63rd Annual Meeting
of the APS Division of Fluid Dynamics

November 21–23, 2010
Long Beach, California

Dynamics of electrically conducting fluids

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Dynamics of electrically conducting fluids

- Evolution equations for electrically conducting fluids
    - Coupling between fluid dynamics and electromagnetism
    - Magnetohydrodynamics
- Examples of electrically conducting fluids
    - Liquid metals
    - Astrophysical systems
    - Plasmas (ITER)
- Numerical results
    - Quasi-static approximation
    - High magnetic Reynolds number flows

Part 1

Evolution equations for electrically conducting fluids

- The evolution of an electrically conducting fluids is characterized by the coupling of
  - **Newton's law**

$$\vec{F} = m\,\vec{a}$$

  - **Maxwell's equations**

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu_0\,\vec{j} + \frac{1}{c^2}\,\frac{\partial \vec{E}}{\partial t}$$

$$\nabla \cdot \vec{E} = \frac{\sigma}{\epsilon_0} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}\,.$$

  - **Lorentz force**

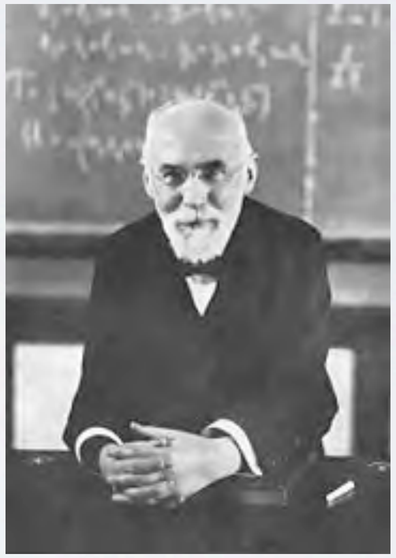
$$\vec{F} = q\,\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

  - **Ohm's law**

$$\vec{j} = \sigma_r\,\vec{E}$$

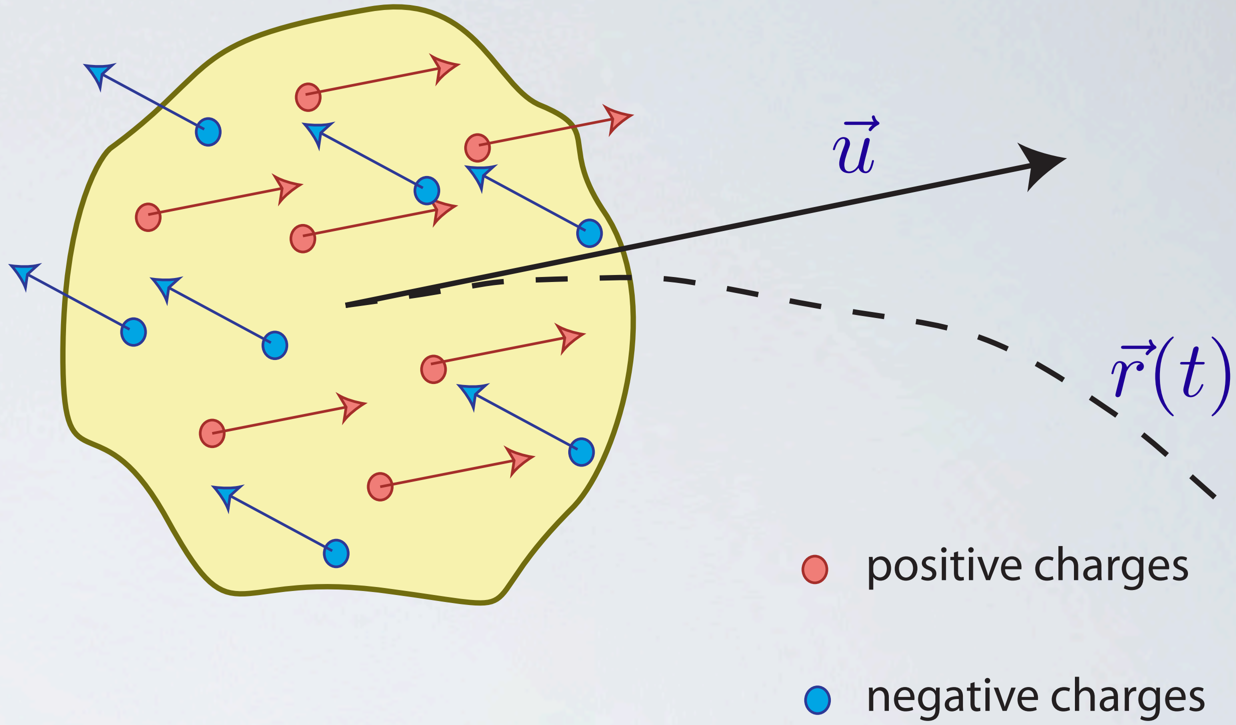

Isaac Newton
(1643-1727)


