Relativistic Self-similar Solutions:
Explosions, implosions and shock breakouts

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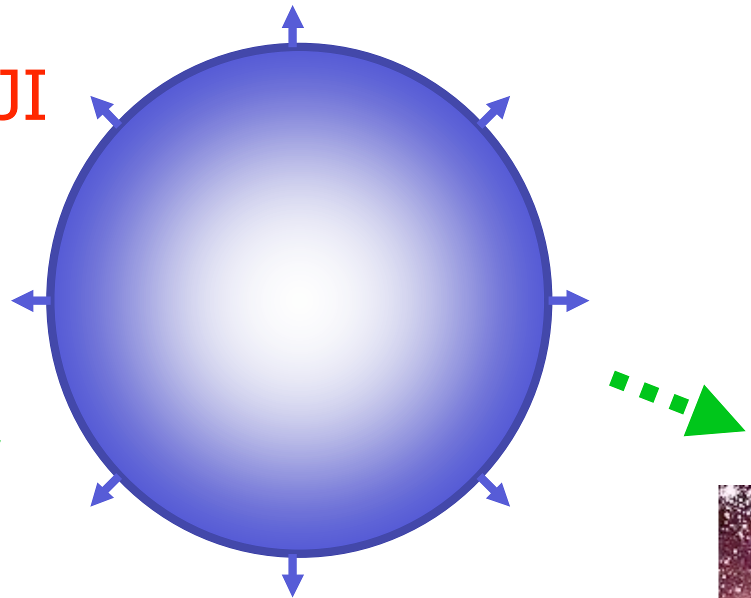
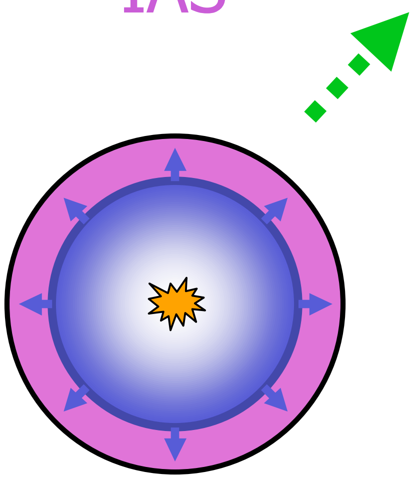
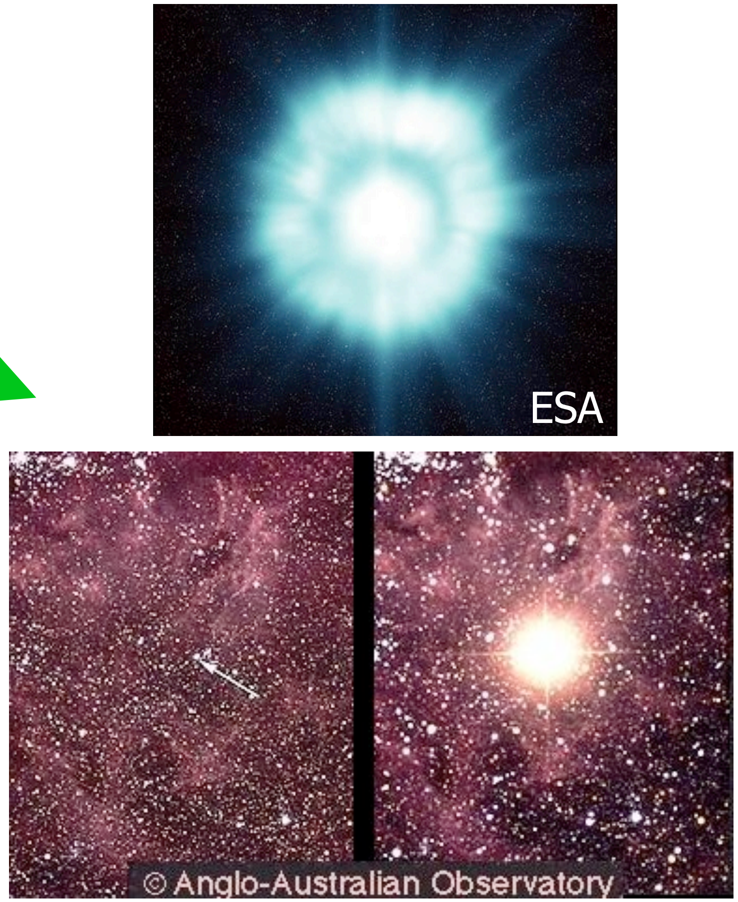
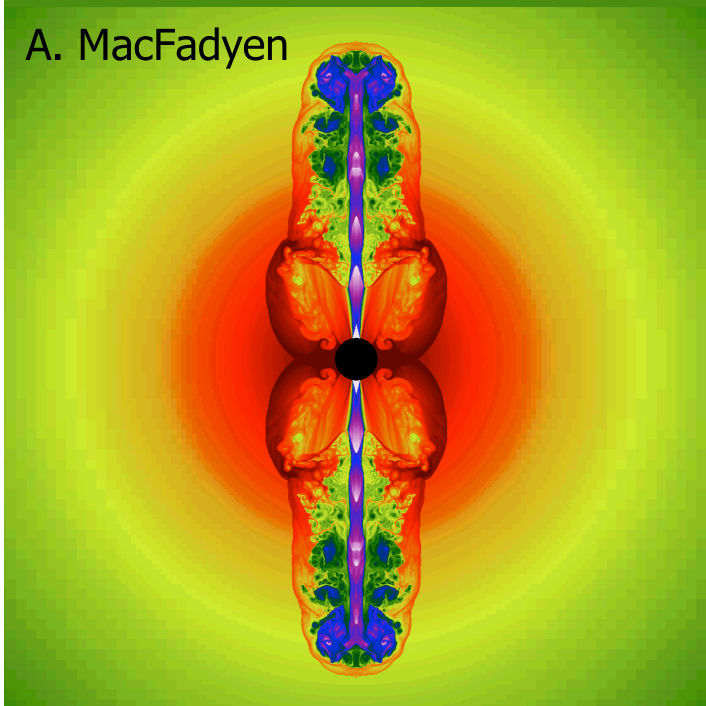
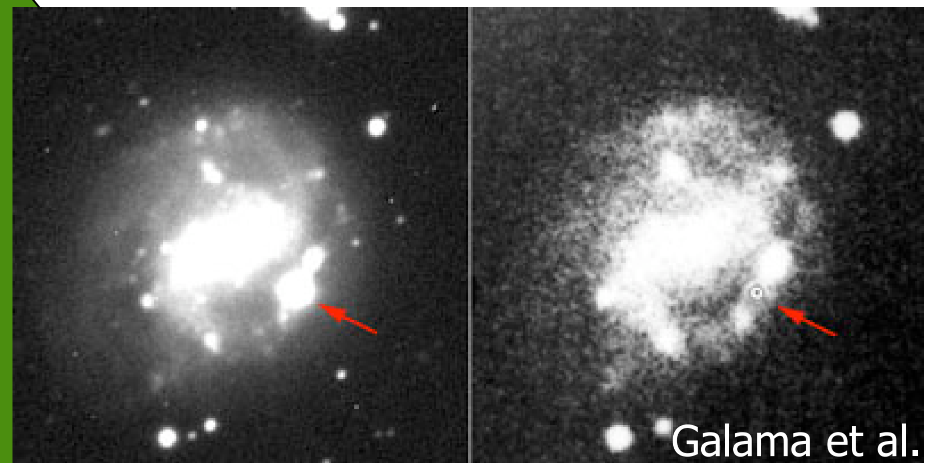


