From Red Cells to Skiing to a New Concept for a Train Track

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Collaborators

Red Cells

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Mechano-transduction

Xiaobing Zhang, City College of New York

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Steve Cowin, City College of New York

Mia Mia Thi, City College of New York

David C. Spray, Albert Einstein Medical College

Skiing and Train track

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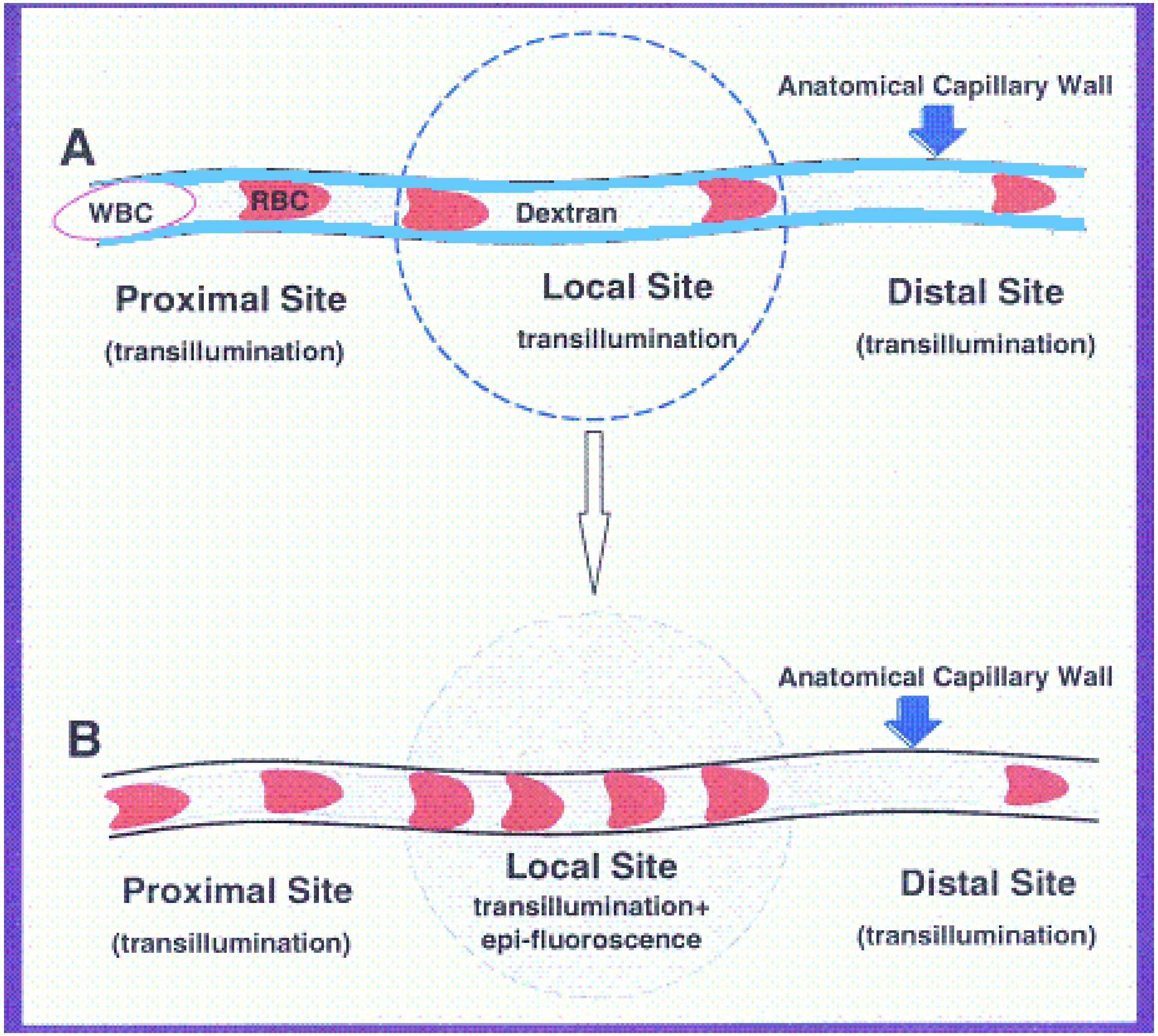
Yiannis Andreopoulos, City College of New York

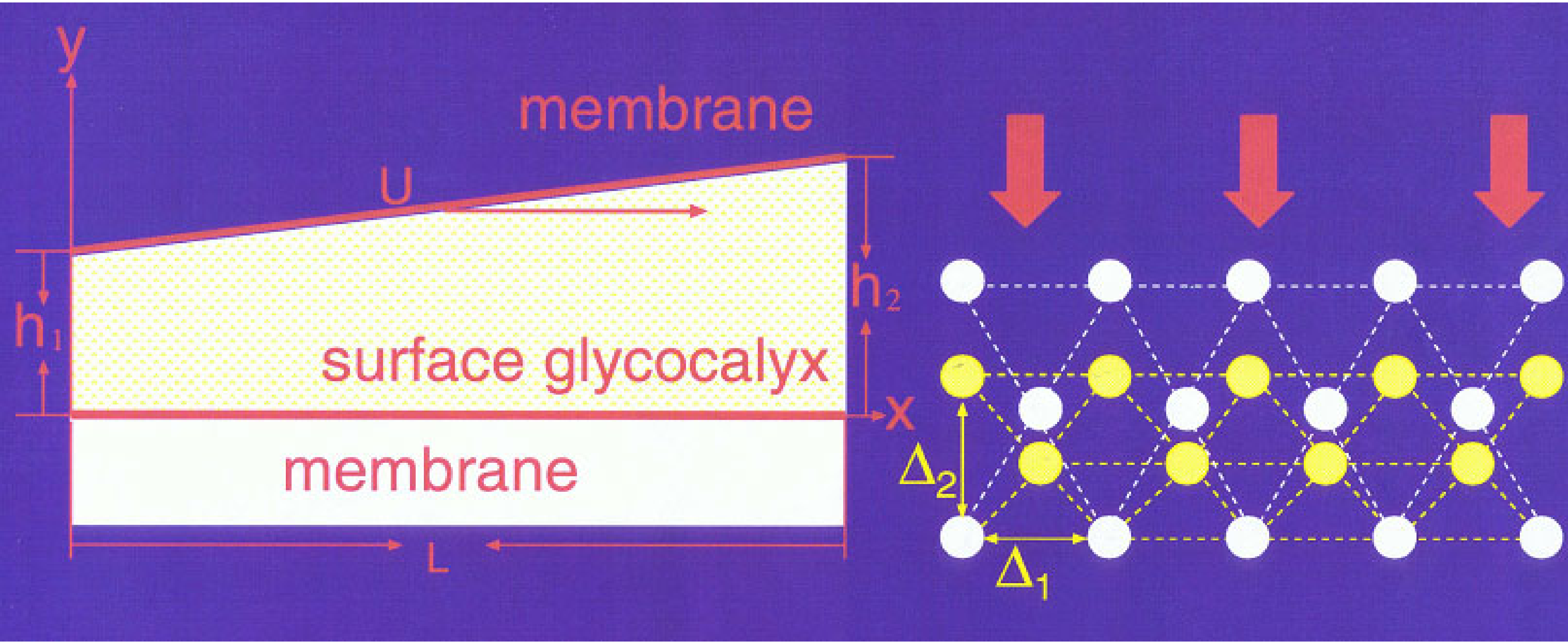
Red and White Cell Motion in Capillaries

Vink and Duling (1996)



Sliding Motion of a Membrane Over a Thin Surface Glycocalyx

Feng and Weinbaum, JFM 422: 281 (2000)



h_2 is fixed in the model.

h_1 changes. $k=h_2/h_1$

$$\alpha = h_2/\sqrt{K_p}$$

Two-Dimensional Lubrication Theory for the Brinkman Medium

Brinkman equation:

$$\nabla p = \mu\left[\nabla^2 - \frac{1}{K_p}\right]V$$

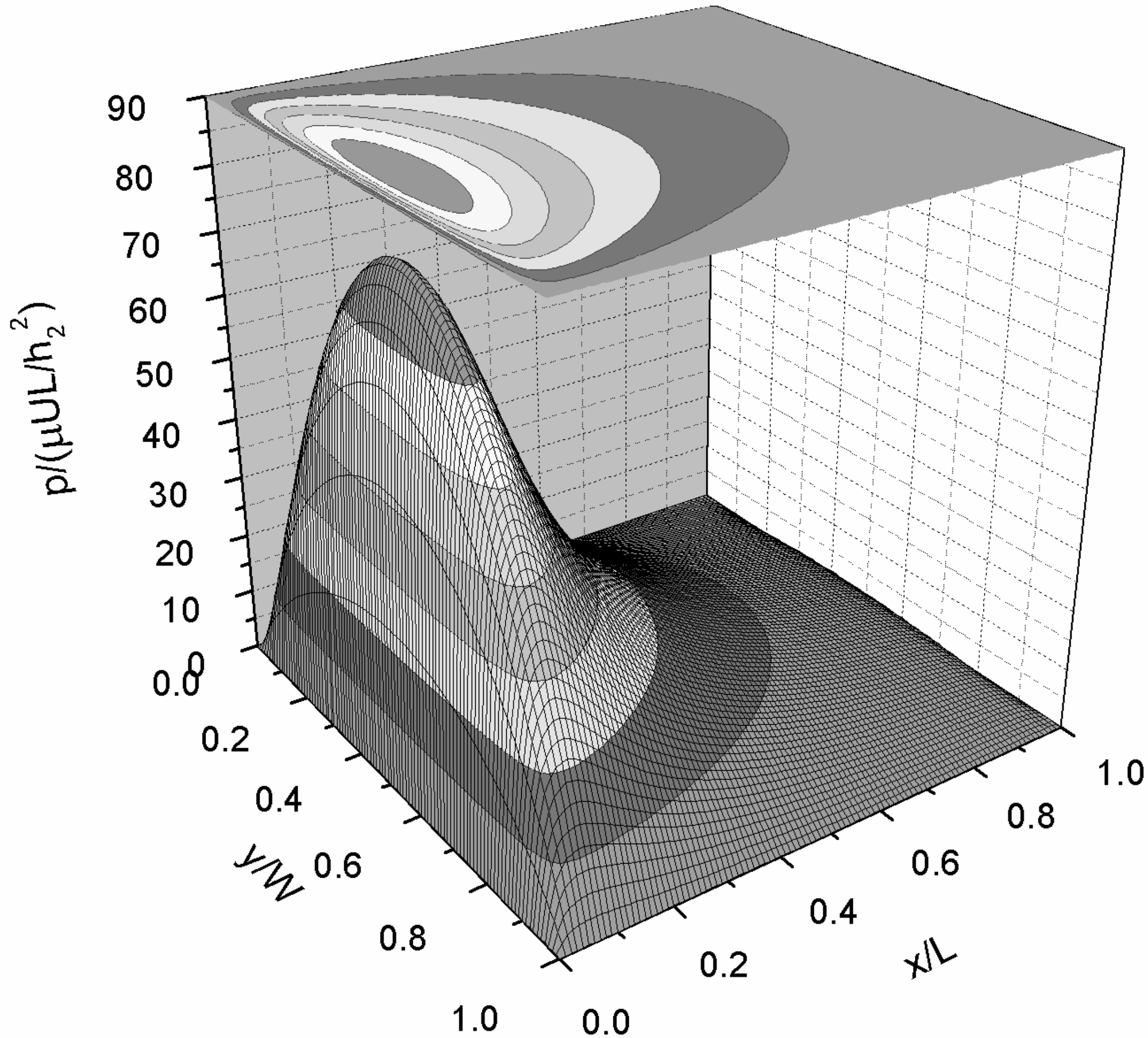
Dimensionless Reynolds-Type Equation:

