Mixing in small scale flows

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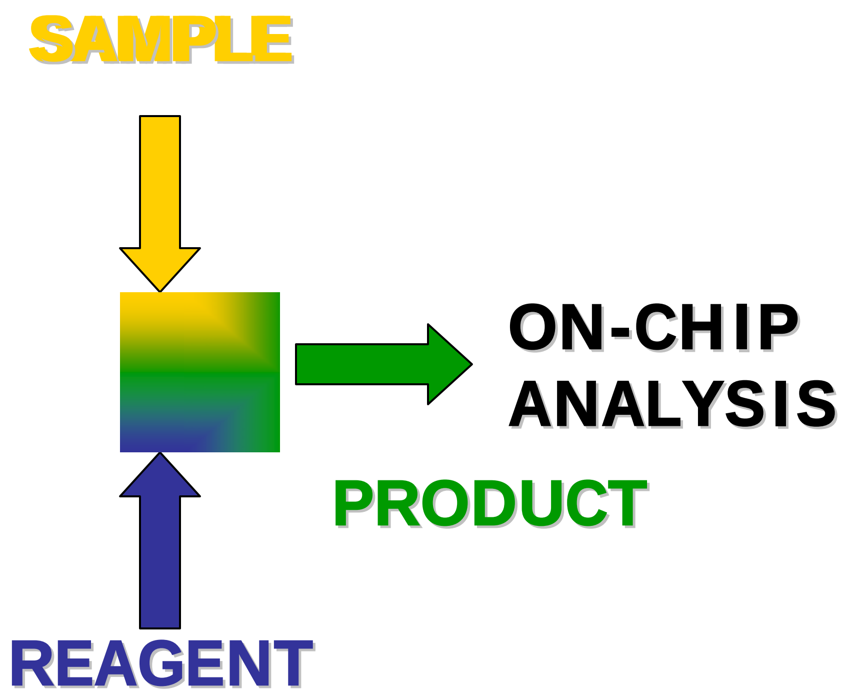
Microscale Mixing

Mixing Applications

- Microlaboratories require fast mixing – Crucial step
- Micro-channels: Small Reynolds numbers Re $\Rightarrow$ No turbulence
- Diffusion: Primary mixing mechanism in straight, smooth channel
- Diffusion of macromolecules such as proteins, peptides is slow



More efficient micromixers needed

Diffusion versus convection

- Peclet Number

$$\left.\begin{aligned} &u \sim 0.1, 1\,\mathrm{cm/s} \\ &h \sim 10^{-3}, 10^{-2}\,\mathrm{cm} \\ &D_m \sim 10^{-8}, 10^{-5}\,\mathrm{cm^2/s} \end{aligned}\right\} \rightarrow \mathrm{Pe} = \frac{uh}{D_m} \sim 10 - 10^6$$

- Convective transport much faster than diffusive transport

- Mixing distance: grows linearly with Pe, which can be of order of meters for proteins, and mixing time takes tens of minutes/hours

Need

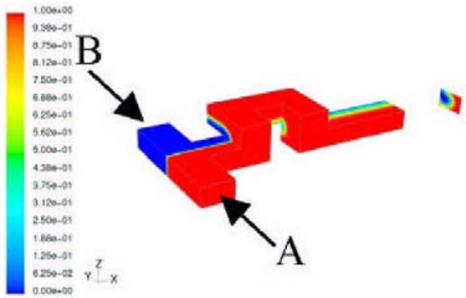
- Increase the interface between initially distinct fluid regions in order to decrease the distance over which diffusion acts to homogenize the fluid.

- Use stretches and folds of material lines typical of chaotic advection (Aref, 1984; Ottino, 1989, 1990): Interface between unmixed regions grows exponentially in time.

“Designing for chaos: Applications of chaotic advection at the microscale” Stremler, Haselton & Aref (2004)

Solutions at Small Scale

- Passive mixers (based on geometry)
  - grooved channels
  - multilamination techniques: splitting and rearranging either channels or flow paths
  - twisted channels



- Active mixers (using forcing)
  - ultrasonics
  - Electrokinetic
  - Electromagnetism
  - time pulsing of cross flows into a main channel

- **Channel geometry**
  - Liu, Stremler, Sharp, Olsen, Santiago, Adrian, Aref & Beebe, 2000 (twisted pipe)
  - Stroock, Dertinger, Whitesides & Adjari, 2002 (grooved channel, pressure driven); Johnson & Locascio, 2002 (grooved channel, EOF)
- **External fields**
  - Rife, Bell, Horowitz & Kabler, 2000 (ultrasonics)
  - Bau, Zhong and Yi, 2001; Yi, Qian & Bau, 2002 (magneto-hydrodynamics)
  - Selverov & Stone, 2001; Yi, Bau & Hu, 2002 (piezoelectric material generating TWs)
  - Oddy, Santiago & Mikkelson, 2001; Lin, Storey, Oddy, Chen, Santiago, 2004; Chen, Lin, Lele, Santiago, 2005 (electrokinetic instability)
- **Perpendicular channels**
  - Volpert, Meinhart, Mezic & Dahleh, 1999
  - Dasgupta, Surowiec & Berg, 2002
  - Tabeling, 2001; Tabeling, Chabert, Dodge, Julien & Okkles, 2004
- **Alternating pumps in T channel**
  - Desmukh, Liepmann & Pisano, 2000

Reviews: Stone, Stroock & Adjari, 2004; Ottino & Wiggins, 2004; Beebe, Mensing & Walker, 2002

This work

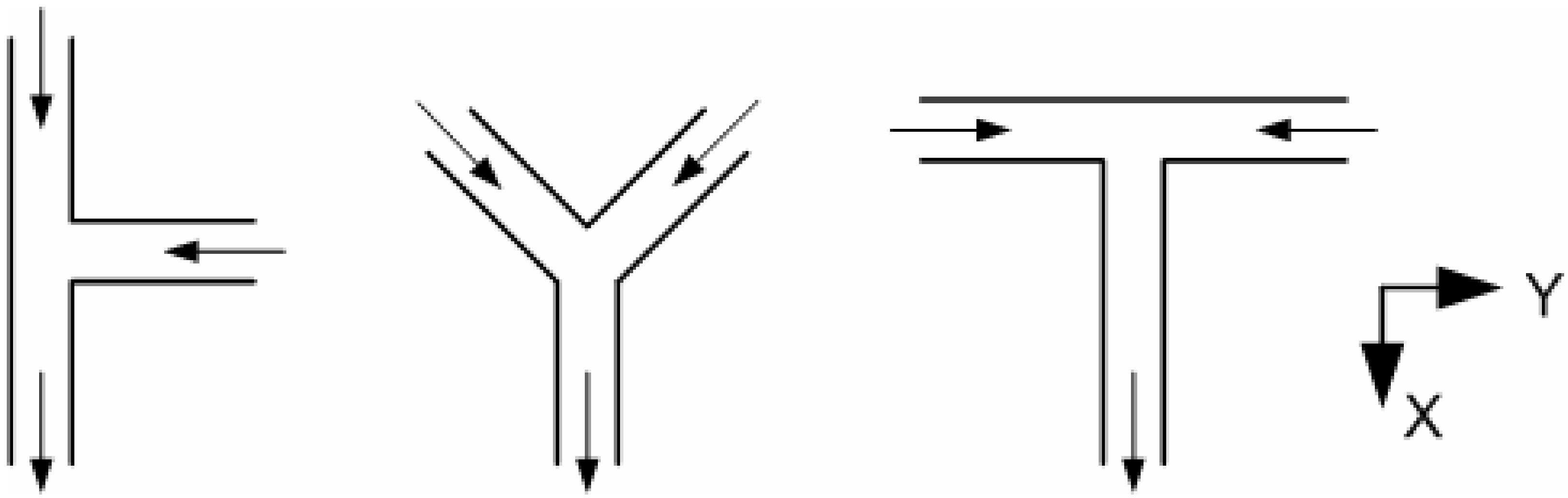
- Use channels of simple geometry, easy to fabricate
- Active micromixers
- Solutions valid at very small Reynolds numbers (Re ~ 10^{-1}, 10^{-2})

Two solutions:

I. Pulsed flow in inlet channels

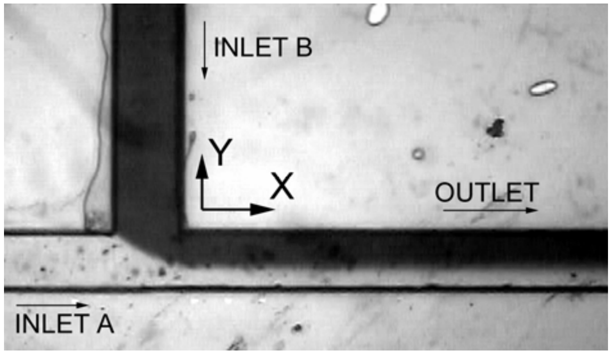
II. Electrohydrodynamic instability with electric field normal to the fluid interface

I. Simple Geometry: two inlets & outlet

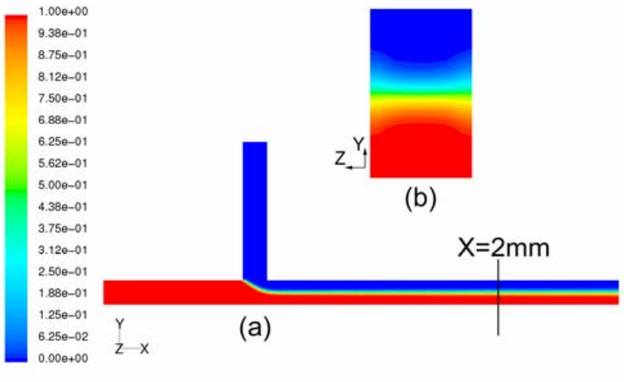


Confluence geometries ("├", "Y", and "T" from left to right) with two inlet and one outlet branches. All three branches are 200 μm wide by 120 μm deep (into the viewgraph).

Side-by-side fluid flows



Physical Model



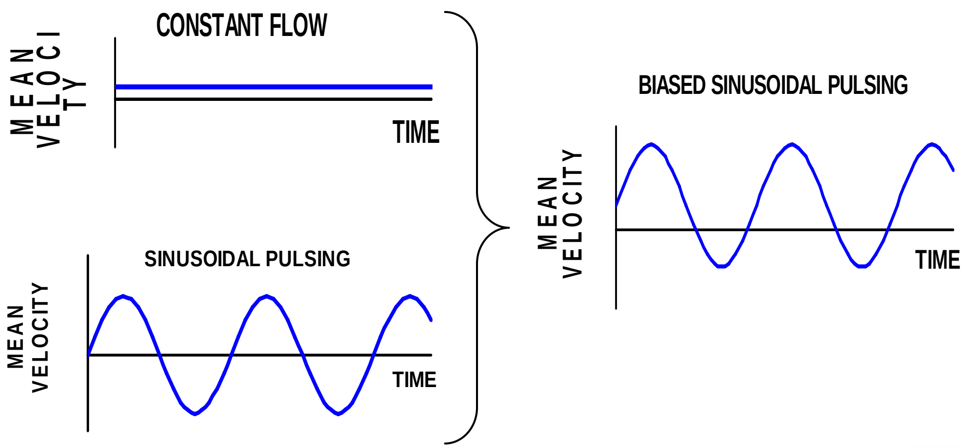
Numerical Simulations
(a) XY-Plane
(b) Cross-section at X=2mm

- *Channels*: 200 μm wide by 120 μm deep
- *Mean velocity*: *V* = 1mm/s from both inlets
- *Volume flow rate after confluence*: 48 nl/s
- *Molecular diffusivity*: $D_m = 1 \times 10^{-10}\ m^2\ s^{-1}$ for small proteins in aqueous solution