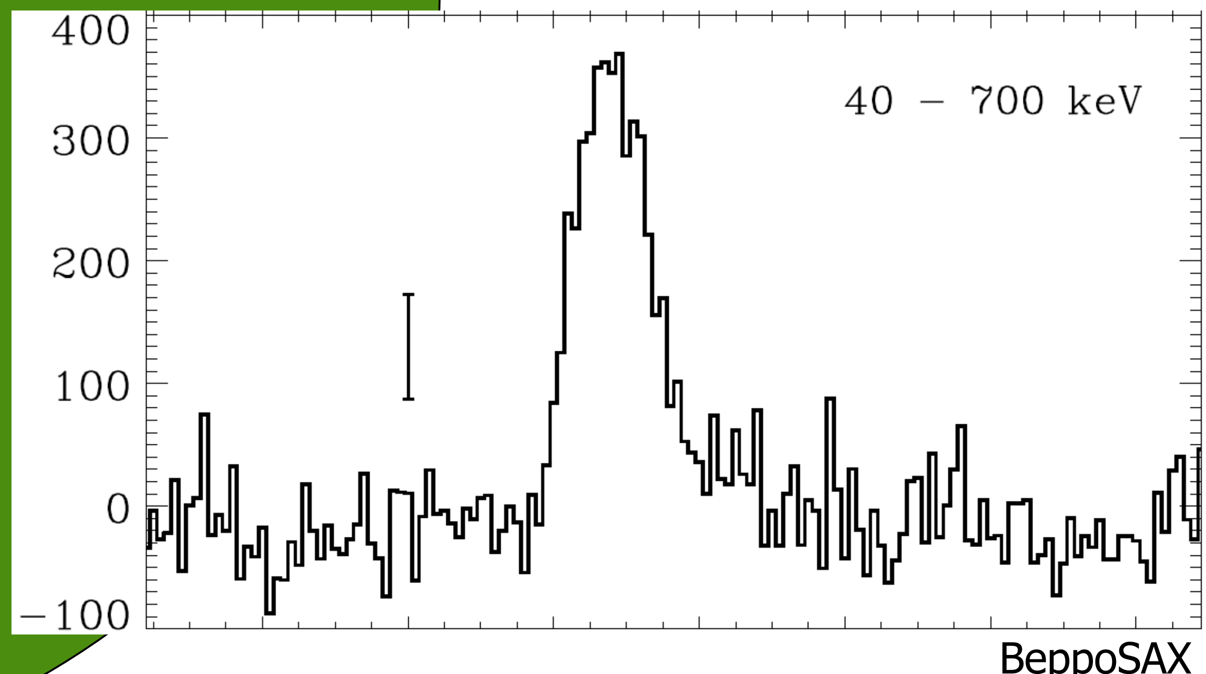




Supernovae & GRBs



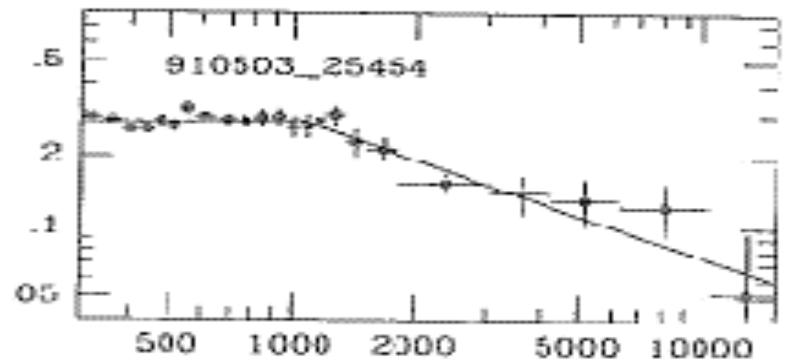




COMPACTNESS PROBLEM

$$\gamma + \gamma \rightarrow e^{+} + e^{-}$$

- dT ~ 1ms $\Rightarrow$ R < $3 \bullet 10^{7}$ cm
- E ~ 10^{51}ergs $\Rightarrow$ 10^{57} photons



high photon density
(many above 500 keV).

- Optical depth σ_T n R~10^{15}>>1
- Inconsistent with the non thermal spectrum!

Spectrum:
Optically thin

? Paradox ?

Size & Energy:
Optically thick

The Solution: Relativistic Motion

- Due to Relativistic Motion:
  - $R = \gamma^2 c\, dT$
  - E_{ph} (emitted) = E_{ph} (obs) / γ

$$\Downarrow$$

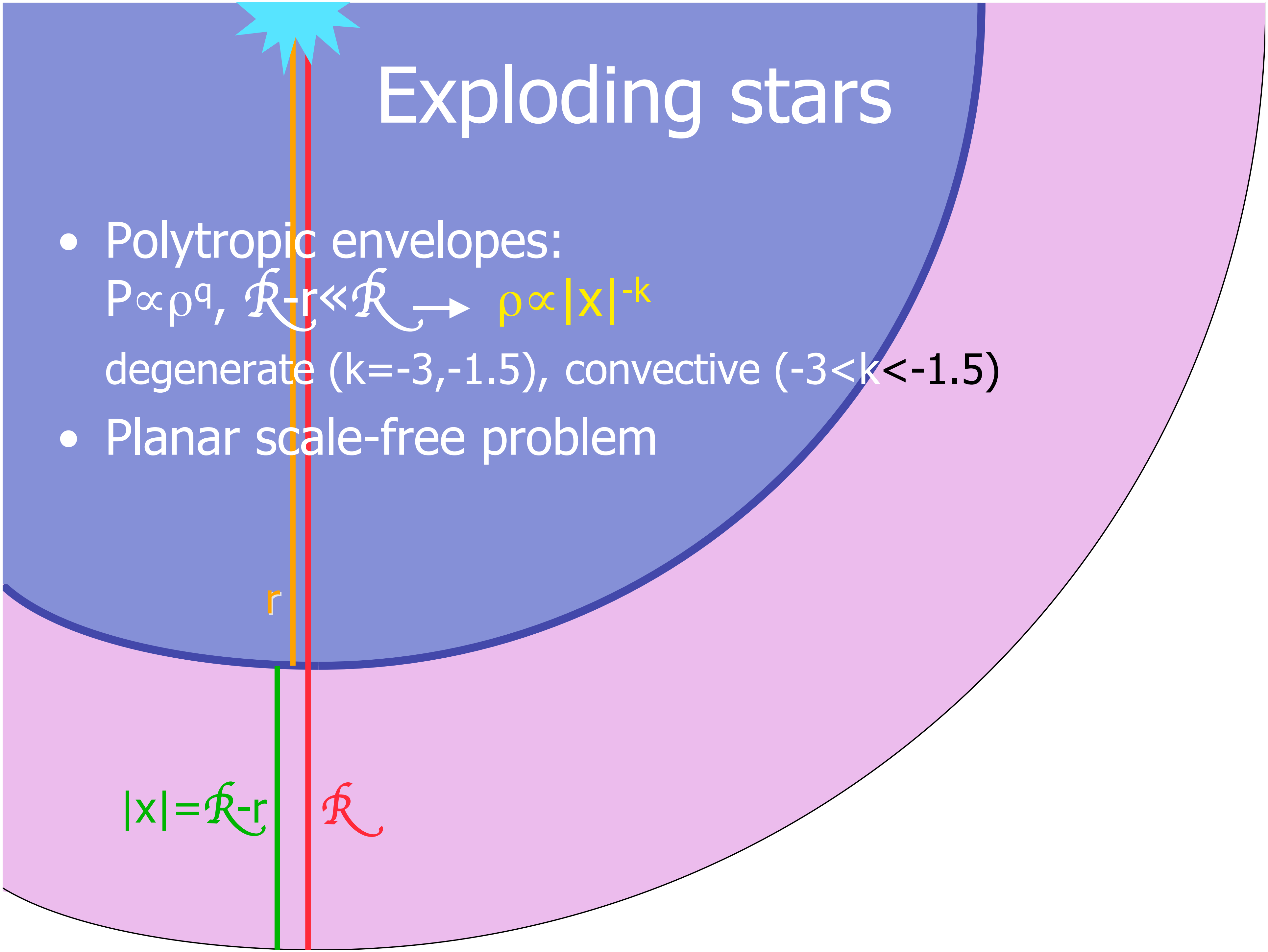
- $\tau_{\gamma\gamma} = \gamma^{-(4+2\alpha)} n\sigma_T R \sim 10^{15}/\gamma^{4+2\alpha}$

$$\boxed{\gamma > 100}$$

(Goodman; Paczynski; Krolik & Pier; Fenimore; Woods & Loeb;Baring &Harding; Piran & Shemi; Lithwick & RS)

