Supersonic Turbulence and Star Formation

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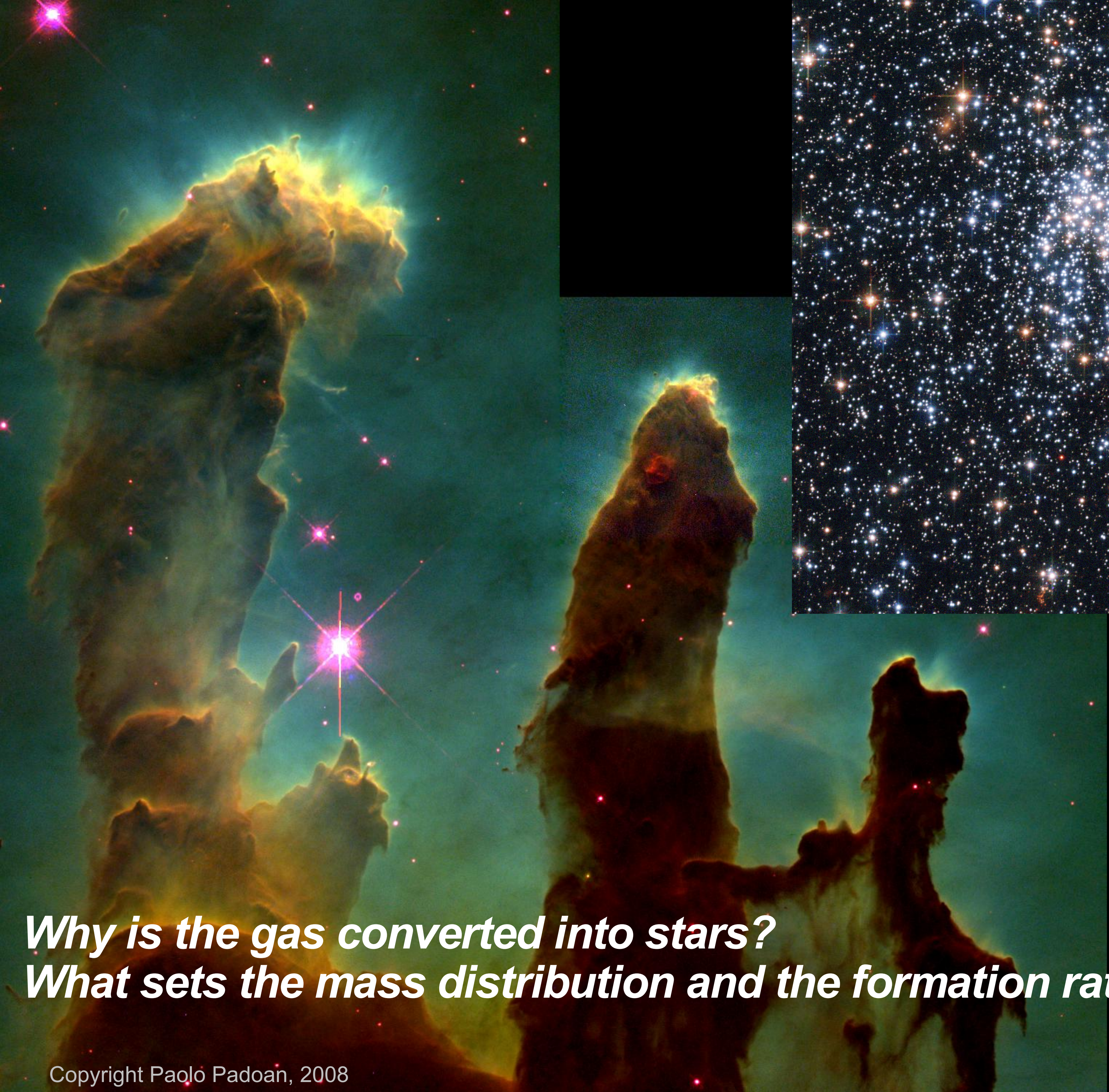


Why is the gas converted into stars?
What sets the mass distribution and the formation rate of stars?

Gravitational Instability

Instability of linear density perturbations of a uniform, isothermal, static gas, extending to infinity (Jeans 1902):

$$\lambda > \lambda_J = \left(\frac{\pi \sigma_{th}^2}{G \rho_0} \right)^{1/2} \quad \Rightarrow \quad M_J = \frac{4}{3} \pi \left(\frac{\lambda_J}{2} \right)^3 \rho_0 = 24\, M_{sun} \left(\frac{n}{200\,\mathrm{cm}^{-3}} \right)^{-1/2} \left(\frac{T}{10\,\mathrm{K}} \right)^{3/2}$$

The cold interstellar medium has a complex hierarchical structure:

$$n \approx 2 \times 10^3 \mathrm{cm}^{-3} \left(\frac{l}{1\,\mathrm{pc}} \right)^{-1}, \qquad T \approx 10\mathrm{K}$$

So clouds of 10 pc size have n~200 cm^{-3}, M_{cl}~10^4 M_{sun}, and M_J~ 24 M_{sun}.

Prediction 1:
The characteristic stellar mass in these molecular clouds is ~24 M_{sun}

At what rate is the gas converted into stars?

Without pressure support, a uniform sphere collapses in a **free-fall time**:

$$\tau_{\rm ff} = \left(\frac{3\pi}{32\, G\rho} \right)^{1/2} = 2.3 \times 10^6\, {\rm yr} \left(\frac{n}{200\, {\rm cm}^{-3}} \right)^{-1/2}$$

(roughly a sound crossing time of the Jeans length).

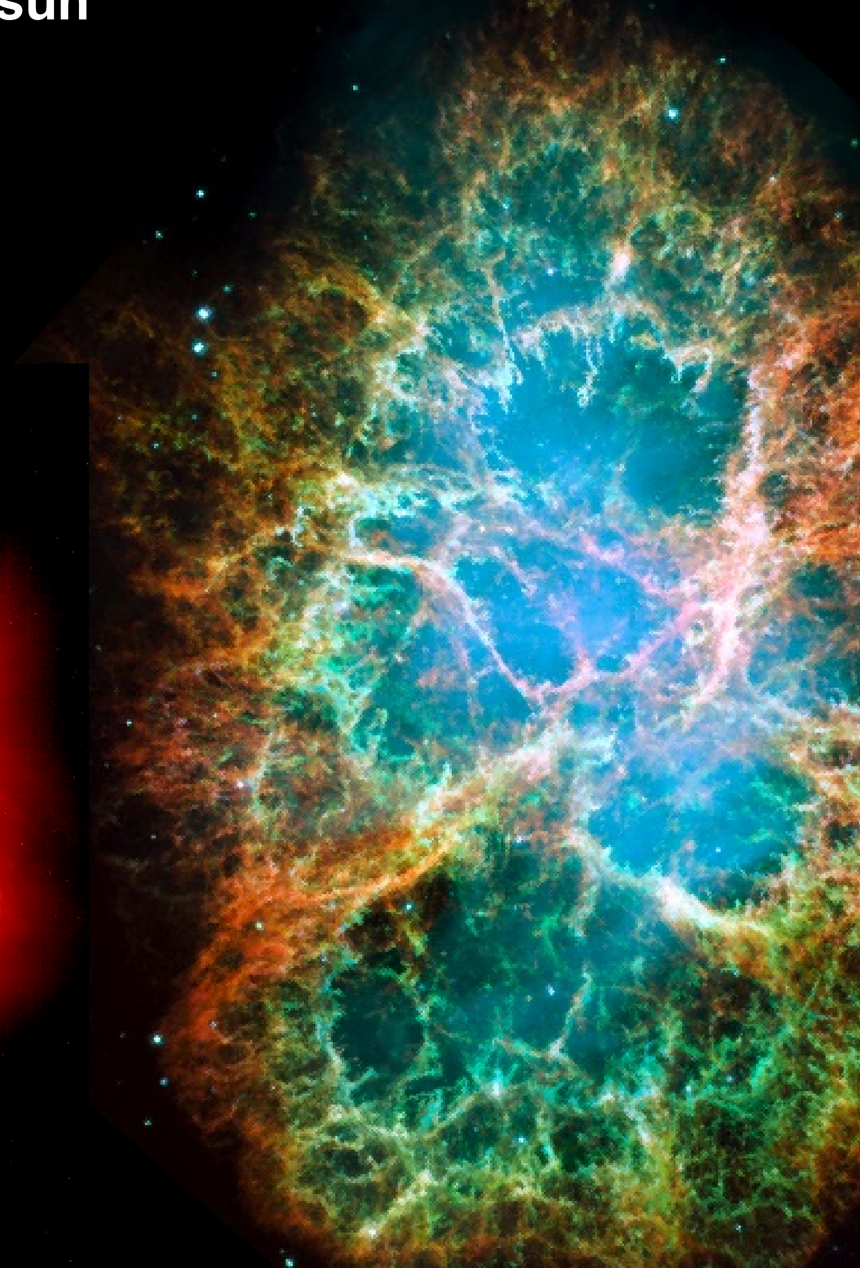
Prediction 2:
Molecular clouds are converted into stars in two million years.