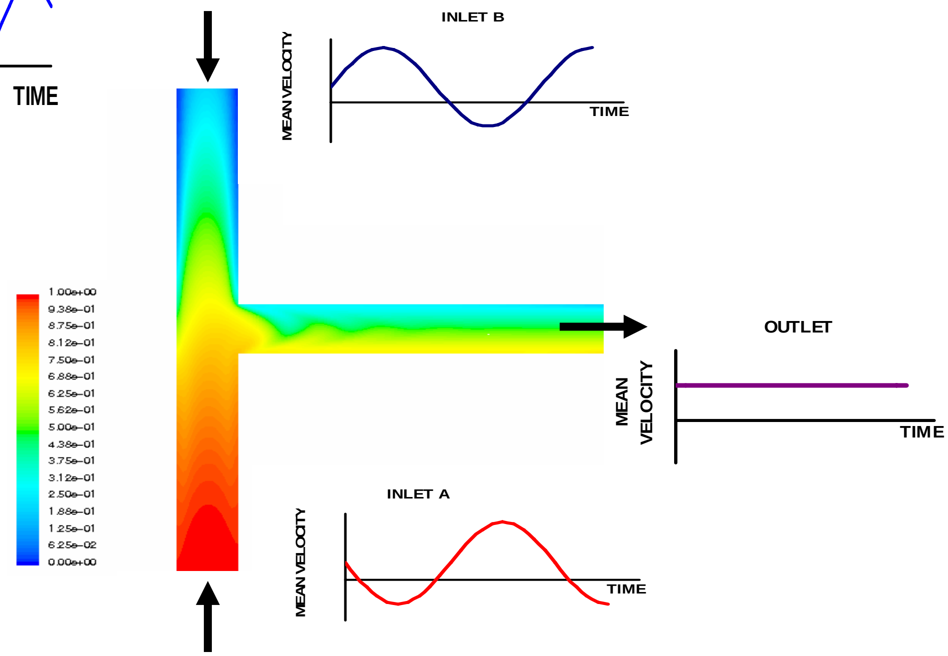
Re = 0.3
St = 0.4 (f=5Hz)
Pe = 3.10^3

Pulsed Flow Mixing - Principle

Out of Phase Pulsing Superimposed Onto Constant Flow



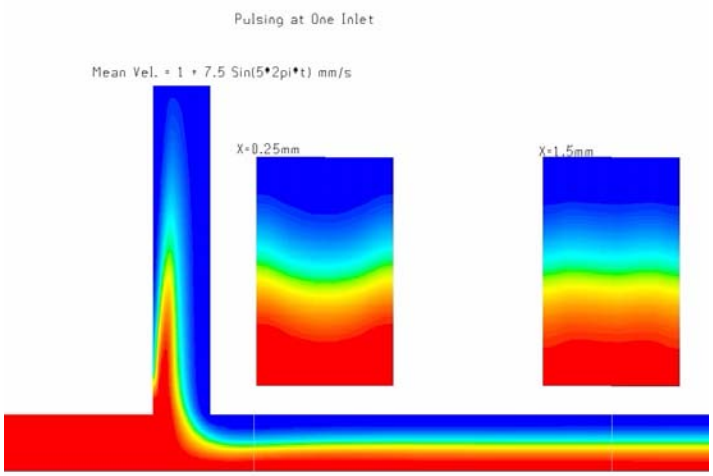
Example: 20 Hz
180º Phase Difference



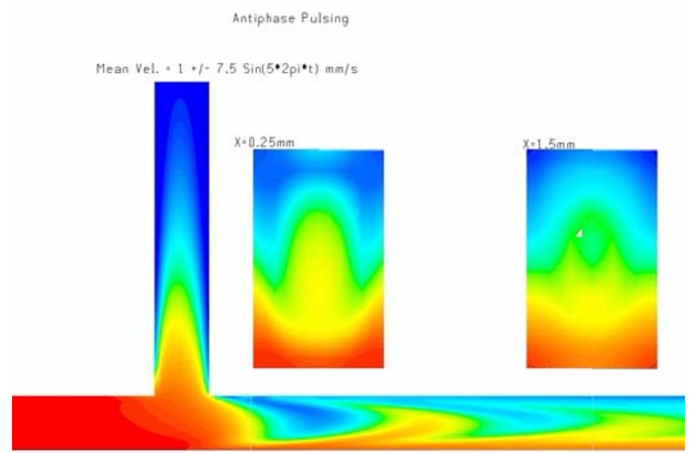
Pulsing by controlling pump or fluid volumes in connecting tubing or device

Nadine Aubry, November 2006

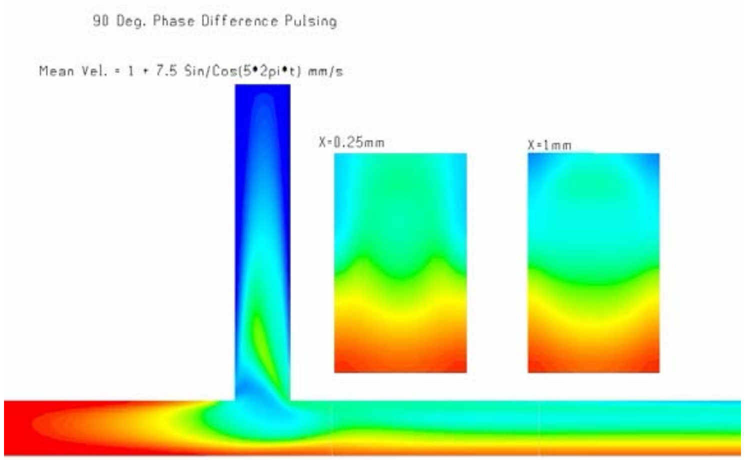
Pulsing – Concentration plots



Pulsing at one inlet only



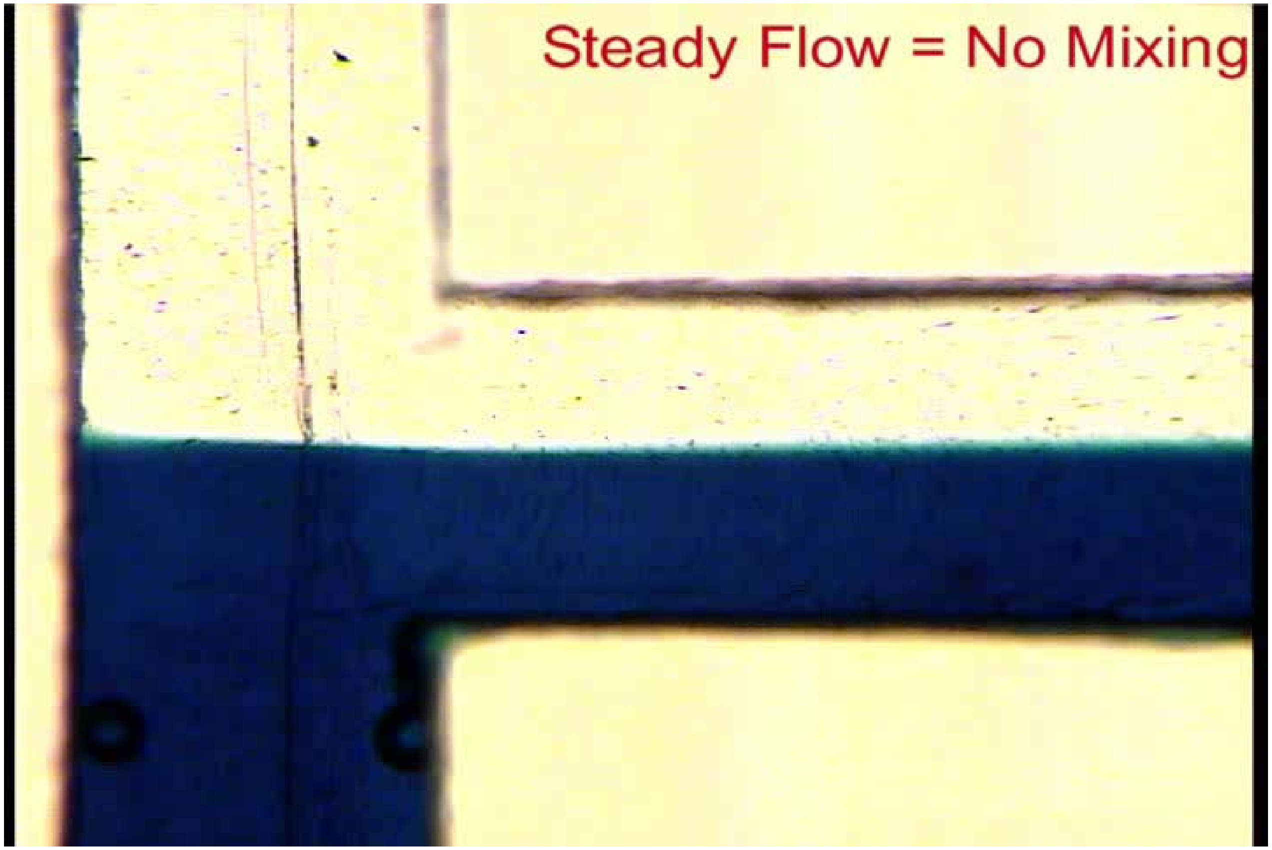
180º Phase Difference Pulsing



90º Phase Difference Pulsing

Refs: Glasgow, NA, 2003
Goullet, Glasgow, NA, 2005, 2006
See also Truesdell et al. 2003, 2005

