である。本報告は 多成分系粒體の 研究の 一歩として，先ず 二成分系粒體の 水平回轉圓筒內に 於ける 粒體運動狀態 及 混合に 就いて 實驗を 行った 結果である。

使用する 實驗裝置は 前報告に 於けると 同樣で ある。即ち 內徑 200 mm，長さ 400 mm の 內面を 丁寧に 仕上たる 鑄鐵製水平回轉圓筒で，一端を 平面硝子板にて 蔽ひ，共れを 通して 內部の粒體の 運動狀態を 觀察し，又は 寫眞に 撮影し得べくなしたる もので ある 實驗に 使用せる 粒體は 石灰石（平均比重 $\rho = 2{\cdot}72$）で，粉碎 篩別し，洗滌 且 充分 乾燥せる もので あり，共の 粒徑は 3·5 mm, 1·35 mm, 0·57 mm の 三種，之を 組合せて 實驗を 行った。

II. 二成分系粒體の 充塡と 混合に 對する 考察

(1) 二成分系粒體の 充塡。

大小徑の 異なる 二種の 同形均一徑粒體の 一定重量の 見掛容積は，粒體を 構成せる 各粒子の 配列組織が 同樣なれば 同一である。故に 此の 二種の 粒體を 一定容器に 判然と 二層に 全然混合せずに 充塡したと すれば，共の 見掛容積は 各單獨に 全重量を 充塡したる 場合と 全く 同樣で ある。

然るに，此の 狀態に 混合 或は 攪拌作用が 與へられる ならば，小徑粒子は 大徑粒子間の 空隙に 入り込んで 共の 空隙を 埋め，粒體組織は 密と なり 全見掛容積は 著しく 小と なる。

Furnas, Westman and Hugill[2] 及 共の 他は，此の 問題を 大徑粒子に 對して，小徑粒子は 無限に 小なりと 云ふ 理想的な 場合を 假想して 取扱って ゐる。

* 大河內研究室報告 第百六十五號
(1) 著者： 理研彙報，第12輯 12號； 第14輯 7號，9號； 第15輯 6號。
(2) C. C. FURNAS: *Ind. Engg. Chem.*, 23 (1931), 1052; *Bureau of Mines Bull.*, 307 74.
A. E. R. WESTMAN and H. R. HUGILL: *J. Am. Ceram. Soc.*, 13 (1930), 767.
A. E. R. WESTMAN: *J. Am. Ceram. Soc.*, 19 (1936), 127.
笠井康一： 理研彙報，11 (昭和 7)，793.

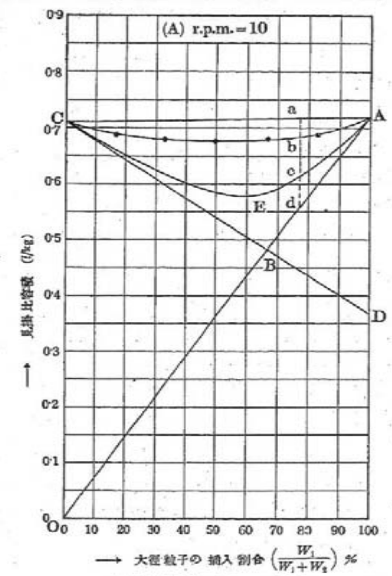
即ち 斯くの 如き 假定を 置く ならば，v_1* なる 空隙率を 有する 大徑粒體中に，v_2 なる 空隙率を 有する 小徑粒群を 順次注いで 一樣に 分布せし場合，小粒群が 大粒子間空隙の 容積を 完全に 埋めつくす までは 大徑粒子群の 見掛の 容積は 全然 増加しないと 考へ得る。故に 斯かる 場合の 混合粒體の 見掛比容積 (V_a) と，粒體插入量割合 $\left(x = \frac{W_1}{W_1 + W_2}\right)$ との 關係は，小徑粒子群が 大徑粒子間の 空隙を 埋めつくす迄は，直線的關係に ある。此處で 見掛比容積 (V_a) とは，s を 粒子の 單位體積當りの 眞の 重量と する ならば，

$V_a = \frac{1}{s(1-v)}$ で 現される 値で ある。

此の 直線的關係を 式で 求むれば

$$V_a = \frac{x}{s_1(1-v_1)} \qquad (1)$$

と なり，第一圖に 於て ABO で 現される 原點 O を 通過する 直線と なる。



第一圖 粒徑比 3·5 mm : 0·57 mm

丁度 大徑粒子間の 空隙が 總て 小徑粒子群にて 埋められた 點 (B 點) に 於ては 粒體見掛單位容積當り，次の 關係が 成立する。即ち

$$w_2 = v_1 \times (1-v_2)s_2$$
$$w_1 = (1-v_1)s_1$$

此處で w は 混合粒體單位容積當りの 各の 粒體の 重量で ある。故に 此の 大徑粒體が 小徑粒體群で 飽和した 時の 二粒體の 插入割合は

* 以下 各記號に 於て 右下に 附したる 小文字數字 1 は 大徑粒子，2 は 小徑粒子の 其れを 示す ものとす。而て 何も 附せざる ものは 混合粒體の 場合とす。

Sleeping Beauty Papers

- "…a publication that goes unnoticed ('sleeps'), gathering less than one citation a year for many years, and then, almost suddenly, attracts a lot of attention (the paper is awakened by a "prince").
  van Raan (*2004*)
- Bridgwater (1976) cites the paper and attributes the reference to Weidenbaum (1958); the next influential reference is Dasgupta et al. (1991), and after that the paper awoke…

NOTICES

OF THE

PROCEEDINGS

AT THE

MEETINGS OF THE MEMBERS

OF THE

Royal Institution of Great Britain,

WITH

ABSTRACTS OF THE DISCOURSES

DELIVERED AT

THE EVENING MEETINGS.

VOLUME XIV.
1893—1895.

LONDON:
PRINTED BY WILLIAM CLOWES AND SONS, LIMITED,
STAMFORD STREET AND CHARING CROSS.
1896.

WEEKLY EVENING MEETING,

Friday, June 2, 1893.

SIR DOUGLAS GALTON, K.C.B. D.C.L. LL.D. F.R.S. Vice-President, in the Chair.

PROFESSOR OSBORNE REYNOLDS, M.A. LL.D. F.R.S.

Study of Fluid Motion by means of Coloured Bands.

IN his charming story of 'The Purloined Letter,' Edgar Allan Poe tells how all the efforts and artifices of the Paris police to obtain possession of a certain letter, known to be in a particular room, were completely baffled for months by the simple plan of leaving the letter in an unsealed envelope in a letter-rack, and so destroying all *curiosity* as to its contents; and how the letter was at last found there by a young man who was not a professional member of the force. Closely analogous to this is the story I have to set before you to-night—how certain mysteries of fluid motion, which have resisted all attempts to penetrate them are at last explained by the simplest means and in the most obvious manner.

Osborne Reynolds, the essential elements of chaos

> In fluids, no less than in cooking, spinning and rolling—this attenuation is only the first step in the process of mixing—all involve the second process, that of folding, piling, or wrapping, by which the attenuated layers are brought together. This

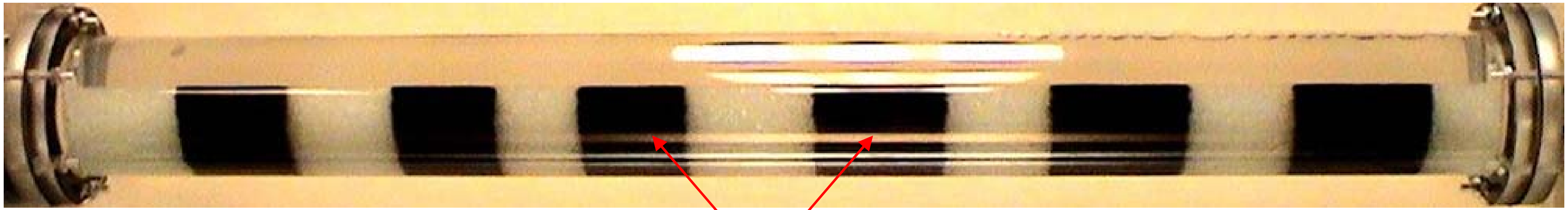
- Reynolds (1894); 16 citations in the period 1955-2004.
- Identification of stretching and folding as basic mechanism for mixing

The work of the Innovator

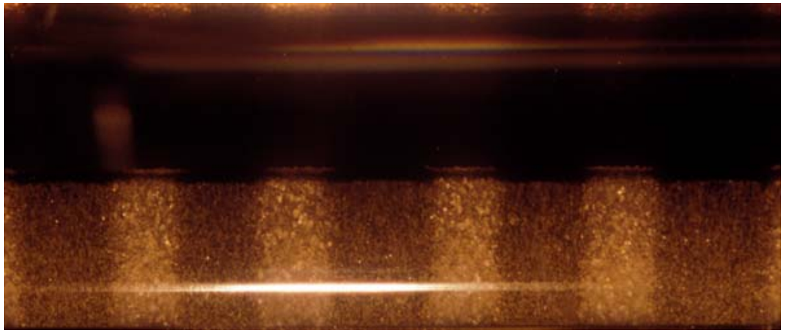
"Never forget what I believe was observed to you by Coleridge, that every great and original writer, in proportion as he is great and original, must himself create the taste by which he is relished"

William Wordsworth (English poet, 1770–1850) in Letter to Lady Beaumont, 21 May 1807; in E. de Selincourt (ed.) *Letters of William and Dorothy Wordsworth* vol. 2; revised by M. Moorman, 1969.

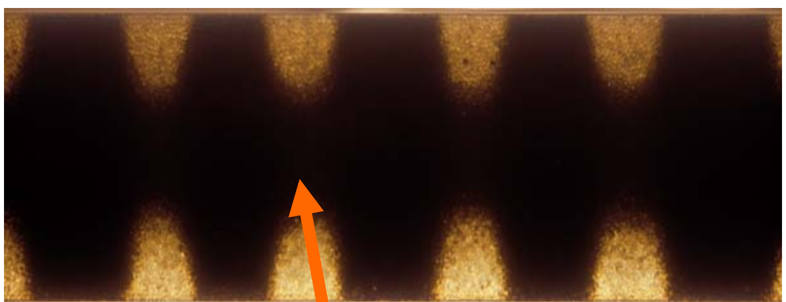
Segregation in Tumblers

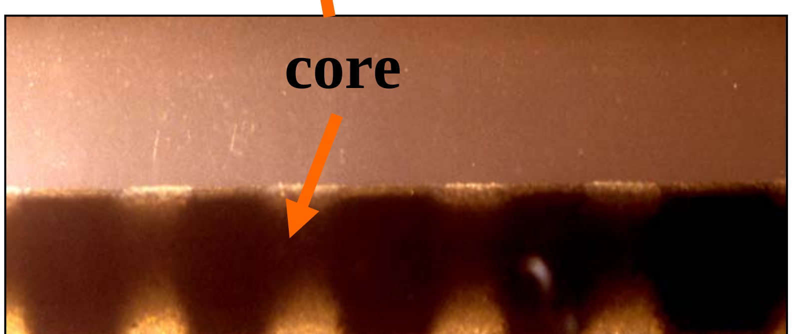


smaller particles

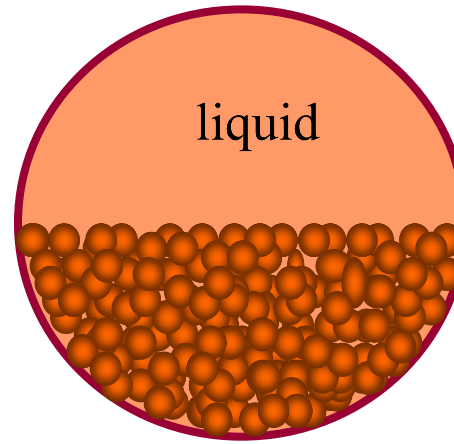


"outside view"





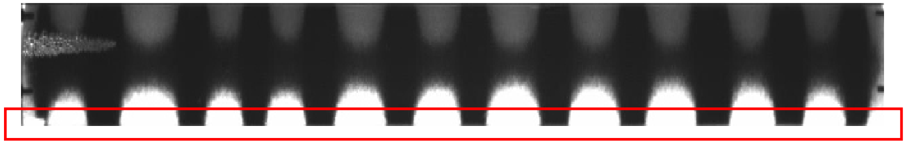
"interior view"