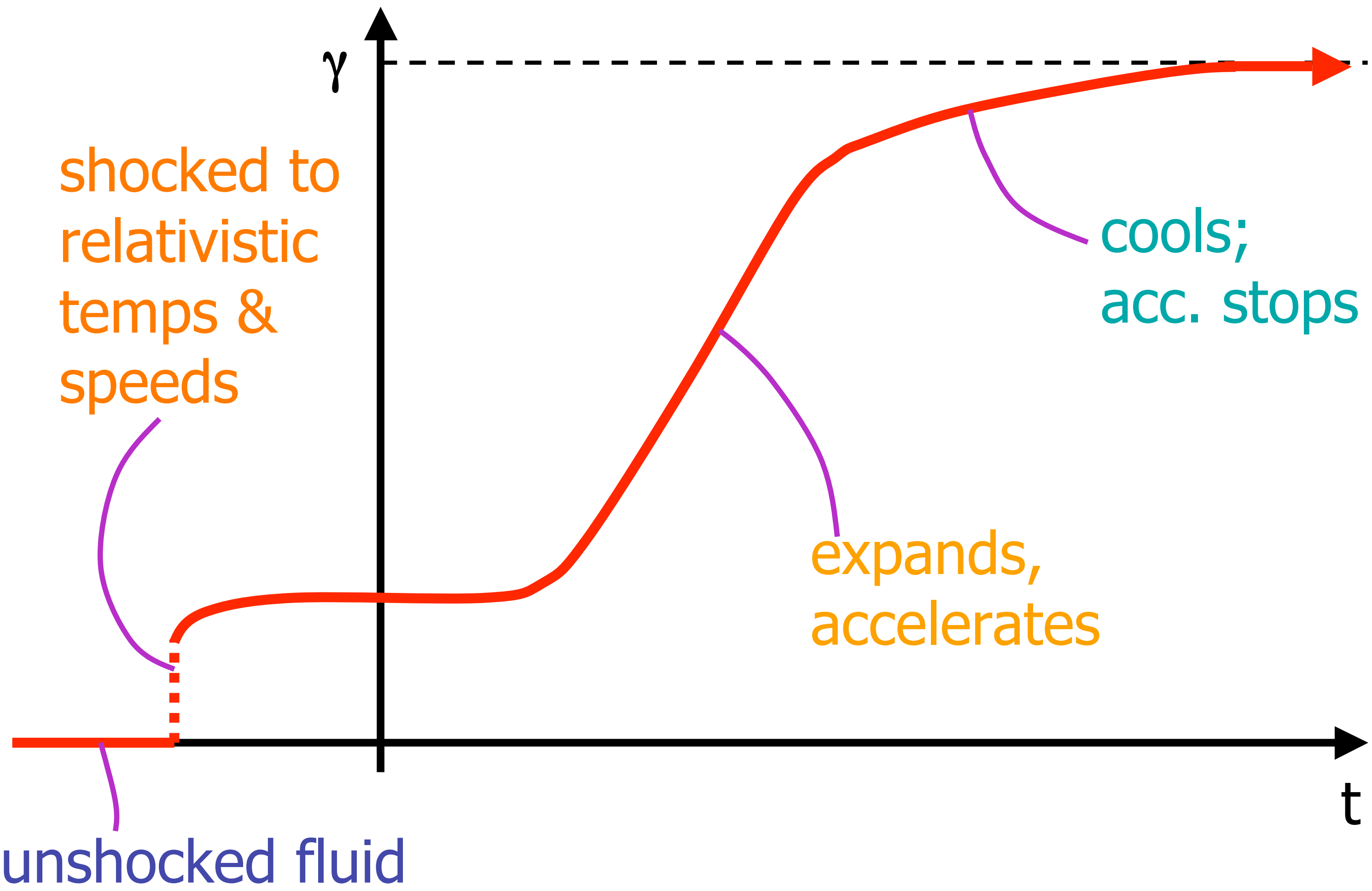
Exploding stars

- Polytropic envelopes:
  $P \propto \rho^q$, $\mathcal{R}-r \ll \mathcal{R} \longrightarrow \rho \propto |x|^{-k}$
  degenerate (k=-3,-1.5), convective (-3<k<-1.5)
- Planar scale-free problem



Exploding stars

Evolution of a single fluid element:



Self-similarity

- For scale-free problems far from initial conditions
- Profiles of hydrodynamic variables
- Decouple t and x evolution:

$$\text{PDEs in } \frac{\partial}{\partial x}, \frac{\partial}{\partial t} \rightarrow \text{ODEs in } \frac{d}{d\chi}$$

Relativistic Self Similar Equations

- Energy equation

$$\frac{\partial}{\partial t}\gamma^2(e+\beta^2 p)+\frac{1}{r^\alpha}\frac{\partial}{\partial r}r^\alpha\gamma^2\beta(e+p)=0,$$

PDEs

- Momentum equation

$$\frac{\partial}{\partial t}\gamma^2\beta(e+p)+\frac{1}{r^\alpha}\frac{\partial}{\partial r}r^\alpha\gamma^2\beta^2(e+p)+\frac{\partial}{\partial r}p=0,$$

$\gamma(t,r) = \Gamma(t)\, g(\chi)/\sqrt{2}$

$p(t,r) = p(t)\, f(\chi)$

$\chi = (t-x)/(t-R)$

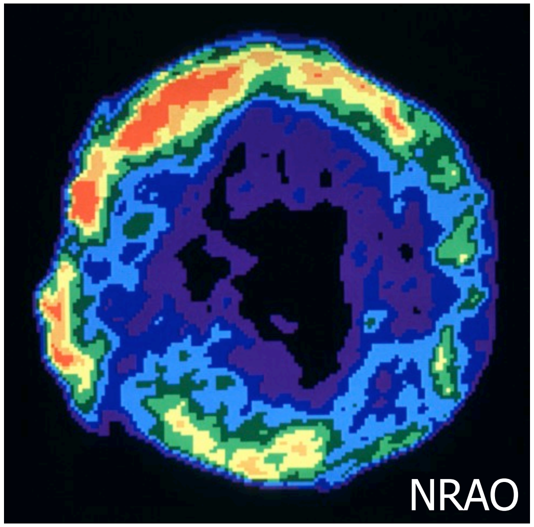
- k - external density profile $\rho \sim r^{-k}$.
- α - 0,1,2 - system's dimension.
- m - temporal evolution, $\gamma \sim t^{-m/2}$

$$\frac{1}{g\chi}\frac{d\log g}{d\log\chi}=\frac{(7m+3k-2\alpha)-(m+\alpha)g\chi}{(m+1)(g^2\chi^2-8g\chi+4)},$$

ODEs

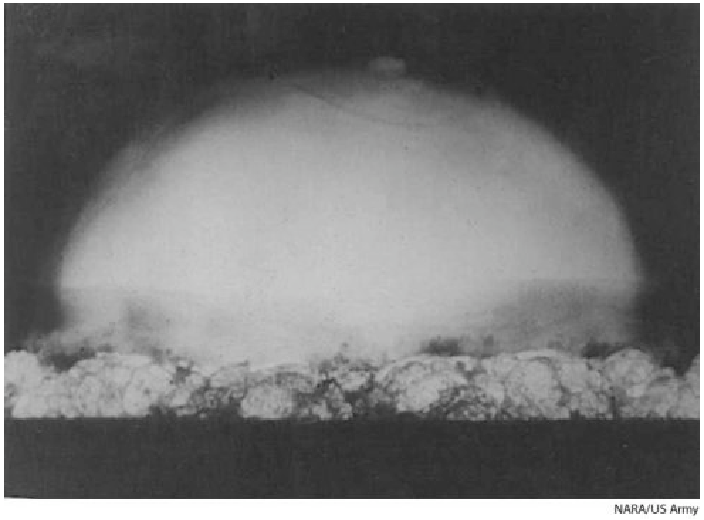
$$\frac{1}{g\chi}\frac{d\log f}{d\log\chi}=\frac{4(2m-\alpha+k)-(m+k-2\alpha)g\chi}{(m+1)(g^2\chi^2-8g\chi+4)},$$

Type I



- Evolution follows from global energy conservation
- Sedov-Taylor (1940s):

$$\text{const } E \sim \rho R^3 v^2$$
$$\sim \rho R^5 t^{-2}$$
$$R \sim t^{2/5}$$
$$v \sim t^{-3/5}$$



- Blandford & McKee 1976:

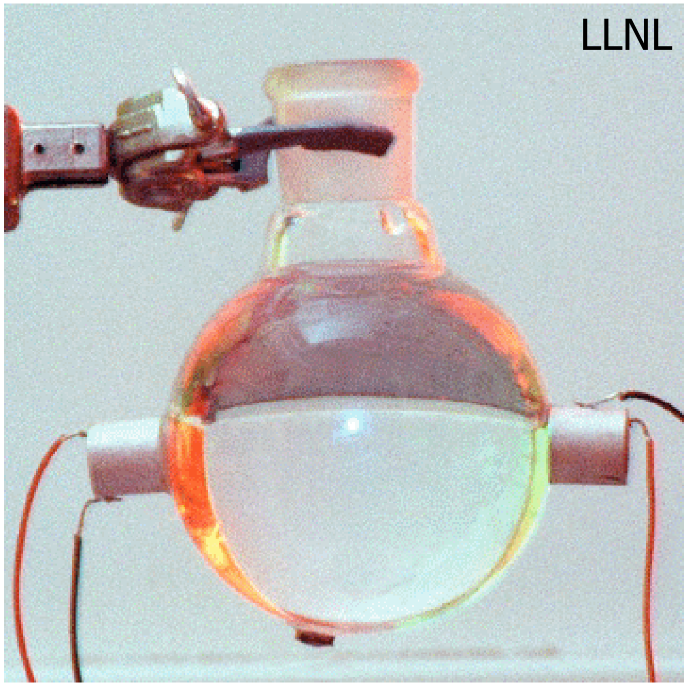
$$\text{const } E \sim \rho R^3 \gamma^2 c^2$$

$$\gamma \equiv \frac{1}{\sqrt{1 - v^2/c^2}} >> 1$$

$$\gamma \sim R^{-3/2} \sim T^{-3/8}$$

Type II

- Implosions
- Shock accelerates
- Boundary conditions from sonic point, not global energy conservation



- Sonic point “protects” the shock from non self-similar region.