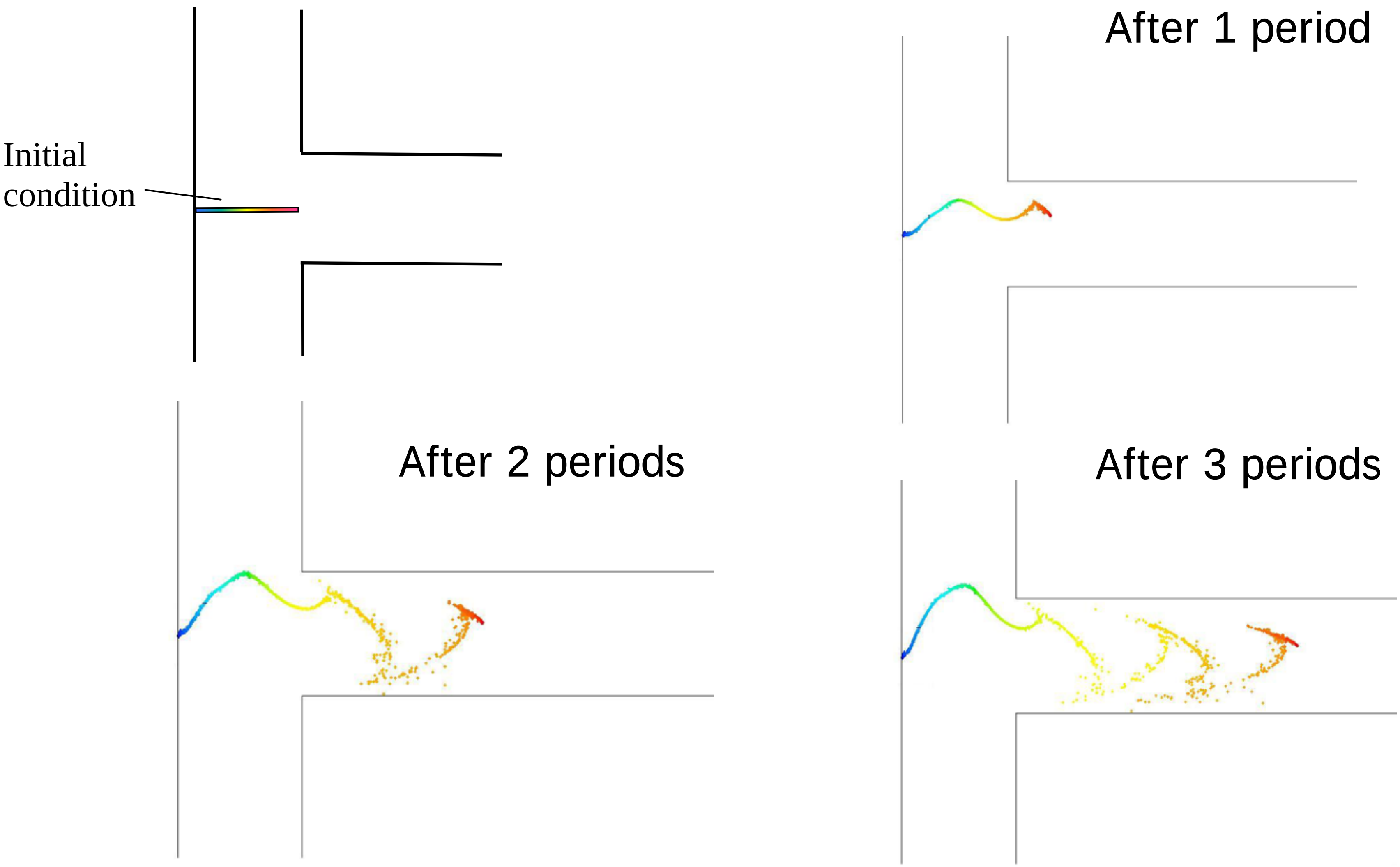
Experiments



Means: controlling peristaltic pumping; or controlling volumes of fluids (alternative compression of the tube); or controlling electro-osmotic flow

Numerical Simulations – 180°

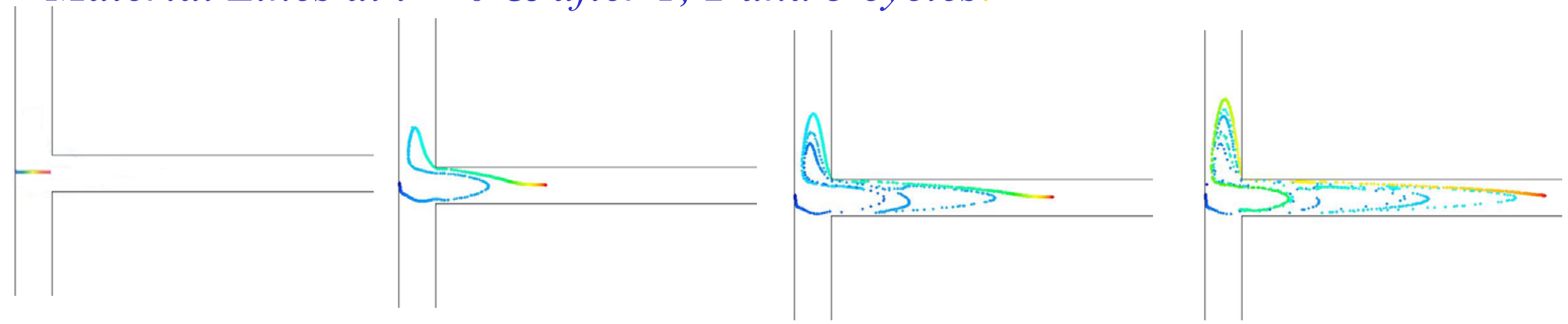
Material lines



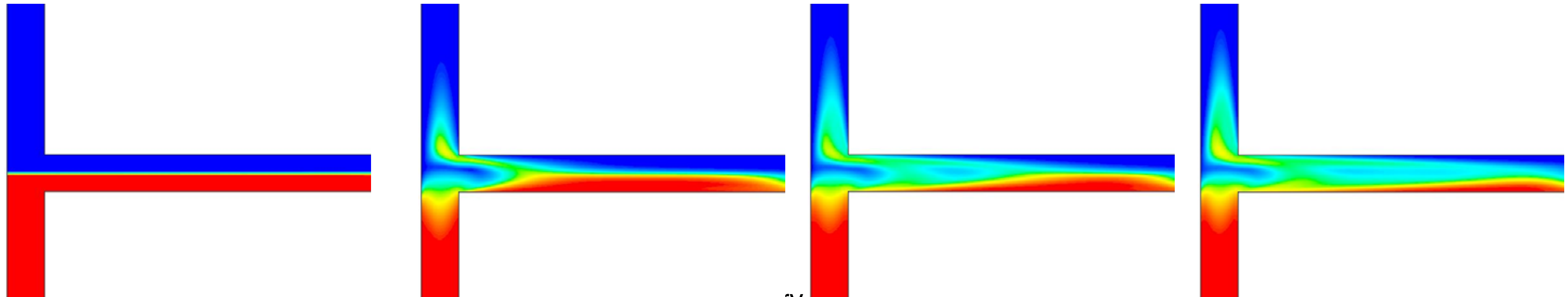
Numerical Simulations - 90º

$$V_1(t) = 1 + 7.5\sin(2\pi 5t) \quad (mm.s^{-1})$$
$$V_2(t) = 1 - 7.5\cos(2\pi 5t) \quad (mm.s^{-1})$$

Material Lines at t = 0 & after 1, 2 and 3 cycles:



Concentration Plots at t = 0 & after 1, 2 and 3 cycles:



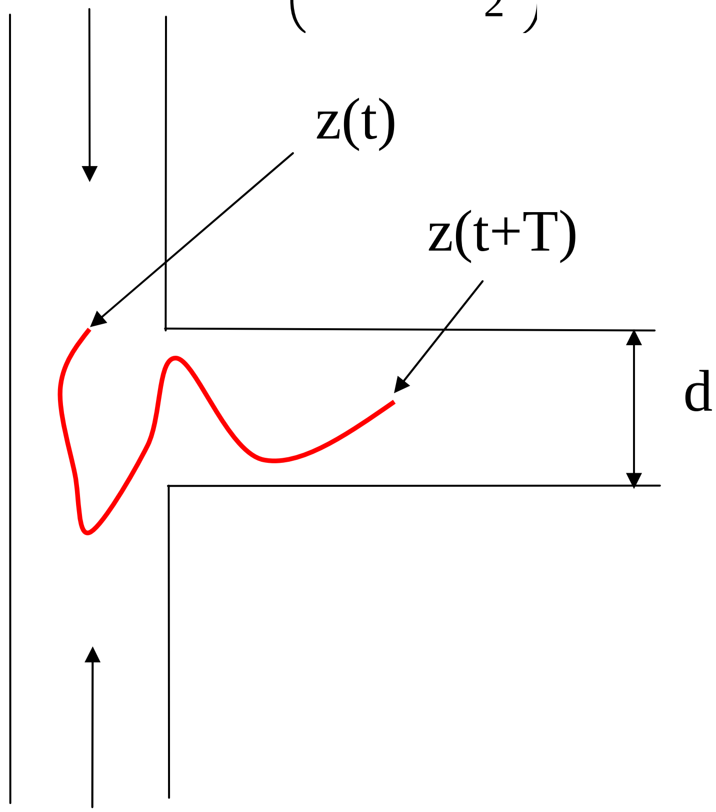
Mixing Mechanism

Is the underlying mechanism

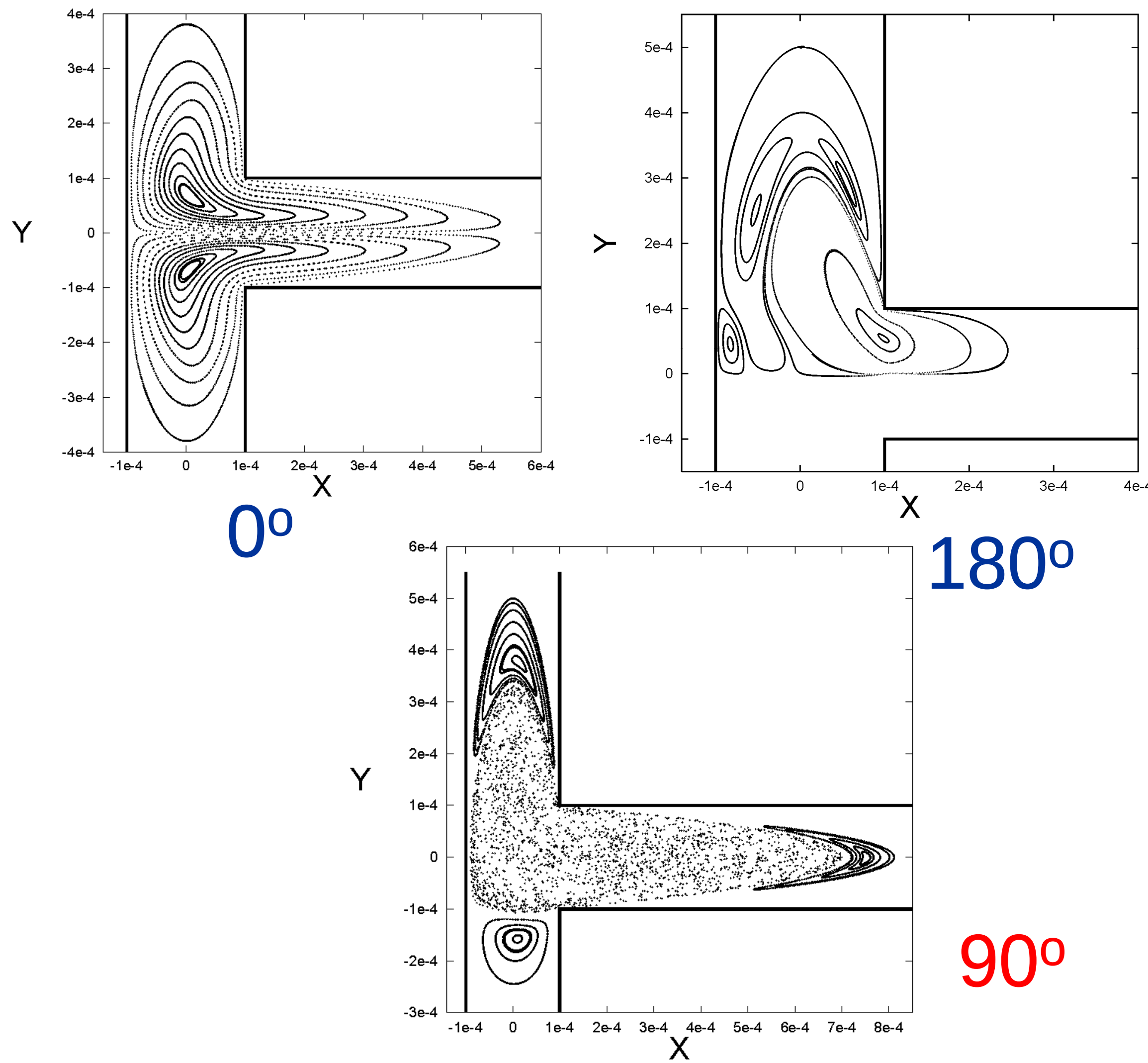
chaotic advection?