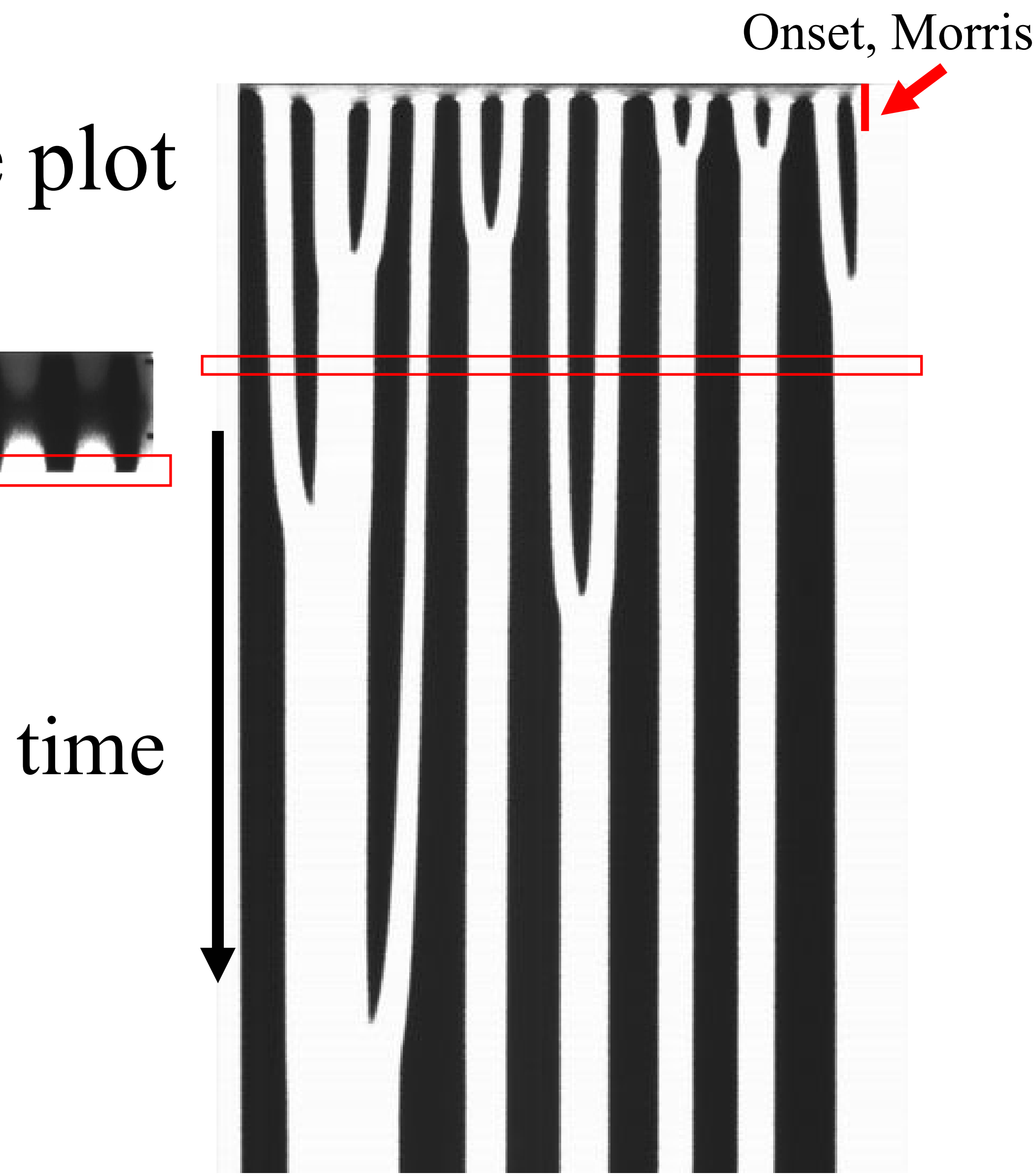


S-LGS system

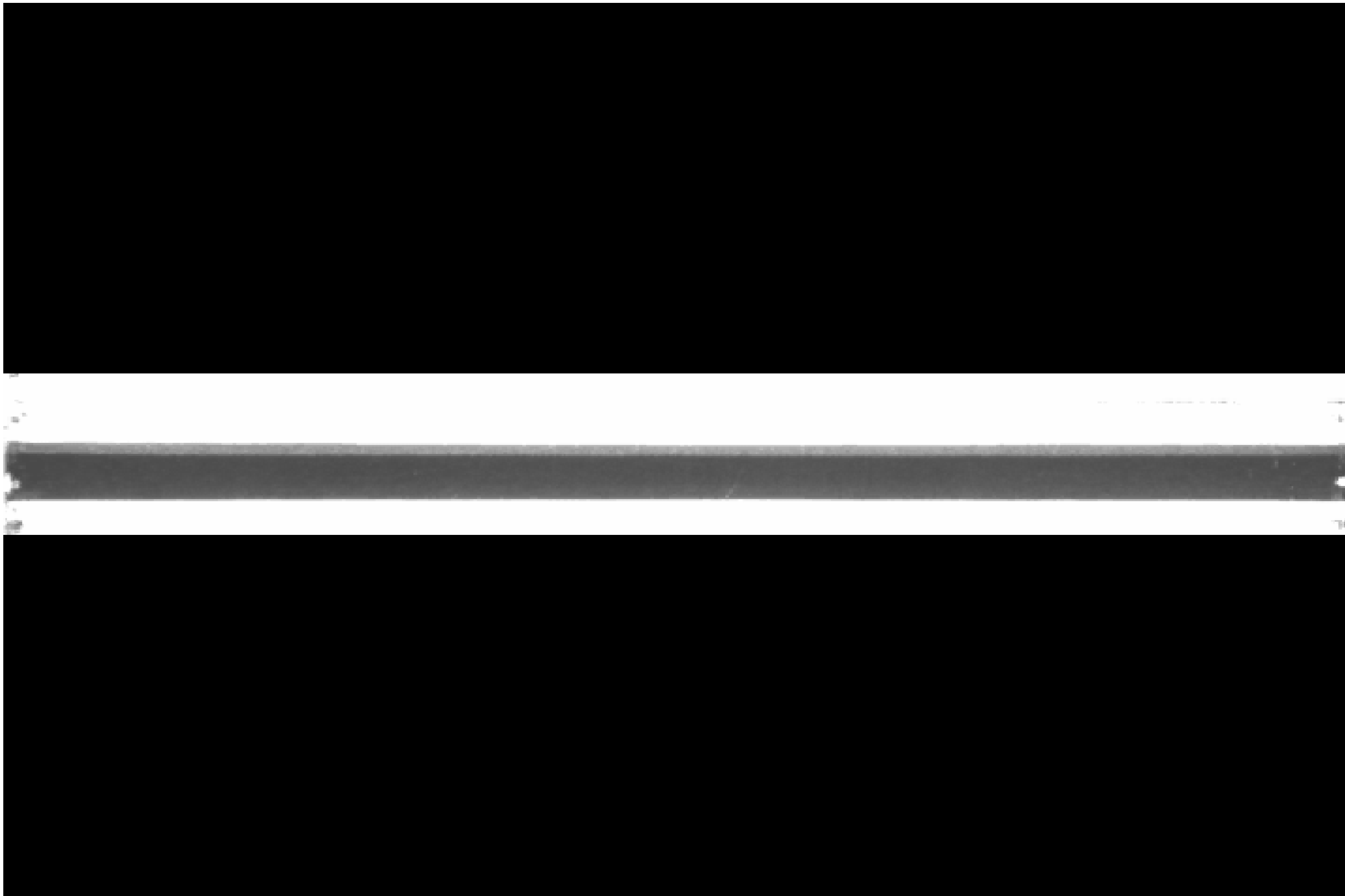
Jain *et al*. *PRL*. **86**, 3771 (2001)

Typical space-time plot

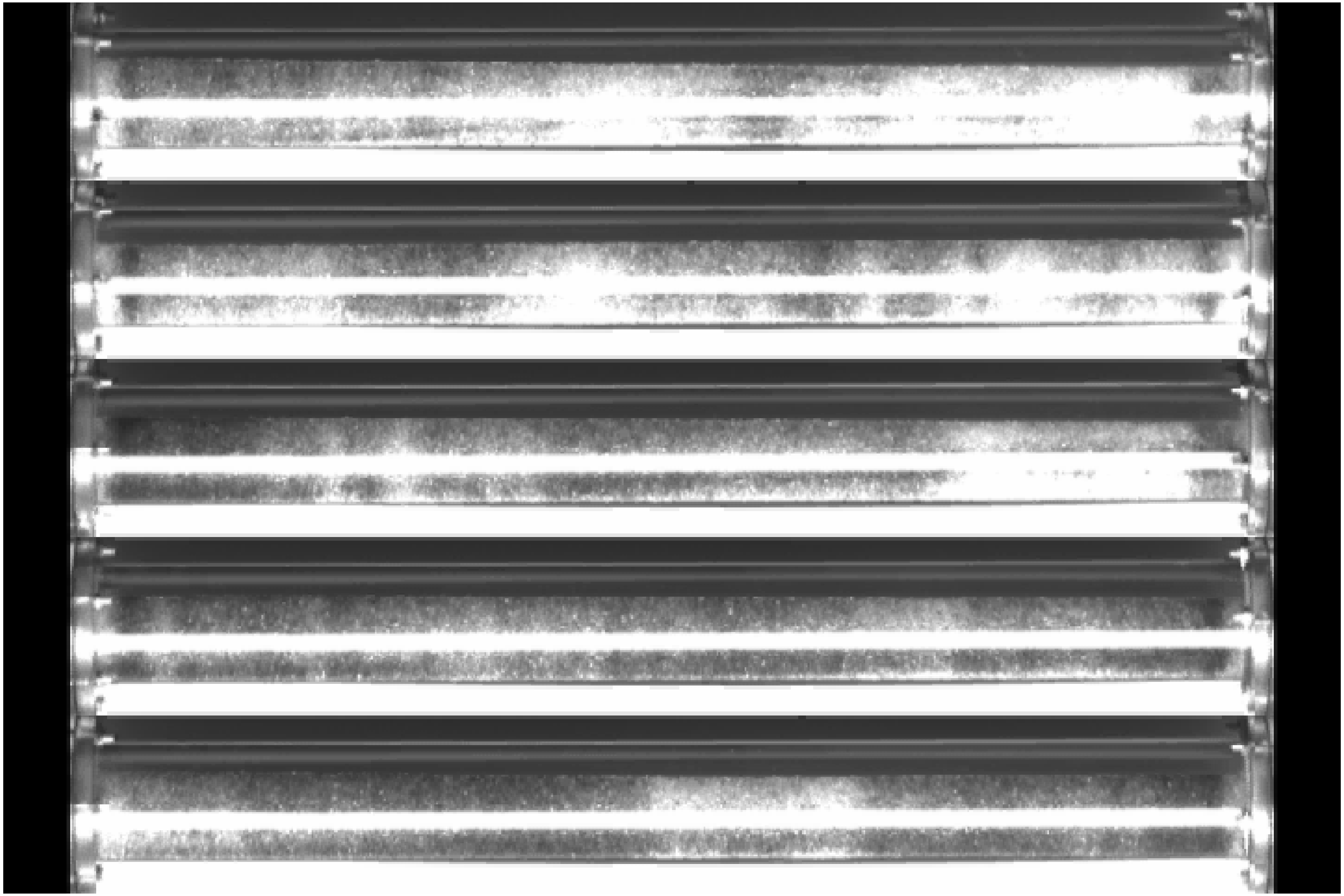




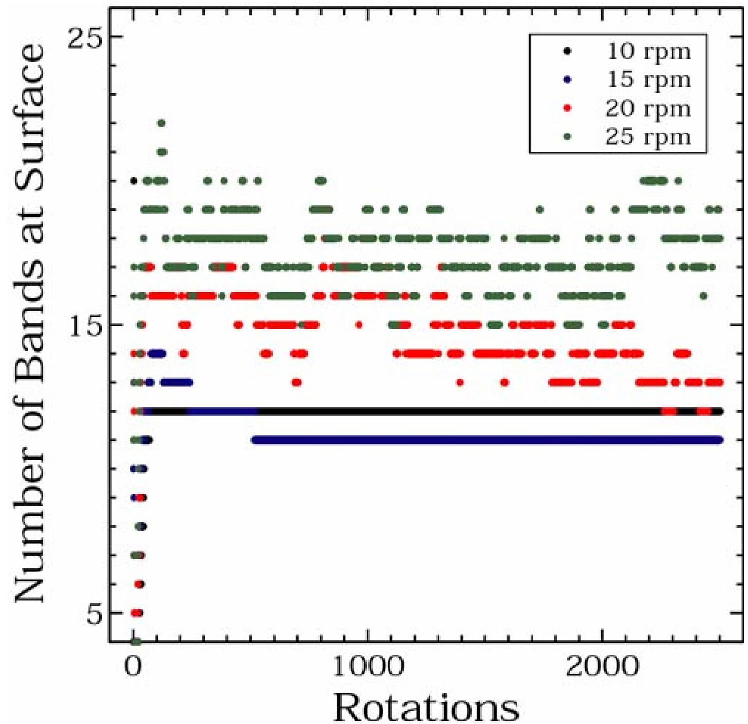
Band Dynamics (LGS)