Both predictions from the linear gravitational instability are quite wrong....

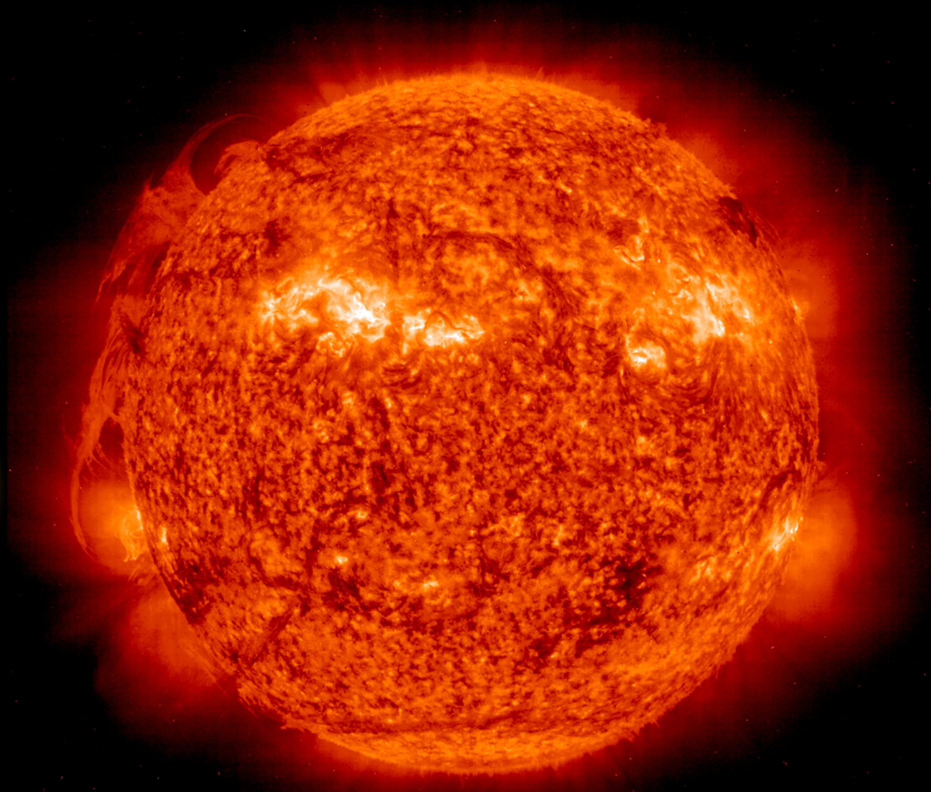
Large range of stellar masses: 0.01 - 100 M_{sun}