Stroboscopic Map (no mean flow)

$$V_2(t) = V_{pulse}\ \sin\left(2\pi f t + \frac{\pi}{2}\right)$$



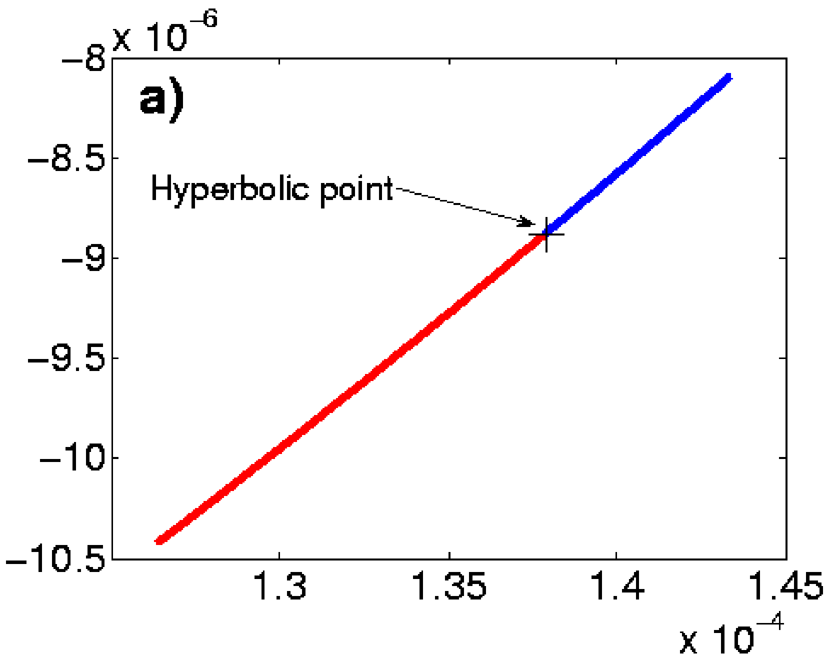
$$V_1(t) = V_{pulse}\ \sin(2\pi f t)$$



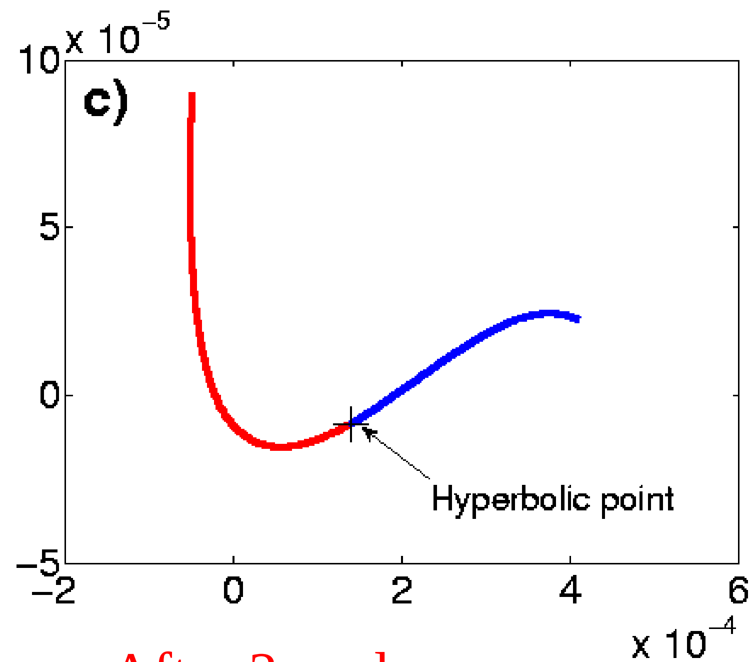
Re=1, St=0.2 (f = 5Hz, V = 5mm/s, d = 200 10^{-6} m, ν = 10^{-6} m^2/s)

New Map to conserve orientation – 90º

$$P: z(t) \mapsto z(t+2T)$$



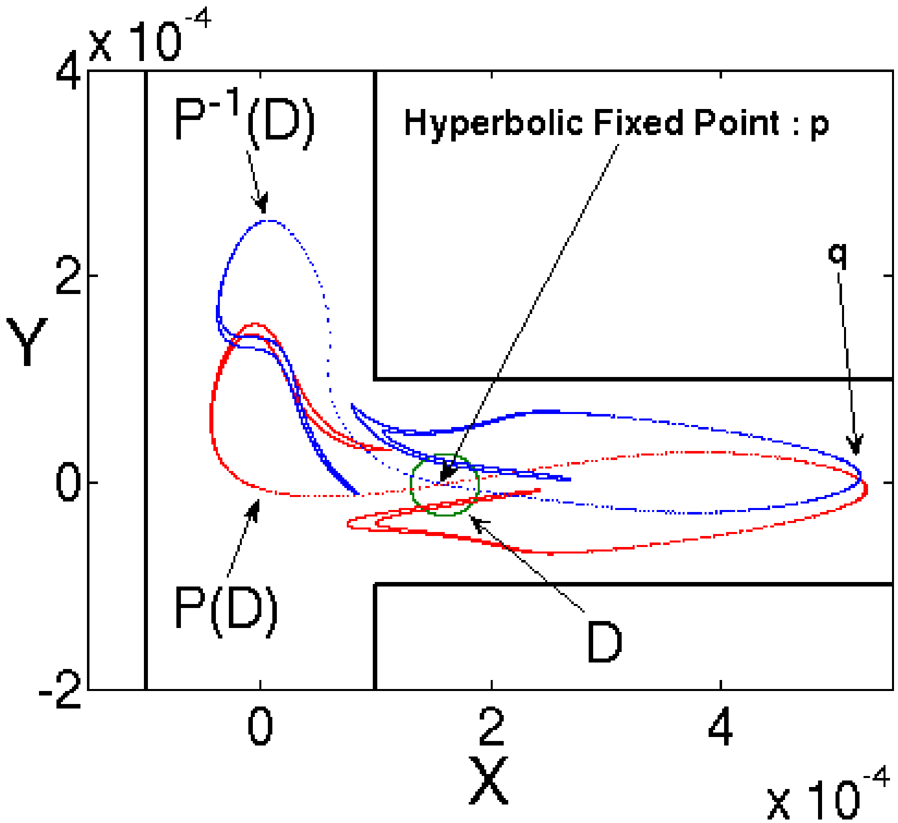
Initial Condition



After 2 cycles or one iteration of map P

The two branches of the unstable manifold

Stable and Unstable Manifolds



Hyperbolic fixed point: *p*

Intersection point: *q*
(Transversal intersection between stable and unstable manifolds of *p*)

Green circle: initial condition

Smale-Birkhoff Homoclinic Theorem: Transverse homoclinic orbit

Summary: pulsed mixing

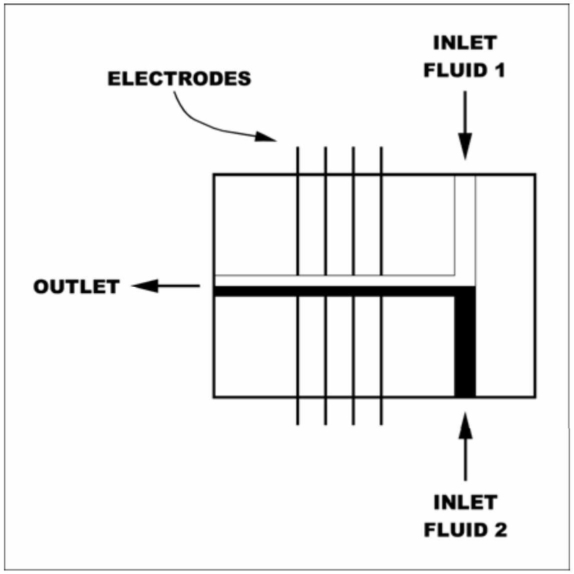
- 90^{o} phase difference pulsing is an efficient, easy to implement mixing means in microchannels

- Chaotic advection was identified in some region of the parameter space: existence of hyperbolic fixed point and transverse homoclinic orbit

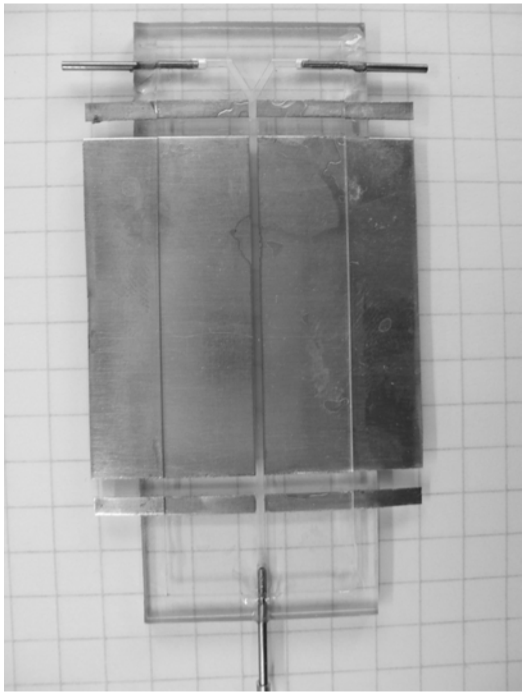
- Regular dynamics also exists in some other region of the parameter space: elliptic point (talk OD.4)

II. Electro-hydrodynamic Instability

2 fluids with different electrical properties
(conductivity, permittivity) + **normal** electric field



250μm x 250μm x 30mm



1.5 mm x 250μm x 70 mm

Image analysis

Images are analyzed on the grey scale levels

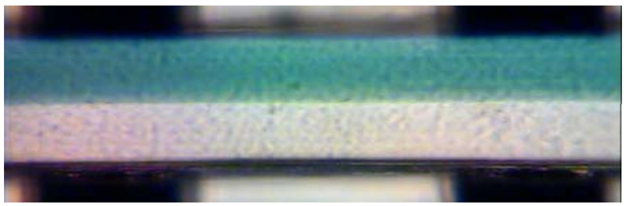
Coefficient of variation, CV=standard deviation divided by the mean

$$Mixing\ index = 1 - \frac{CV_{elect} - CV_{bkgnd}}{CV_{nofield} - CV_{bkgnd}}$$

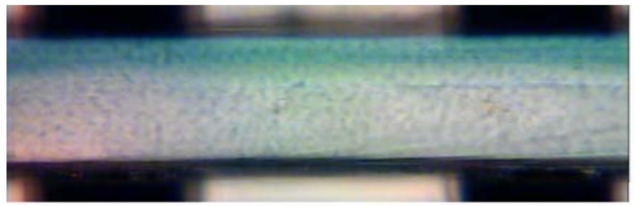
0 no mixing
1 total mixing

Ould El Moctar, Batton, NA, 2003

Miscible fluids

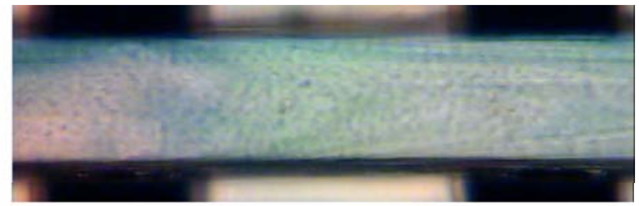


no electric field



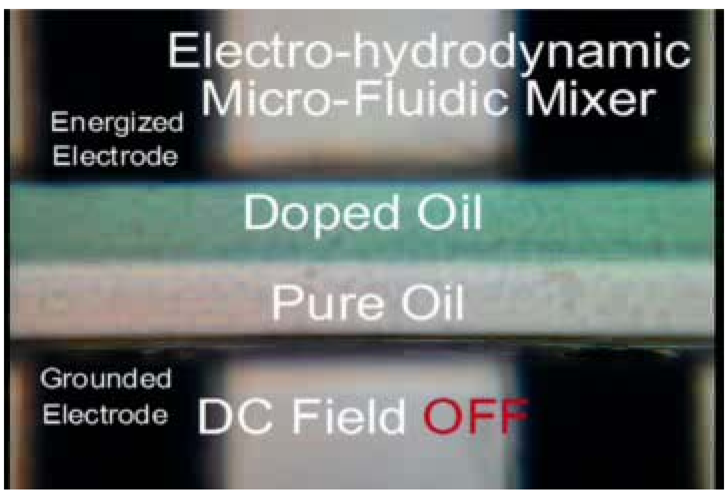
(a)