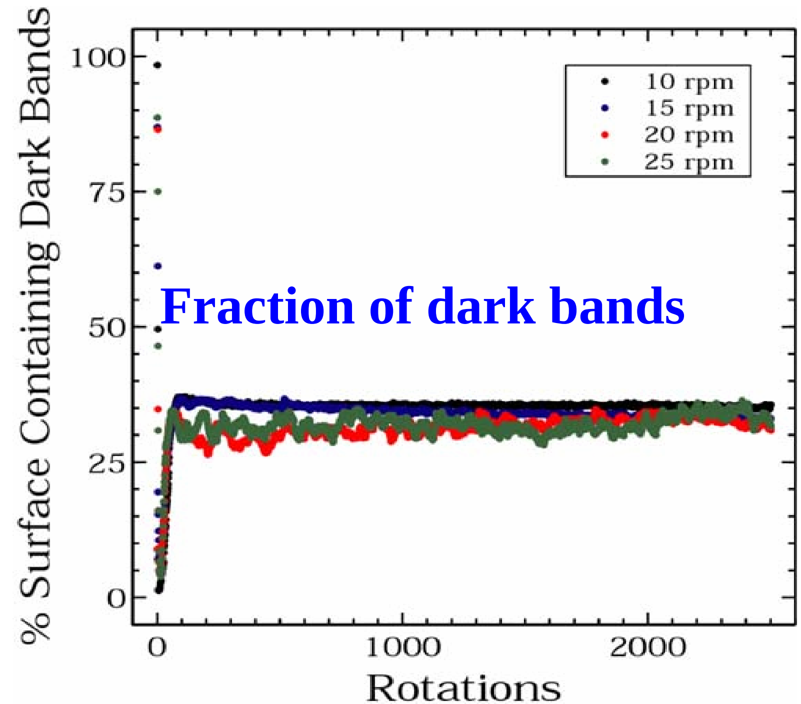


Effect of rotation rate - DGS



Effect of rotation rate DGS



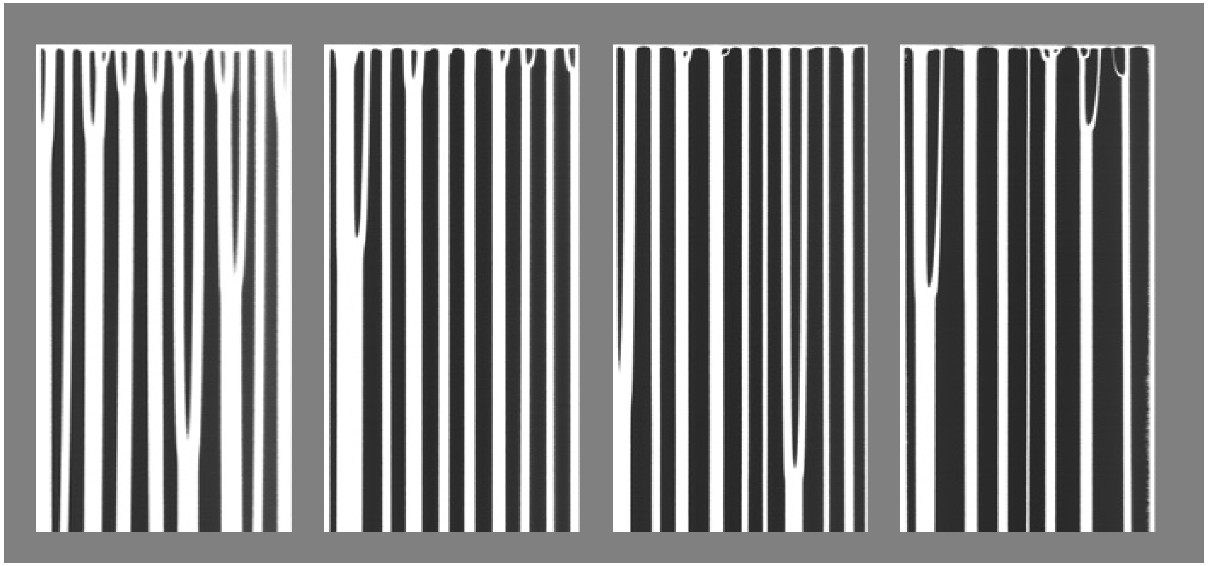


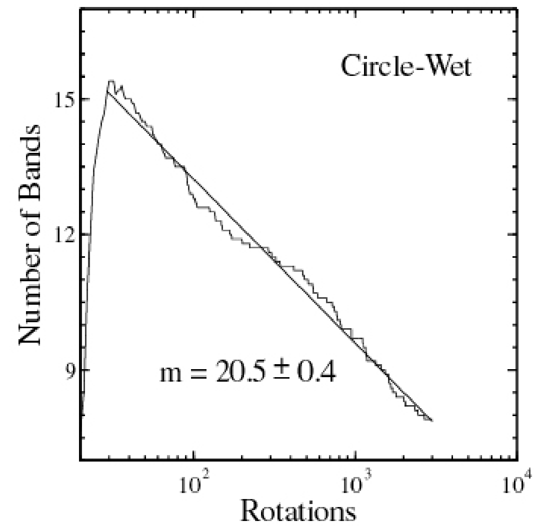
In all cases "mass" conserved (circular, square cross sections)