James Clerk Maxwell
(1831–1879)


Hendrik Antoon Lorentz
(1853-1928)


Georg Simon Ohm
(1789-1854)

- In addition, electrically conducting fluids are assumed to be locally neutral



$$\sigma \approx 0$$

<u>First consequence</u> : Conservation of charge (from Maxwell's equations) implies that the current field should be divergence free

$$\frac{\partial \sigma}{\partial t} = -\nabla \cdot \vec{j}$$

$$\sigma \approx 0$$

$$\nabla \cdot \vec{j} \approx 0$$

Similar to the incompressible flow approximation

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho\, \vec{u})$$

$$\rho \approx \text{constant}$$

$$\nabla \cdot \vec{u} \approx 0$$

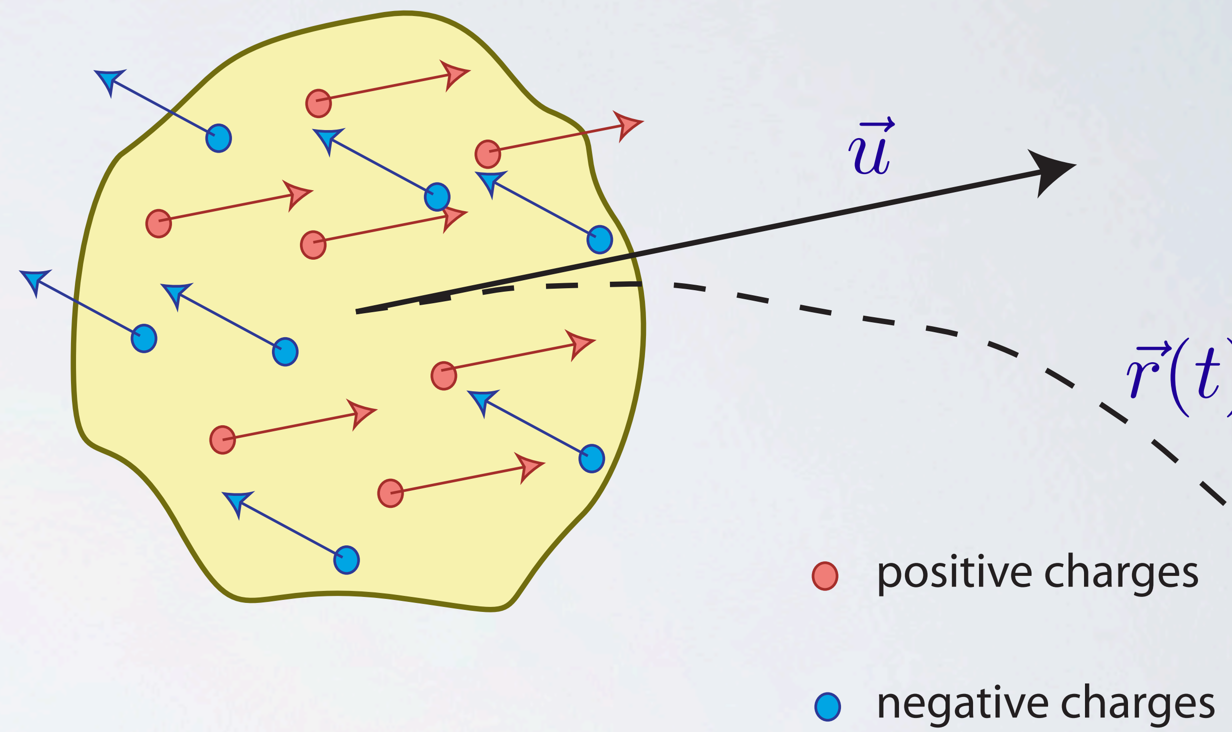
Second consequence: The Lorentz force can be simplified.

$$\frac{\vec{F}}{V} = \frac{1}{V}\sum_{\ell} q_\ell \left(\vec{E} + \vec{v}_\ell \times \vec{B}\right)$$

$$= \frac{1}{V}\sum_{\ell} (q_\ell)\ \vec{E} + \sum_{\ell} (q_\ell \vec{v}_\ell) \times \vec{B}$$