$$\frac{\partial}{\partial x}\left[fU + \frac{1}{\alpha^2}\frac{\partial p}{\partial x}(2f - h)\right] + \frac{L^2}{W^2}\frac{\partial}{\partial y}\left[fU + \frac{1}{\alpha^2}\frac{\partial p}{\partial y}(2f - h)\right]$$

$$= U\frac{\partial h}{\partial x} + V\frac{\partial h}{\partial y} - W,$$

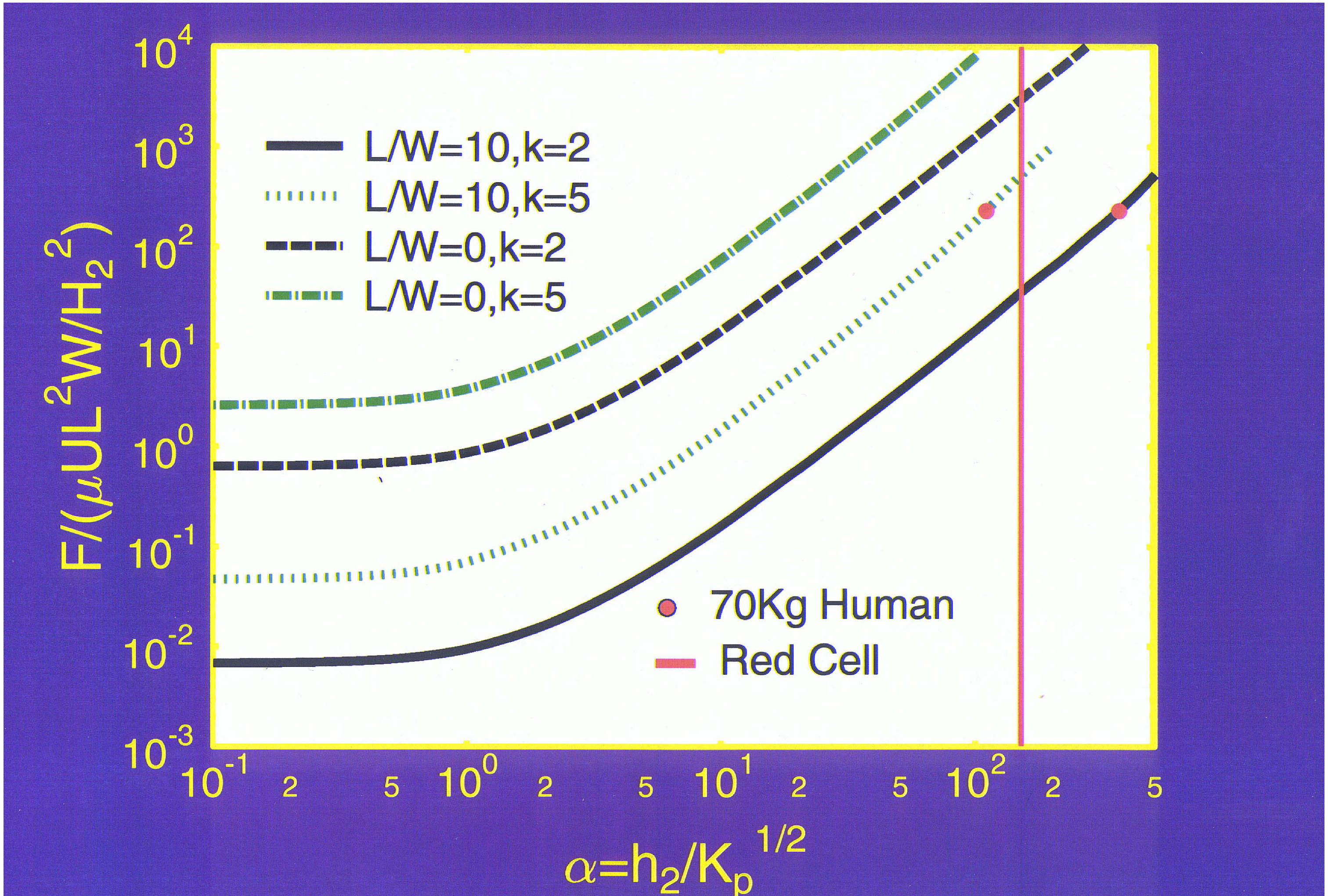
$$f = \frac{\cosh \alpha h - 1}{\alpha \sinh \alpha h}, \qquad \alpha = \frac{h_2}{\sqrt{K_p}}$$

Pressure Distribution and Equal Pressure Contours Under a Snowboard ($L/W = 10$, h_2 = 2cm, $\alpha(h_2) = 100$)