Fiedor & Ottino, PRL 2003

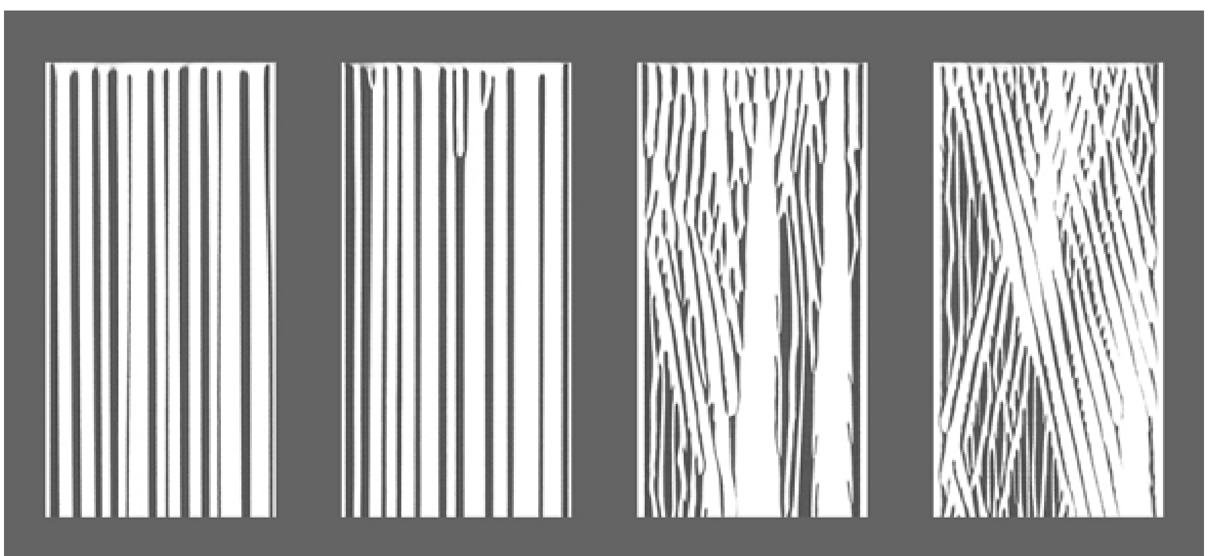
Logarithmic decay

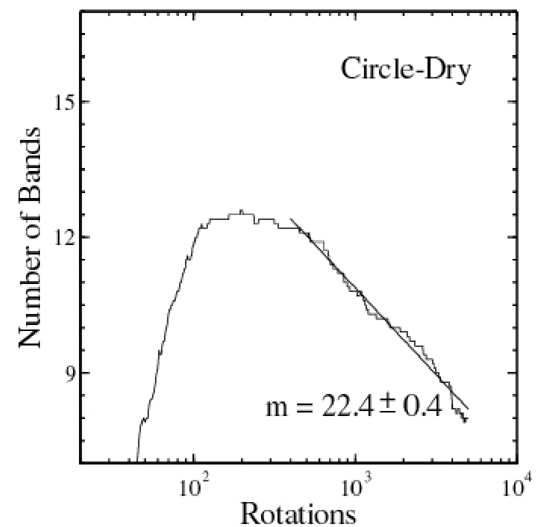
LGS



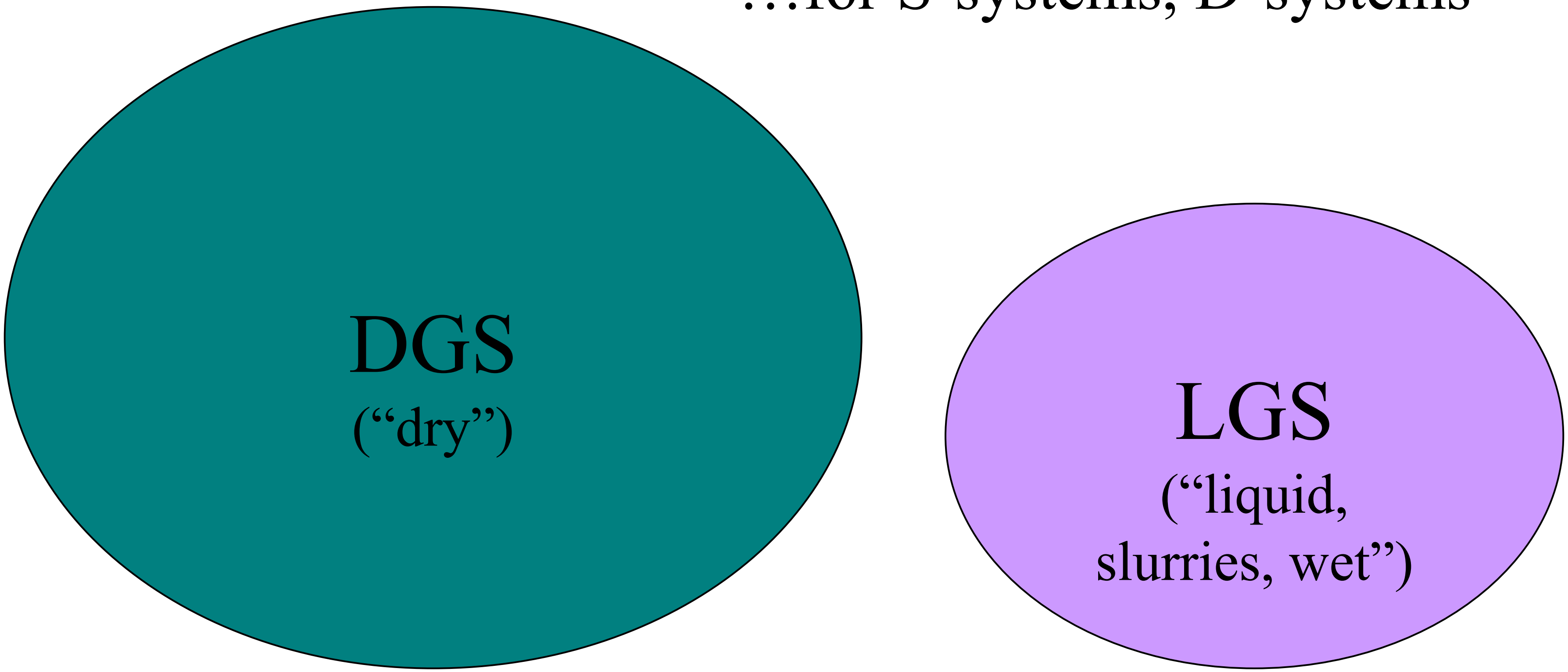


DGS





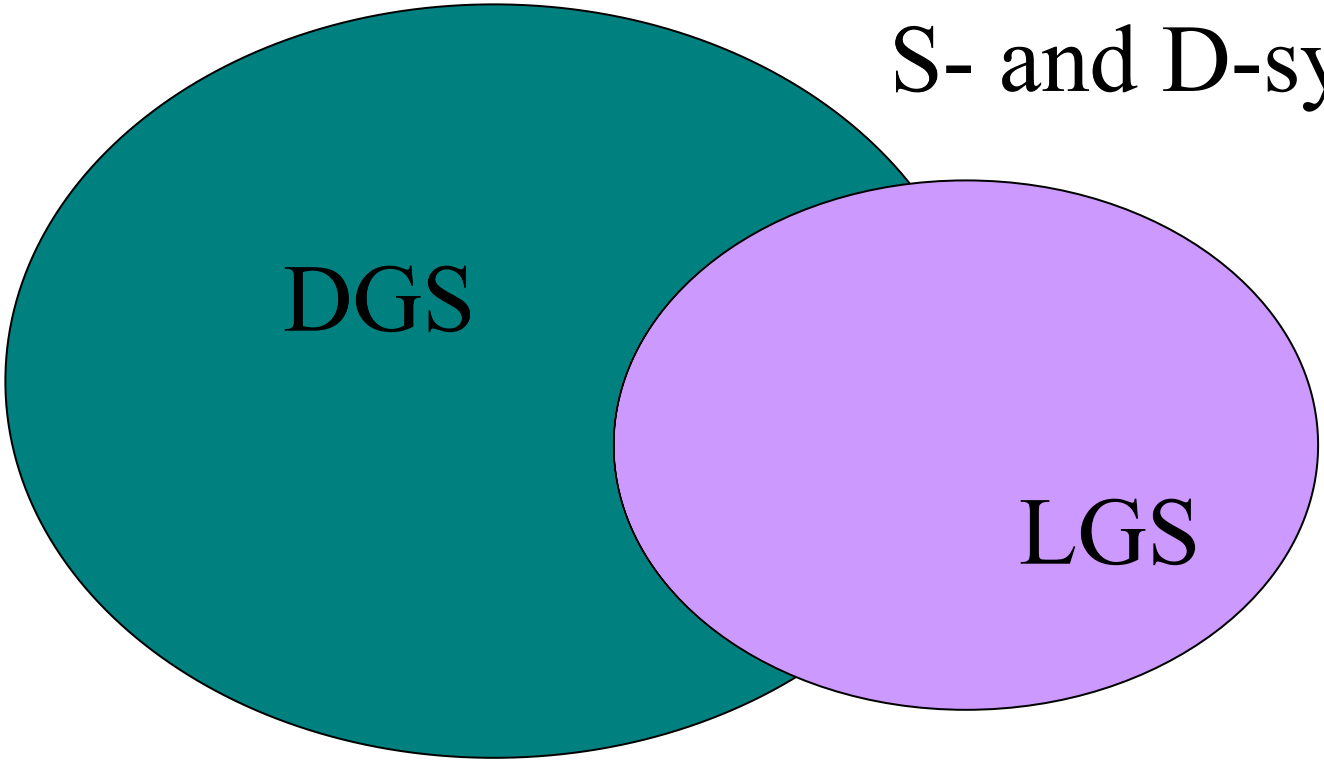
…for S-systems, D-systems



Set of all possible behaviors of DGSs under tumbling

Set of all possible behaviors of LGSs under tumbling

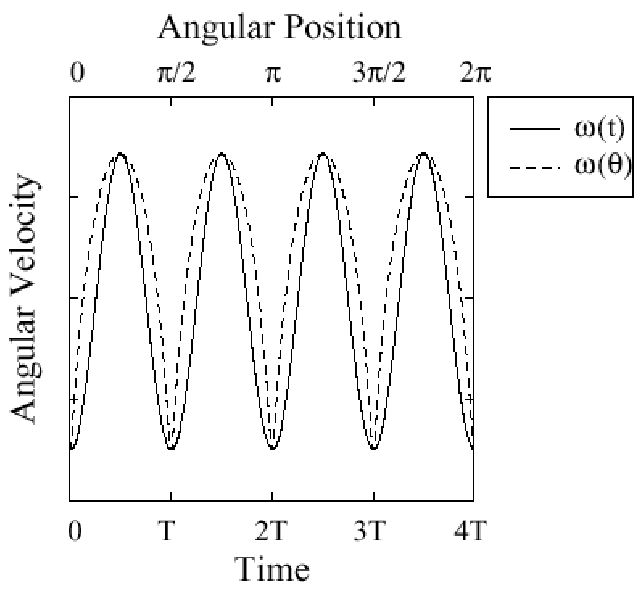
S- and D-systems

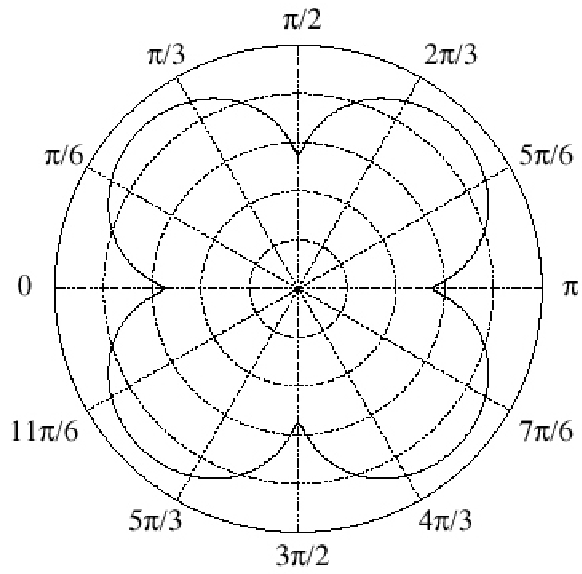


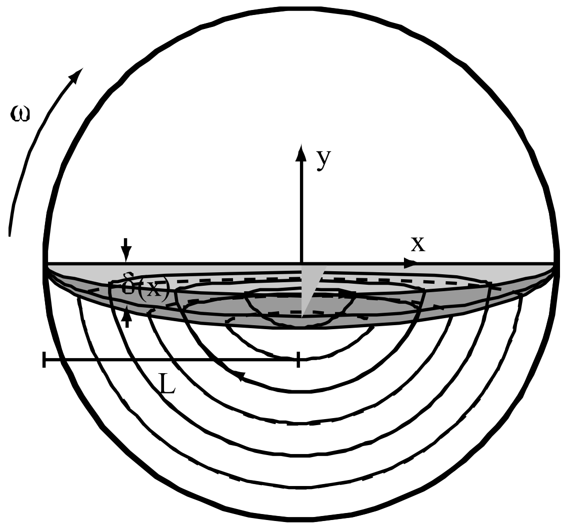
Set of all possible behaviors of DGSs under tumbling

Set of all possible behaviors of LGSs under tumbling

Periodic Forcing in Quasi-2D Tumblers



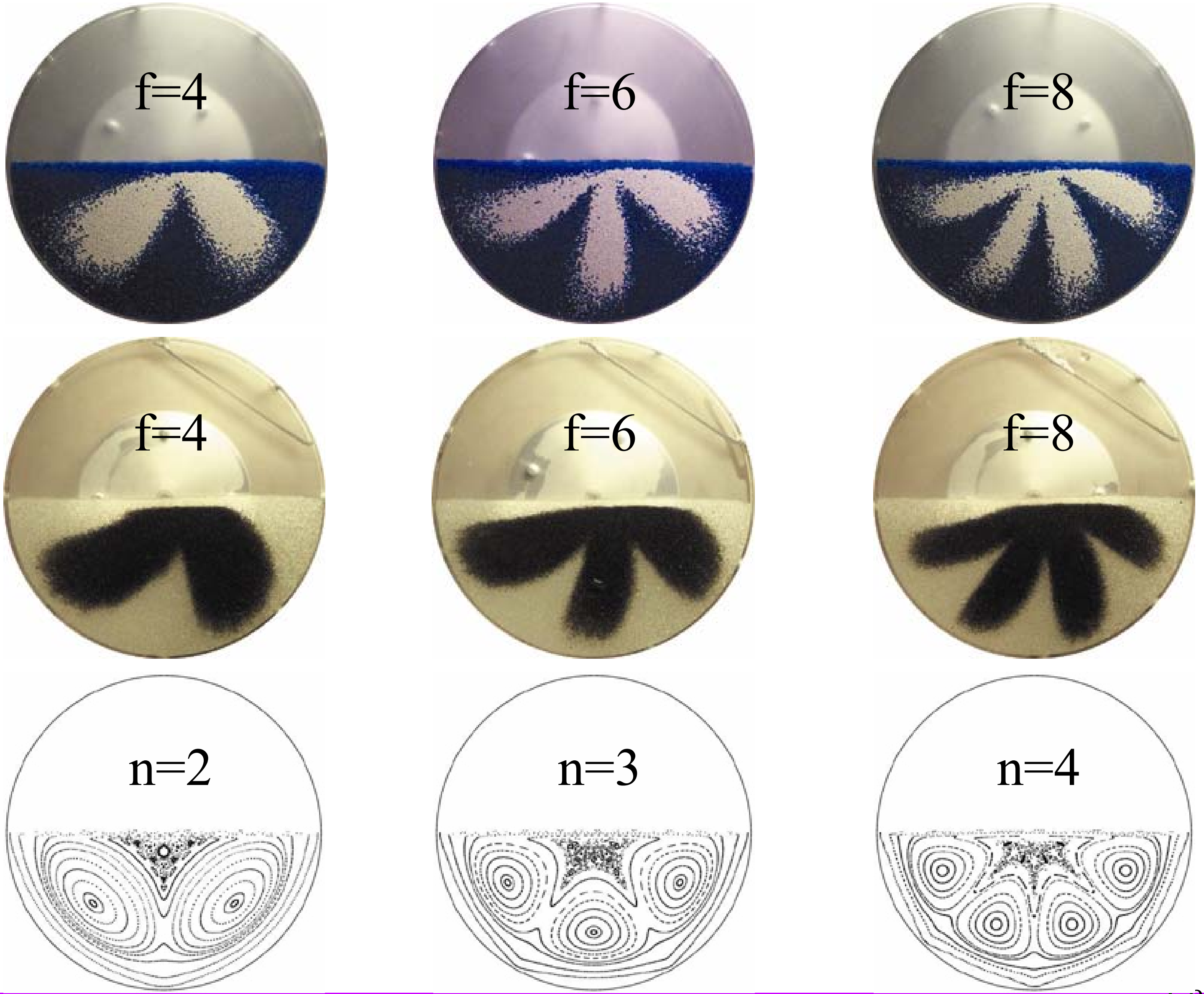




Fiedor and Ottino 2004 (unpublished)

(related work Hill et al. 2004)