Solutions

$$\frac{1}{g\chi}\frac{d\log g}{d\log\chi} = \frac{(7m+3k-2\alpha)-(m+\alpha)g\chi}{(m+1)(g^2\chi^2-8g\chi+4)},$$

f(1)=g(1)=1

$$\frac{1}{g\chi}\frac{d\log f}{d\log\chi} = \frac{4(2m-\alpha+k)-(m+k-2\alpha)g\chi}{(m+1)(g^2\chi^2-8g\chi+4)},$$

First type

- Energy conservation: m=1+α-k

$$g=\chi^{-1},$$

$$f=\chi^{(4k-7-5\alpha)/[3(2+\alpha-k)]}$$

- Calculate the energy in the solution

$$\int fg\,d\chi \sim \chi^{(4k-7-5\alpha)/[3(2+\alpha-k)]}$$

- Finite for:
  - Explosions k<(7+5α)/4
  - Implosions k>(7+5α)/4

Second type

- Sonic point: $g\chi = 4-2\sqrt{3}$
- Smooth solution at sonic point:

$$m=-2\alpha(5-3\sqrt{3})+(3-2\sqrt{3})k$$

- Solve:

$$g=\left[\frac{g\chi(\alpha-1)+4\alpha(\sqrt{3}-1)-2k\sqrt{3}+4+2\sqrt{3}}{(\alpha-1)+4\alpha(\sqrt{3}-1)-2k\sqrt{3}+4+2\sqrt{3}}\right]^{(3-2\sqrt{3})(k-3\alpha)/(\alpha-1)}$$

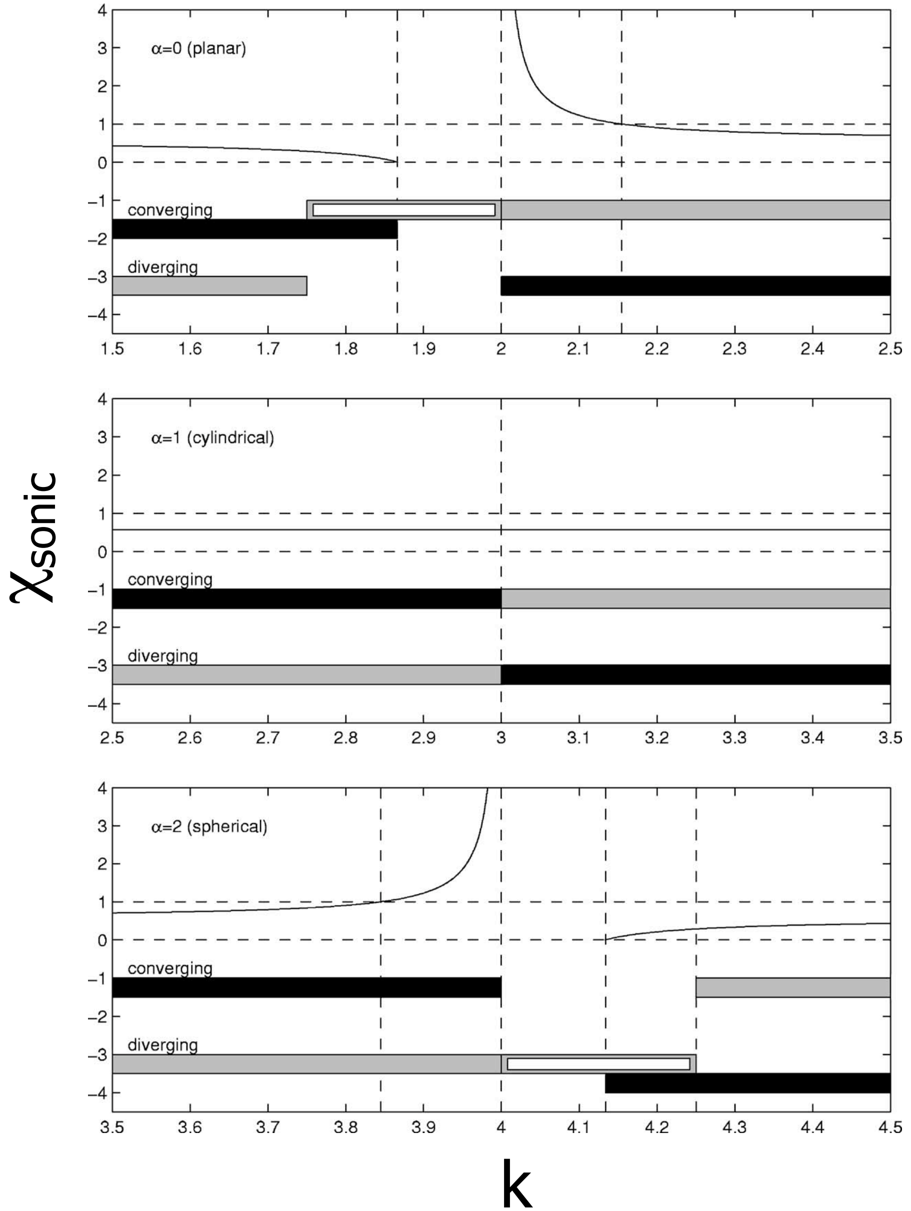
$$f=\left[\frac{g\chi(\alpha-1)-4\alpha+4\alpha\sqrt{3}-2k\sqrt{3}+4+2\sqrt{3}}{\alpha-1-4\alpha+4\alpha\sqrt{3}-2k\sqrt{3}+4+2\sqrt{3}}\right]^{(4-2\sqrt{3})(k-3\alpha)/(\alpha-1)}$$

- Sonic point exists exists if:
  - Explosions k>5-(3/4)$^{1/2}$
  - Implosions k<4