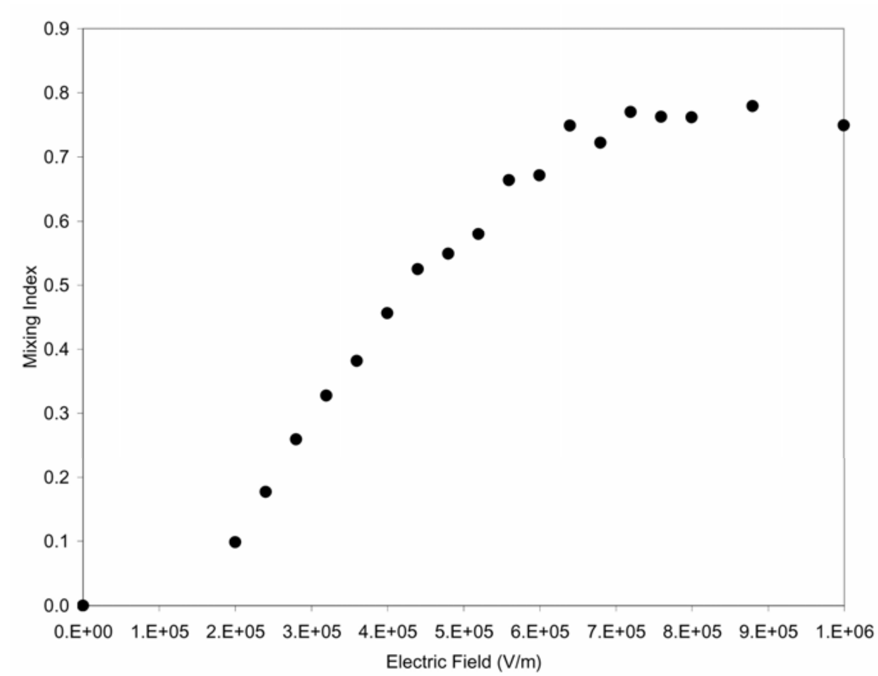
$E = 4\text{x}10^5$ V/m



(b)

$E = 6\text{x}10^5$ V/m





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Using drops for micromixing

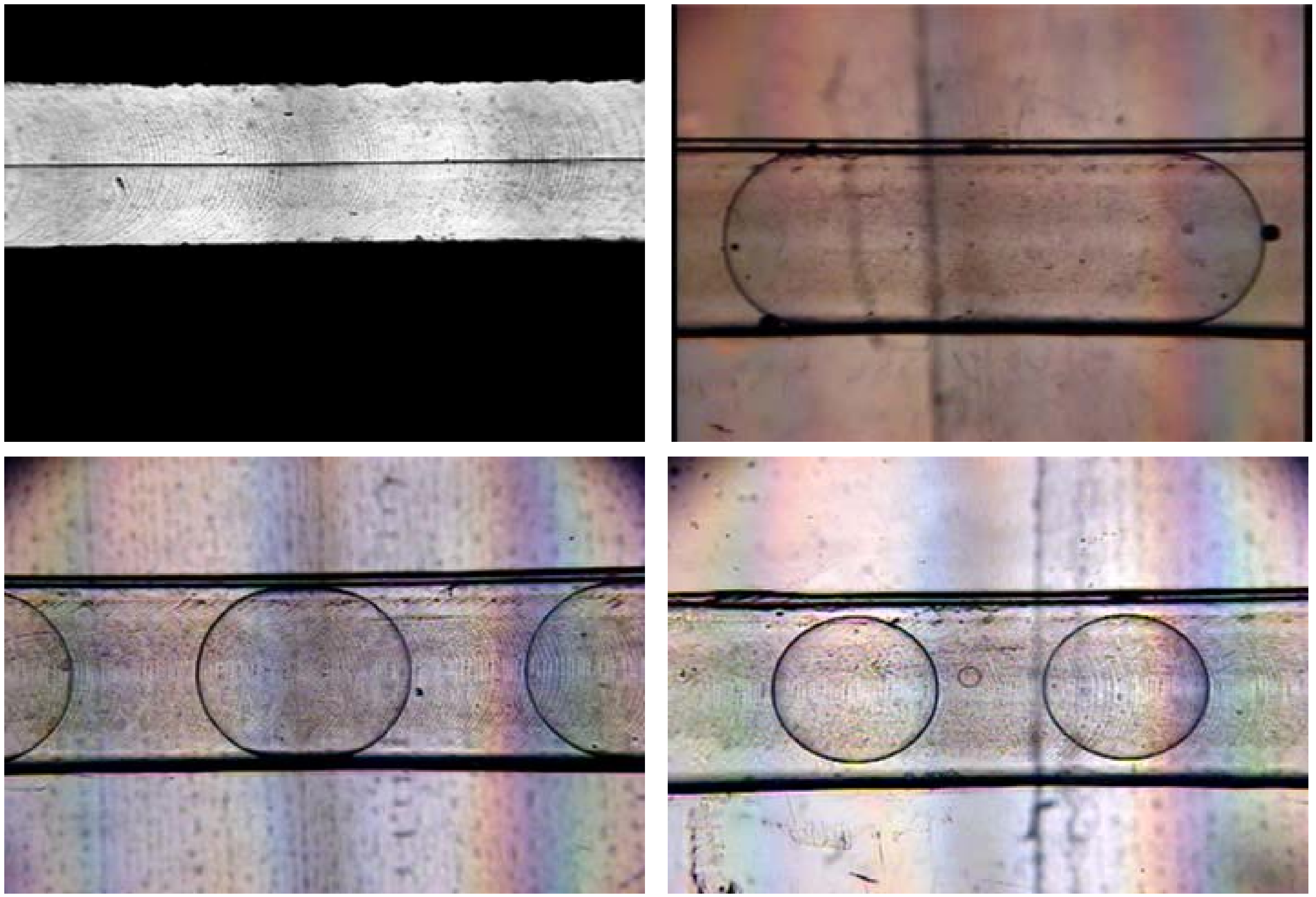
- Drops as individual "chemical reactors"
- "Discrete" or "Digital" microfluidics
- Steps/Issues (translating drops)
  - Step 1: Generate drops of controlled size
    - Using geometry: e.g. flow focusing - Anna, Bontoux, Stone, 2003
    - Can one generate drops in a straight microchannel?
  - Step 2: Generate internal flow within drops
    - Passively (curved channels) - Song, Tice & Ismagilov, 2003
    - Actively (electric field) – Lee, Im & Kang, 2000; Ward & Homsy (2001)

Step 1: Formation of Drops

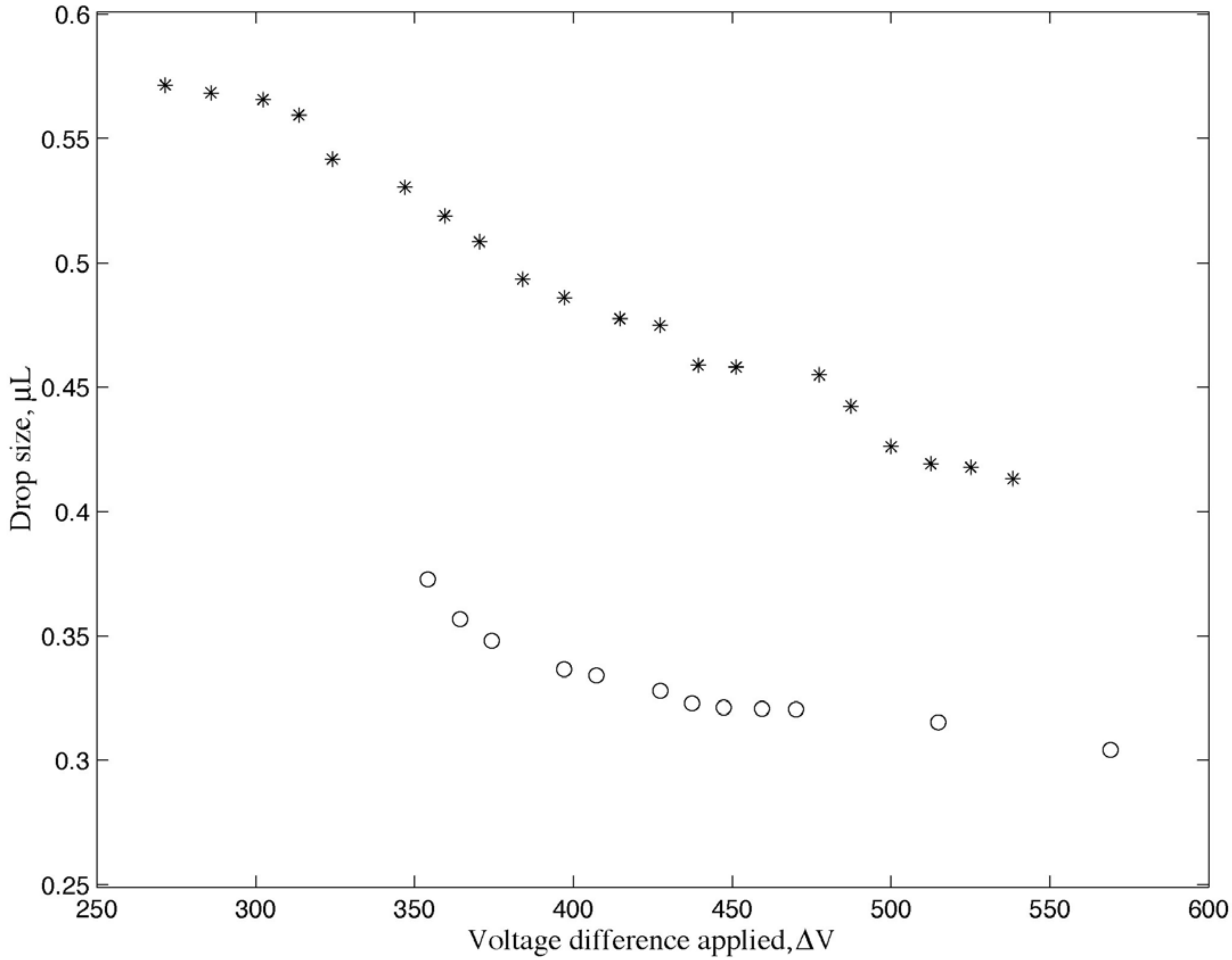
Straight channel, using electrodes in walls