Half Full – Even Frequencies

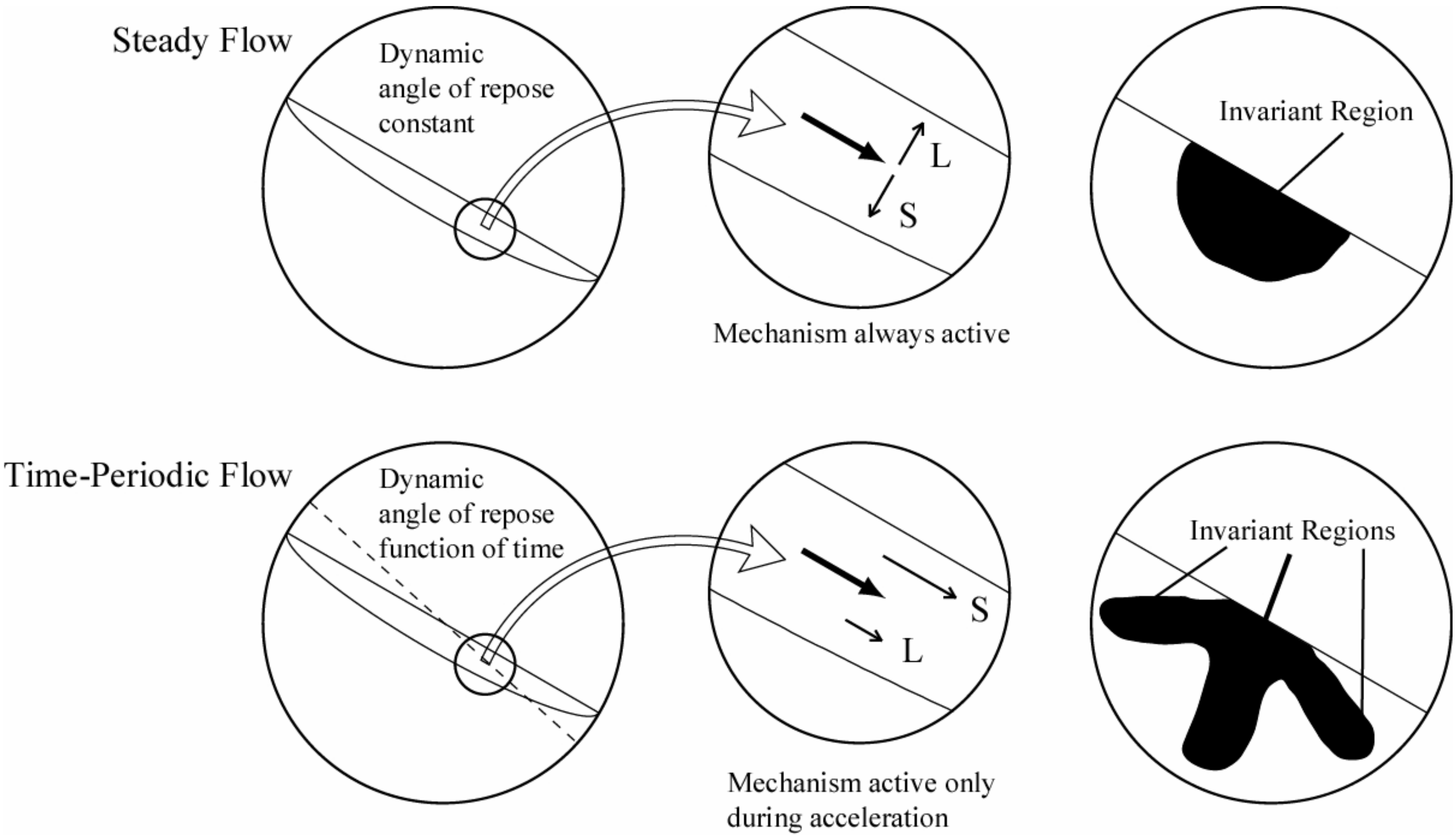


DGS

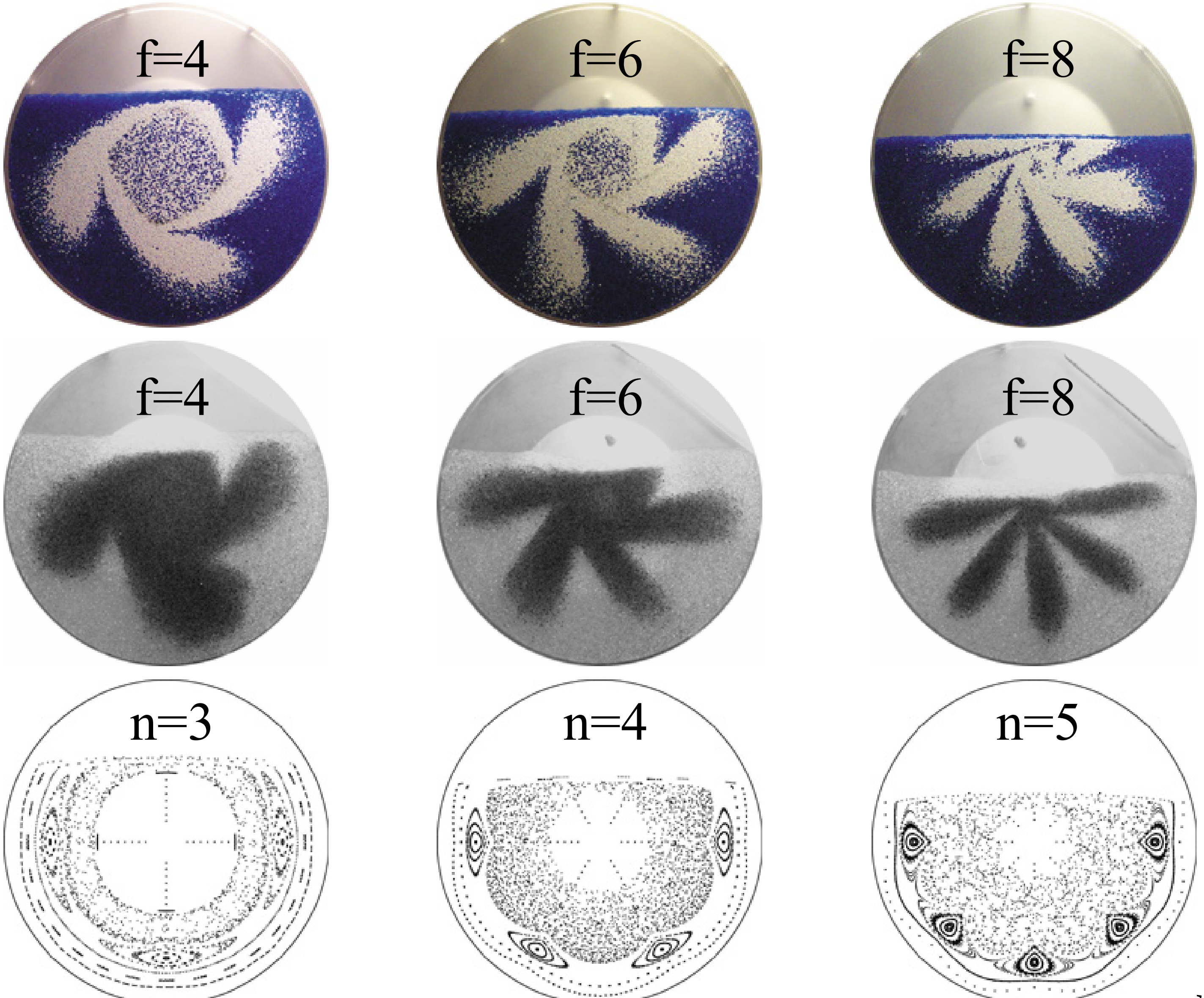
LGS

Poincaré Sections

How to invariant regions form?



More Than Half Full – Even Freqs



DGS

LGS

Poincaré Sections

Stokes Number:

$$St = \frac{\text{particle inertia}}{\text{viscosity}}$$

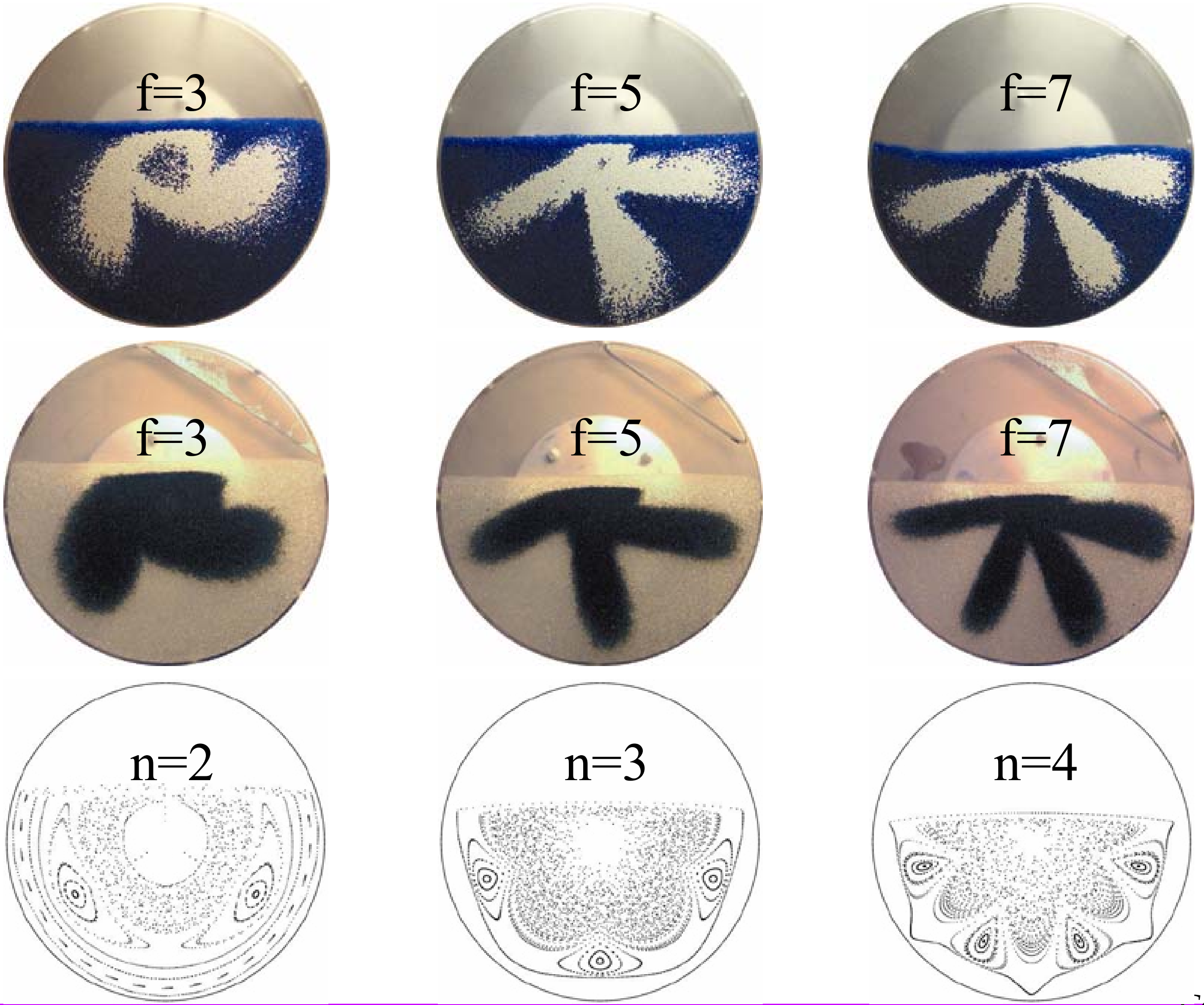
DGS:
St >> 1

LGS

(smallest particles St~10)

Brady 2004

More Than Half Full – Odd Freqs

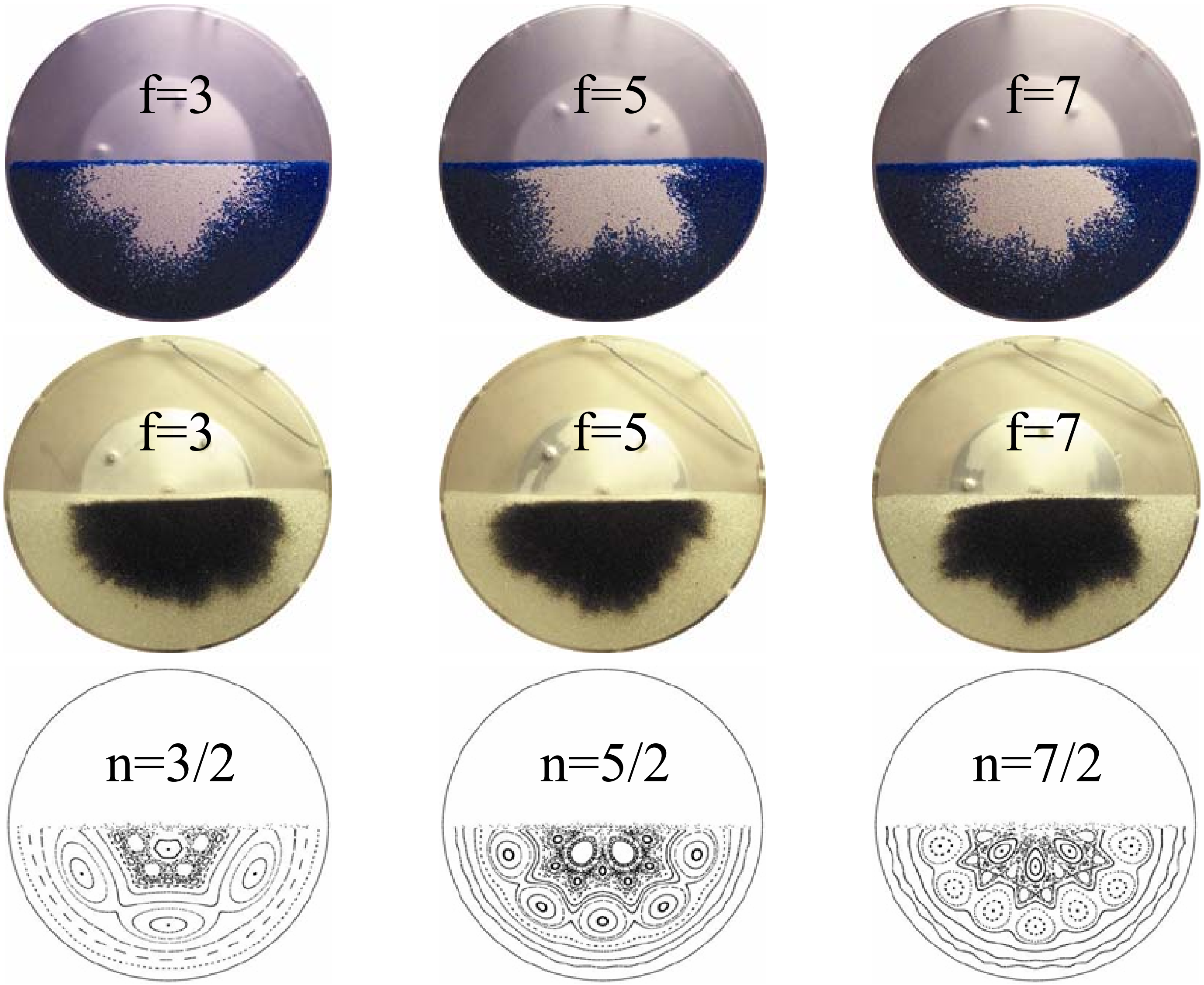


DGS

LGS

Poincaré Sections

Half Full – Odd Frequencies (Disagreement?)