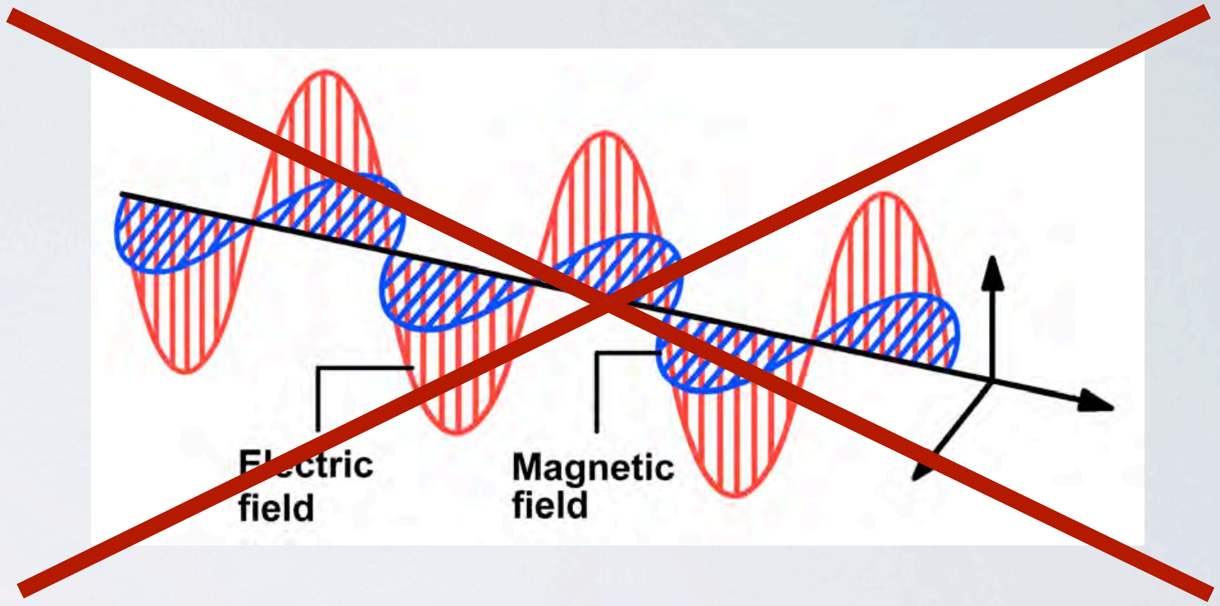
$$= \sigma\,\vec{E} + \vec{j} \times \vec{B}$$

$$\approx \vec{j} \times \vec{B}$$



- The displacement current is also assumed to be negligible

$$\frac{1}{\epsilon_0}\frac{\partial \vec{E}}{\partial t} \ll \vec{j}$$



No electromagnetic wave propagation

This approximation is compatible with the assumed electro-neutrality

$$\vec{j} = \frac{1}{\mu_0}\nabla \times \vec{B} \qquad \Rightarrow \nabla \cdot \vec{j} = 0$$

- The Ohm's law has to be adapted to moving frame

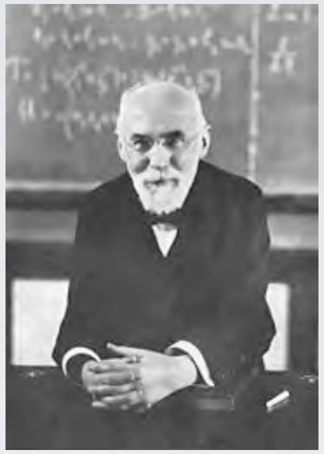
$$\vec{E} \rightarrow \vec{E} + \vec{u} \times \vec{B} \qquad \vec{j} = \sigma_r \left(\vec{E} + \vec{u} \times \vec{B}\right)$$

Newton

+

Maxwell

+



Lorentz

+

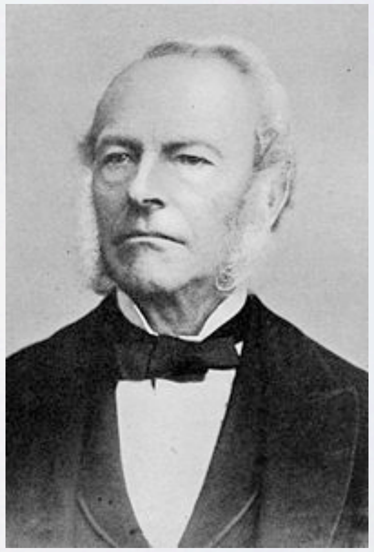
Ohm

\+ Electro-neutrality
\+ Negligible displacement current
\+ Moving frame

\+ Navier

Claude Louis Marie Henri Navier, (1785-1836)

\+ Stokes



George Gabriel Stokes (1819 -1903)

$$\rho\left(\frac{\partial \vec{u}}{\partial t} + \vec{u}\cdot\nabla\,\vec{u}\right) = -\nabla p + \mu_0^{-1}\,\left(\nabla\times\vec{B}\right)\times\vec{B} + \mu\nabla^2\,\vec{u}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times\left(\vec{u}\times\vec{B} - \frac{1}{\mu_0\,\sigma_r}\nabla\times B\right)$$