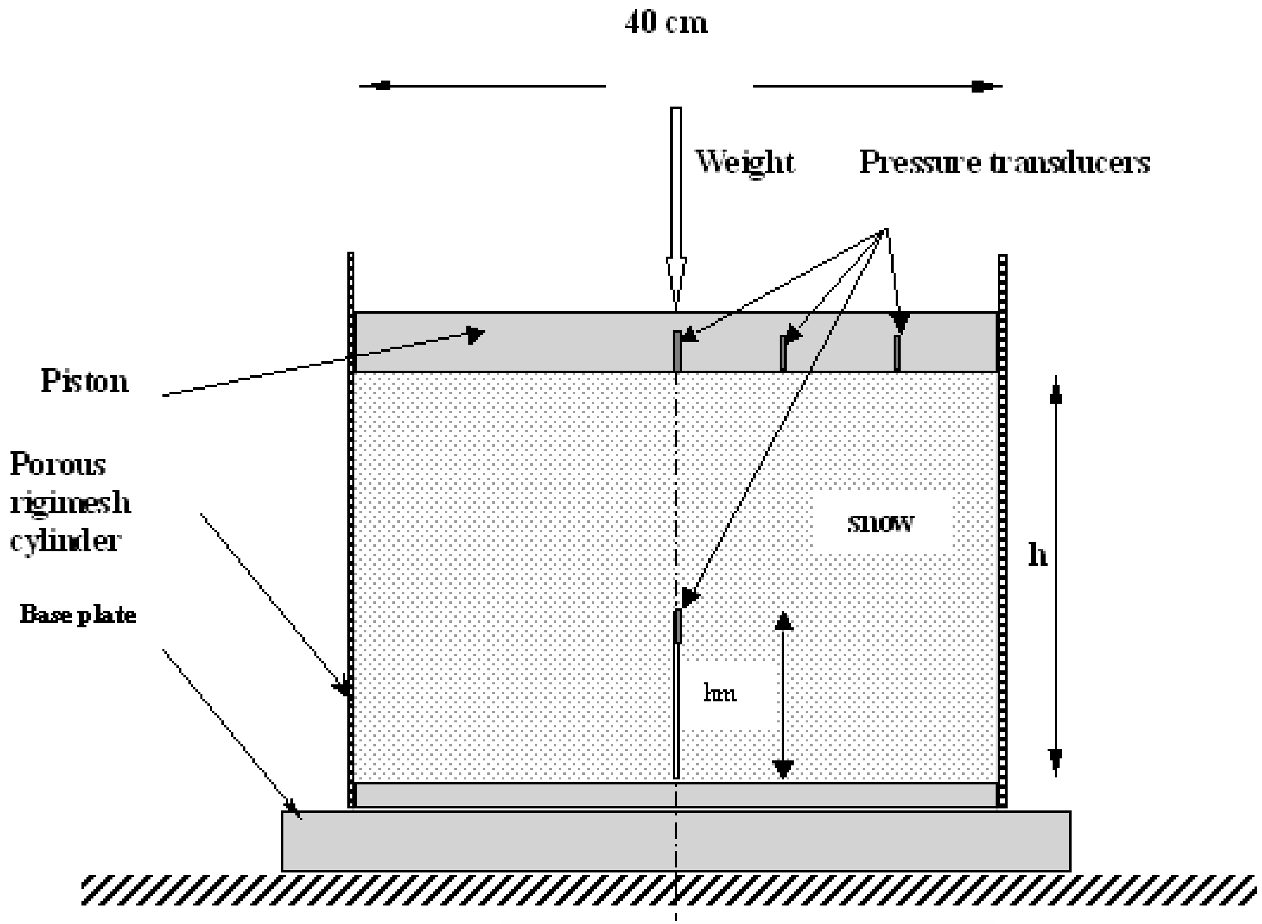


Veinbaum, 2003

Comparison of a Red Cell and SnowBoard

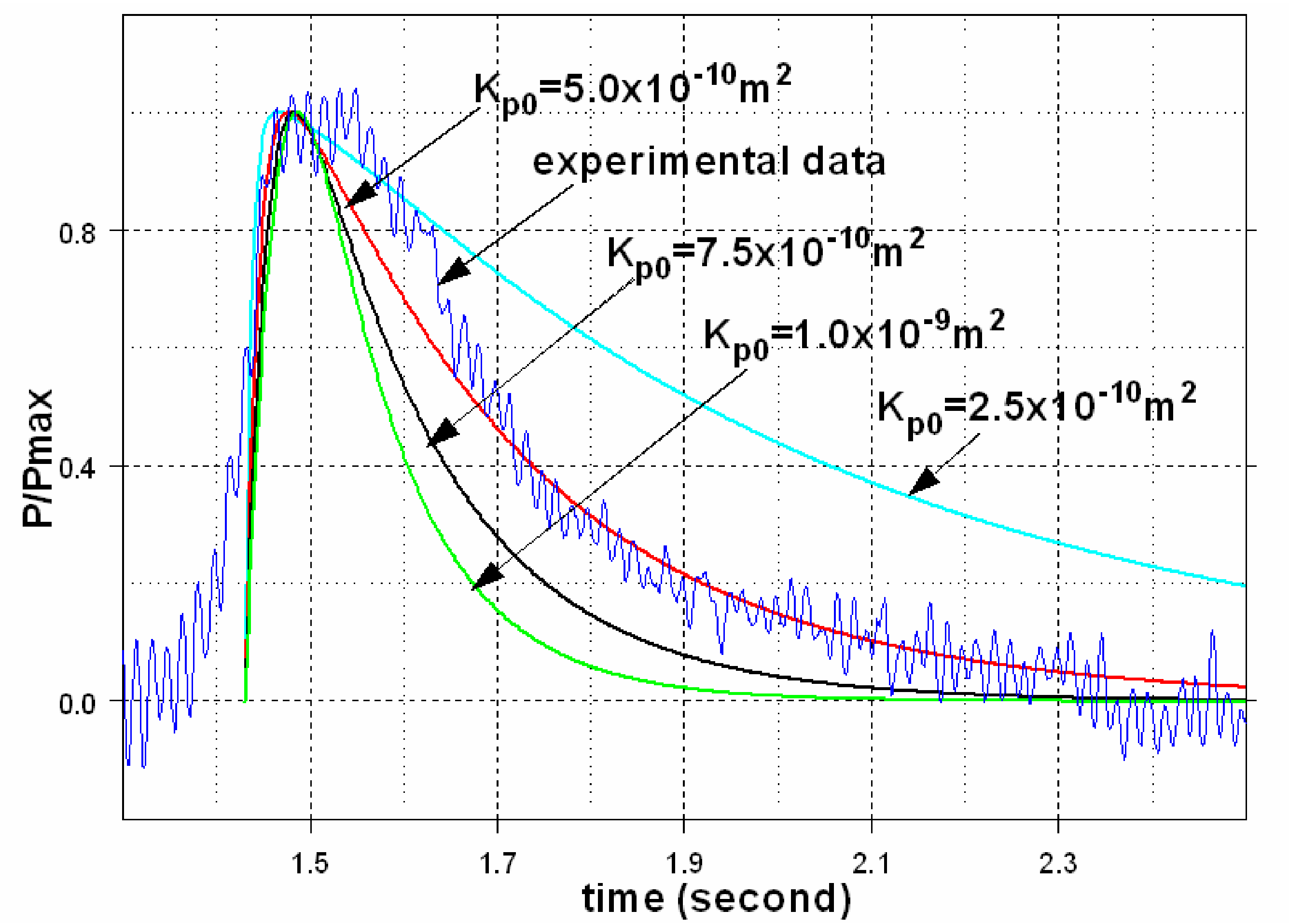


Schematic of Dynamic Snow Compression Apparatus



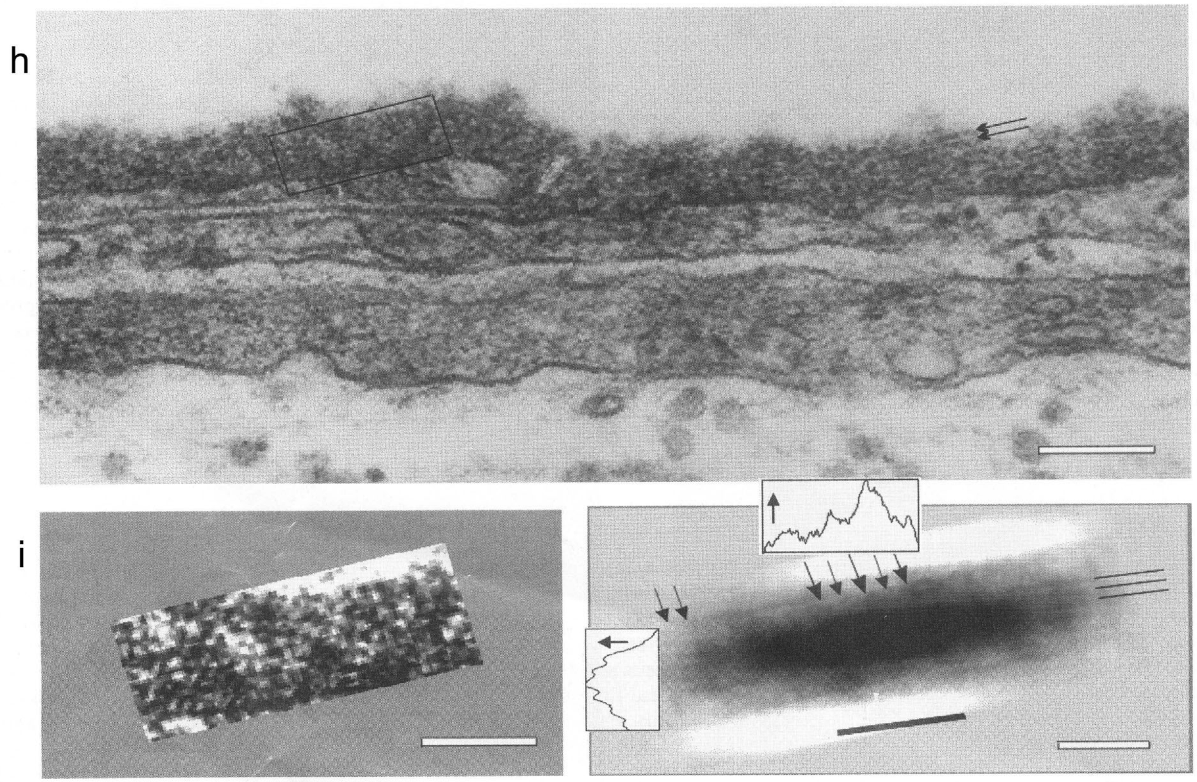
$$m\frac{d^2h}{dt^2} = -mg + F_{air} + F_{snow}$$

Comparison Between Theoretical and Experimental Pressure Profiles



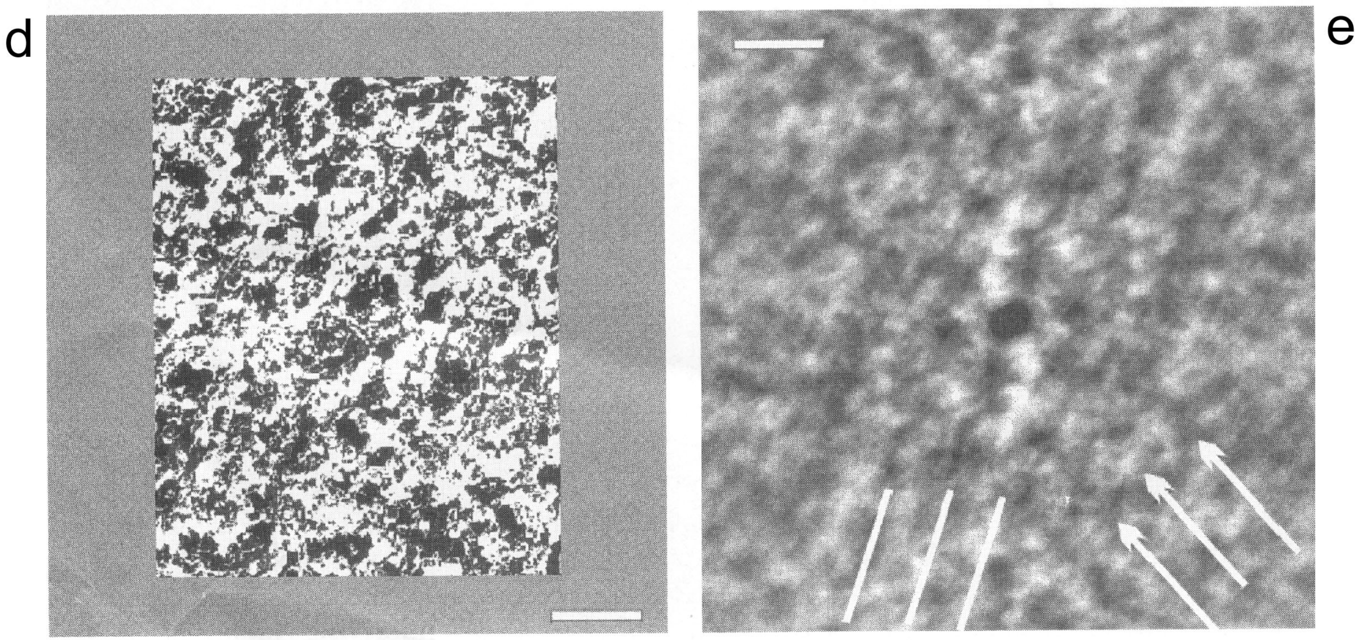
Periodic Structure of the Endothelial Glycocalyx

Squire, Chew, Nenji, Neal, Barry and Michel
J. Struct. Biol. 136, 239 (2001)



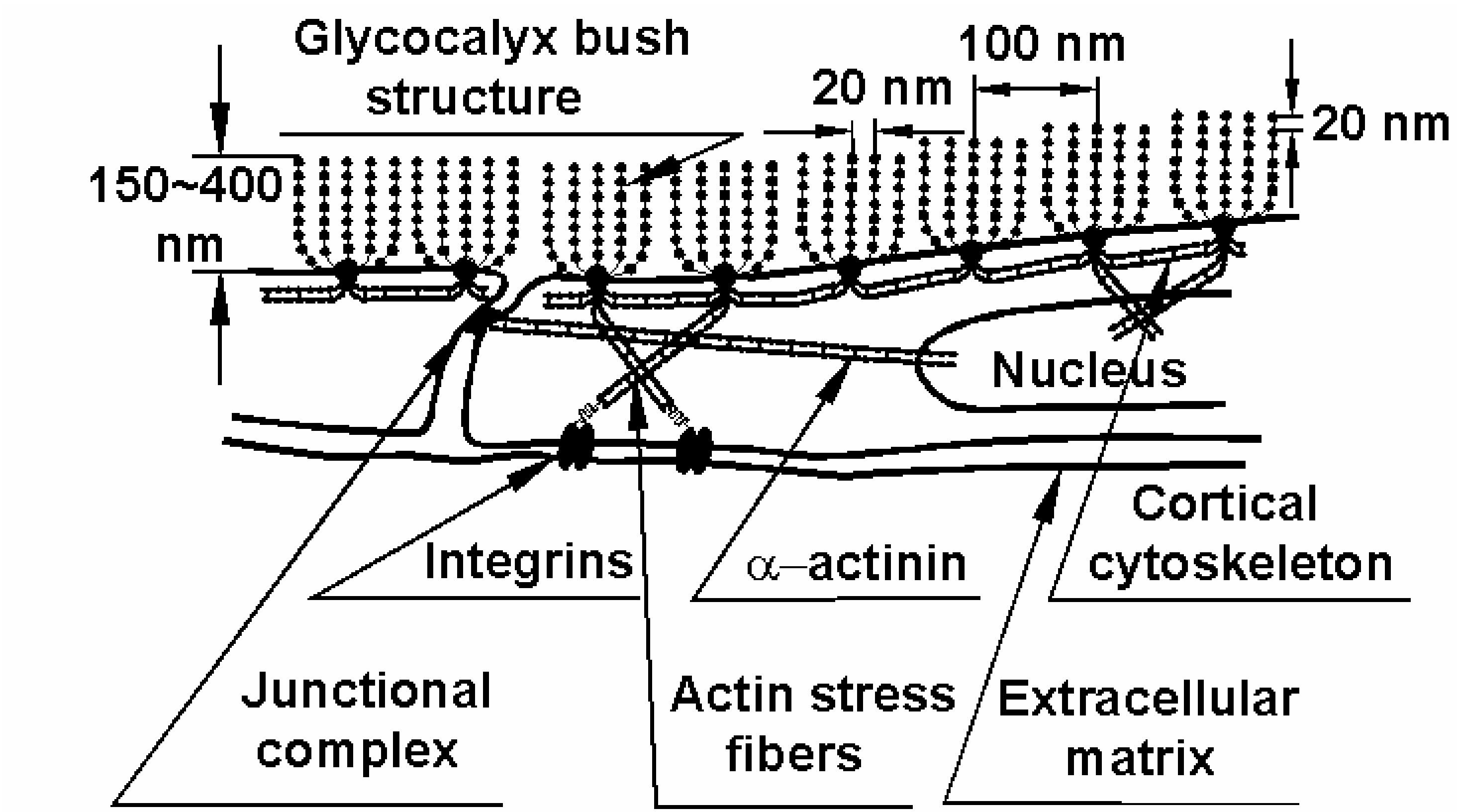
Hexagonal Array seen near Inner Surface Of Glycocalyx in Freeze-fracture