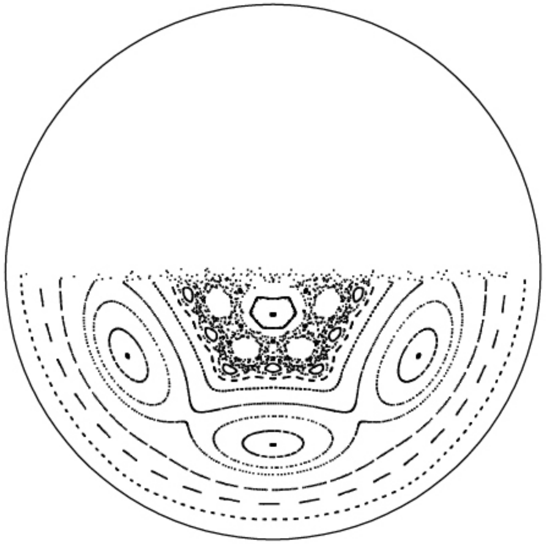
DGS

LGS

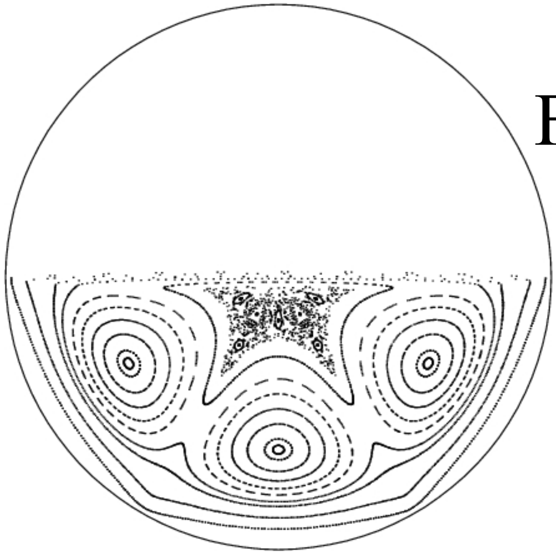
Poincaré Sections

Matching Model-Experimental Images

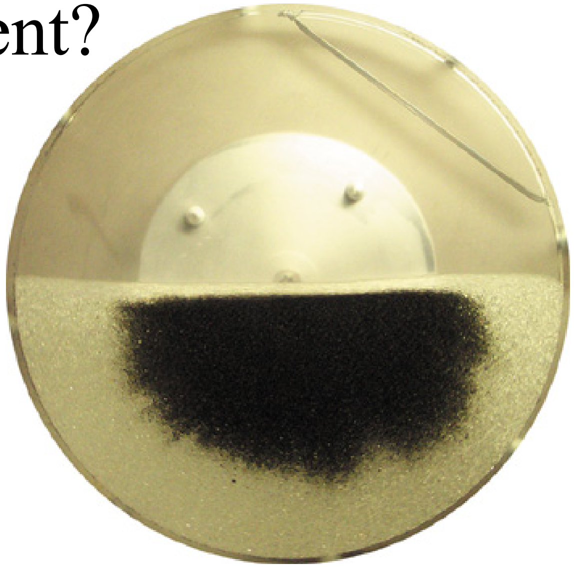
Freq=3



Freq=6



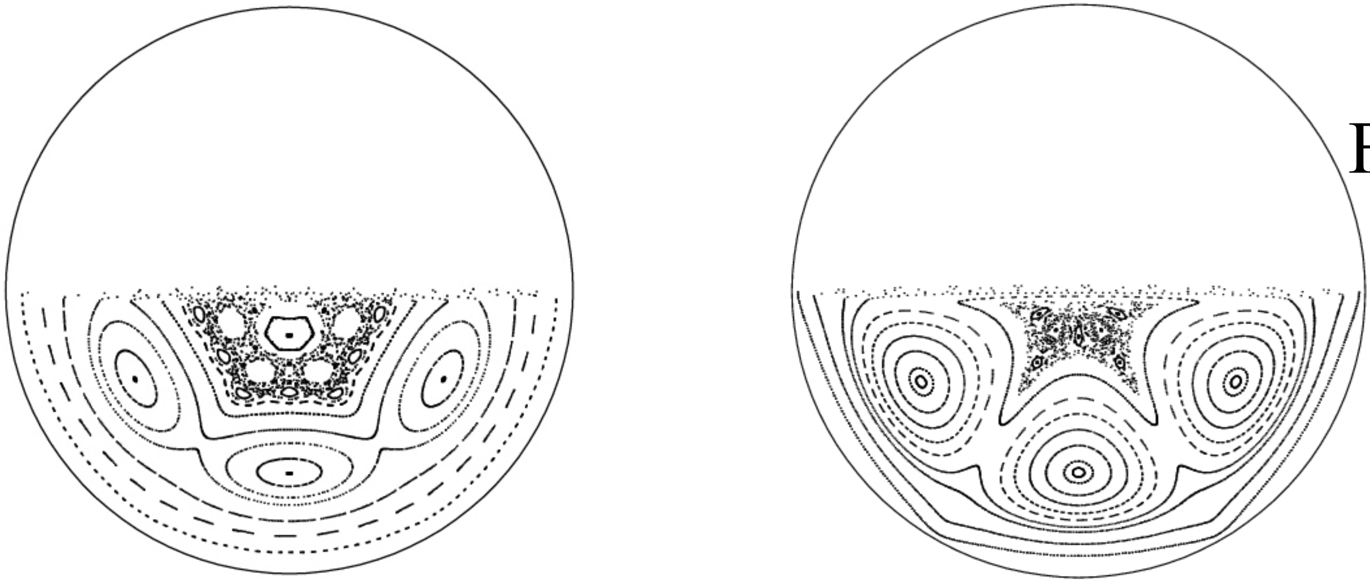
Bad Agreement?



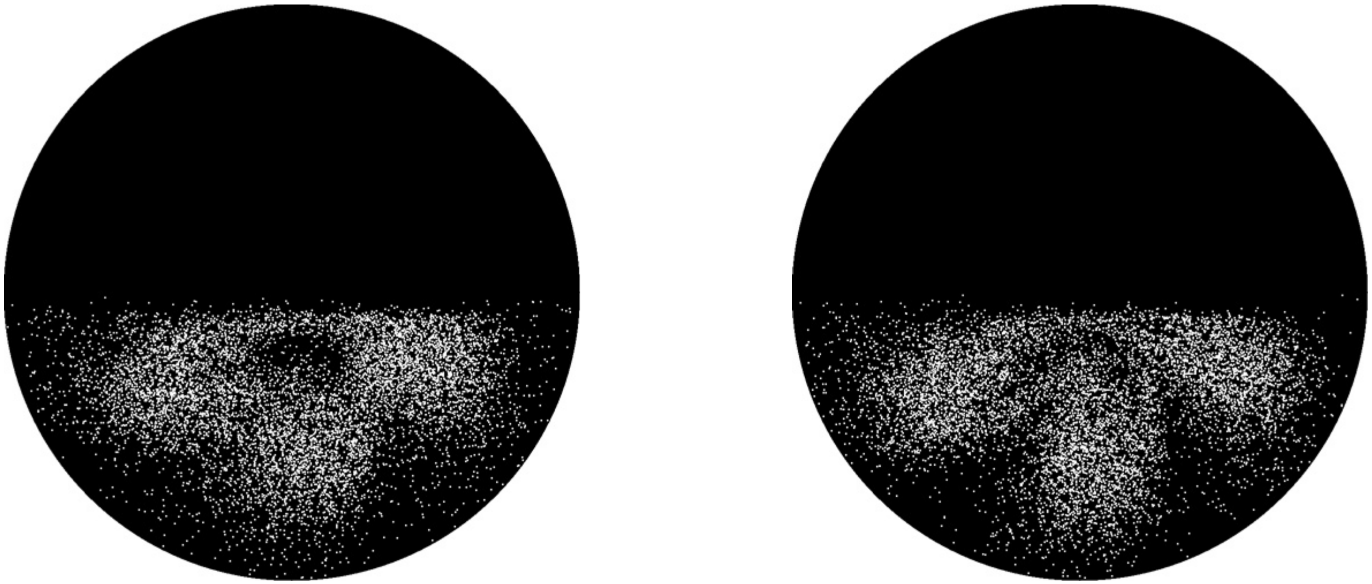
Exploring the nature of the islands

Freq=3

Freq=6

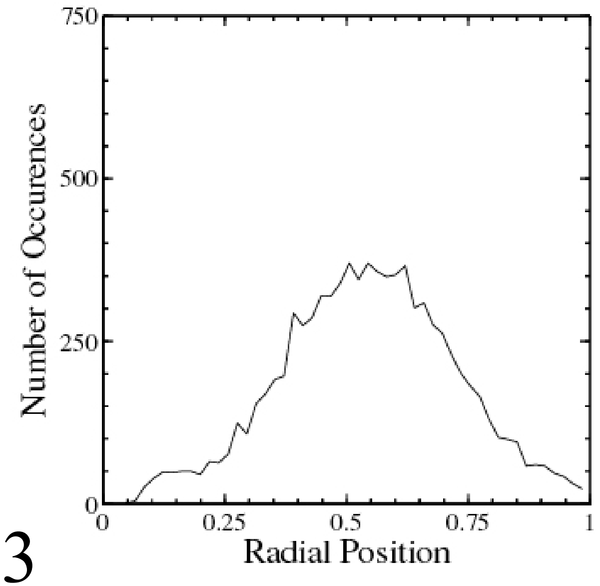


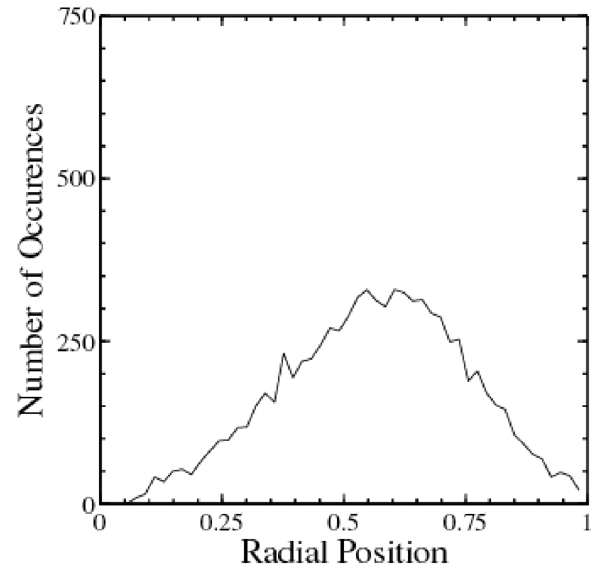
tracer particle positions



Probability Distributions

Freq=3





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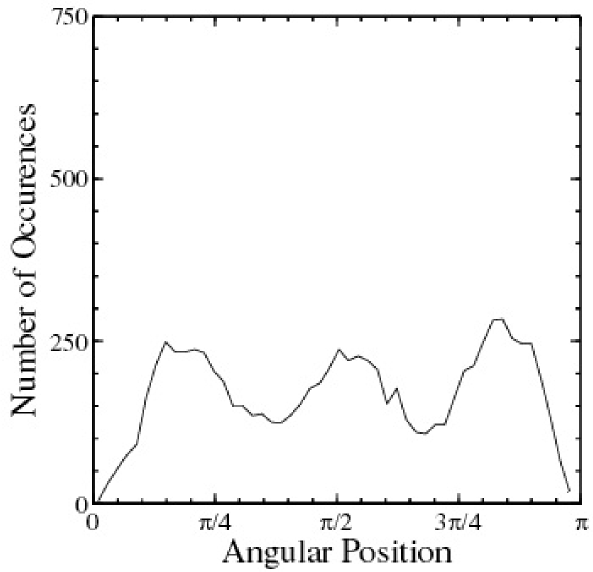
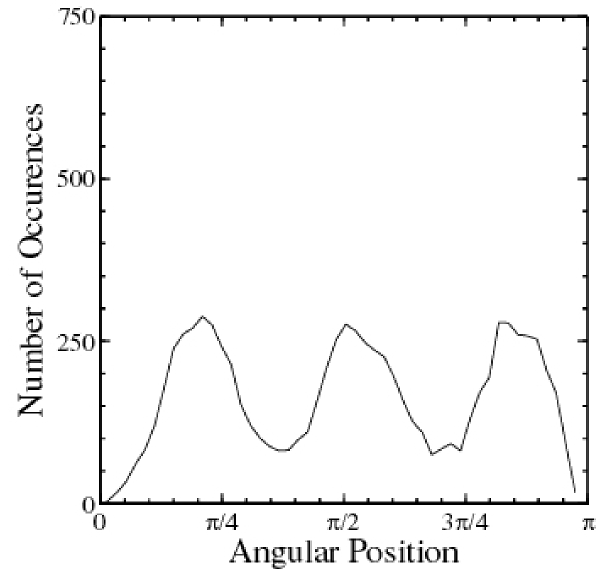
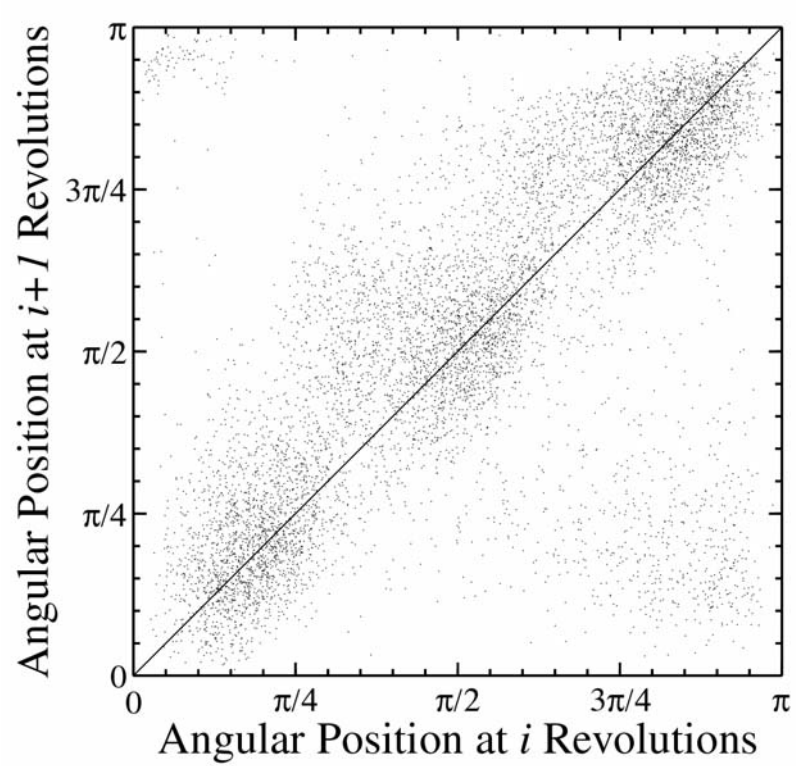