Characteristic stellar mass: 0.2 M_{sun}

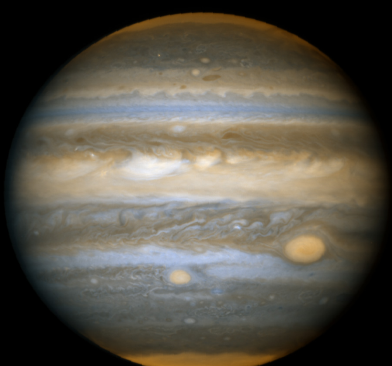
100 M_{sun}

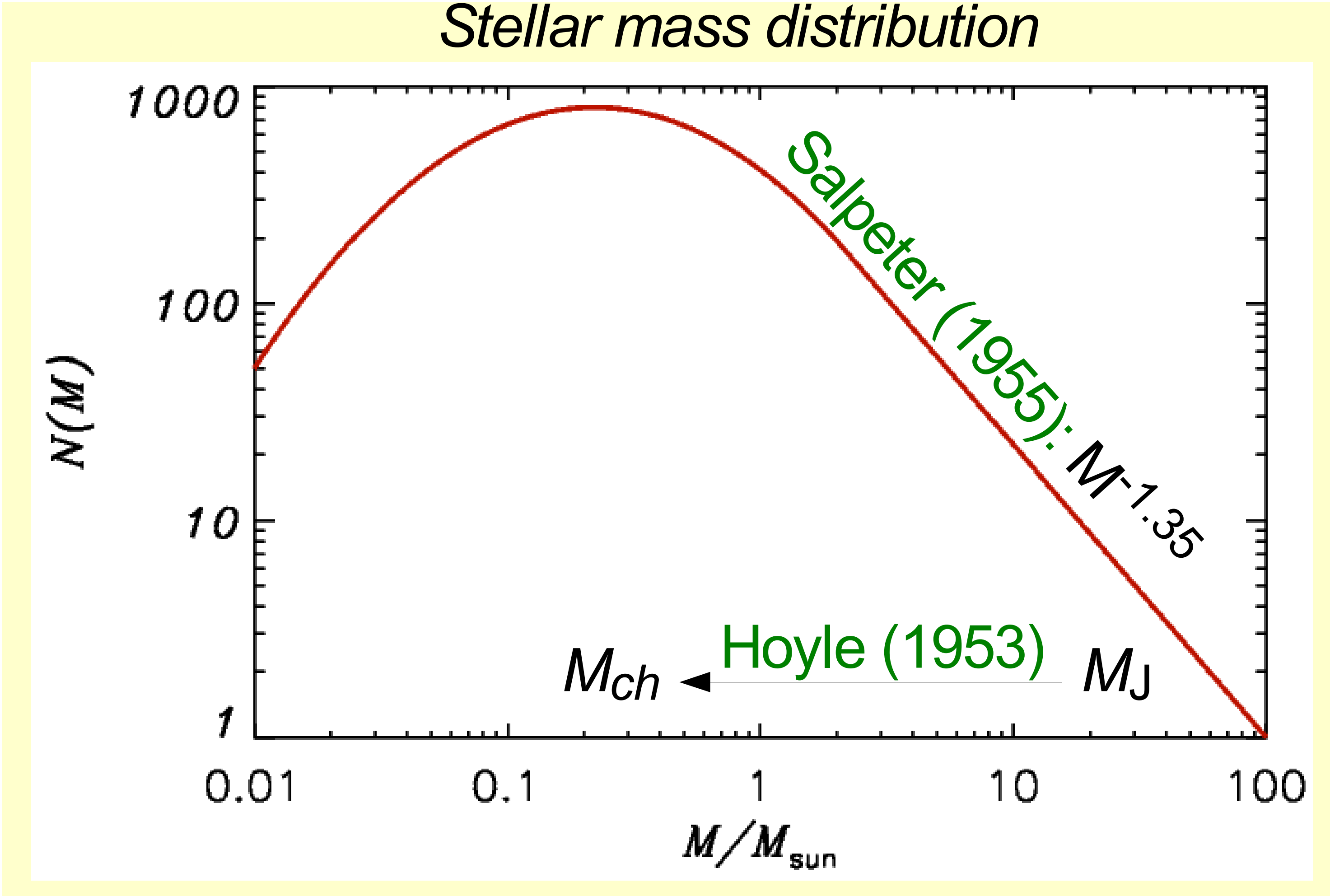


1 M_{sun}



10^{-3} M_{sun}





1. Broad range of masses, characteristic mass $M_{ch} << M_J$

2. Gas conversion into stars ~ 2% per free-fall time

Why are the predictions from the gravitational instability so wrong?

The cold ISM is highly turbulent, $Re = \frac{U\,L}{\nu} \sim 10^8$

The turbulence is supersonic, $\mathscr{M}_s \sim 30$

--> Highly non-linear velocity and density

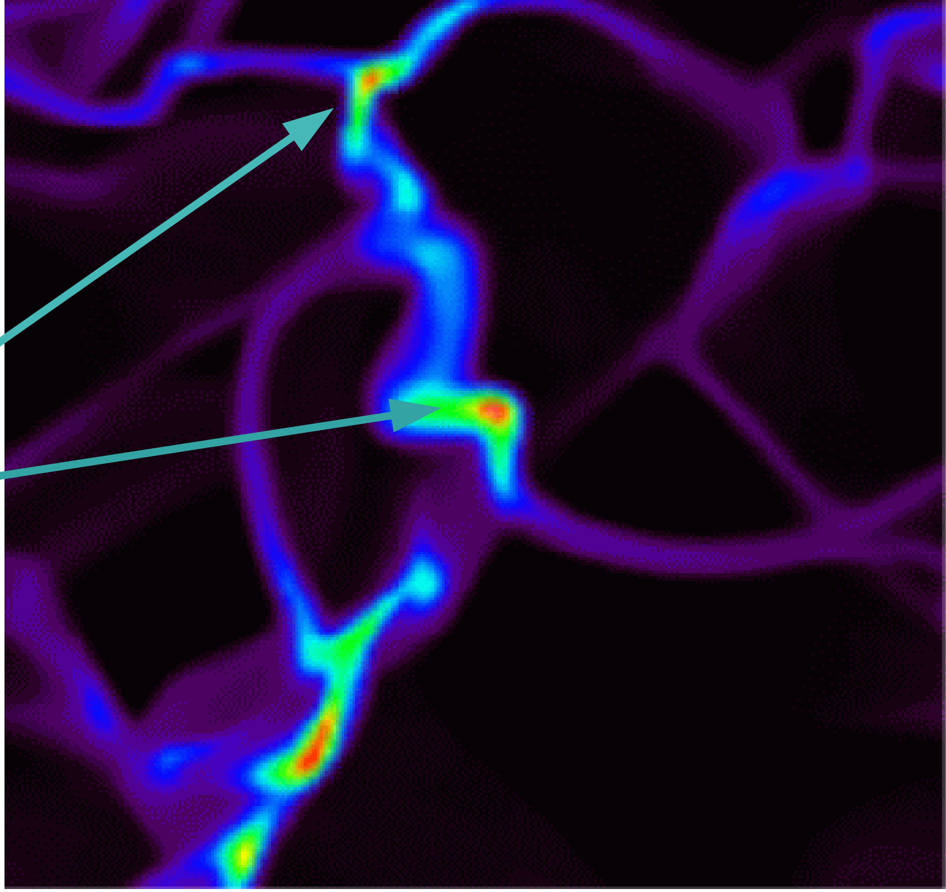
Turbulence Solution to Star Formation

1) Mass range of stars:

Stellar masses are set by turbulence, not by self-gravity ($M > M_J$ is possible).

Density peaks that become stars are pieces of postshock gas.

Their size scales with the thickness of the postshock gas, set by shock jump conditions and velocity scaling.



$$\text{MHD shocks:} \quad \lambda = l / \mathcal{M}_A(l), \quad \rho(l) = \rho_0 \mathcal{M}_A \Rightarrow M \sim \lambda^3 \rho \sim l^3 \rho_0 / \mathcal{M}_A^2$$

$$\text{Velocity scaling:} \quad \mathcal{M}_A(l) \sim u(l) \sim l^{\zeta_2/2} \Rightarrow M \sim l^{3-\zeta_2} \sim l^2$$

$$\Rightarrow M_{max} / M_{min} = \left(L_0 / l_1 \right)^{3-\zeta_2} = \mathcal{M}_{A,0}^{-2+6/\zeta_2} = \mathcal{M}_{A,0}^{4}$$

$$\mathcal{M}_{A,0} = 10 \Rightarrow M_{max} / M_{min} = 10^4$$

2) Characteristic stellar mass:

Bonnor-Ebert mass: isothermal sphere confined by external pressure (Ebert 1955; Bonnor 1956; McCrea 1957).

Thermal pressure:

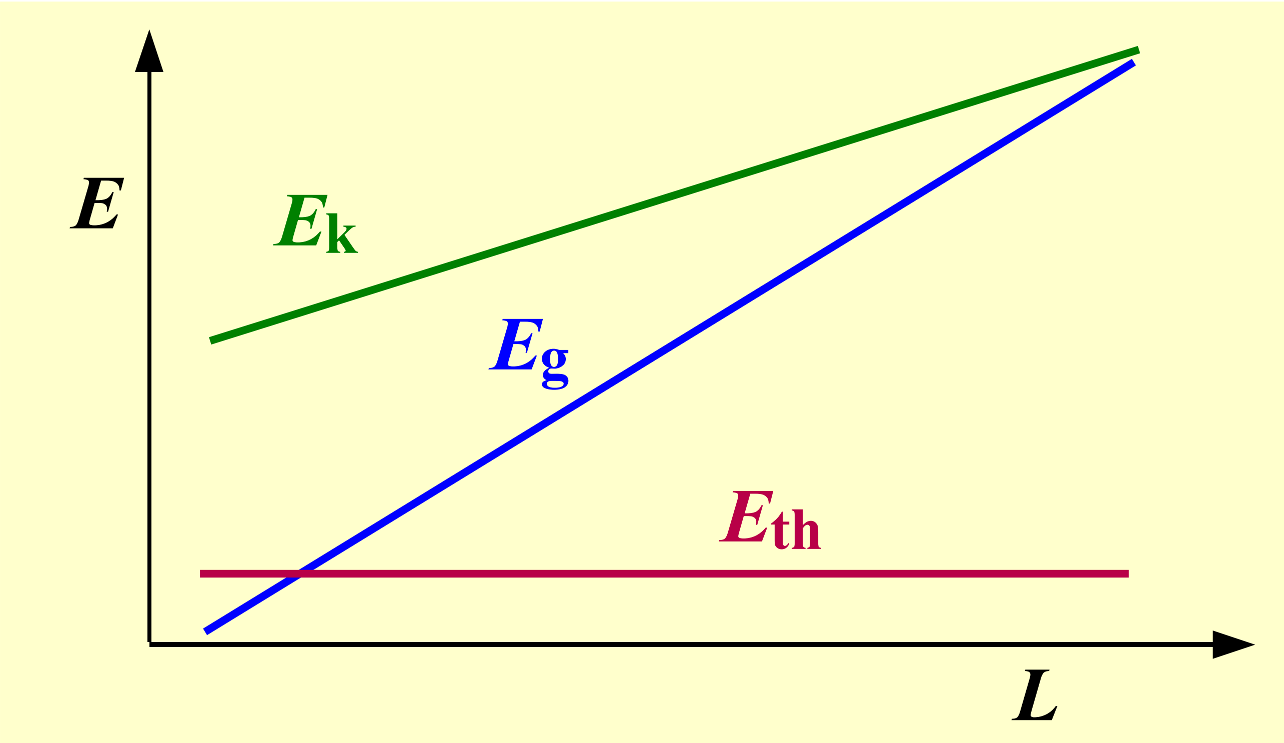
$$M_{\rm BE} \approx \frac{\sigma_{\rm th}^4}{G^{3/2} \boldsymbol{P}_{\bf th,0}^{1/2}} \approx \frac{\sigma_{\rm th}^3}{G^{3/2} \rho_0^{1/2}} \approx 10\, M_{\rm sun} \left(\frac{n}{200\,{\rm cm}^{-3}}\right)^{-1/2} \left(\frac{T}{10\,{\rm K}}\right)^{3/2} \approx \frac{M_{\rm J}}{2.47}$$

Dynamic pressure of turbulence (shocks --> nonlinear density jump):

$$\boldsymbol{M}_{\bf BE,t} \approx \frac{\sigma_{\rm th}^4}{G^{3/2} \boldsymbol{P}_{\bf dyn,0}^{1/2}} \approx \frac{\sigma_{\rm th}^3}{G^{3/2} \rho_0^{1/2}} \left(\frac{\sigma_{\rm th}}{\sigma_{\rm v}}\right) = \frac{\boldsymbol{M}_{\bf BE}}{\mathcal{M}_{\bf s}} \approx \mathbf{0.4}\, \boldsymbol{M}_{\bf sun} \quad (\text{for } \mathcal{M}_{\rm s} = 25)$$

$$\left(\text{Notice that} \quad M_{\rm BE,t} \sim n^{-1/2} T^2 \sigma_{\rm v}^{-1}\right)$$

3) Rate of star formation:



Thermal energy

$$u_0 \gg C_S \quad \Rightarrow \quad E_{k,0} \gg E_{th}$$

Isothermal shocks create a complex filamentary density structure.

Gravitational energy

$$\frac{E_k}{E_g} \sim \frac{u^2}{(\rho L)^2}, \quad u \sim L^{1/2}, \quad \frac{E_{k,0}}{E_{g,0}} \sim 1 \quad \Rightarrow \quad \frac{E_k(L)}{E_g(L)} = \left(\frac{L}{L_0}\right)^{-1}$$

The turbulence can prevent the gravitational collapse.

Star formation occurs only where the density is enhanced and the turbulence is dissipated, few % of the total mass.

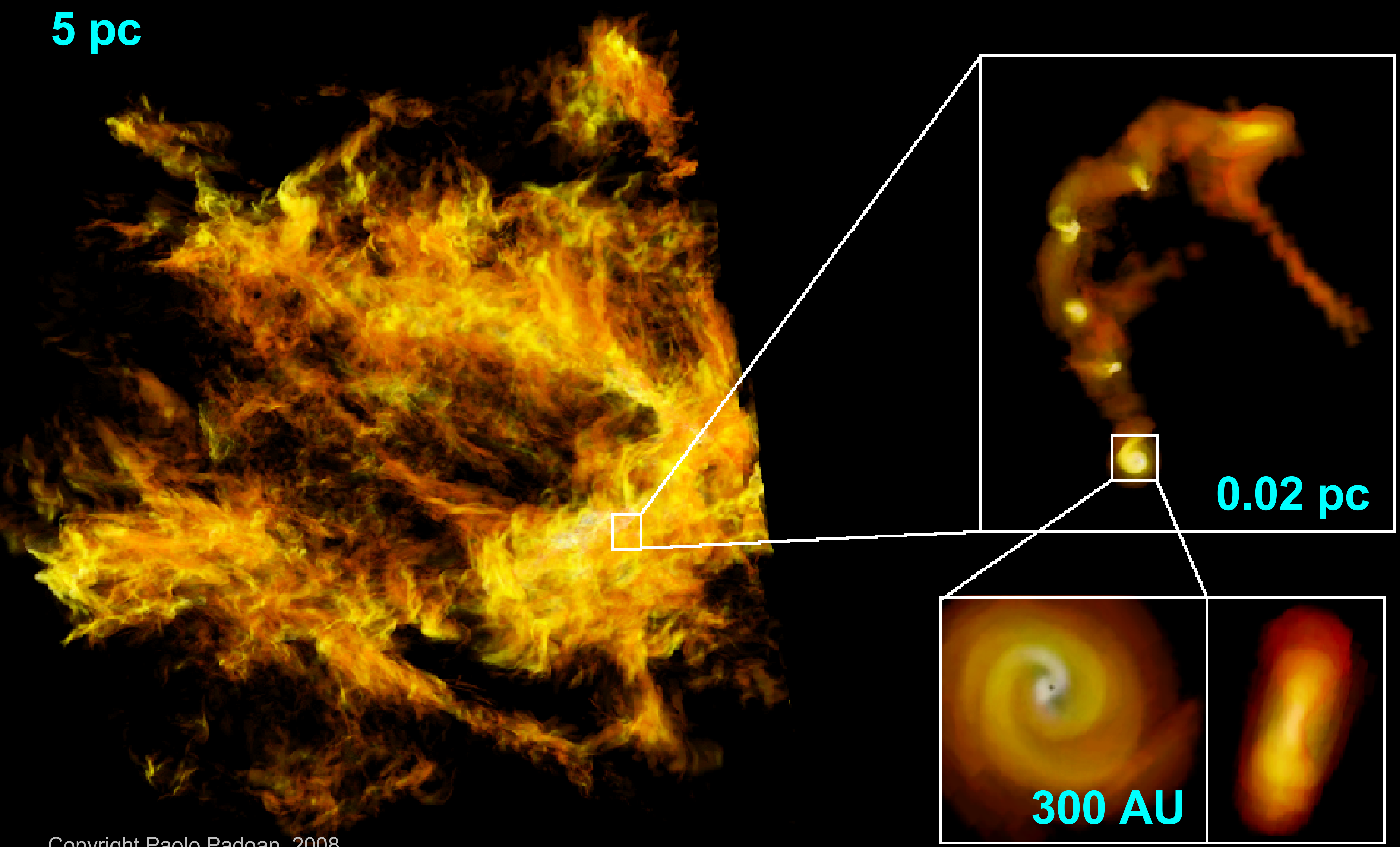
Supersonic turbulence is ubiquitous and energetically dominant in star-forming regions.