Magneto-Hydro-Dynamics (MHD)

- For incompressible flows, these equations can be re-written as follows:

$$\frac{\partial \vec{u}}{\partial t} = -\vec{u}\cdot\nabla\,\vec{u} + \vec{b}\cdot\nabla\,\vec{b} - \nabla p' + \nu\nabla^2\,\vec{u} \qquad \nabla\cdot\vec{u} = 0$$

$$\frac{\partial \vec{b}}{\partial t} = -\vec{u}\cdot\nabla\,\vec{b} + \vec{b}\cdot\nabla\,\vec{u} + \kappa\,\nabla^2\,\vec{b} \qquad \nabla\cdot\vec{b} = 0$$

Hannes Olof Gösta Alfvén
(1908-1995)

- Where the Alfvén units have been introduced for the magnetic field

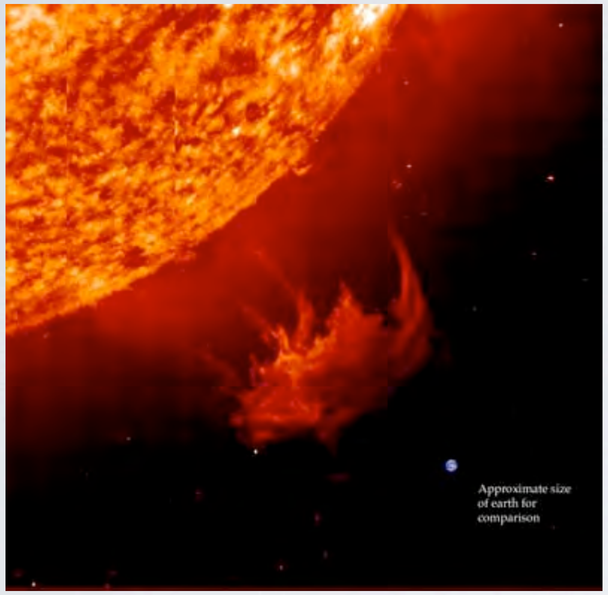
$$\vec{b} = \frac{\vec{B}}{\sqrt{\rho\,\mu_0}} \qquad p' = \frac{p}{\rho} + \frac{b^2}{2} \qquad \nu = \frac{\mu}{\rho} \qquad \kappa = \frac{1}{\mu_0\,\sigma_r}$$

Part 2

Examples of electrically conducting fluids

Examples of electrically conducting fluids:

Solar wind



http://son.nasa.gov/tass/content/solarwind.htm

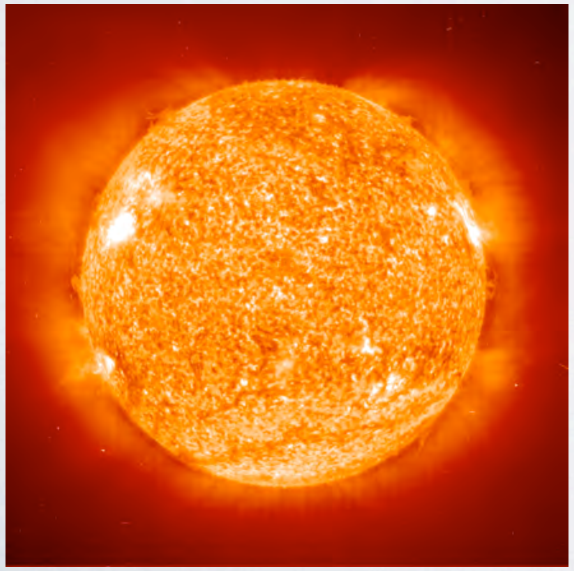
Fusion plasmas

http://www.jet.org

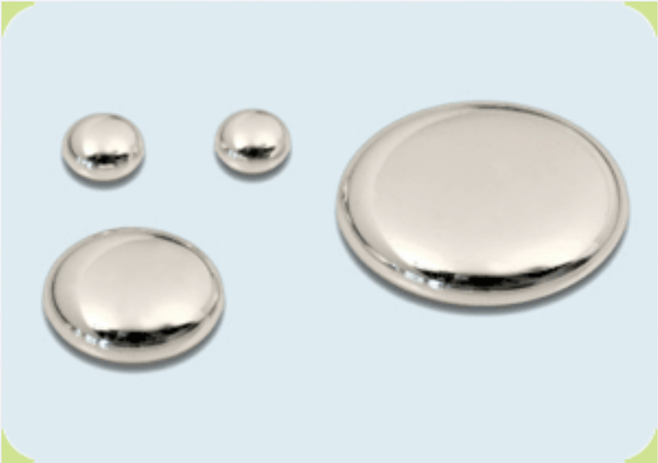
Steel industry

http://www.forbesmarshall.com

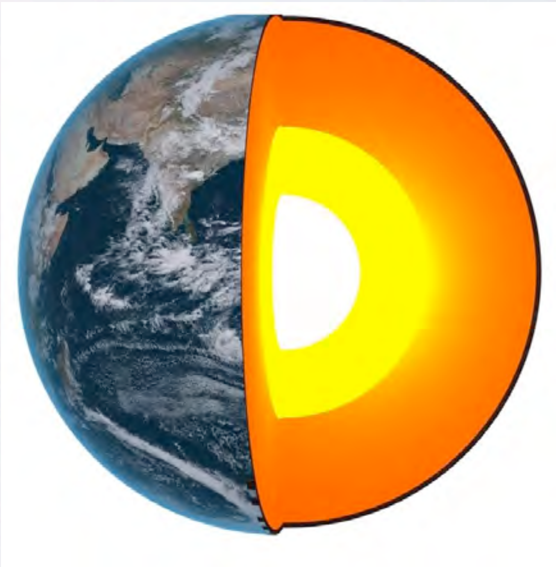