Squire, Chew, Nenji, Neal, Barry and Michel (2001)

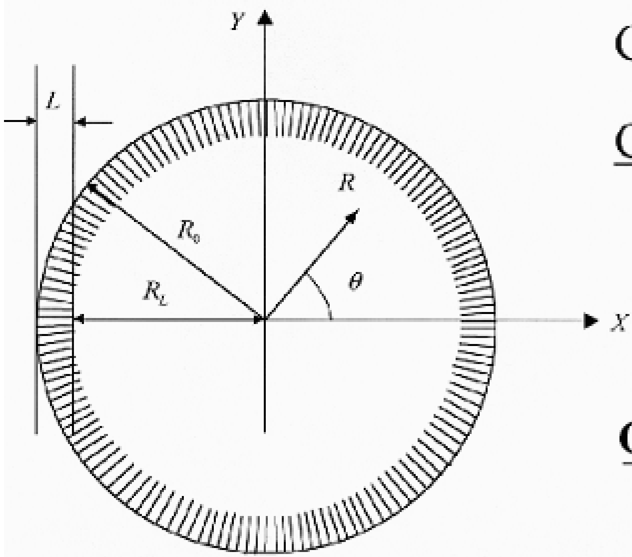


Model for Mechanotransduction

Weinbaum et al. Proc. Natl. Acad. Sci. 100, 7988-7996 (2003)



Model for Flow in Capillary



Cross-section of capillary

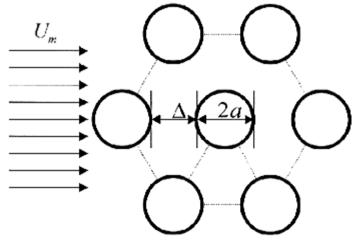
Governing Equations

Core – Navier-Stokes Equation

$$\frac{dP}{dZ} = \mu \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U_c}{\partial R}\right)$$

Glycocalyx-Brinkman Equation

$$\frac{dP}{dZ} = \mu \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial U_m}{\partial R}\right) - \frac{\mu}{K_p}U_m$$



Core protein array

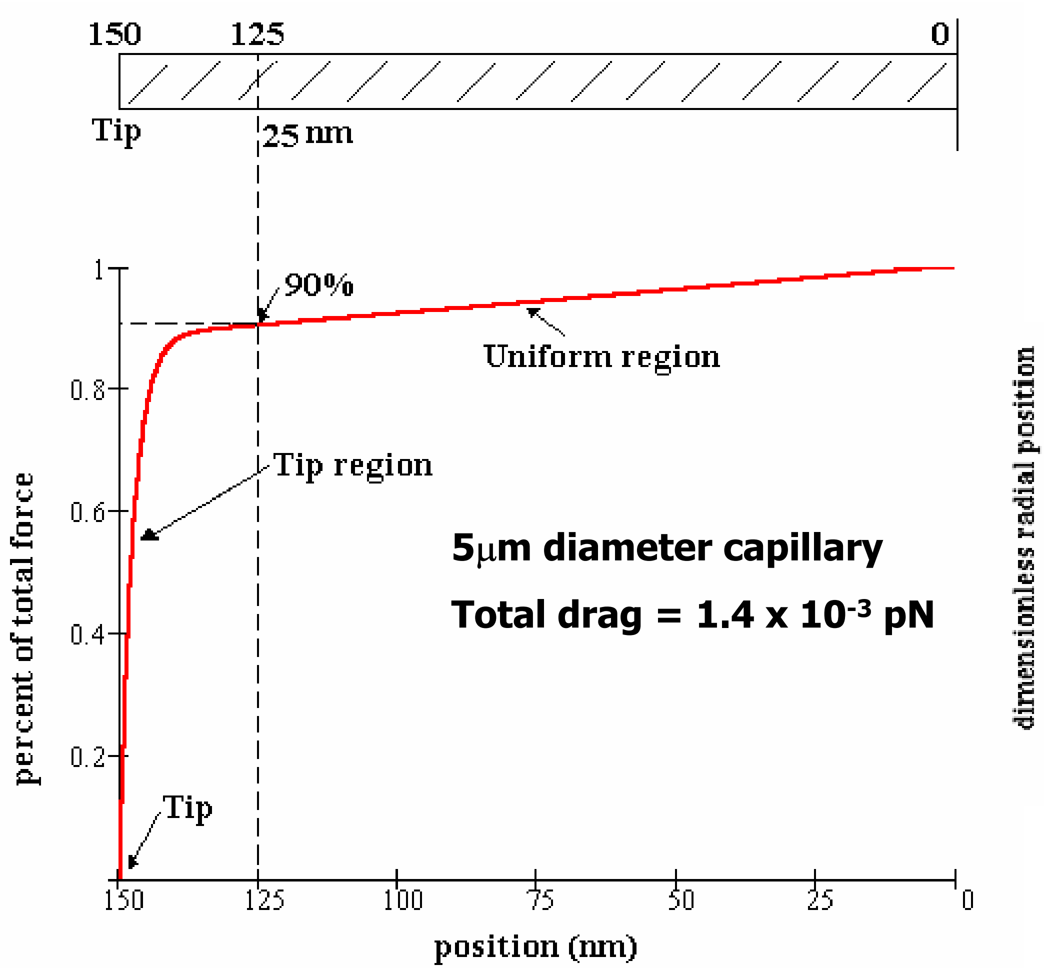
Sangani and Acrivos (1982)

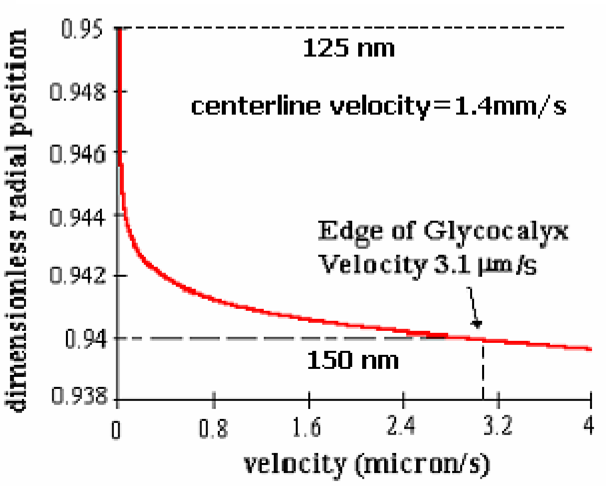
$$\frac{K_p}{a^2} = \frac{-\frac{1}{2}\ln(c) - 0.745 + c - \frac{1}{4}c^2 + O(c^4)}{4c}$$

c solid fraction

m, 2003

Drag Force Distribution on Each Core Protein





Deflection of Core Protein

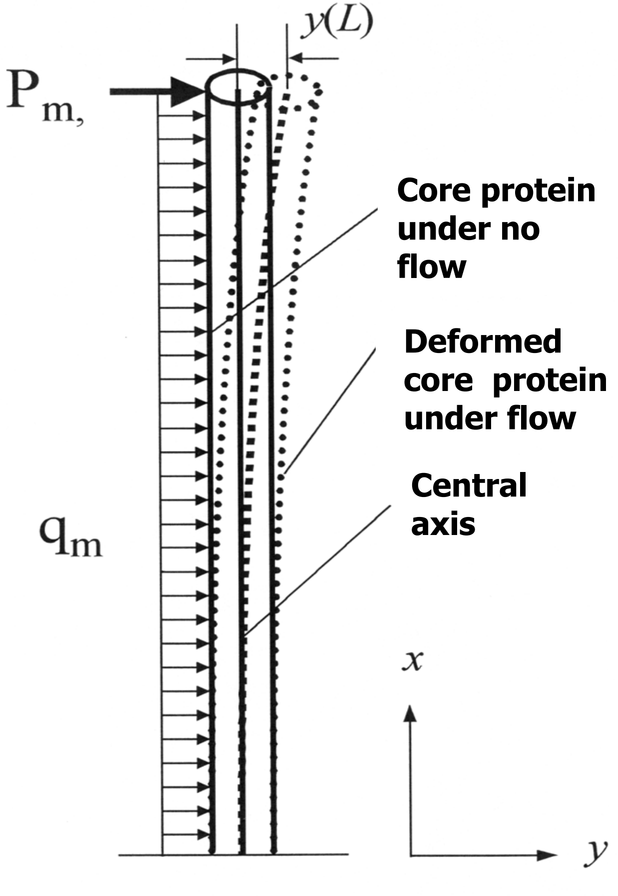


Diagram of loading on core protein

<u>Beam equation</u>

$$EI\frac{d^4y}{dx^4} = Q(x)$$

<u>Loading</u>

$$Q(x) = P_m\delta(x-L) + q_m(x)$$

P_m - Concentrated force on tip

q_m - Distributed force along core protein

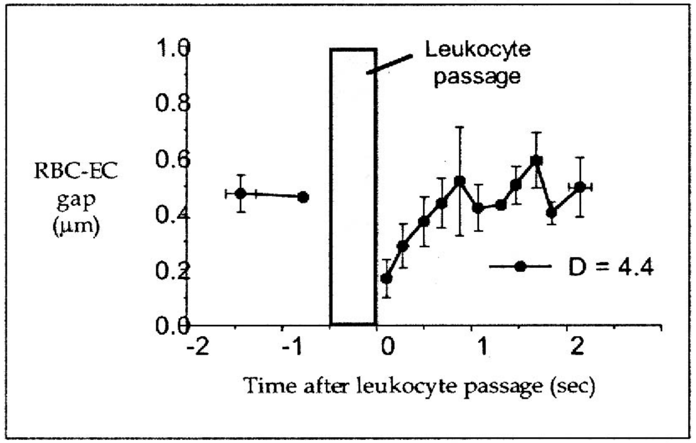
<u>Boundary conditions</u>

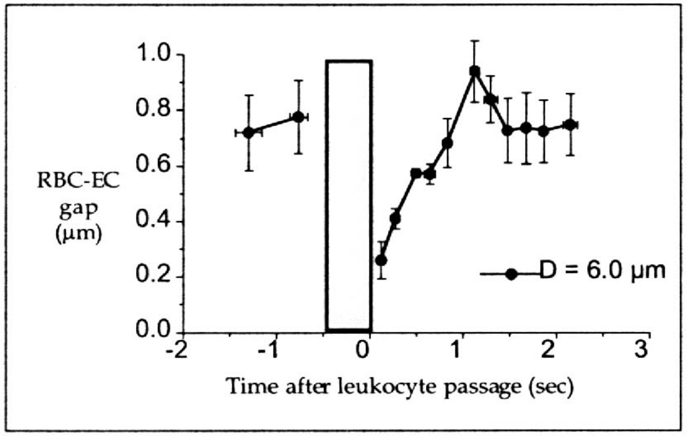
$$y(0) = 0; \qquad \left.\frac{dy}{dx}\right|_{x=0} = 0$$

$$EI\left.\frac{d^2y}{dx^2}\right|_{x=L} = 0; \quad EI\left.\frac{d^3y}{dx^3}\right|_{x=L} = P$$