First & Second Type Solutions

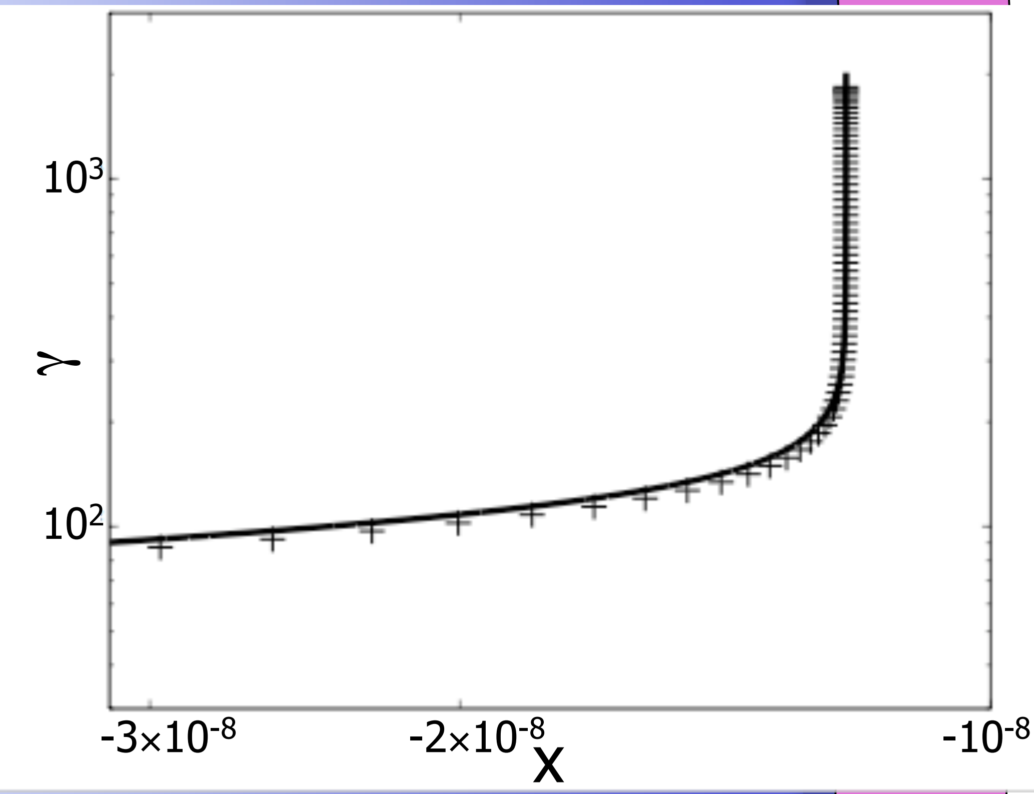
- Explored all three geometries

- Overlaps & gaps - mysteries:
  - Is there a 3rd type self similar solutions to fill the gaps?
  - What selects between the two solution in the overlap regions?

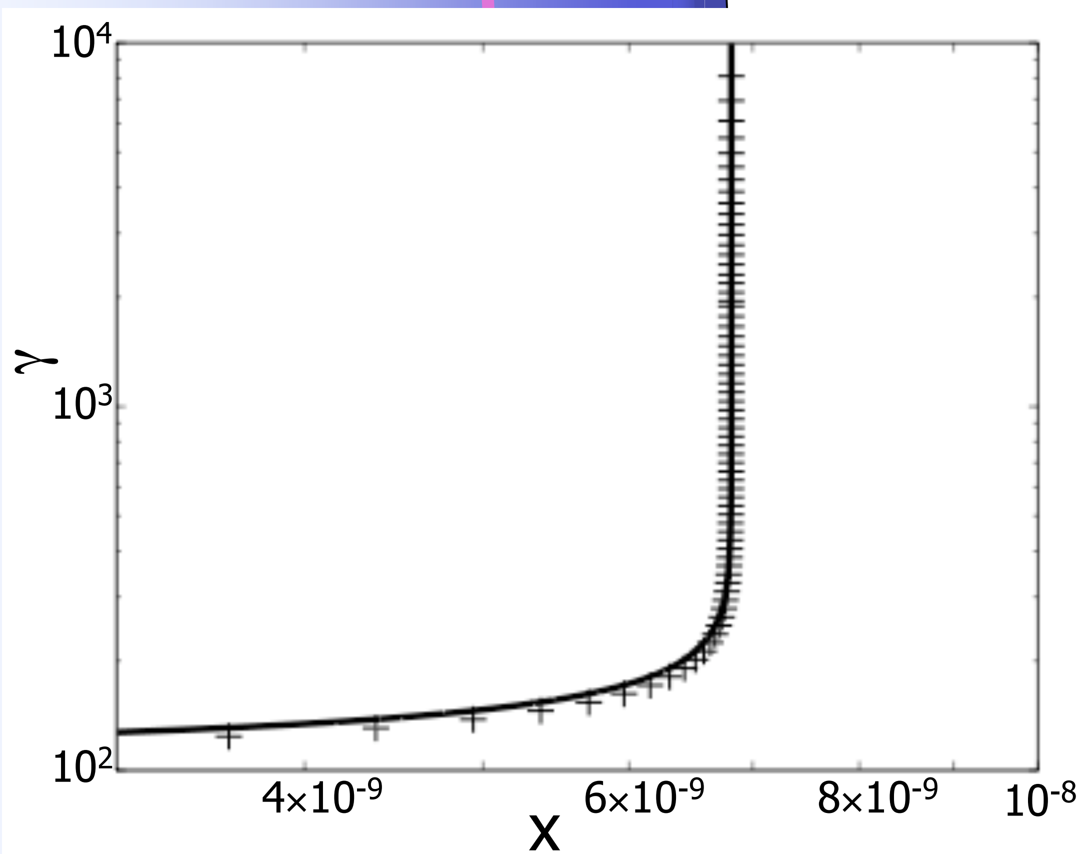


Pre-breakout (Sari 06)

- t<0, R<0
- Shock accelerates toward edge: Type II
- R = position of shock

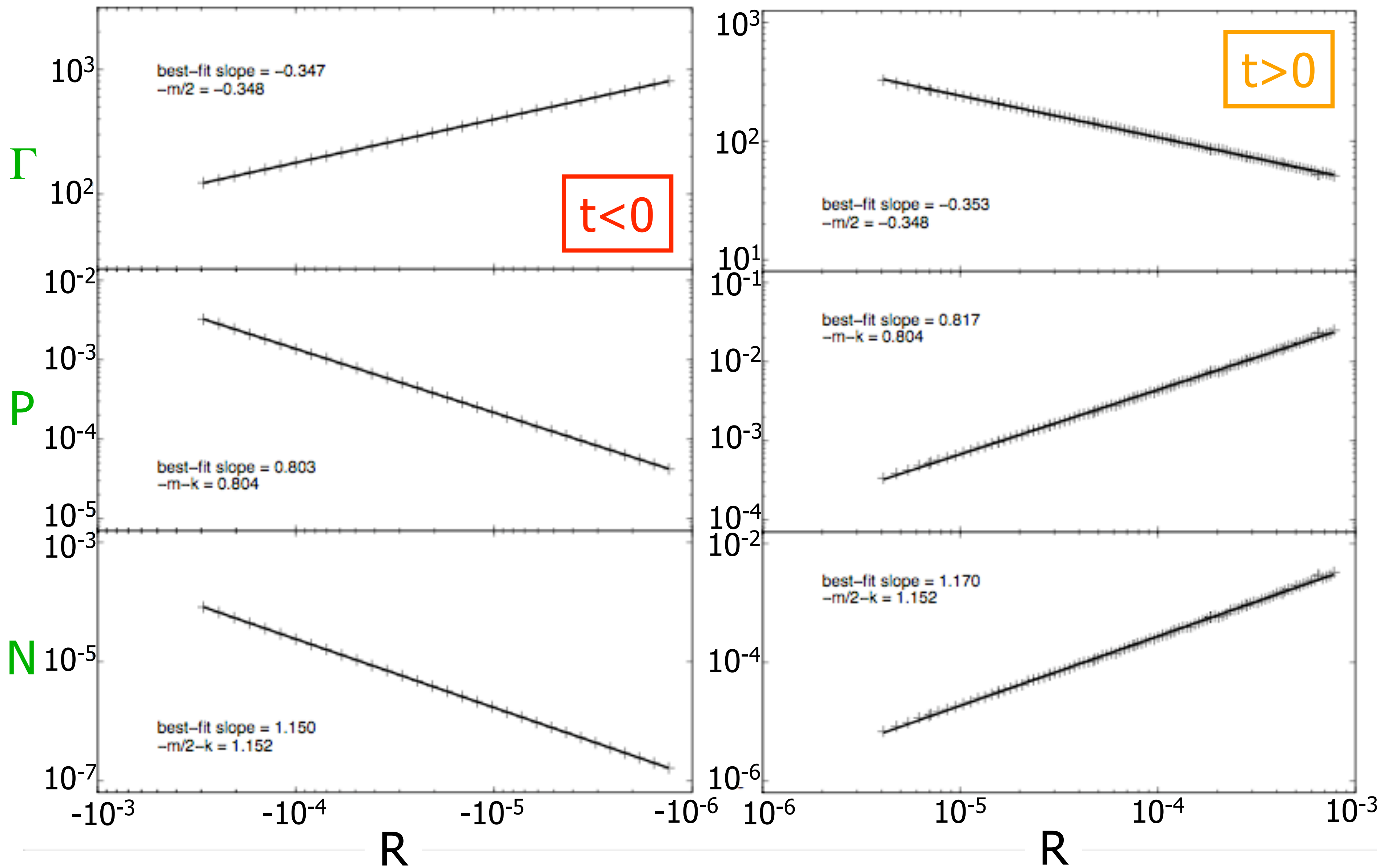


Post-breakout (Pan & Sari 06)