Sun



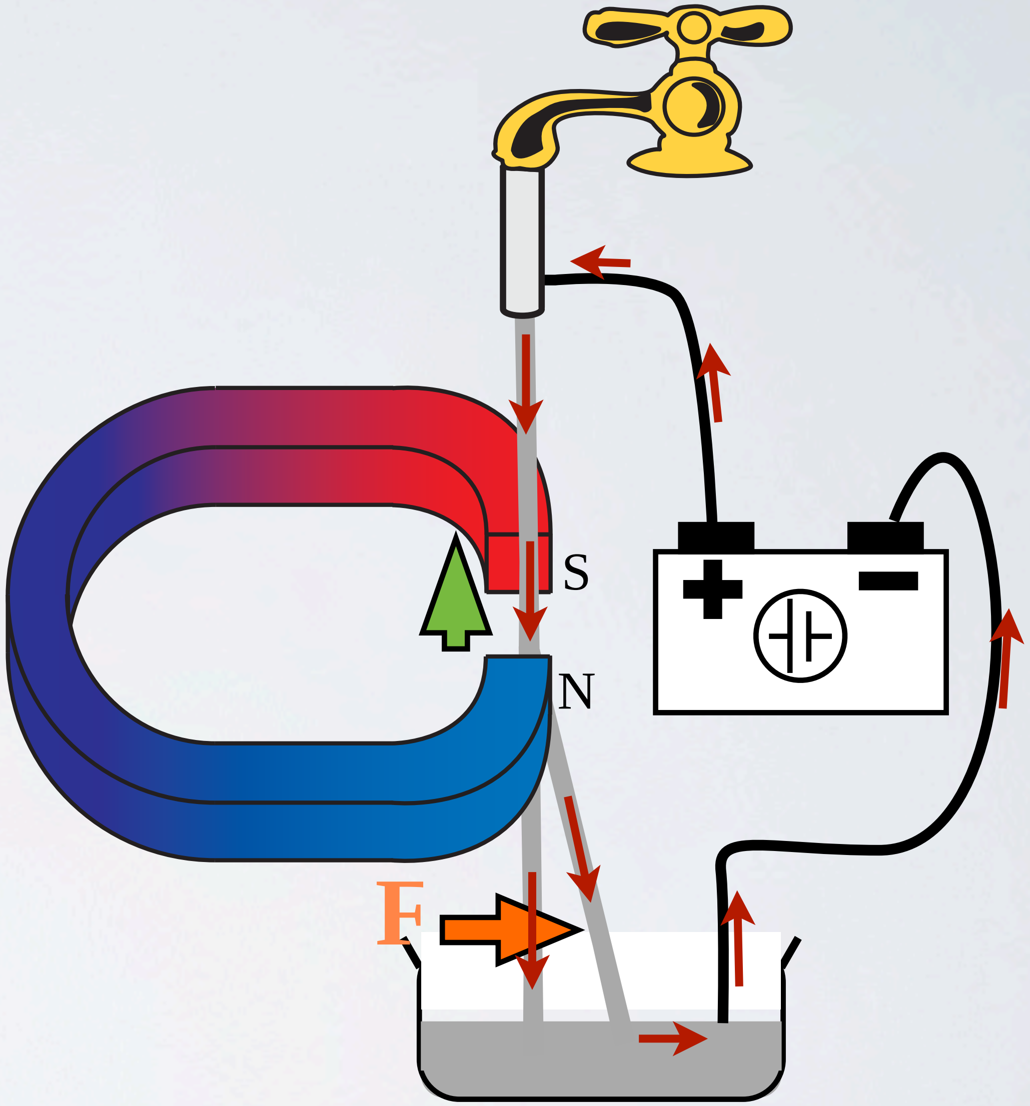
Mercury



Earth's core

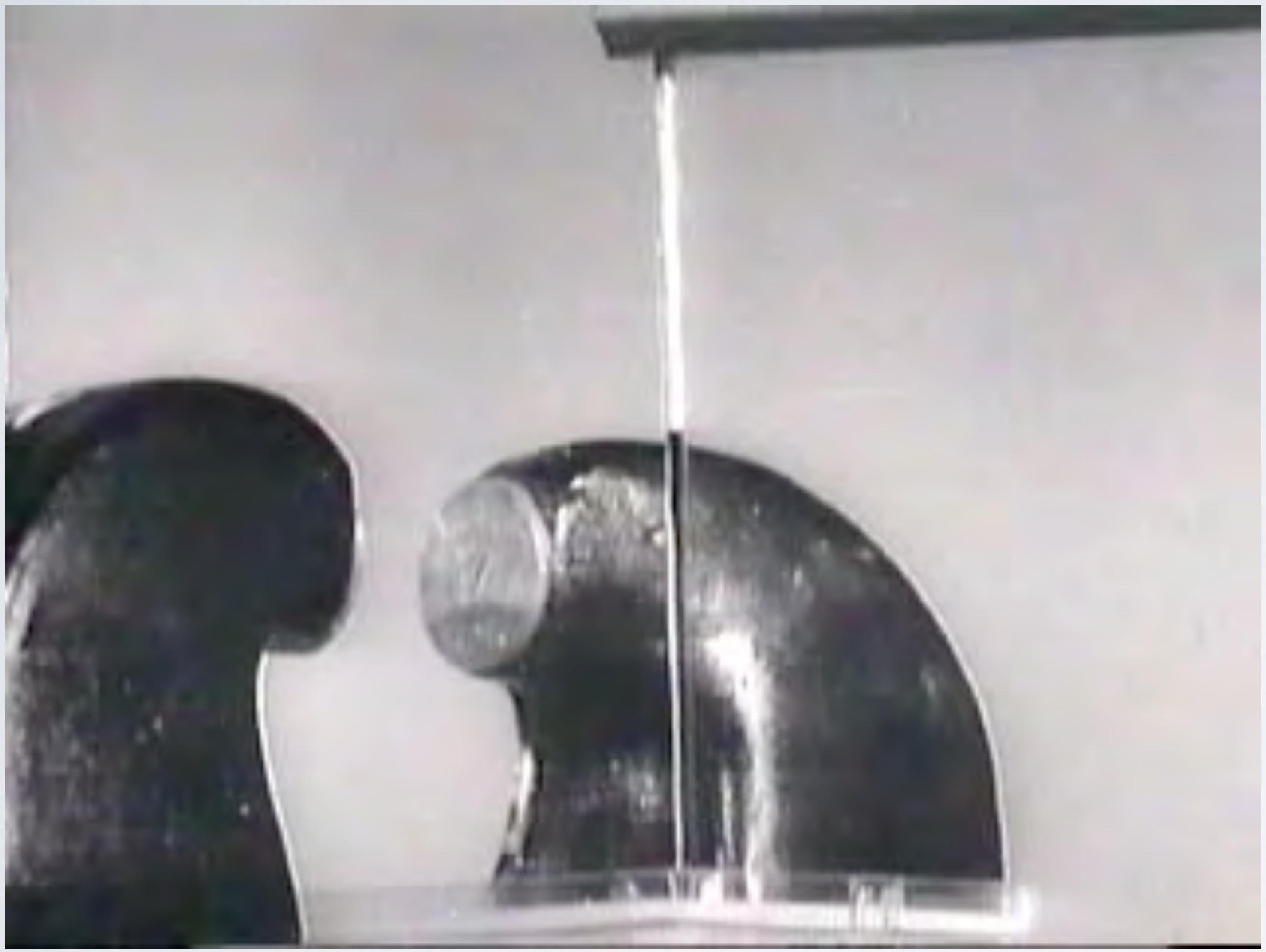


Examples of electrically conducting fluids:



http://www.mit.edu

Examples of electrically conducting fluids:



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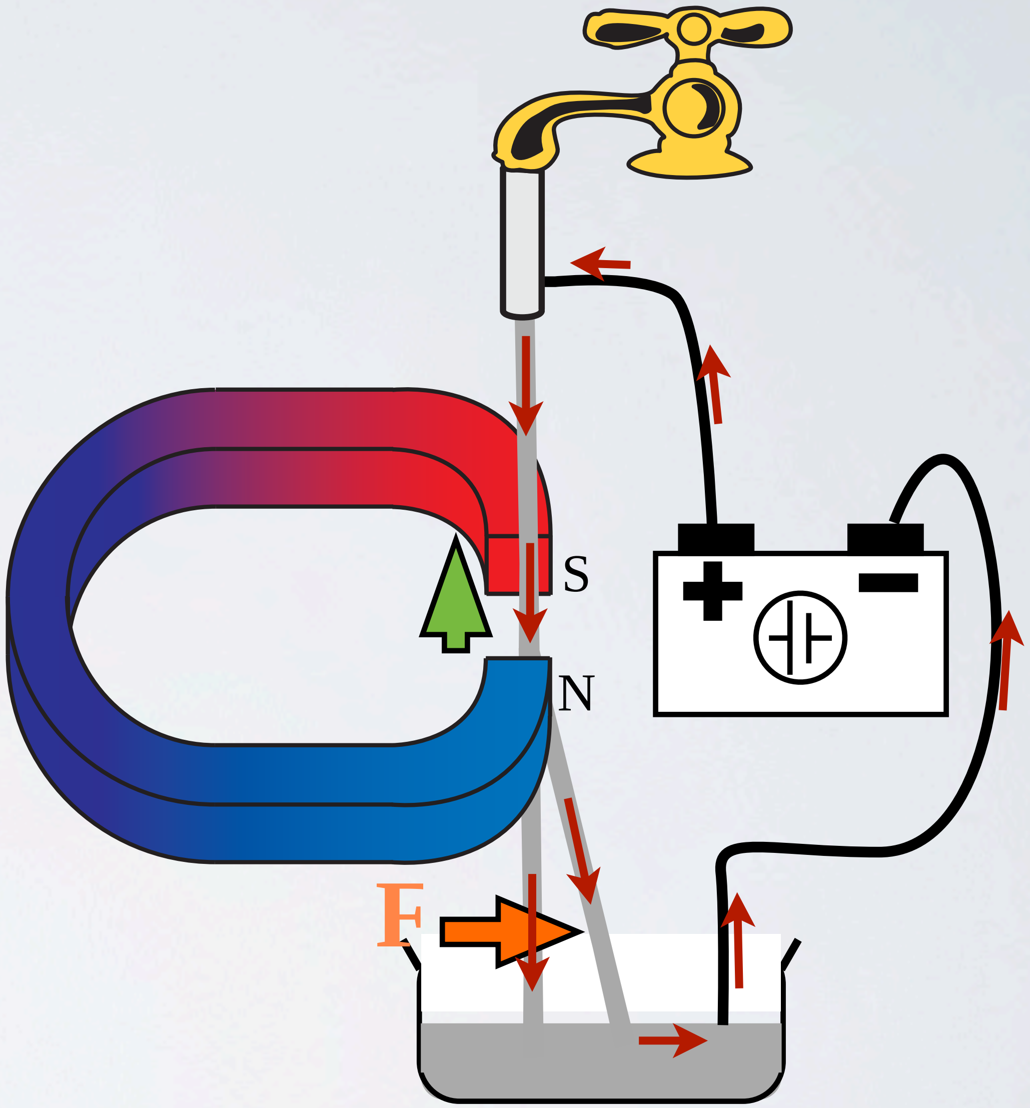


Q: How can these examples be so different?

A: They correspond to very different parameter ranges.

$$\frac{\partial \vec{u}}{\partial t} = -\vec{u}\cdot\nabla\,\vec{u} + \vec{b}\cdot\nabla\,\vec{b} - \nabla p' + \nu\nabla^2\,\vec{u}$$

$$\frac{\partial \vec{b}}{\partial t} = -\vec{u}\cdot\nabla\,\vec{b} + \vec{b}\cdot\nabla\,\vec{u} + \kappa\,\nabla^2\,\vec{b}$$

Reynolds number

$$R_c = \frac{|\vec{u}\cdot\nabla\,\vec{u}|}{|\nu\,\nabla^2\vec{u}|}$$

Magnetic Reynolds number

$$R_m = \frac{|\vec{u}\cdot\nabla\,\vec{b}|}{|\kappa\,\nabla^2\vec{b}|}$$

Magnetic Prandtl number

$$P_m = \frac{R_m}{R_c} = \frac{\nu}{\kappa}$$

Osborne Reynolds
(1842–1912)

Ludwig Prandtl
(1875-1953)

Interaction number

$$N = \frac{|\vec{b}\cdot\nabla\,\vec{b}|}{|\vec{u}\cdot\nabla\,\vec{u}|}$$

Hartmann number

$$H_a^2 = \frac{|\vec{b}\cdot\nabla\,\vec{b}|}{|\nu\,\nabla^2\vec{u}|}$$

$$H_a = \sqrt{N\,R_c}$$

Q: Can we simplify the equations if one of these parameters is small?