Relaxation Of Endothelial Surface Layer

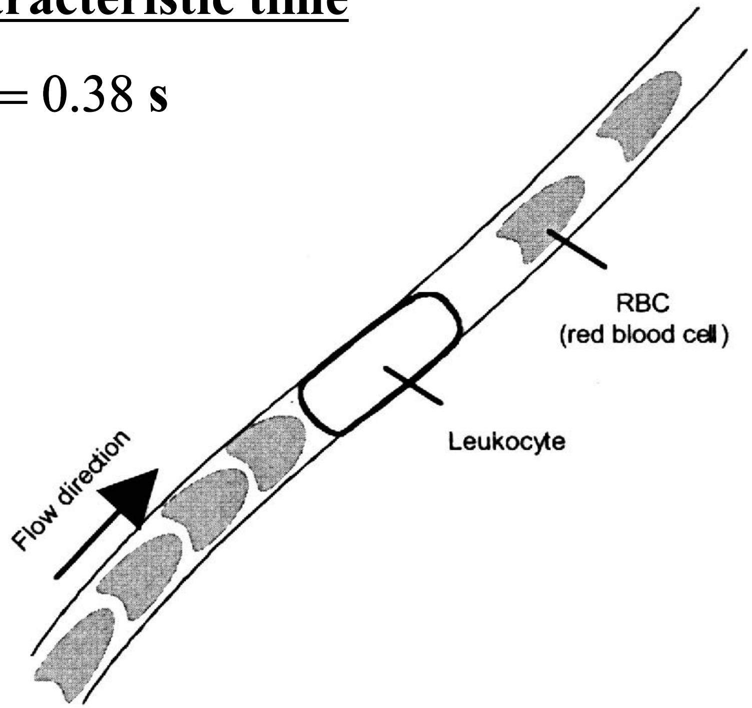
Vink, Duling and Spaan (2001)





Characteristic time

$T_2^* = 0.38$ s



Flexural Rigidity of Core Protein

Novel Beam Equation:

$$EI \cdot \frac{\partial^4 y}{\partial x^4} = -\frac{\pi}{c} \cdot \frac{\mu a^2}{K_p} \cdot \frac{\partial y}{\partial t}$$

c--solid fraction

Characteristic Times:

$$T_1^* = 0.0044 \cdot \frac{\pi}{c} \cdot \frac{\mu a^2}{K_p} \cdot \frac{L^4}{EI}$$ (short time)

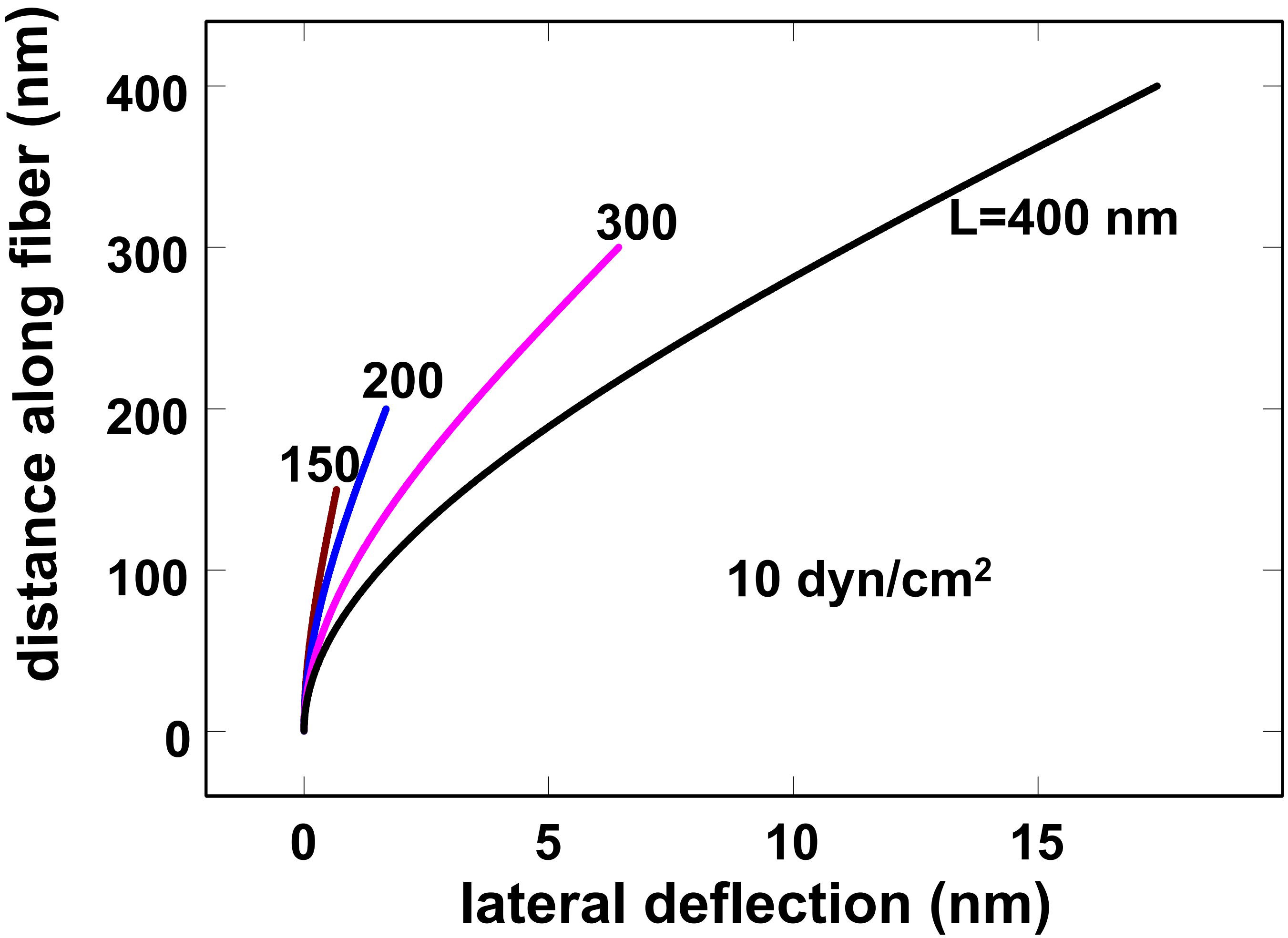
$$T_2^* = 0.0789 \cdot \frac{\pi}{c} \cdot \frac{\mu a^2}{K_p} \cdot \frac{L^4}{EI}$$ (long time)

Two time constants found by series solution to beam equation

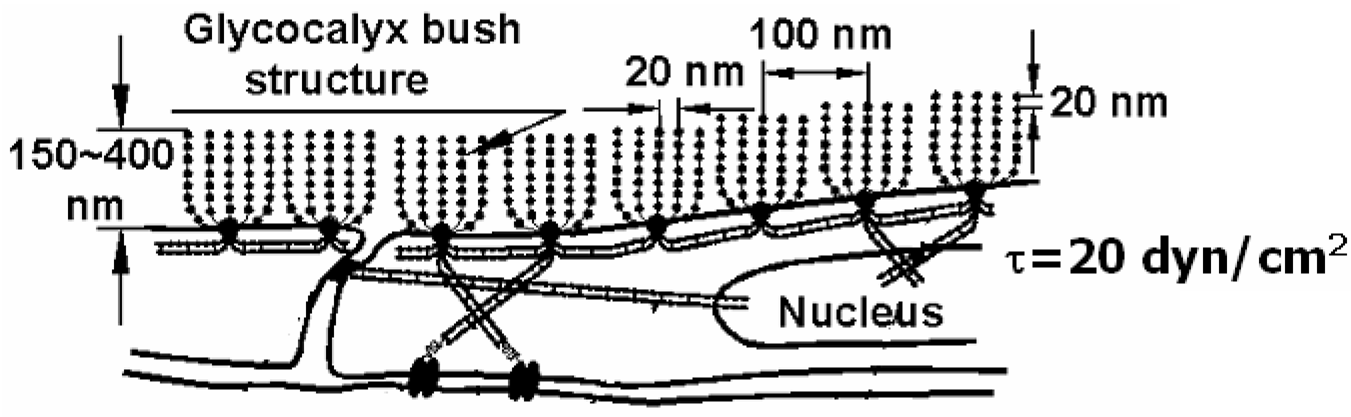
Predicted *EI* Vink's Experiment: $EI = 700\ pN \cdot nm^2$

Measured *EI*: $EI = 17 \times 10^3\ pN \cdot nm^2$ actin (Satcher and Dewey, 1996)

Deflection of Core Protein



Force Amplification



Optical trap: **0.1~0.5 pN (transform receptor)**
Drag core protein: **1.4×10^{-3} pN**
Drag 27 fiber bush: **3.8×10^{-2}pN**
Vertical shear force actin filament: 0.09 pN