Ozen, NA, Papageorgiou, Petropoulos, 2006

Formation of droplets



Drop size vs. Voltage



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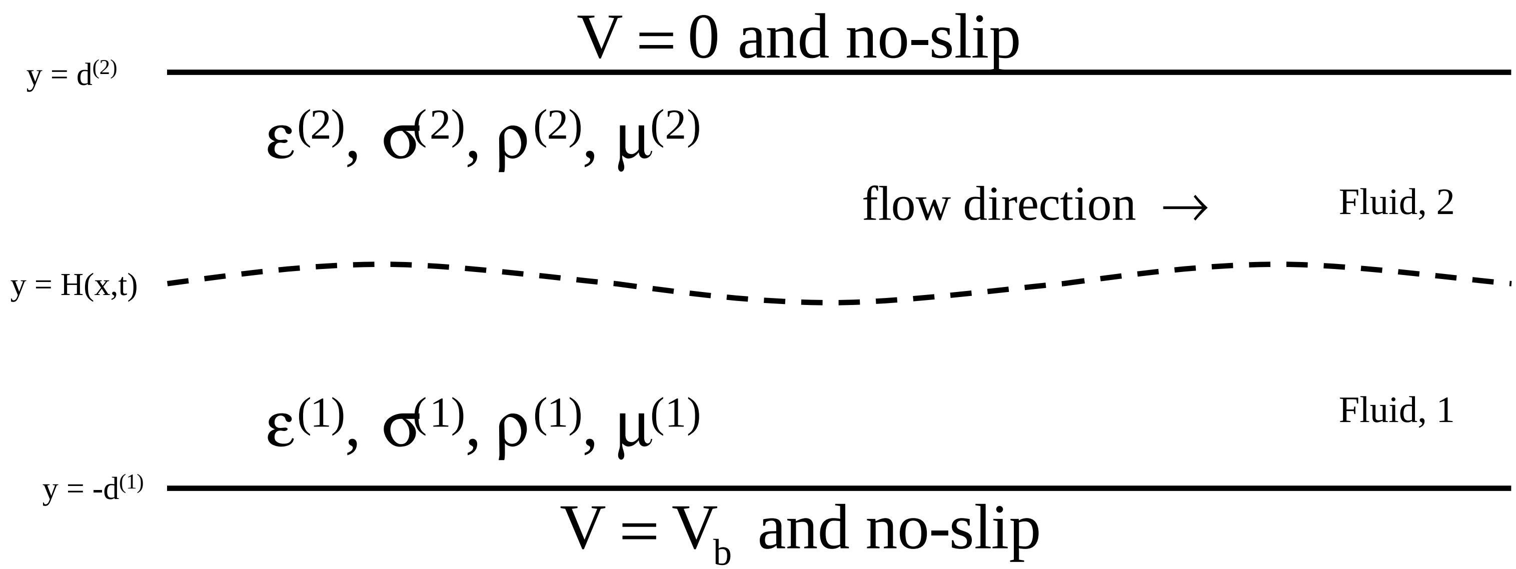
Instability: Model



- No electrical body forces – fluid dynamics and electric field are only coupled at the interface
- Linear stability of the system of unperturbed interface at y = 0, and its growth rate vs. wavenumber
- Interface shape through evolution equations

Mathematical model

DOMAIN EQUATIONS

Navier-Stokes equations

Continuity equation

Laplace equations

INTERFACIAL CONDITIONS

No mass transfer

No slip

Continuity of tangential electrical field

Gauss' Law

Tangential stress balance
Normal stress balance
} Coupling of electric field and fluid dynamics

Conservation of interfacial charge

Ozen, NA, Papageorgiou, Petropoulos, 2006
Li, Ozen, NA, Papageorgiou, Petropoulos, 2007

Dimensionless groups

Reynolds number, $\mathrm{Re} = \dfrac{\rho^{(1)} U_{int} d^{(1)}}{\mu^{(1)}}$ ≤ 1

Electric Weber number, $E_b = \dfrac{\varepsilon_0 V_b^2}{\mu^{(1)} U_{int} d^{(1)}}$ 1 to 10^3

Capillary number, $\mathrm{Ca} = \dfrac{\mu^{(1)} U_{int}}{\gamma}$ 10^{-4} to 1

$$S = \frac{\text{Fluid time-scale}}{\text{Electric charge time-scale}} = \frac{d^{(1)} / U_{int}}{\varepsilon_0 / \sigma^{(1)}}$$ 10^{-7} to 10^{7}

Depth ratio, $d = \dfrac{d^{(2)}}{d^{(1)}}$ Viscosity ratio, $\mu = \dfrac{\mu^{(2)}}{\mu^{(1)}}$ Density ratio, $\rho = \dfrac{\rho^{(2)}}{\rho^{(1)}}$

Electrical permittivity ratio, $\varepsilon = \dfrac{\varepsilon^{(2)}}{\varepsilon^{(1)}}$ Electrical conductivity ratio, $\sigma = \dfrac{\sigma^{(2)}}{\sigma^{(1)}}$

Linear stability analysis

Normal mode expansion

$$u_1 = \overline{u_1}(y) e^{\omega t} e^{ikx} + c.c.$$

- Perturbed equations
- Perturbed interfacial conditions
- Eigenvalue problem solved numerically using Chebyshev spectral tau method
- Solved for a broad range of values of S. However, simplification for large S values (charge relaxation time scale much faster than fluid time scale)

Analytical results

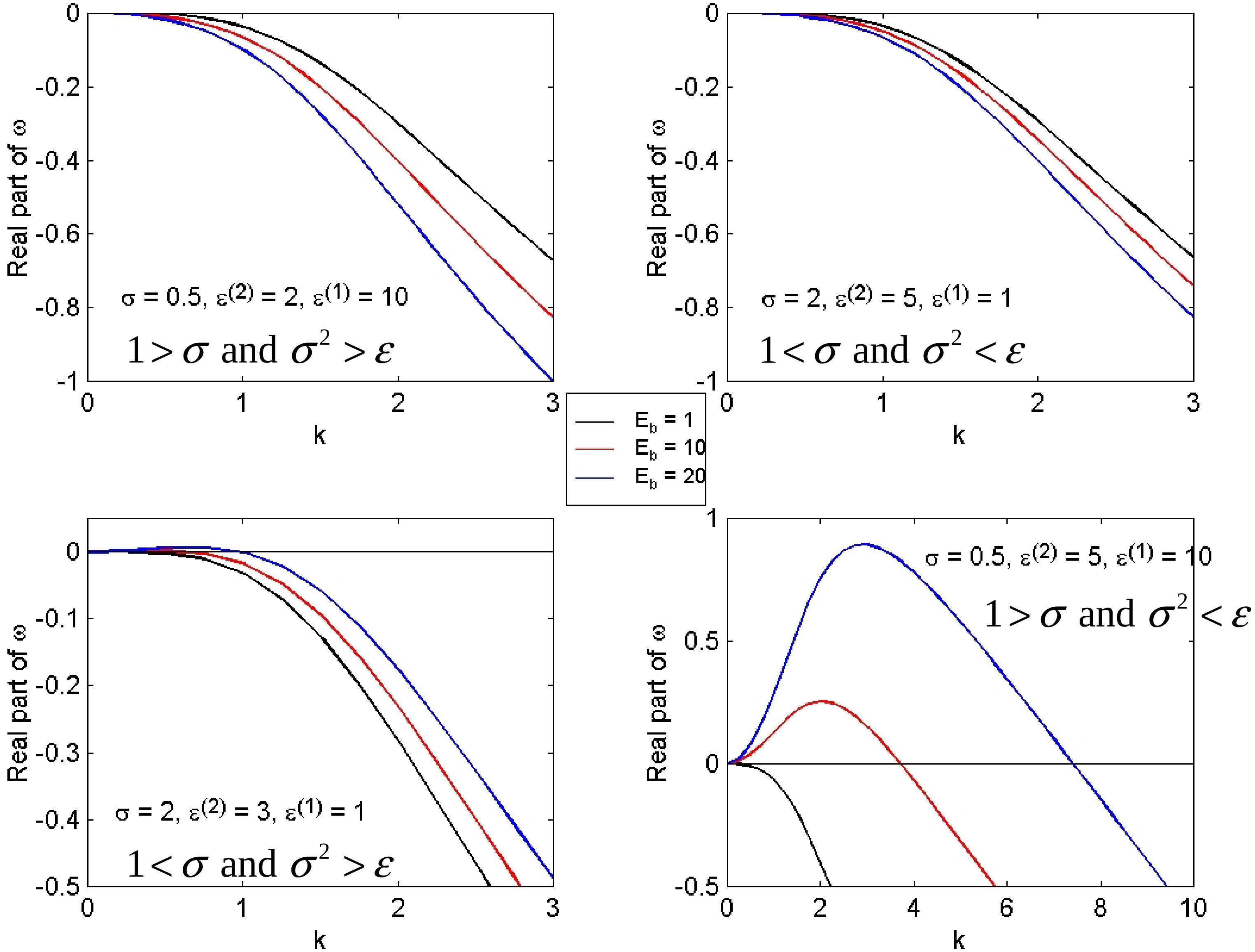
- S large

$$(\sigma^2 - \varepsilon)(1-\sigma) > 0 \quad \text{E stabilizing}$$

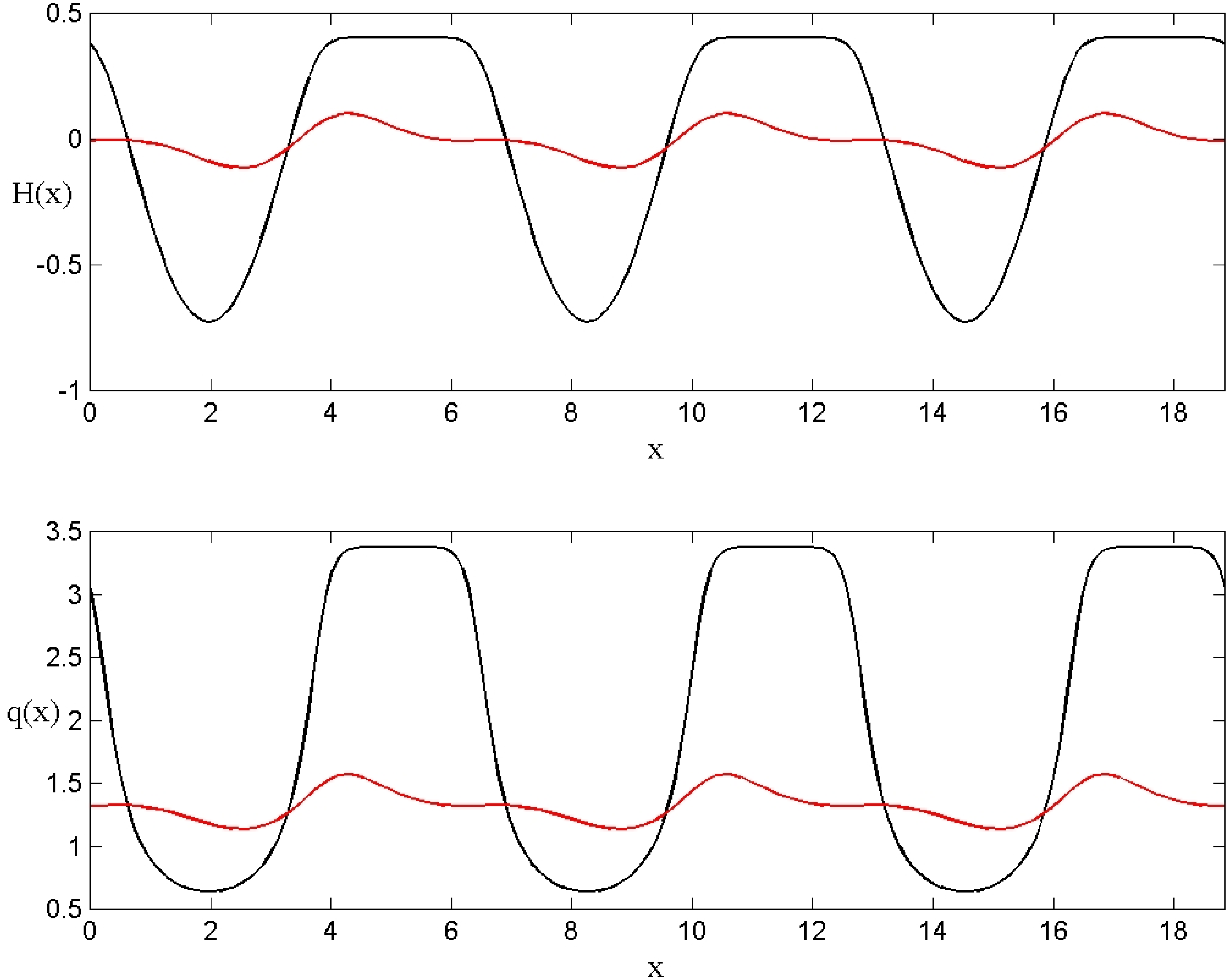
$$(\sigma^2 - \varepsilon)(1-\sigma) < 0 \quad \text{E destabilizing}$$