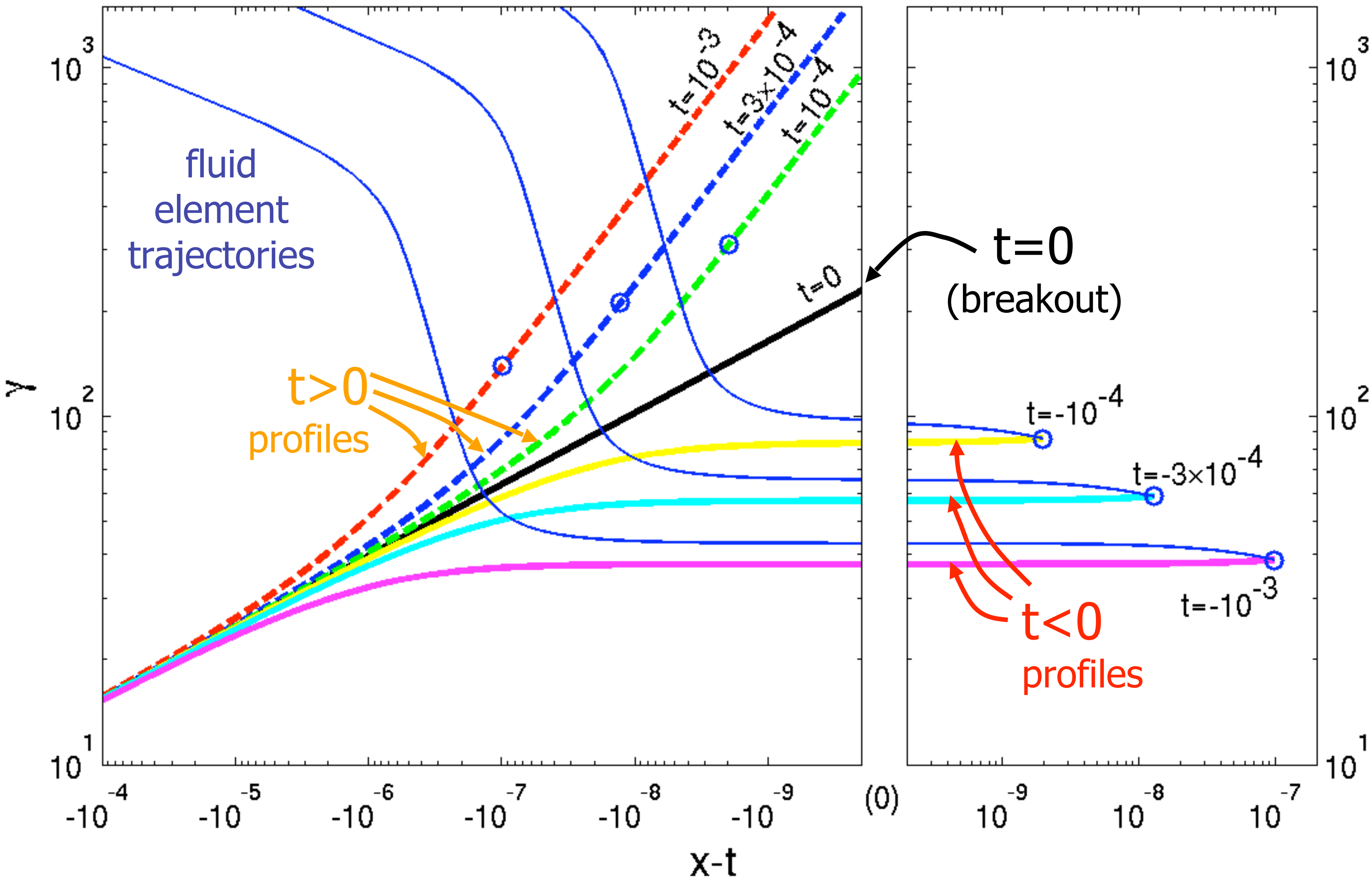


- $t>0$, $R>0$
- No new scales: still self-similar!
- R = position of fluid element which has expanded by factor of ~few
- Same time evolution as $t<0$