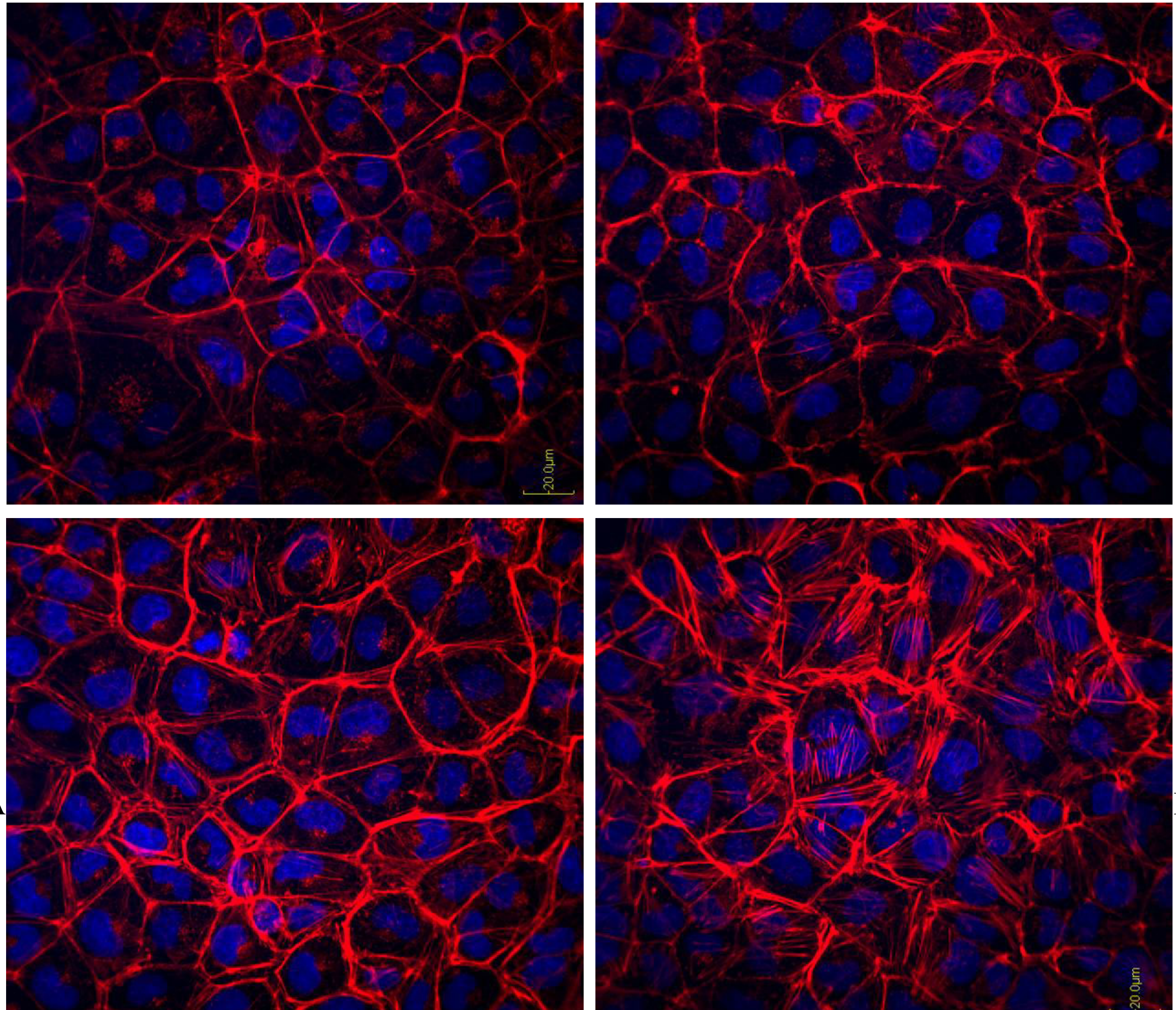
Results: Uniform Laminar flow region

Thi, Weinbaum and Spray (2003)

F-actin

Control DMEM

τ = 10dyn/cm^2 for 5 h with DMEM,

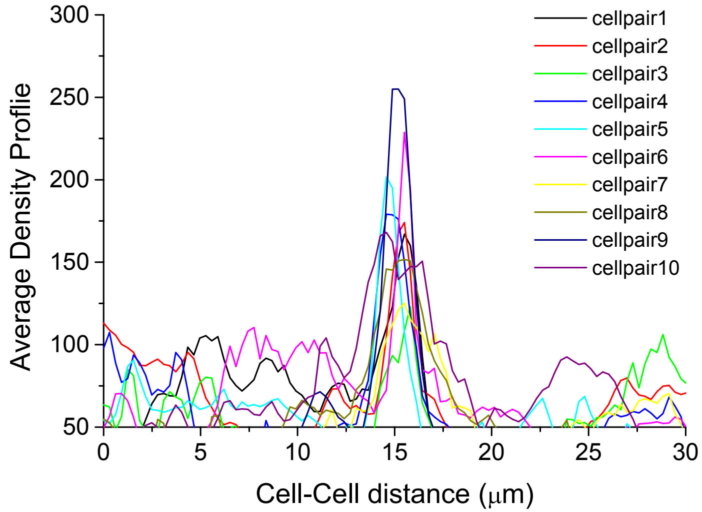


τ = 10dyn/cm^2 for 5 h with DMEM+1% BSA

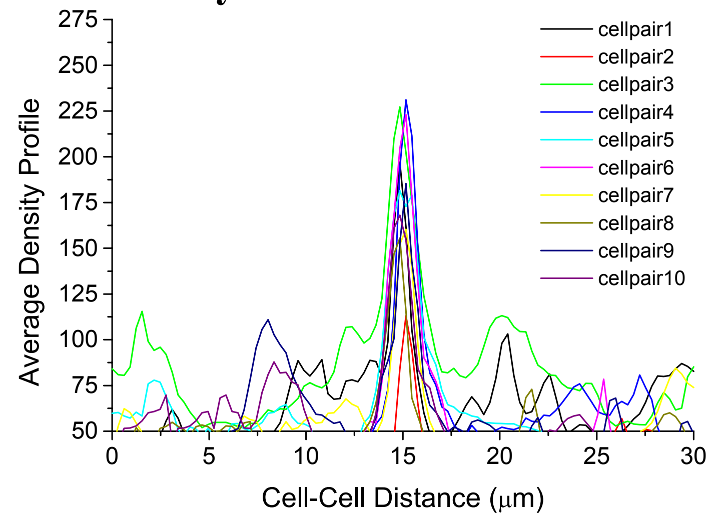
τ = 10dyn/cm^2 for 5 h with DMEM+10% FBS

F-actin is redistributed (more stress fibers throughout cell)

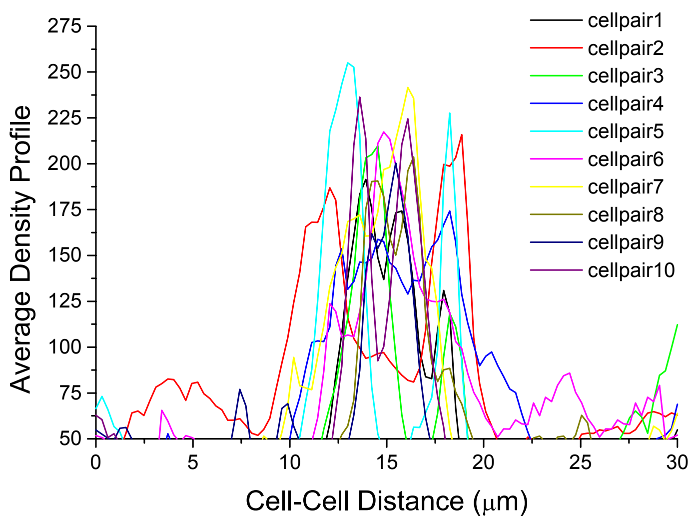
Control (no flow)



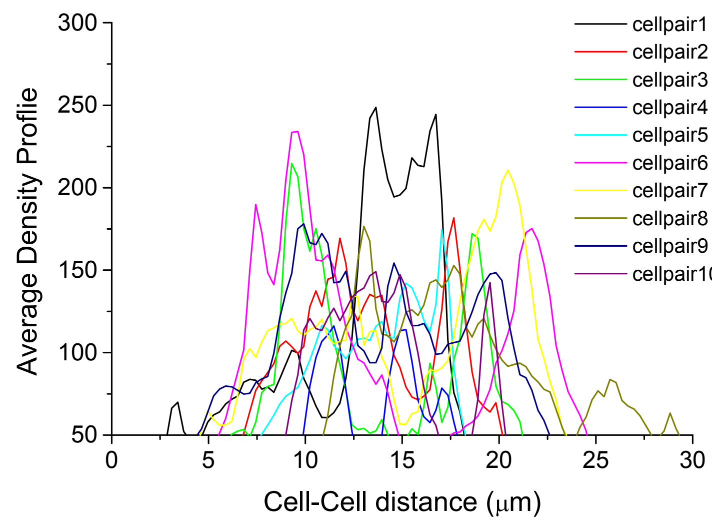
τ = 10dyn/cm² for 5 h with DMEM



τ = 10dyn/cm² for 5 h with DMEM + 1% BSA



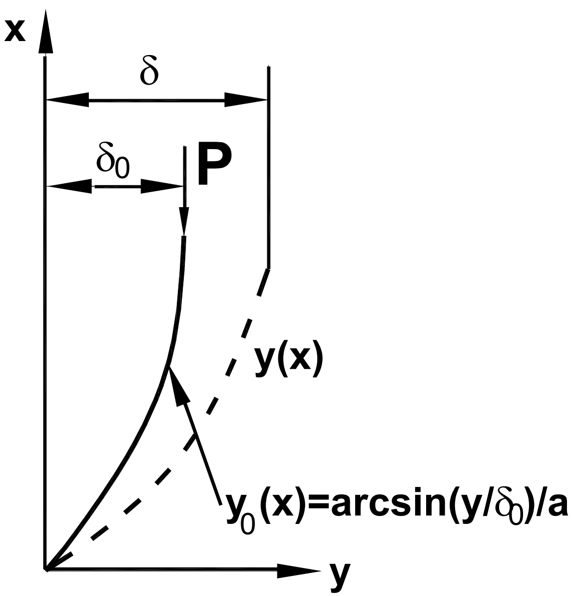
τ = 10dyn/cm² for 5 h with DMEM + 10% FBS