Stabilizing	Destabilizing
$1-\sigma > 0 \text{ and } \sigma^2 - \varepsilon > 0$	$1-\sigma > 0 \text{ and } \sigma^2 - \varepsilon < 0$
$1-\sigma < 0 \text{ and } \sigma^2 - \varepsilon < 0$	$1-\sigma < 0 \text{ and } \sigma^2 - \varepsilon > 0$

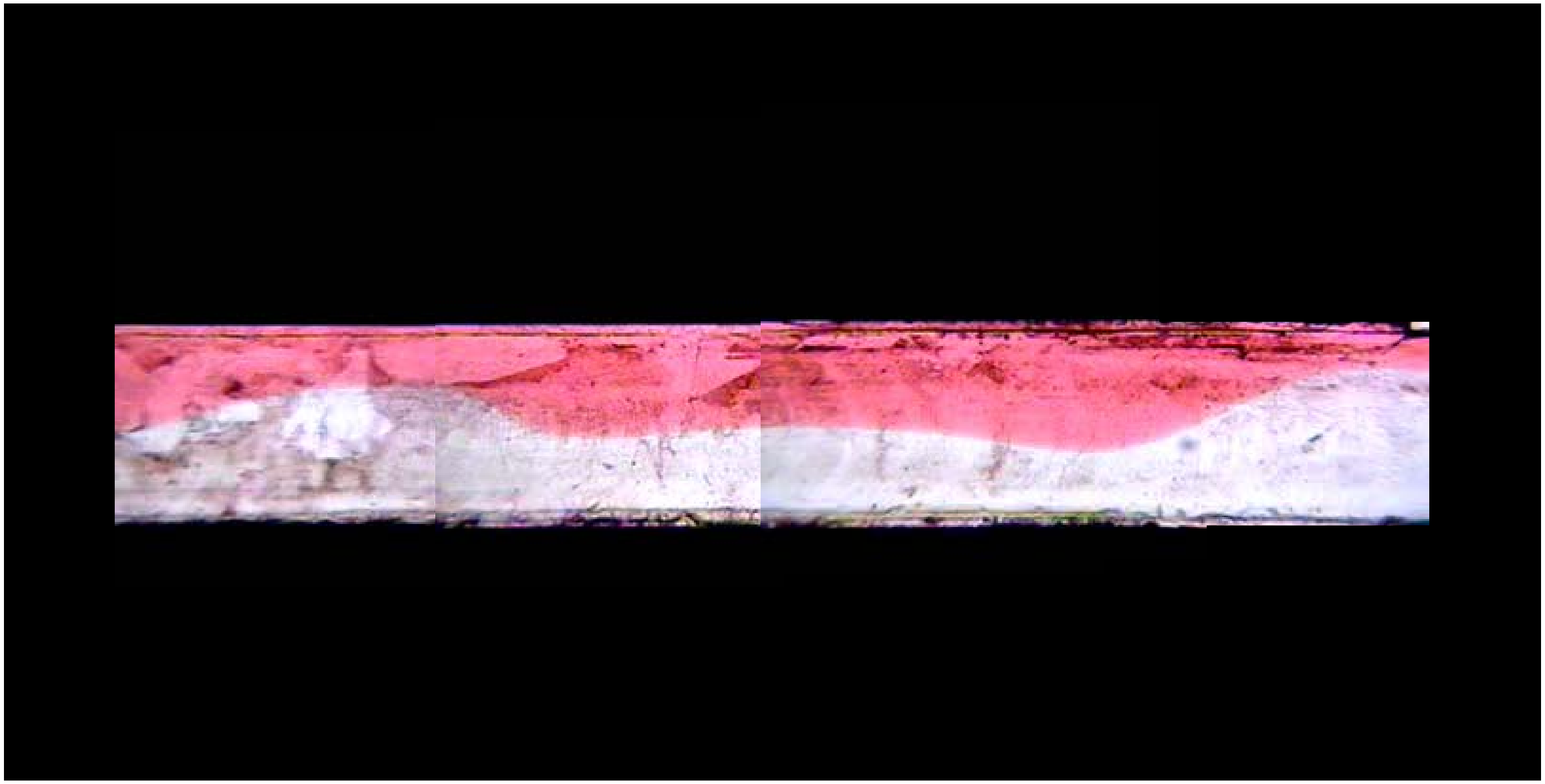
Comparison – Numerical results



Microfluidics – Interface



Experimental result



Step 2: Mixing within Drops

- Drops subjected to both translation and rotation

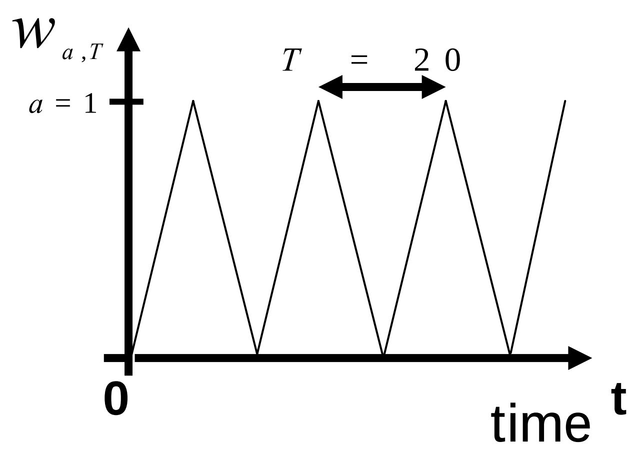
- Solution: Stokes flow in infinite domain; drop size small; drop remains spherical

- Previous studies: cst. translation, cst. rotation - Bajer, Moffatt (1990); Stone, Nadim & Strogatz (1991); Kroujiline & Stone (1999)

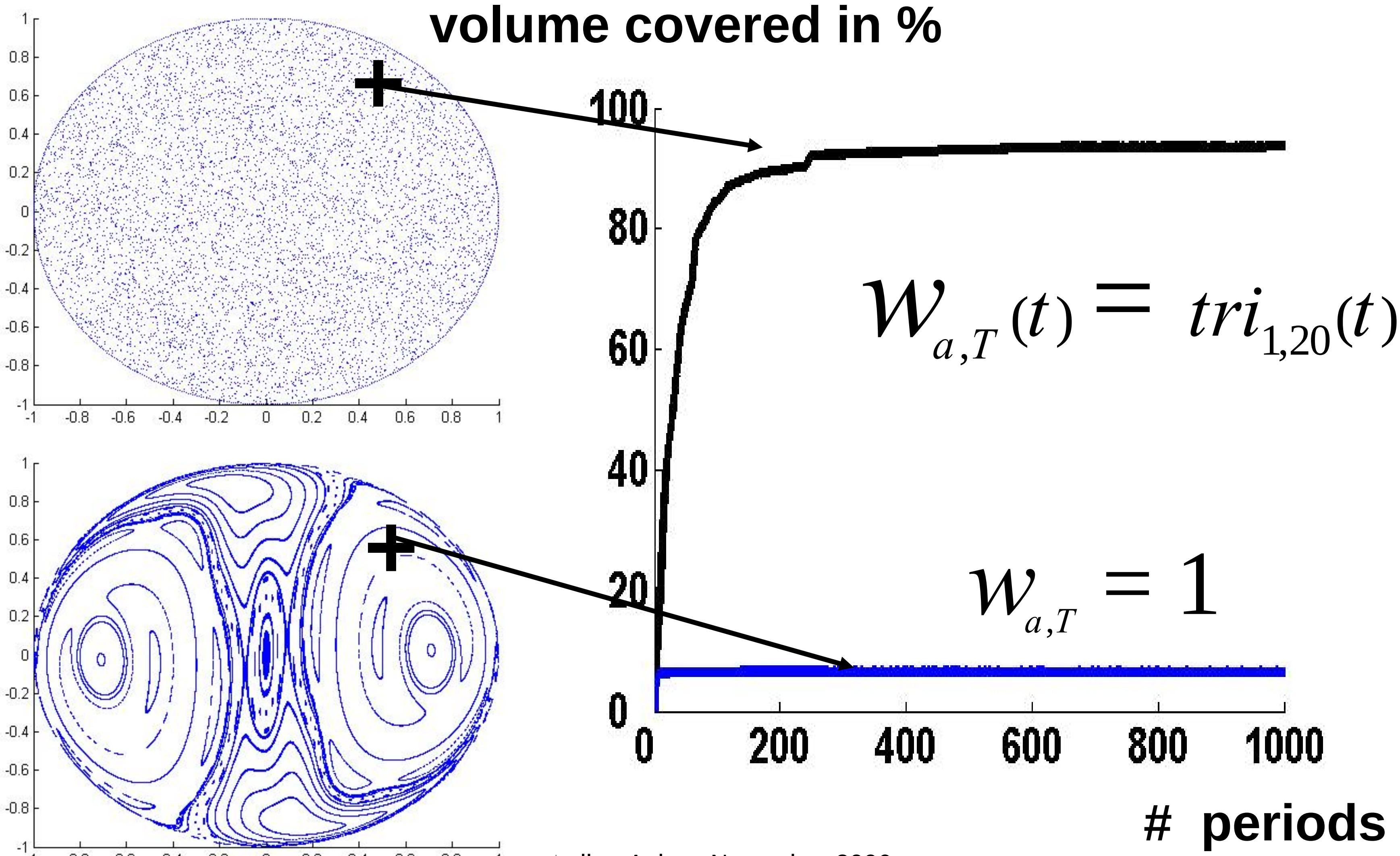
  Chaotic advection when axis of rotation differs from translation direction

- This work: cst translation, time dependent rotation

  (Talk AF.8)



Volume covered by a fluid particle as function of time



Next: Traveling Wave Dielectrophoresis

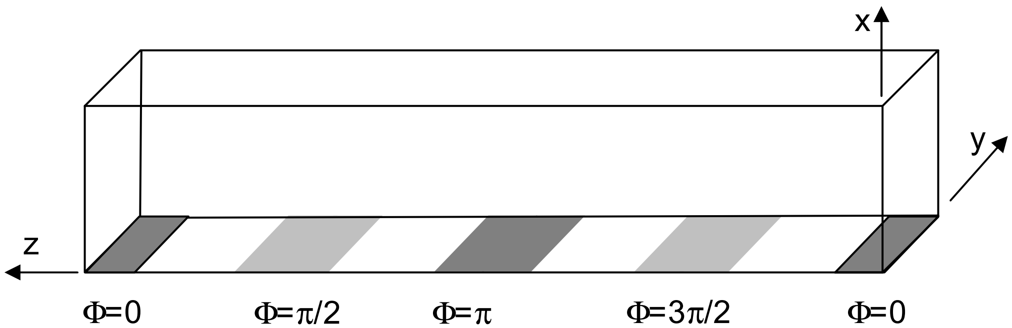
- Electrodes embedded in channel wall(s) periodically placed

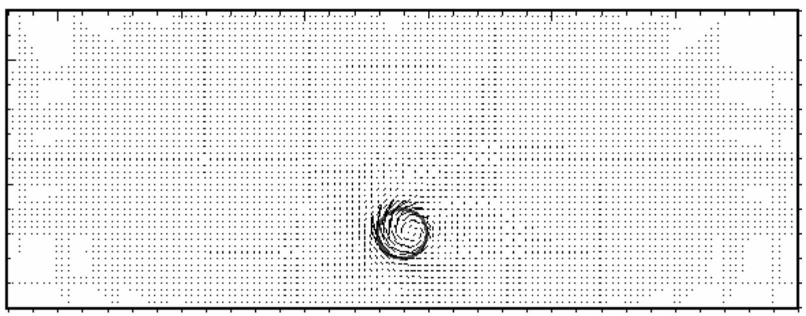
- 90° phase difference between voltages of adjacent electrodes