Time evolution



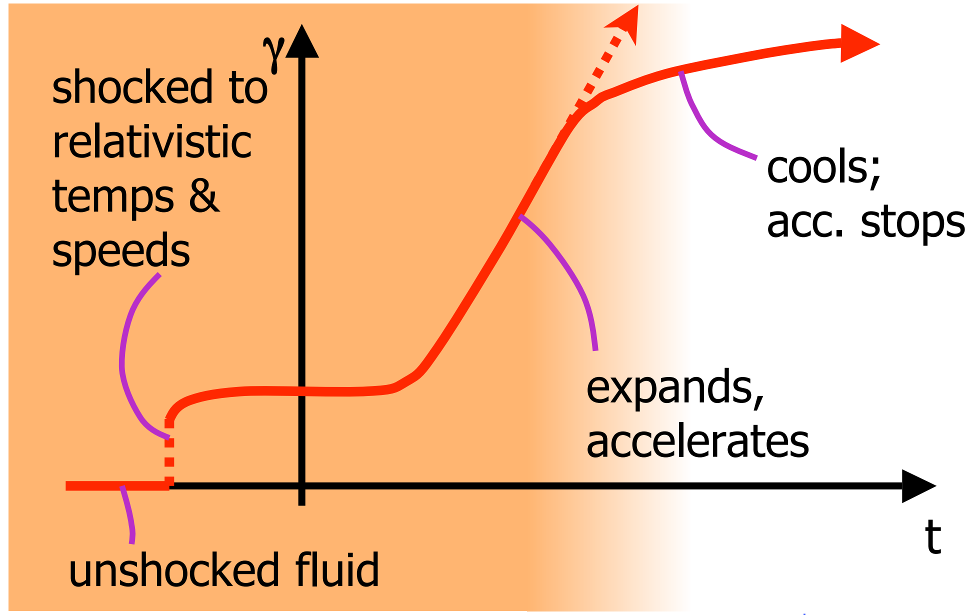
Summary - pre & post breakout

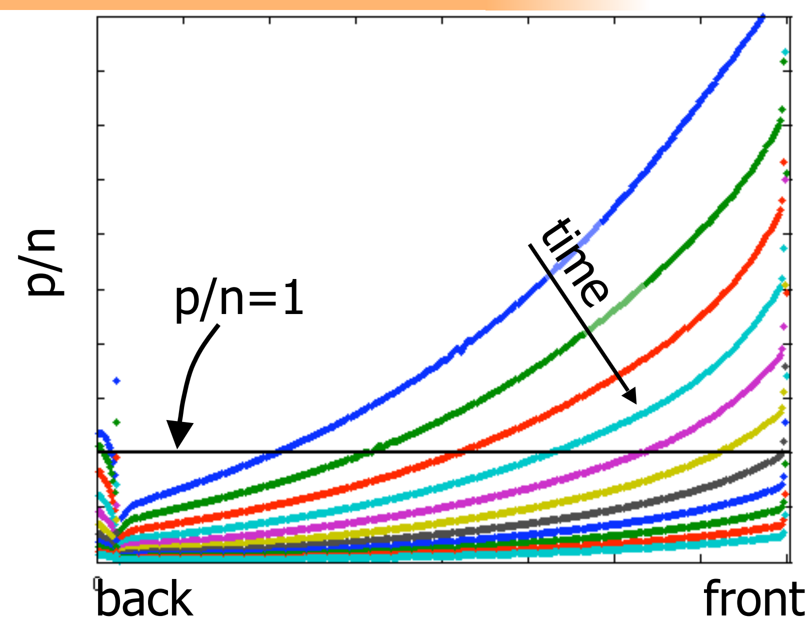


Fluid elements at late times

- p=e/3: assumed hot fluid

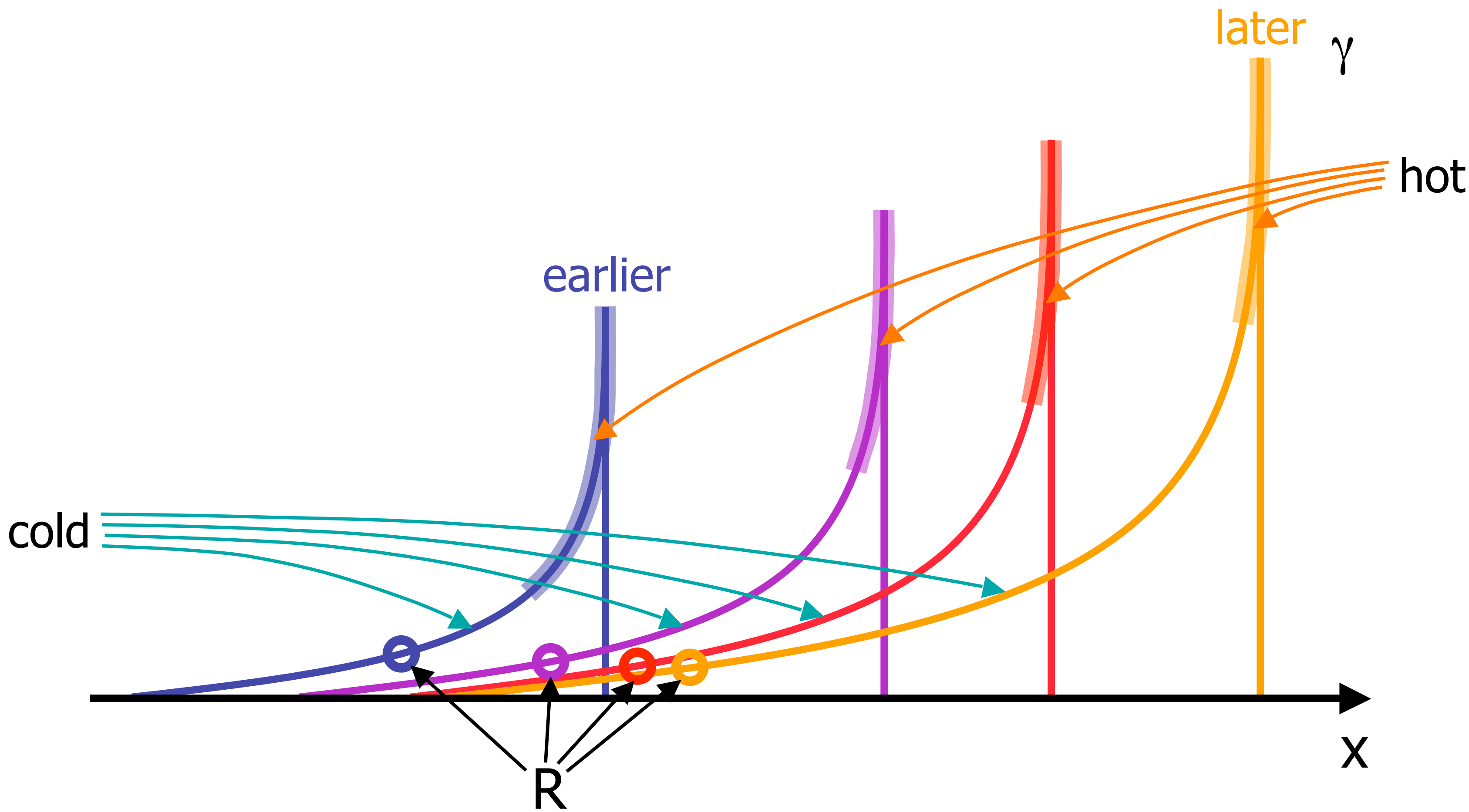
- To get final γ, cut off the flow when $p/n \approx 1$





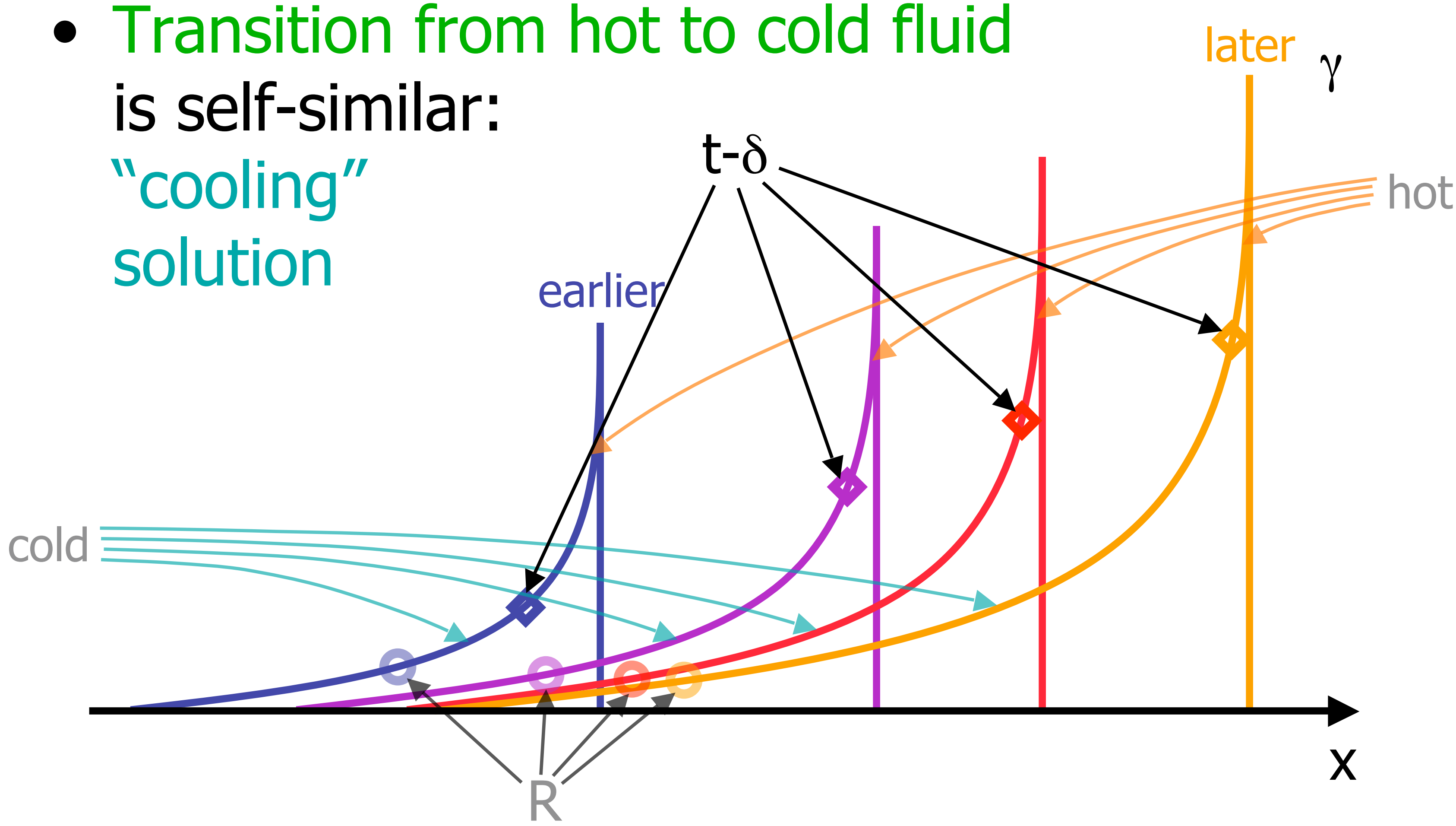
How to deal with cold fluid?

- Cold fluid breaks self-similarity



How to deal with cold fluid?

- Cold fluid not self-similar in the "hot" solution
- Transition from hot to cold fluid is self-similar: "cooling" solution



Hot solution vs cooling solution

- $p=e/3$
- R: fluid which has expanded by factor of few
- $\chi=(t-x)/(t-R)$
- Valid at early times & for fluid near the front

- $p=(e-nmc^2)/3$
- $t-\delta$: fluid which has just become cold ($p/n=1$)
- $\xi=(t-x)/\delta$
- Valid at late times, when $t-R \gg \delta$