How do we study its role in the process of star formation?

Two different numerical approaches......

1. Brute-force approach: AMR simulations of star-formation

5 pc --> 0.5AU, 512^3 --> $(2\times10^6)^3$



2. Idealized experiments of supersonic turbulence

Statistics of turbulence (universal) --> Statistical theory of star formation

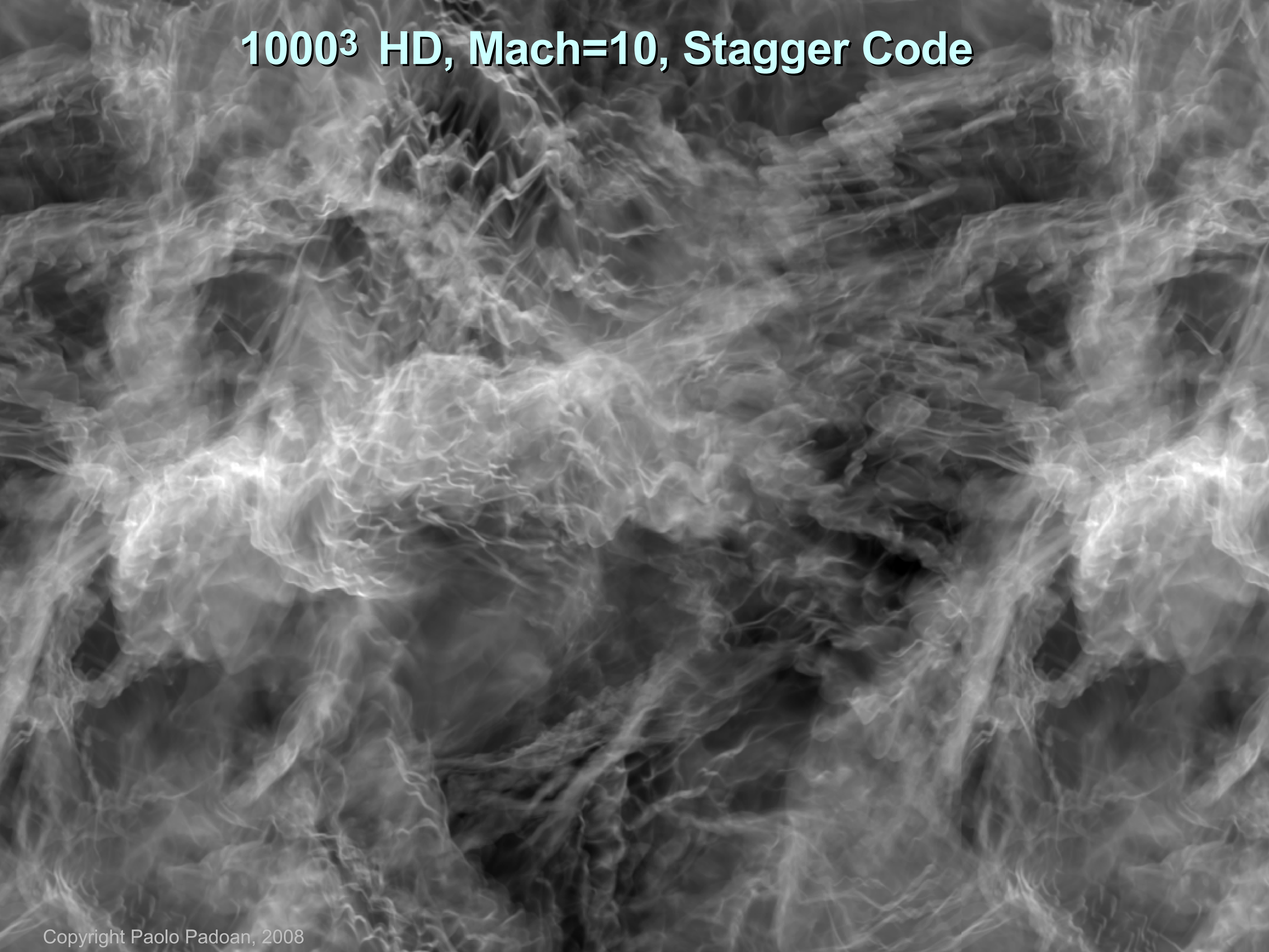
Experiment setup:

- Isothermal E.O.S.
- Periodic B.C.
- Uniform I.C. (rho, B)
- Random I.C. (u)
- Random acceleration ($1 < k < 2$)
- No gravity
- Up to $2{,}048^3$ (or larger with AMR)
- The flow is relaxed for several $t_{\rm dyn}$ before computing statistics

Euler Codes:

- PPM (Colella and Woodward 1984)
- PPML (Popov and Ustyugov 2007, 2008)
- Stagger (Nordlund)