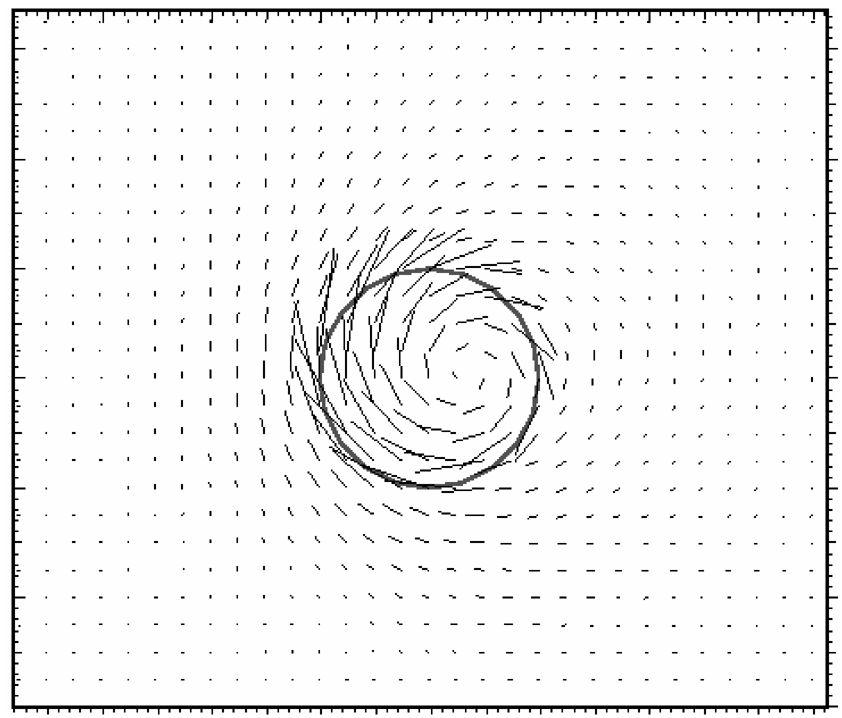
- Particles/drops experience traveling wave dielectrophoretic force and torque, thus enabling both translation along the channel and rotation

- One practical way to generate drop translation and rotation within a microchannel

Traveling Wave Dielectrophoresis







Rotation and translation of a rigid particle in a traveling wave electric field

NA & Singh, 2006; Nudurupati, NA & Singh, 2006; Talk FC.2

Governing equations for DNS

$$\nabla \bullet \mathrm{u} = 0$$

Surface tension — Electric stress

$$\rho\left[\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{u}\right] = -\nabla p + \nabla.(2\eta\mathbf{D}) + \gamma\kappa\delta(\phi)\mathbf{n} + \nabla.\boldsymbol{\sigma}_M$$

$\mathbf{u} = \mathbf{u}_{\mathbf{L}}$ on domain boundary

u is the velocity, p is the pressure
η is the viscosity, ρ is the density, **D** is the symmetric part of the velocity gradient tensor, **n** is the outer normal, γ is the surface tension, κ is the surface curvature, ϕ is the distance from the interface

$\boldsymbol{\sigma}_M$ = Maxwell stress tensor

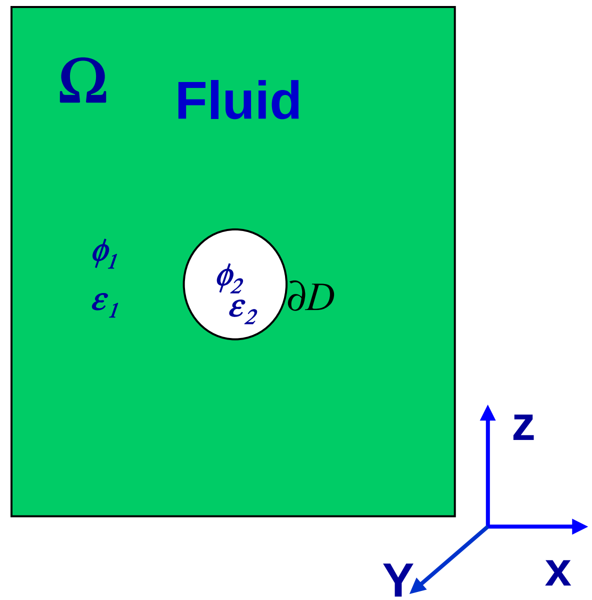
Electric Force Calculation

Electric Potential

$$0 \quad \text{in } \Omega$$

Boundary conditions

$$\phi_1 = \phi_2,\ \varepsilon_c \frac{\partial \phi_1}{\partial n} = \varepsilon_p \frac{\partial \phi_2}{\partial n} \quad \text{on} \quad \partial D(t)$$



- Maxwell Stress Tensor (MST)

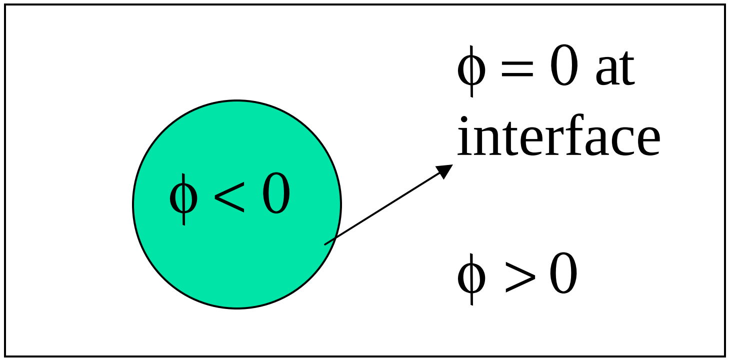
$$\boldsymbol{\sigma}_M = \varepsilon \mathbf{E}\mathbf{E} - \frac{1}{2}\varepsilon(\mathbf{E} \bullet \mathbf{E})\mathbf{I}, \qquad \mathbf{E}$$

Singh & NA,2005

Direct Numerical Simulations (DNS)

- Full DNS: Governing equations of motion are solved exactly: Flow and electric field are resolved at scales finer than particle size; No model used

- Interface is tracked using the level set method

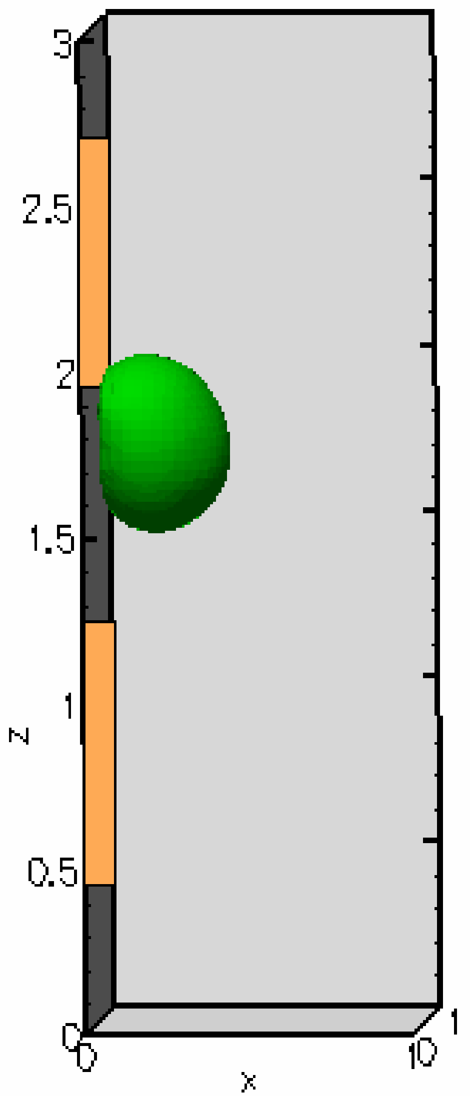


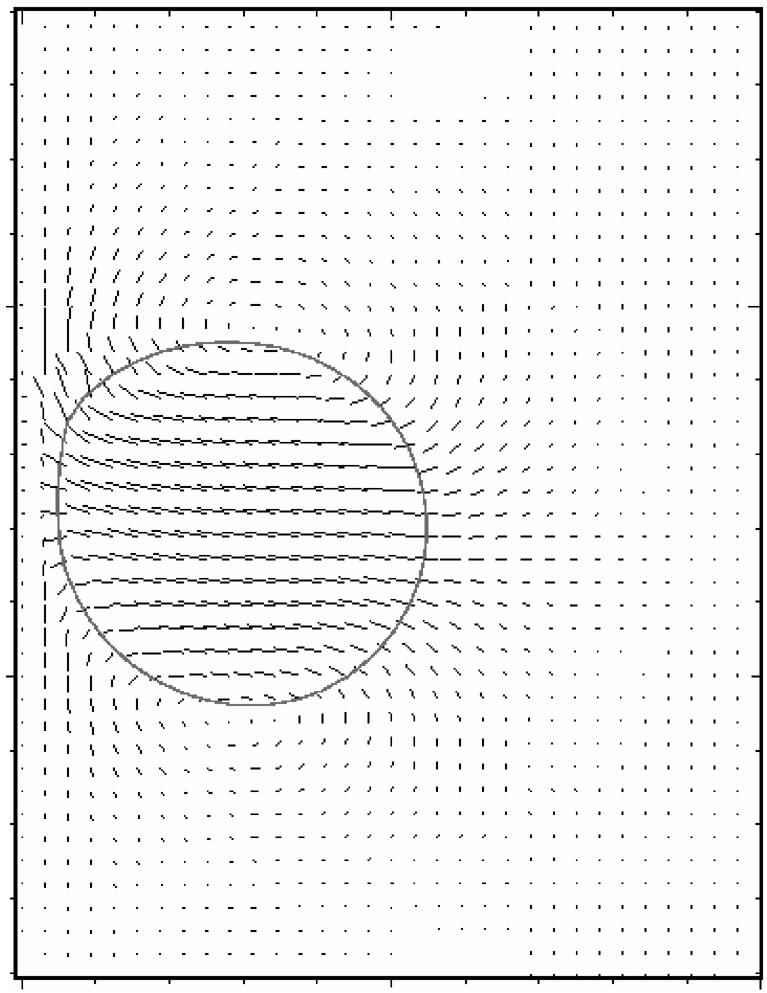
ϕ = distance from the interface

$$\frac{\partial \phi}{\partial t} + u.\nabla\phi = 0$$

Singh, Joseph, Hesla, Glowinski, Pan, 2000; Kadaksham, Singh, NA, 2004; Sussman, Smereka and Osher, 1994; Pillaipakkam and Singh, 2001; Singh and NA, 2006, 2007

Electric stress induced motion/deformation of a drop





Example of calculation (Without mixing)

Drop is attracted to the electrode edge (Dielectrophoresis)

Singh & NA, 2006; Singh & NA, 2007; Talk EF.2

Summary

Mixing in small scale flows is crucial for applications

- In micro-channels of simple geometry
  - Pulsed flow mixing (chaotic advection)
  - Electro-hydrodynamic instability (E normal to interface)
- Within drops ("digital microfluidics")
  - Generation of monodisperse drops: electro-hydrodynamic instability (E normal to interface)
  - Creation of internal flow within the drop (chaotic advection) – using electric field
  - DNS to study this problem (no model: confined geometry, deformation of drop, influence of drop on electric field, etc.)