Cooling solution ($\bar{g},\bar{f},\bar{h}$)

e,p,n conservation

$$\frac{\partial}{\partial t}\left[\gamma^2(e+\beta^2 p)\right]+\frac{\partial}{\partial x}\left[\gamma^2\beta(e+p)\right]=0$$

$$\frac{\partial}{\partial t}\left[\gamma^2\beta(e+p)\right]+\frac{\partial}{\partial x}\left[\gamma^2(\beta^2 e+p)\right]=0$$

$$\frac{\partial}{\partial t}(\gamma n)+\frac{\partial}{\partial x}(\gamma\beta n)=0$$

$$p=(e-n)/3$$

new EOS

$$\gamma^2(x,t)=\frac{1}{2}\bar{\Gamma}^2(t)\bar{g}(\xi)$$

$$p(x,t)=\bar{P}(t)\bar{f}(\xi)$$

$$n(x,t)=\bar{N}(t)\frac{\bar{h}(\xi)}{\bar{g}^{1/2}(\xi)}$$

similarity

ODE solver

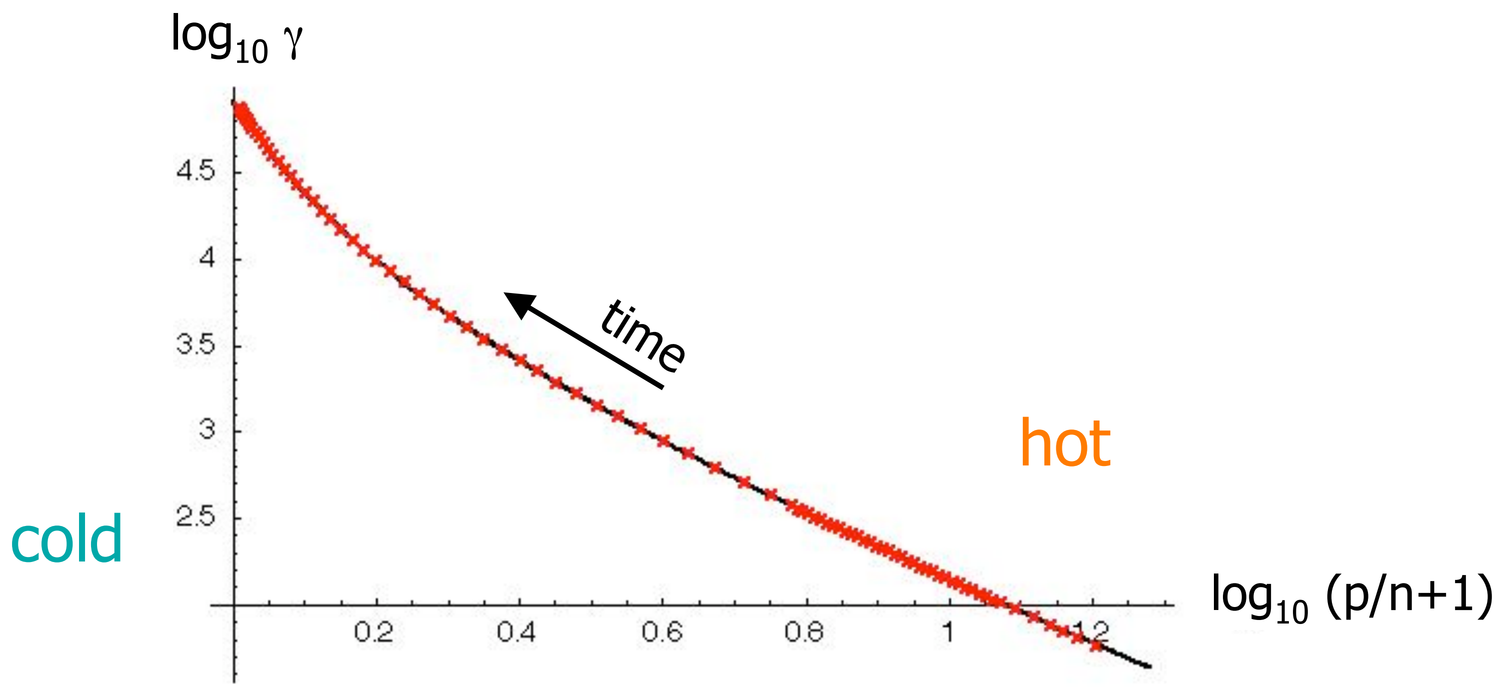
self-similar form

$$\begin{aligned}0 &= b\left(2\bar{f}+\frac{\bar{h}}{\bar{g}^{1/2}}\right)+\frac{t}{\delta\bar{\Gamma}^2}\left[-\bar{g}'\left(4\frac{\bar{f}}{\bar{g}^2}+\frac{3}{2}\frac{\bar{h}}{\bar{g}^{5/2}}\right)+\frac{4\bar{f}'}{\bar{g}}+\frac{\bar{h}'}{\bar{g}^{3/2}}\right]\\ &\quad -\xi\frac{t\dot{\delta}}{\delta}\left[-\frac{\bar{g}'}{2}\frac{\bar{h}}{\bar{g}^{3/2}}+2\bar{f}'+\frac{\bar{h}'}{\bar{g}^{1/2}}\right]\end{aligned}$$

$$\begin{aligned}0 &= (2a+b)\left(2\bar{g}\bar{f}+\frac{\bar{h}\bar{g}^{1/2}}{2}\right)+\frac{t}{\delta\bar{\Gamma}^2}\left[-\frac{\bar{g}'}{4}\frac{\bar{h}}{\bar{g}^{3/2}}+\bar{f}'+\frac{\bar{h}'}{2\bar{g}^{1/2}}\right]\\ &\quad -\xi\frac{t\dot{\delta}}{\delta}\left[\bar{g}'\left(2\bar{f}+\frac{\bar{h}}{4\bar{g}^{1/2}}\right)+2\bar{f}'\bar{g}+\frac{\bar{h}'}{4}\bar{g}\right]\end{aligned}$$

$$0=(a+b)\bar{h}+\frac{t}{\delta\bar{\Gamma}^2}\left[-\bar{g}'\frac{\bar{h}}{\bar{g}^2}+\frac{\bar{h}'}{\bar{g}}\right]-\xi\frac{t\dot{\delta}}{\delta}\bar{h}'$$

Fluid elements at late times



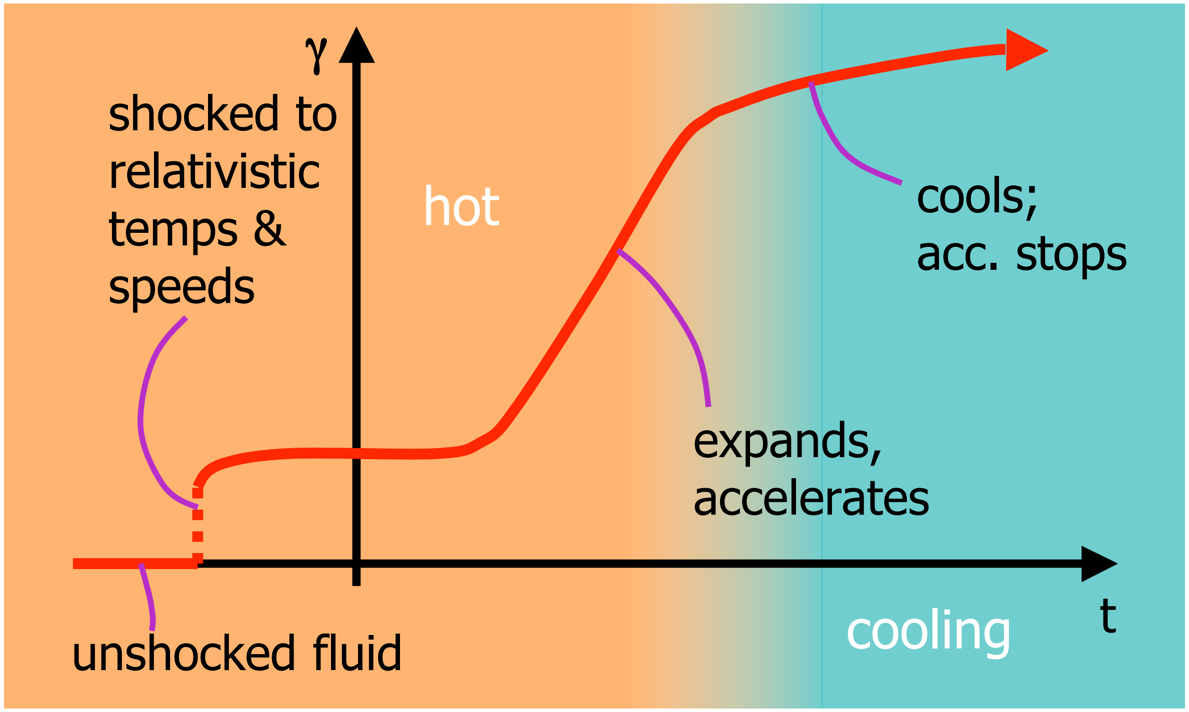
- 2 solutions together give

  $\gamma_{final}=1.95\gamma_0^{1+\sqrt{3}}$, k=-3

- Tan, Matzner, & McKee: $\gamma_{final}=2.6\gamma_0^{1+\sqrt{3}}$

Summary

- 2 self-similar solutions for post-breakout flow which time evolve with different power laws



- Together give accurate expression for final Lorentz factors of fluid elements