1000^3 HD, Mach=10, Stagger Code

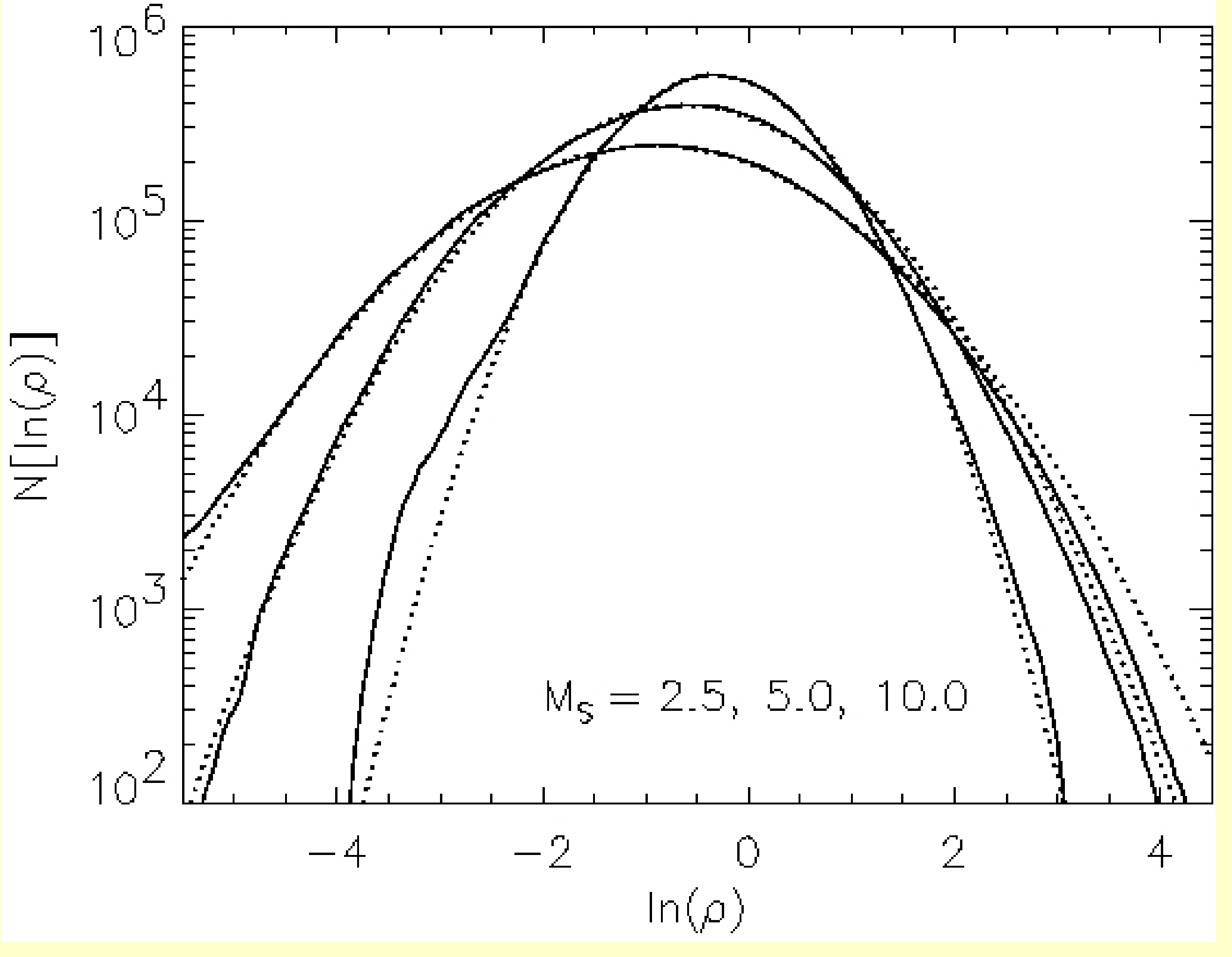


1000^3 ideal MHD, Mach=10, Stagger Code

Lognormal PDF of gas density

Nordlund and Padoan (1999):

$$\sigma_\rho \approx M_S/2 \qquad \sigma^2_{\ln\rho} \approx \ln(1+M_S^2/4)$$



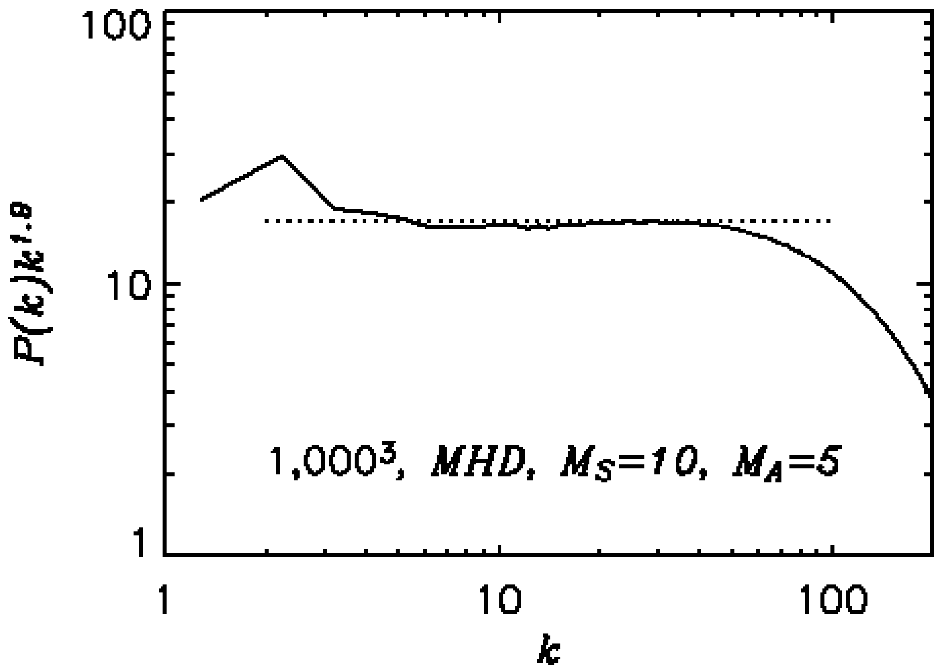
Consistent with observations (Alyssa Goodman et al. 2008)

Power-law velocity power spectrum:

Padoan et al. (2007):

$$E(k) \propto k^{-1.9}$$

Kolmogorov: $k^{-5/3}$
Burgers: k^{-2}



Is there an energy cascade in supersonic turbulence?

Supersonic turbulence as **inertial motions** ending into **oblique shocks**:

$u_\perp$ is dissipated by the shock

$u_\parallel$ goes into postshock shear

$$\Rightarrow \frac{E_C}{E_S} \approx \frac{\epsilon_C}{\epsilon_S} \approx 1/2$$

So there is a solenoidal cascade, but the dissipative flow geometry is primarily sheets (postshock regions), not filaments (vortices).

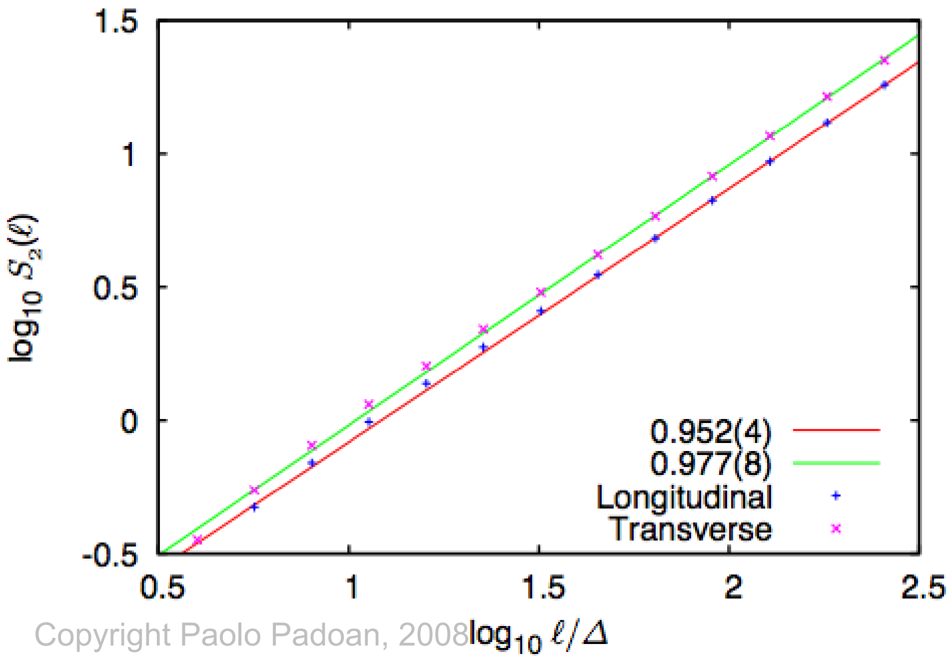
Energy cascade in incompressible turbulence