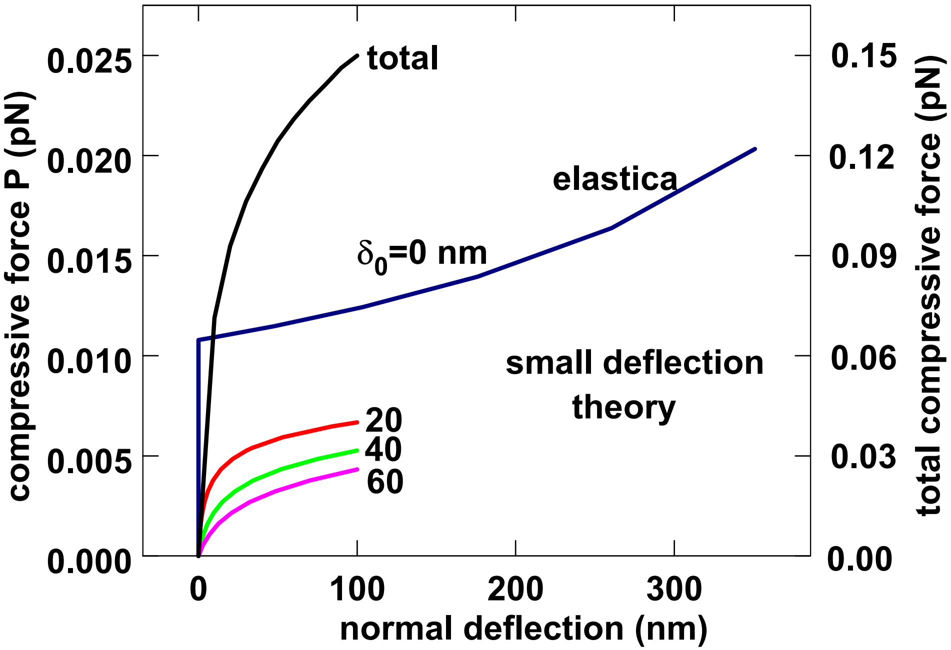


Buckling of Initially Curved Beam

Revised Beam Equation

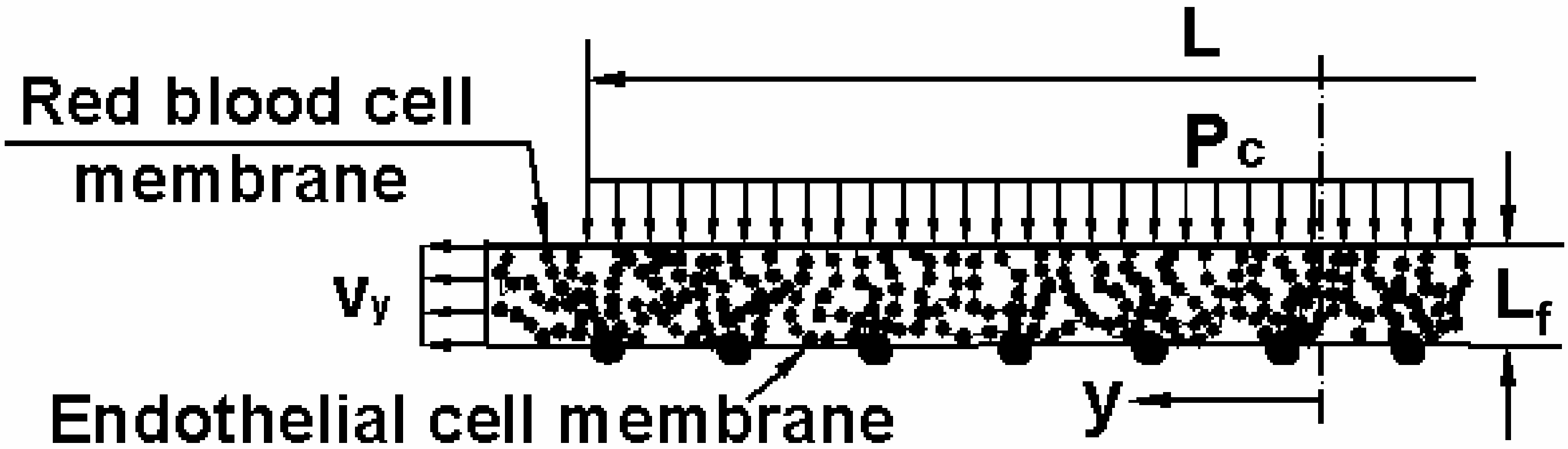
$$EI \cdot \frac{d^2}{dx^2}(y - y_0) = P(\delta - y)$$





$P_f = 160 dyn/cm^2$

ESL Drainage Due to RBC Arrest



Drainage Time

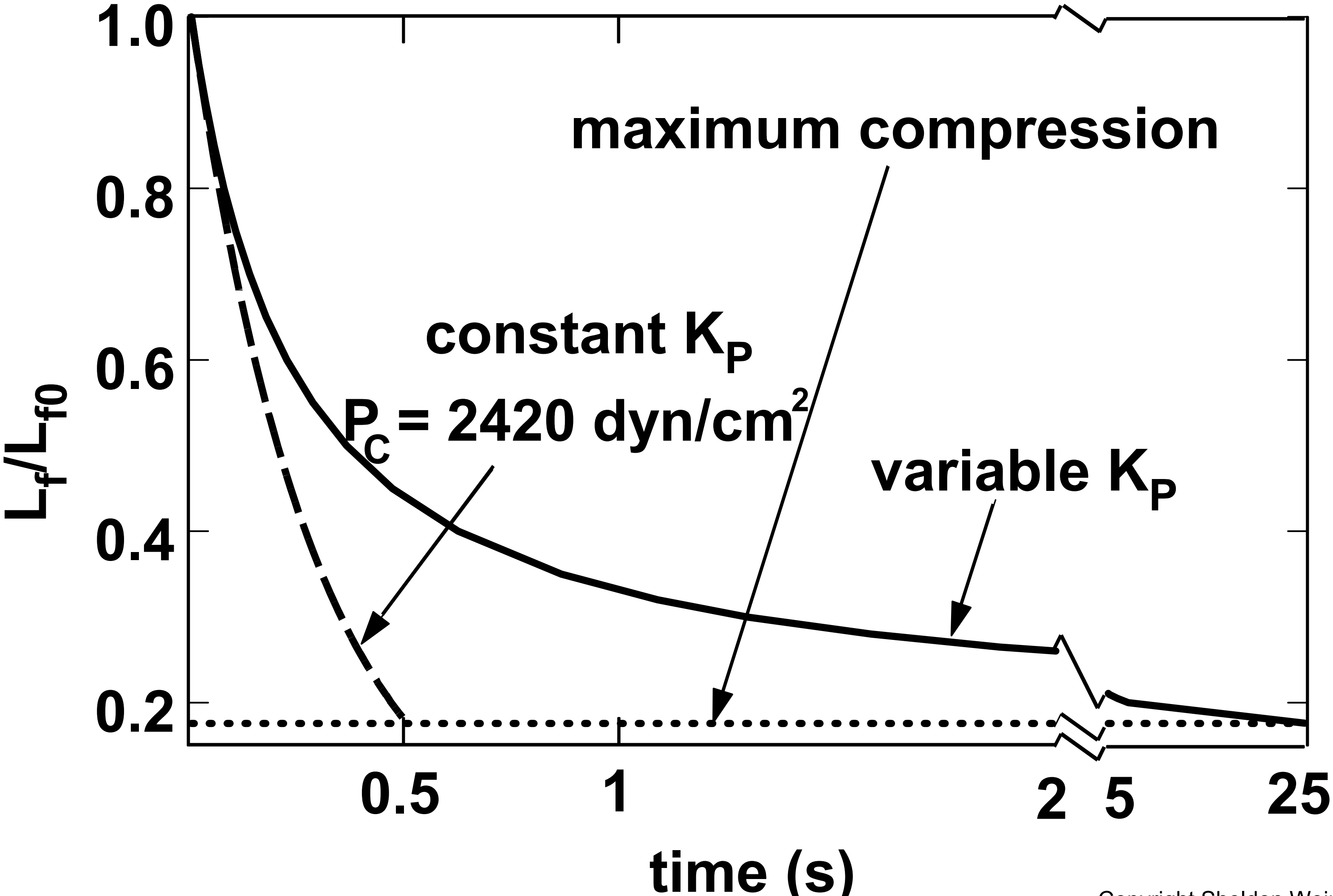
$$t = \frac{\mu L^2}{12 P_c} \int_{L_{f0}}^{L_f} \frac{-dL_f}{K_p L_f}$$

Variable K

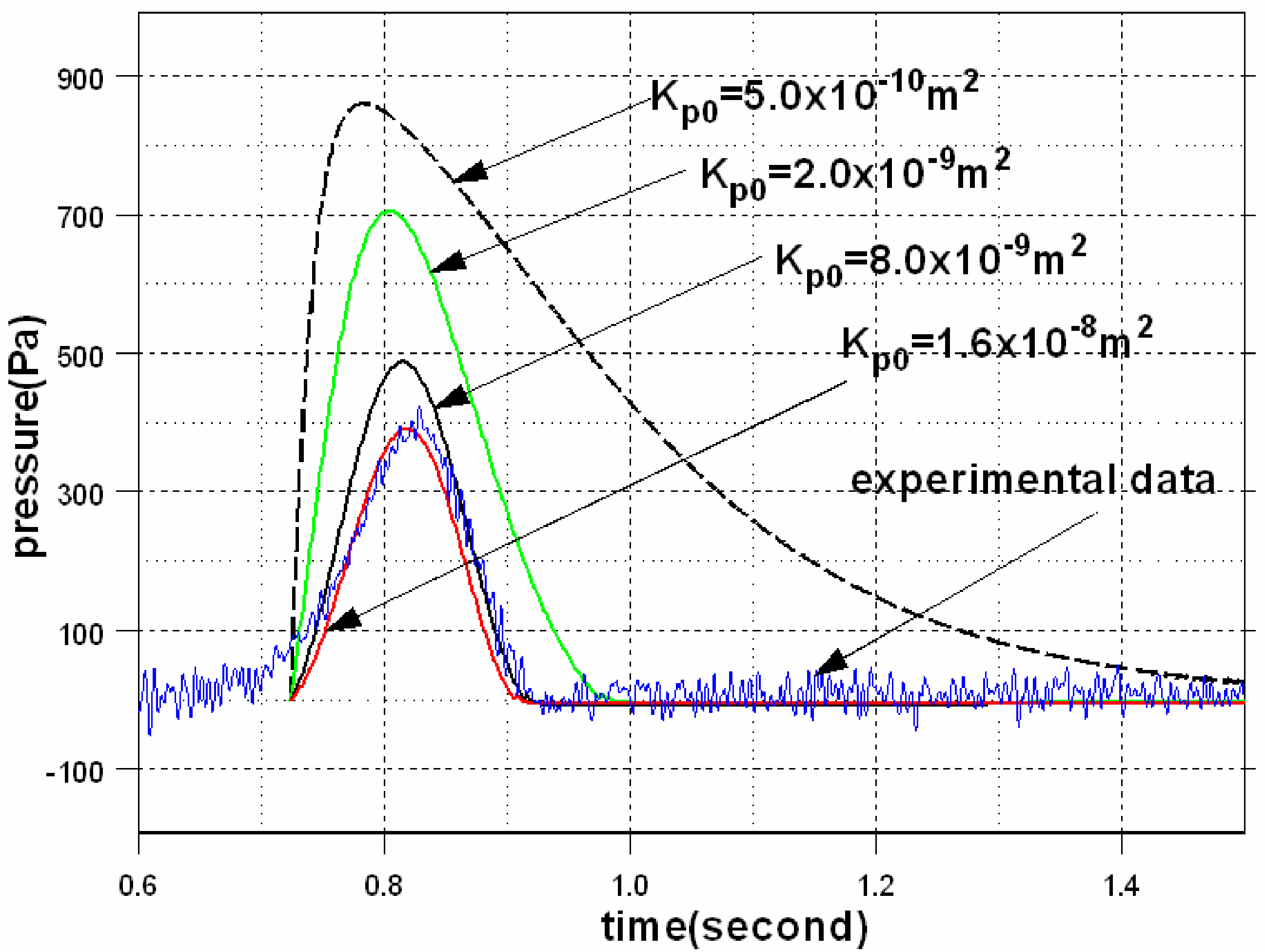
Sangani and Acrivos (1982)