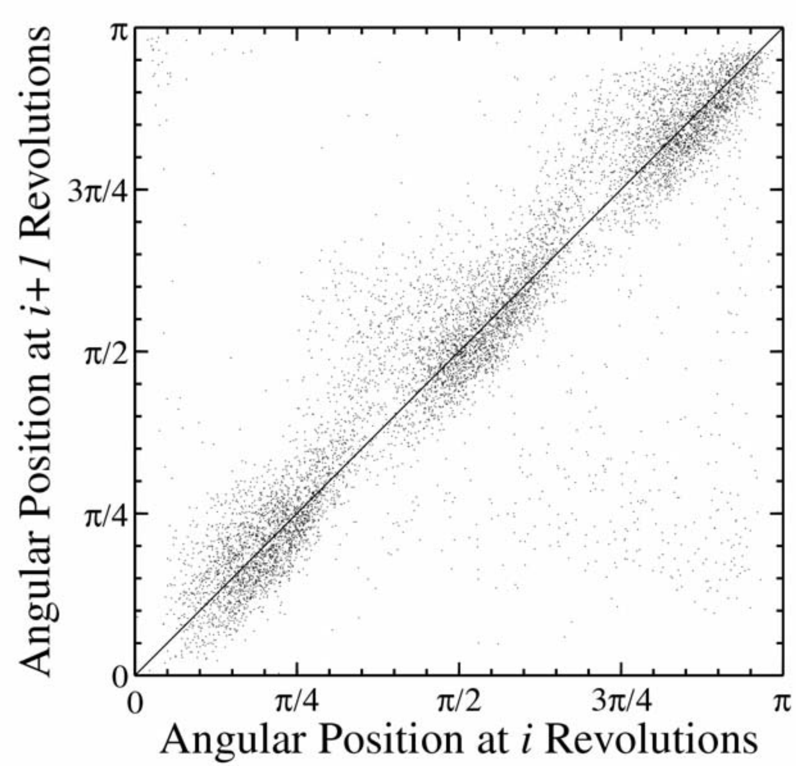


Return Map

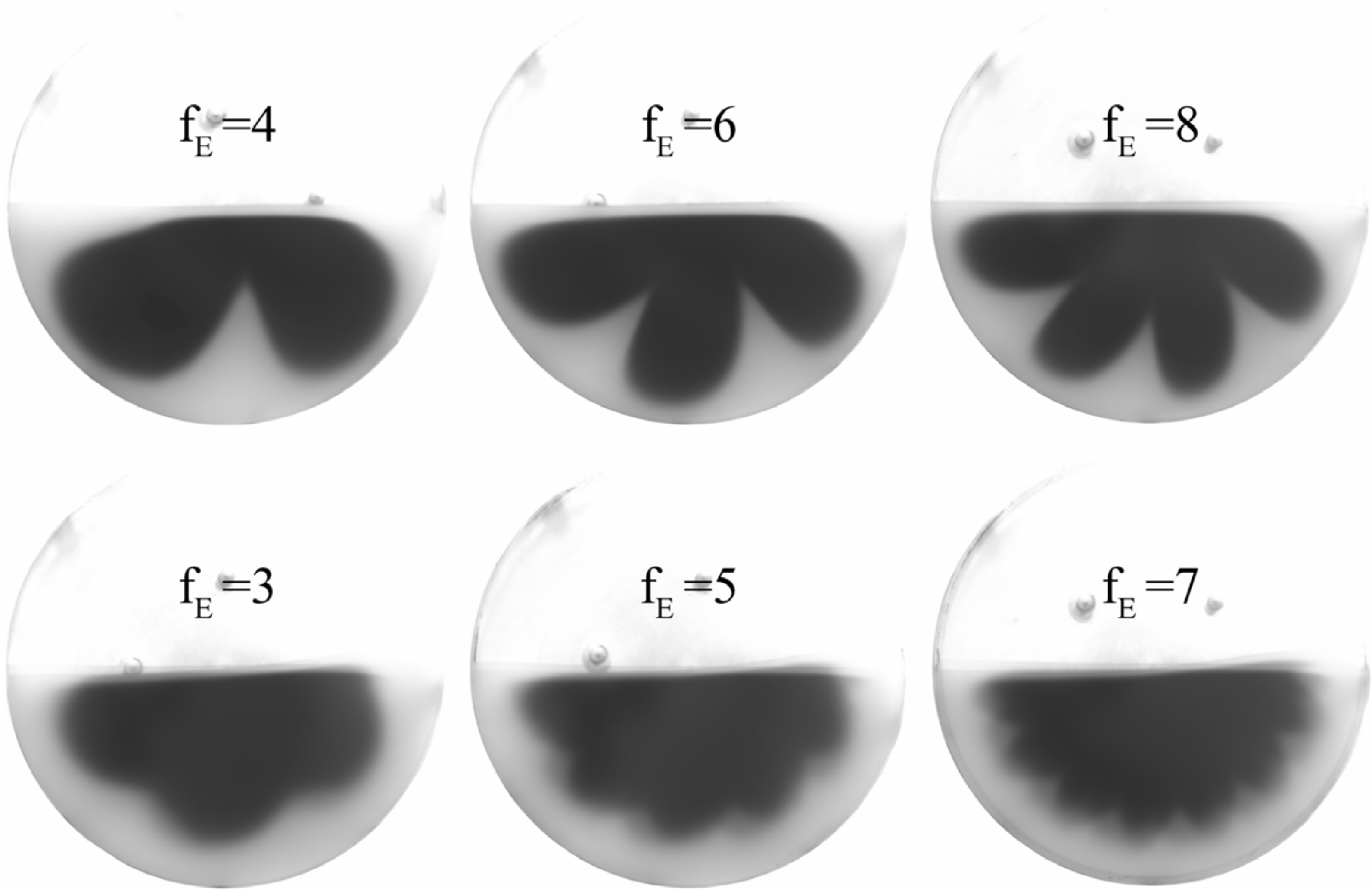
Freq=3

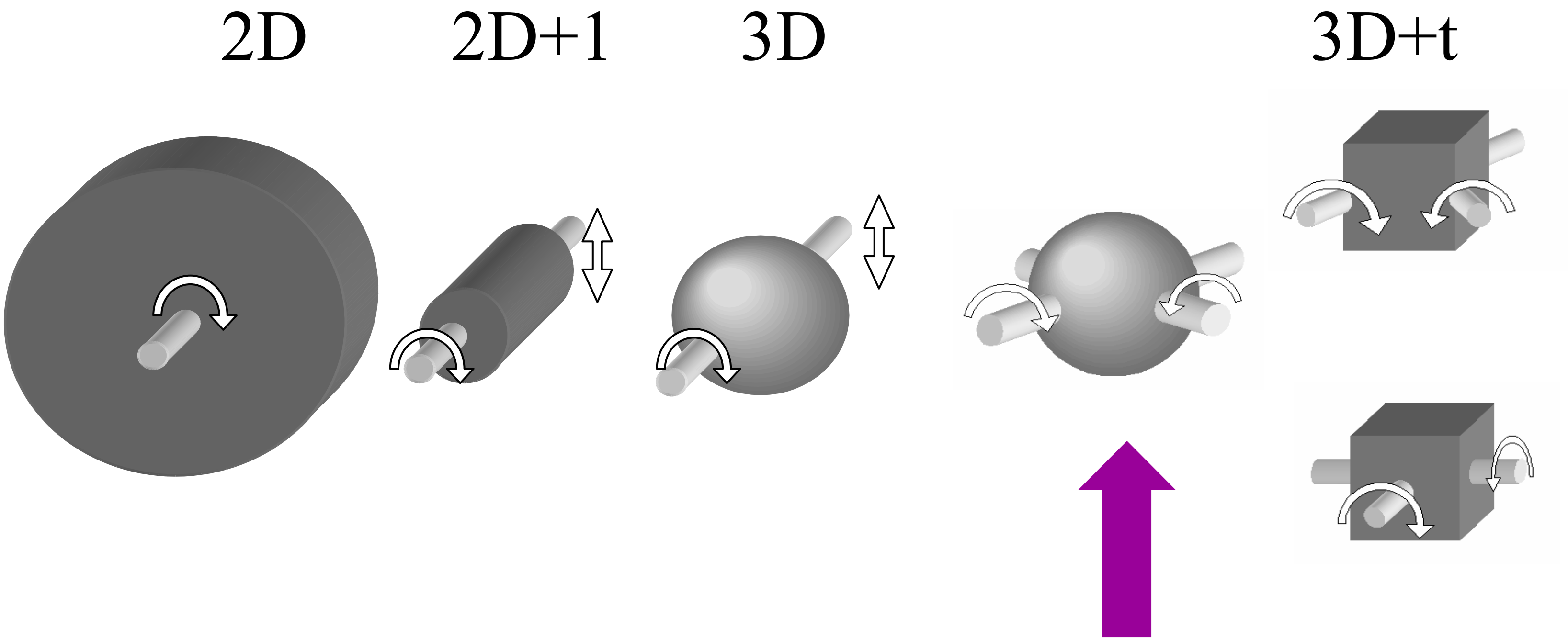


Freq=6



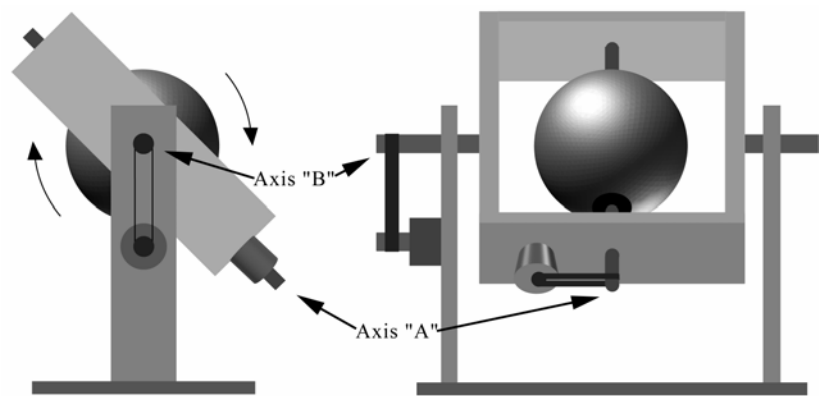
Long Term Behavior - Averaged Experimental Images