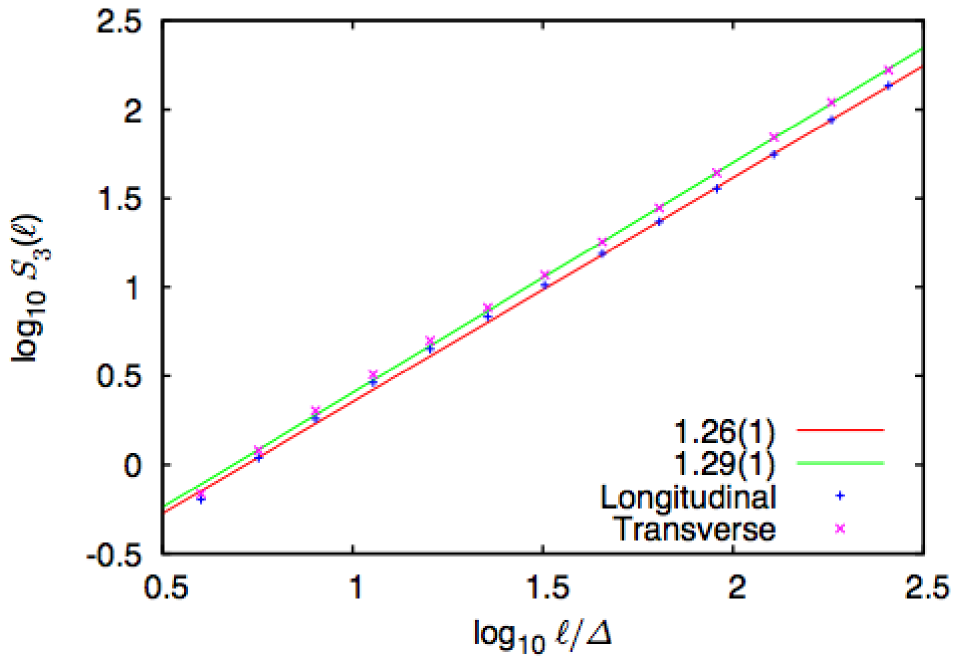
Kolmogorov (1941): $\delta u^2 \left(\frac{\delta u}{l} \right) = \text{constant} \quad \Rightarrow \quad \boldsymbol{\delta u^3 \propto l} \quad \Rightarrow \quad \delta u^p \propto l^{p/3}$

What are the scaling exponents in supersonic turbulence?

Kritsuk et al. (2007): $1{,}024^3$ and $2{,}048^3$ PPM simulations:

2nd order SFs: $\zeta_2^{\parallel} = 0.95, \zeta_2^{\perp} = 0.98$ 3rd order SFs: $\zeta_3^{\parallel} = 1.26, \zeta_3^{\perp} = 1.29$