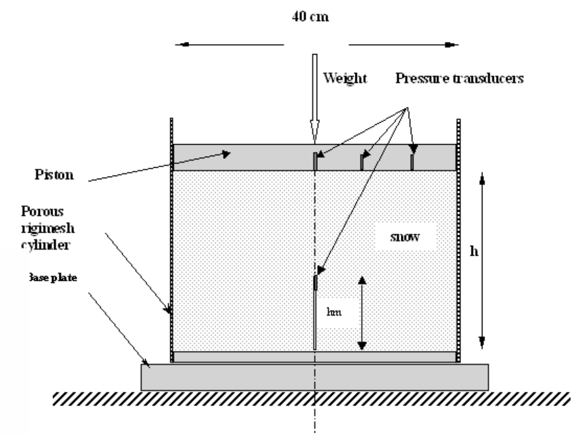
$$K_p = \frac{2}{9} \cdot \frac{r^2 L_f / c_0 L_{f0}}{\sum_{s=0}^{30} q_s \left[\left(c_0 L_{f0} / c_{max} L_f \right)^{1/3} \right]^s}$$

ESL Drainage



Dynamic Compression with Goose Down





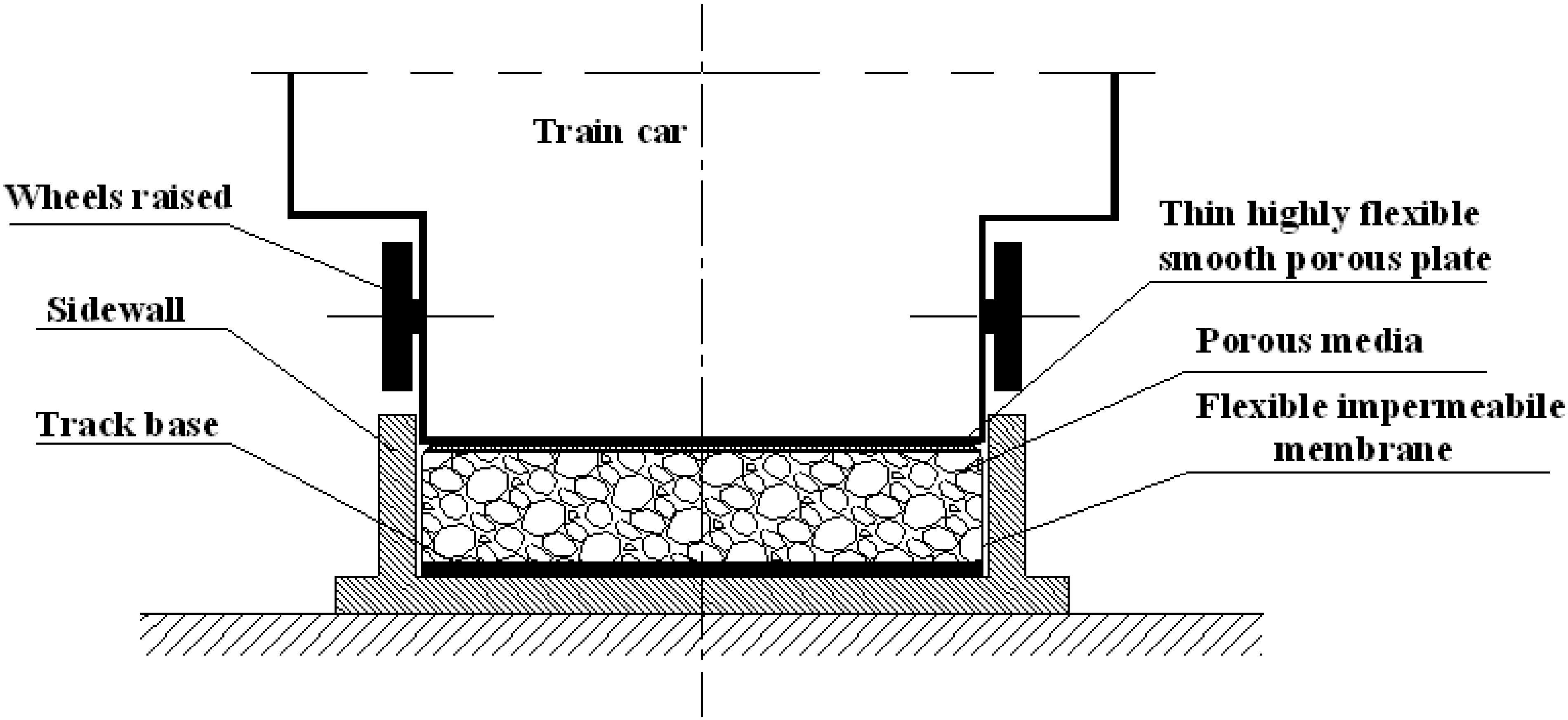
Feasibility of Supporting a Train Car

	DYNAMIC COMPRESSION WITH GOOSE DOWN (CASE 1)	ENHANCED LIFT TRAIN TRACK MODEL (L=25m, W =2m) (CASE 2)
Darcy permeability K_p	1.6×10^{-8} m^2	1.6×10^{-8} m^2
Characteristic time t_c	0.1s	3s(v=8.3m/s)
Characteristic length L_c	0.40m	25m
Pressure P_c	400Pa	P_{c2}

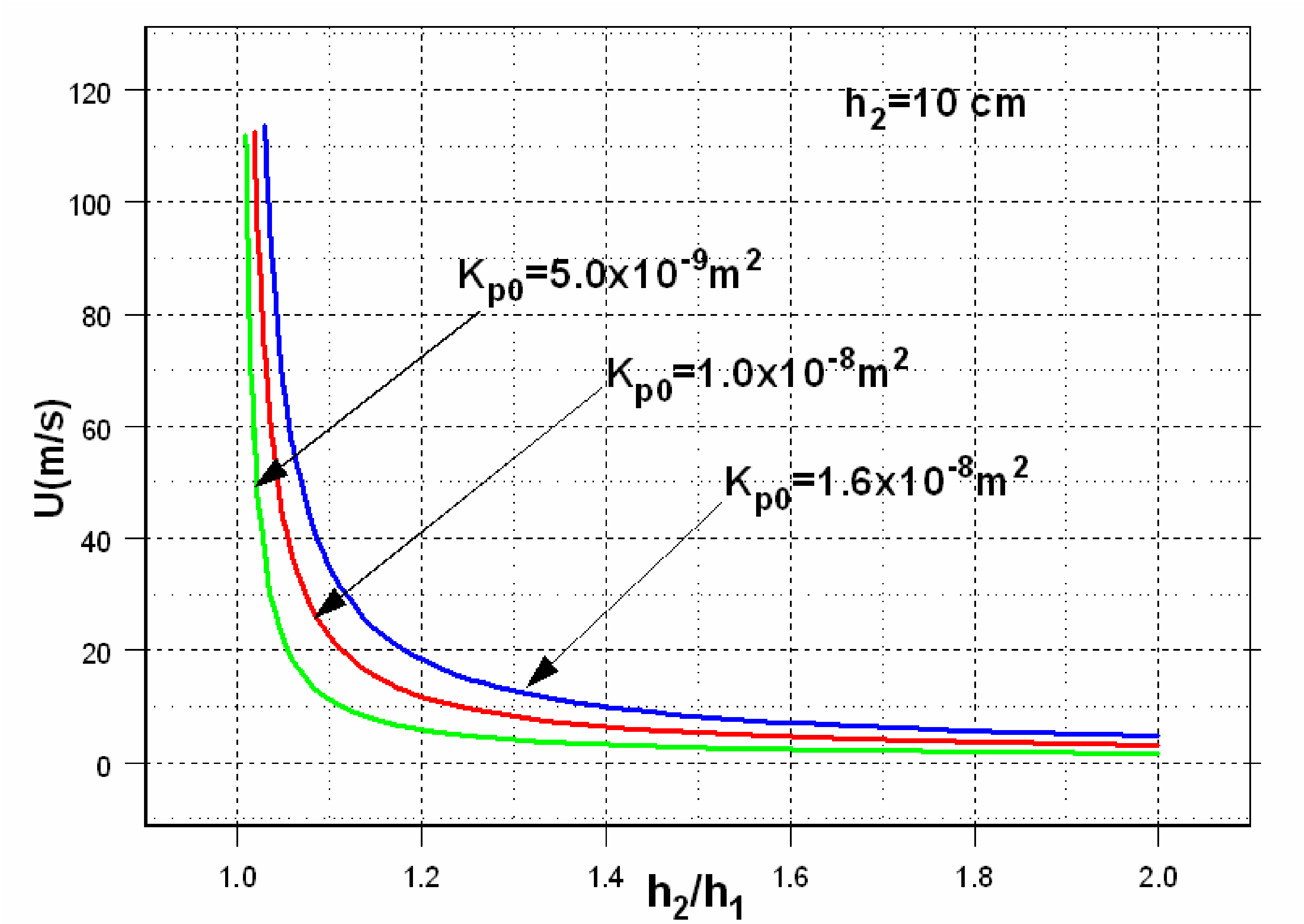
$$\frac{P_{c2}}{P_{c1}} = \left(\frac{L_2}{L_1}\right)^2 \frac{t_{c1}}{t_{c2}} \frac{K_{p1}}{K_{p2}} \Rightarrow$$

$P_{c2}=5.2\times10^4$Pa   Lift force = 260 tons

Sketch of the New Train Model in Transverse Plane



Performance of the Enhanced Lift 50 Ton Train Car



ɘinbaum, 2003

Conclusions

- **T**here is a remarkable dynamic similarity between a red cell gliding on the endothelial glycocalyx and a human skiing though they differ in size by $O(10^{15})$.
- **F**or a given planform without lateral leakage lift increases as the square of the Brinkman permeability parameter $\alpha = h/K_p^{1/2}$
- **F**or two-dimensional planforms with lateral leakage the lift decreases as $(W/L)^2$.
- **T**he endothelial glycocalyx is an extraordinary structure whose fibers are stiff enough to transmit fluid shear stress to the actin cytoskeleton in initiating intracellular signaling. However, they would easily buckle during red cell arrest were it not for the fluid draining pressure which carries most of the normal load.
- **T**he small elastic restoring force of the fibers allows for a huge reduction in the sliding friction due to the solid phase.
- **A** highly compressible track with the mechanical properties of goose down is capable of supporting a 50 ton train car traveling at even relatively low speeds with minimal sliding friction. At high speeds there would be little deformation.