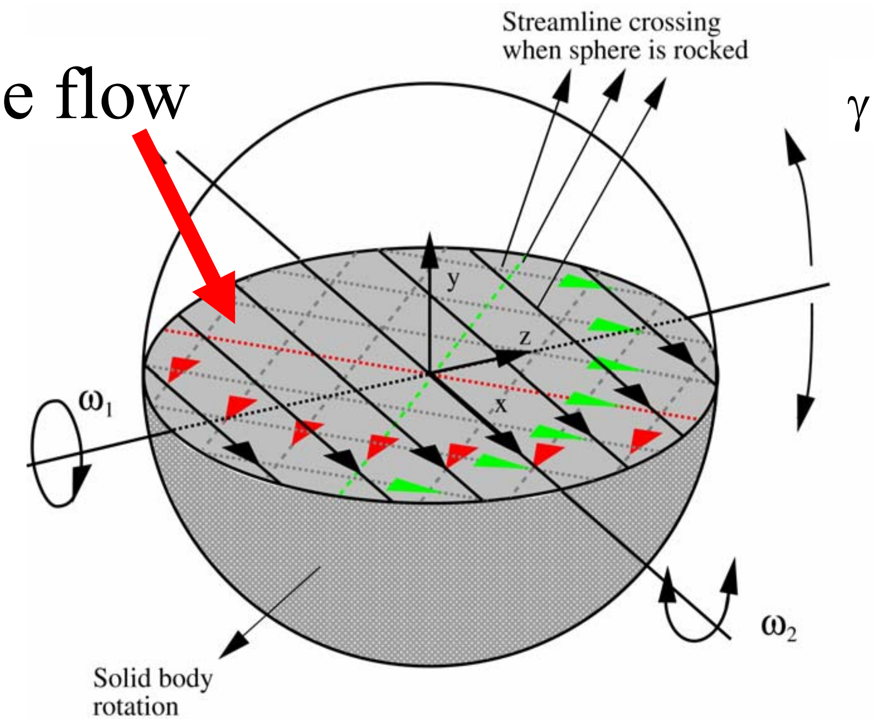


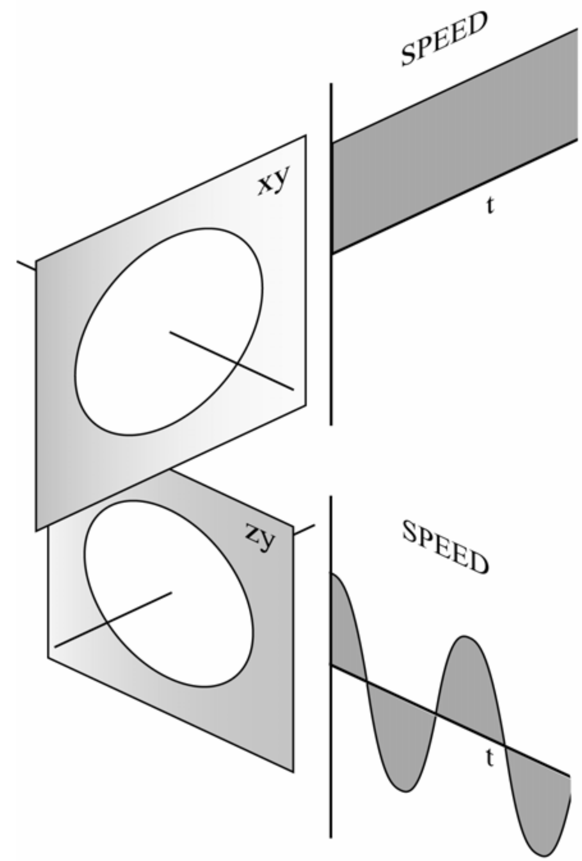
Meier 2004

3D Experimental Setup



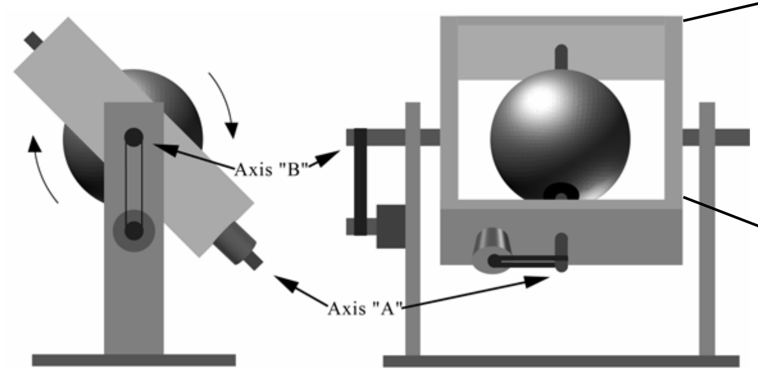
Surface flow



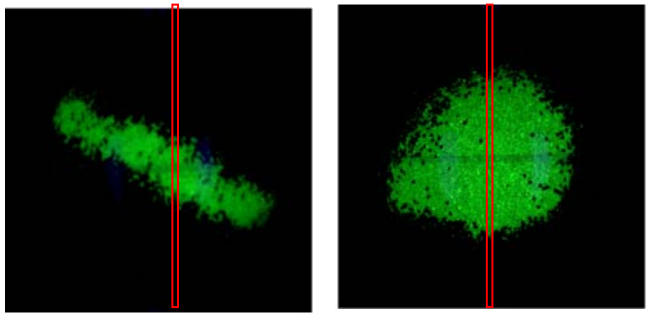


Gilchrist & Ottino PRE 2004

Space-time plots

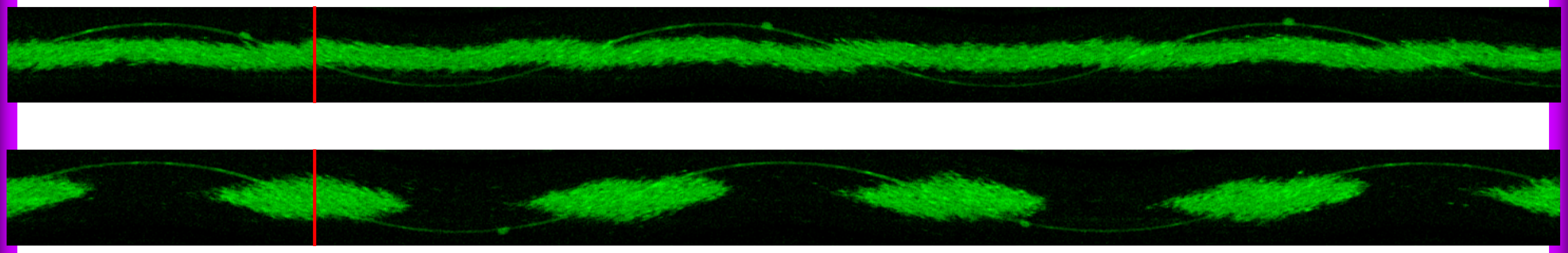


Bottom View



Large beads: Black
Small beads: Fluorescent

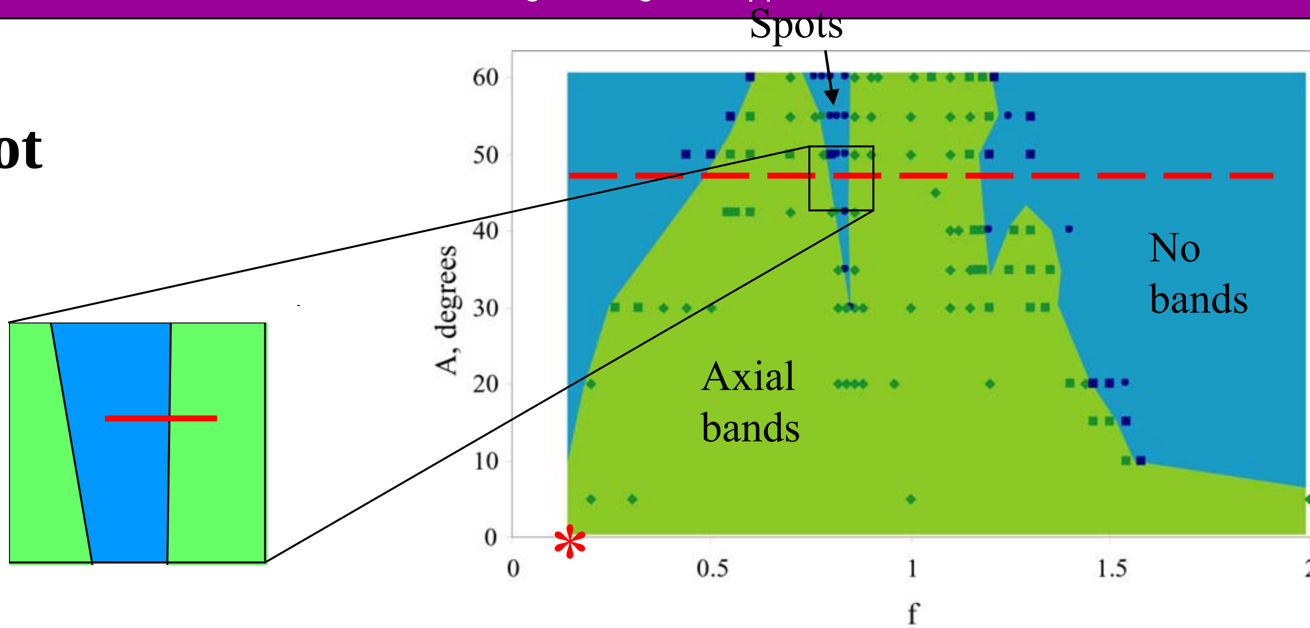
Time

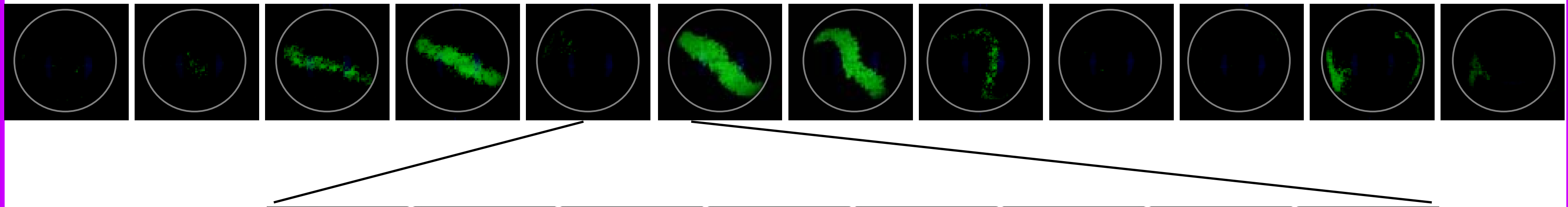


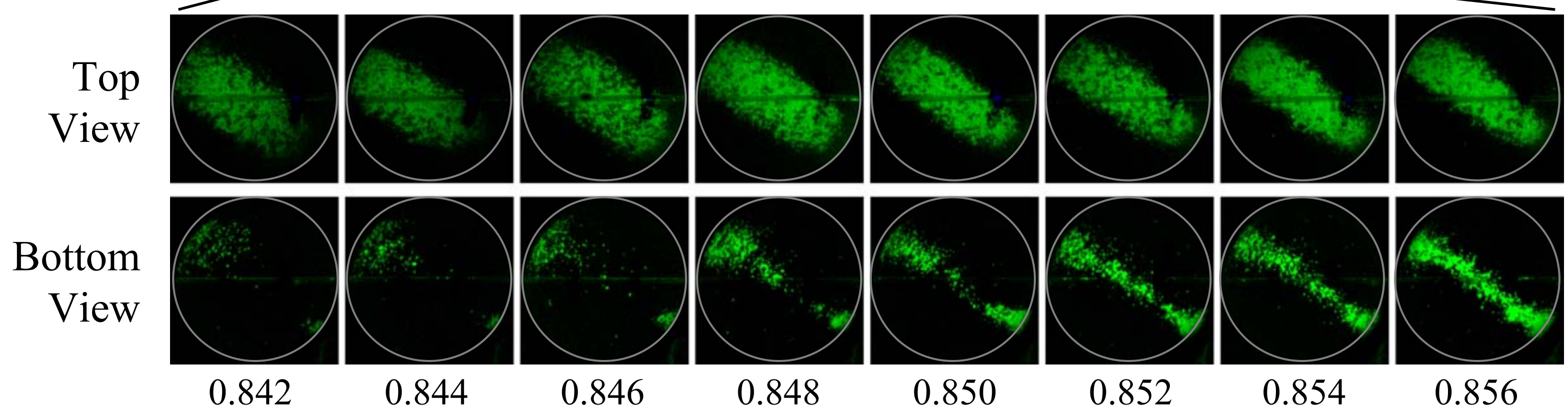
Gilchrist & Ottino PRE 2003

Phase Plot

Gilchrist & Ottino PRE 2003

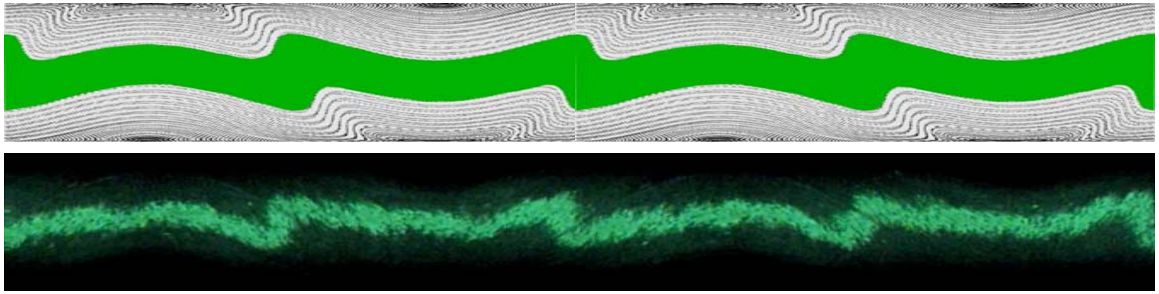






Continuum Model (template)

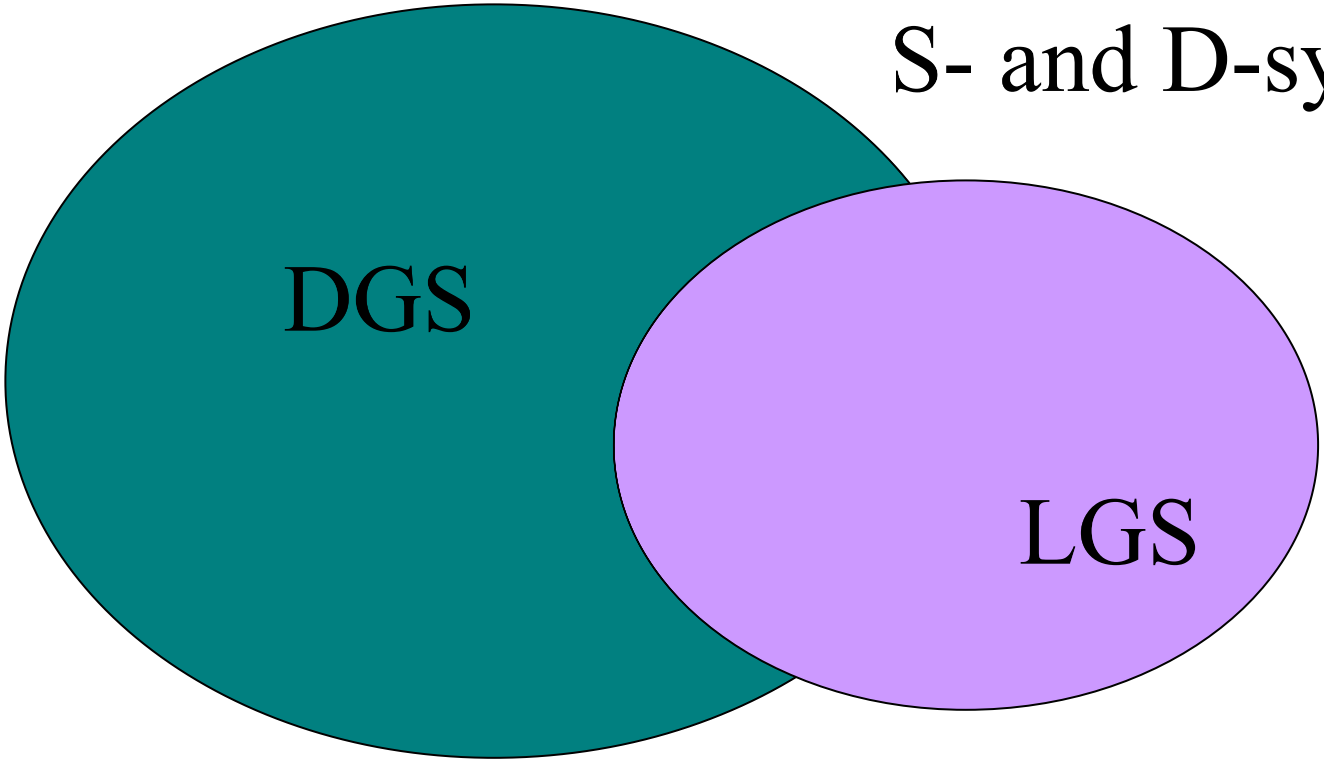
Experiment-Model
(without segregation)



Time →

Gilchrist & Ottino PRE 2003

S- and D-systems



Set of all possible behaviors of DGSs under tumbling

Set of all possible behaviors of LGSs under tumbling

Some open issues

- Framework…skeleton, symmetries
- Granular matter/suspensions, exploitation of LGS
- Surface flows
- Math 3D mixing-segregation
- Segregation
  - Fluidity
  - Combined SD
  - Friction
- Surface properties