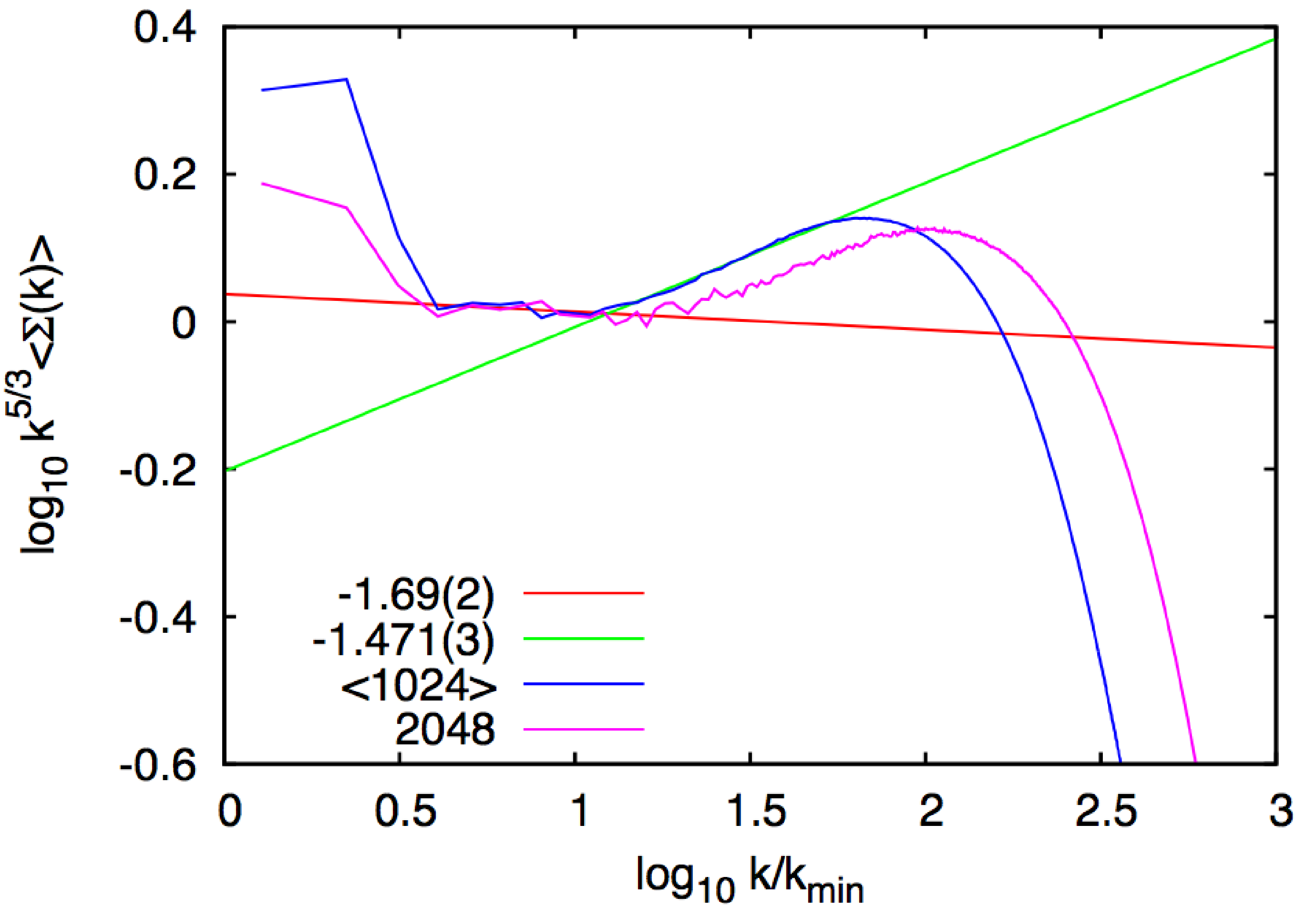




Energy cascade in supersonic turbulence

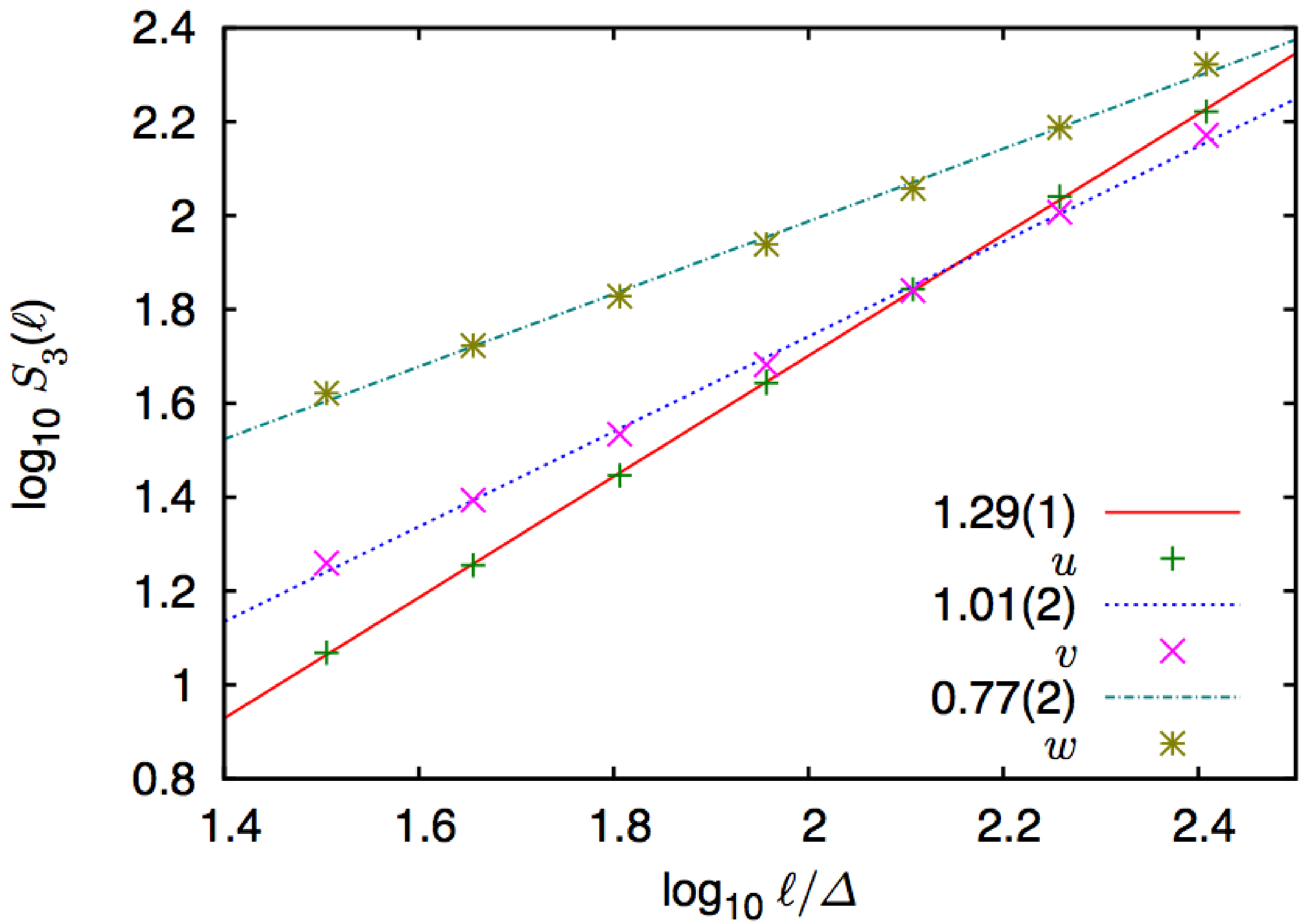
Lighthill (1955): $\rho\,\delta u^2 \left(\frac{\delta u}{l}\right) = \text{constant} \quad \Rightarrow \quad \boldsymbol{\rho\,\delta u^3 \propto l}$

$$v \equiv \rho^{1/3} u \quad \Rightarrow \quad \delta v^p = \delta(\rho^{1/3} u)^p \propto l^{p/3}$$



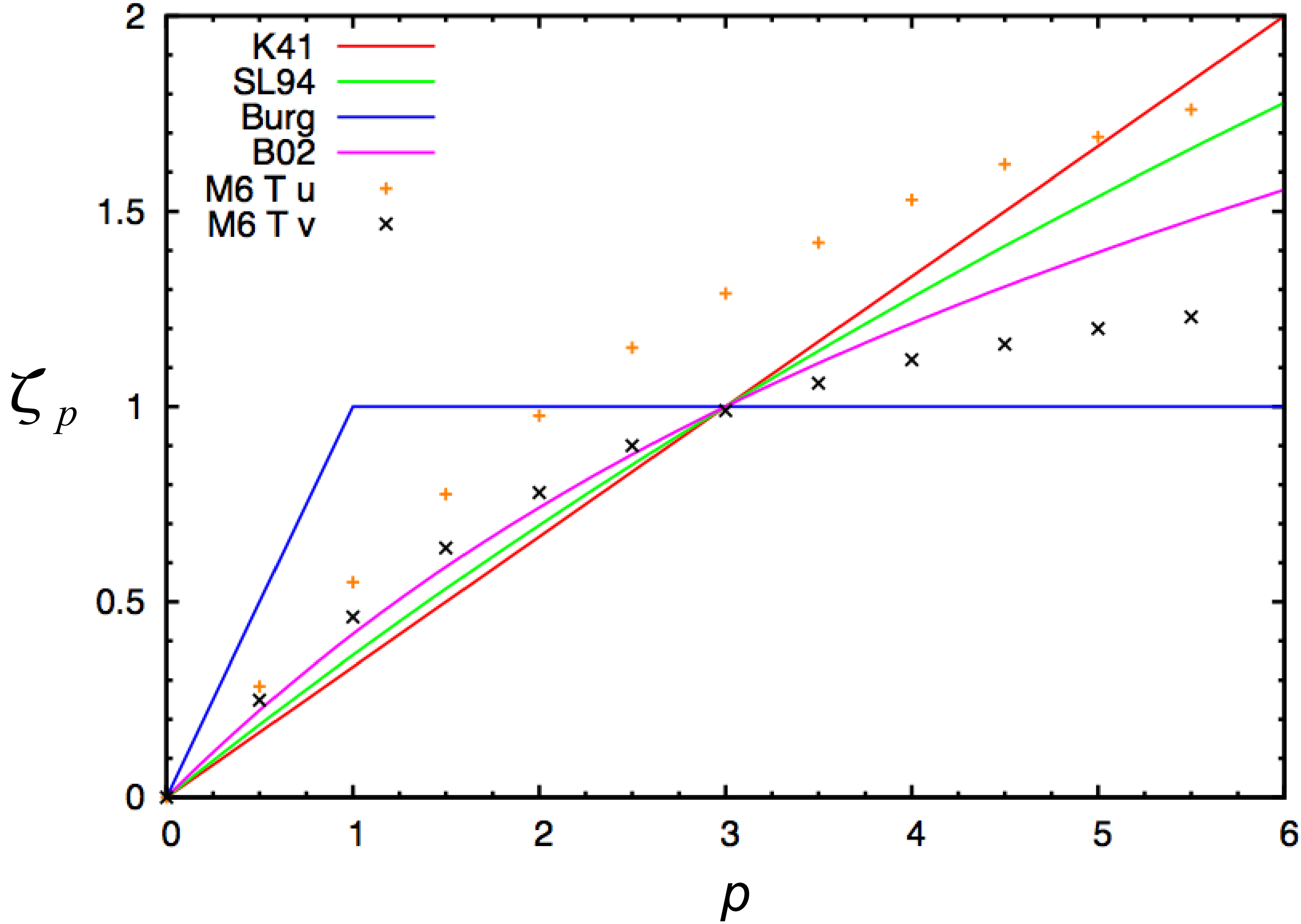
Kolmogorov scaling for $\boldsymbol{v}$: $\Sigma(k) \sim k^{-1.7}$

Velocity u, $v \equiv \rho^{1/3}u$, and $w \equiv \rho^{1/2}u$



Kolmogorov scaling for $\boldsymbol{v}$: $S_3(\ell) \equiv \langle|\delta \boldsymbol{v}|^3\rangle \sim \ell$

Structure function exponents of $v \equiv \rho^{1/3} u$



Summary

- Supersonic turbulence can explain masses and formation rate of stars.
- A statistical theory of star formation can be derived from the statistics of supersonic turbulence.
- The pdf of gas density is a Lognormal and its standard deviation is a function of the rms Mach number.
- The energy spectrum is a power law, with slope ~1.9, and $E_s/E_c \sim 2$.
- The “energy cascade” concept applies to supersonic turbulence, in the sense that the average kinetic energy density rate does not depend on scale.
- The Log-Poisson intermittency model works well in supersonic turbulence, and its parameters (scaling exponent and dimension of the most dissipative structures) have the correct physical meaning.