A: Yes!

$H_a \ll 1 \quad N \ll 1$ The fluid ignores the magnetic field

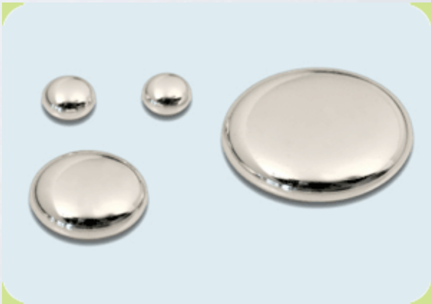
$R_m \ll 1$ The magnetic field is almost unaffected by the flow.

However, imposed magnetic field may strongly affect the flow.

Quasi-static approximation :

$$\frac{\partial \vec{u}}{\partial t} = -\vec{u} \cdot \nabla \vec{u} + \vec{b_0} \cdot \nabla \vec{b}' - \nabla p + \nu \nabla^2 \vec{u}$$

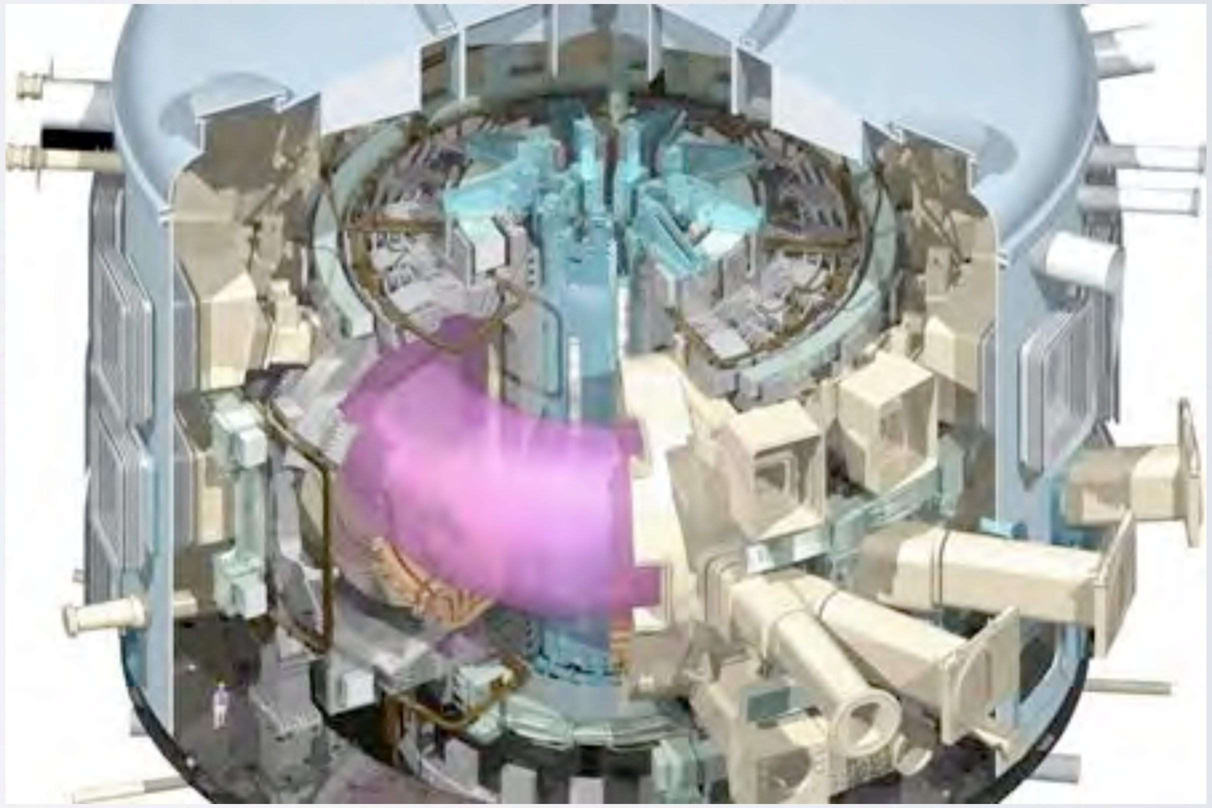
$$\kappa \nabla^2 \vec{b}' = -\vec{b}_0 \cdot \nabla \vec{u}$$



Q: Can we really represent all the former examples with the "simple" MHD equations?

A: No!

Like in standard fluid dynamics, the flow can be **compressible**. Additional equations for the pressure and the temperature are then needed.



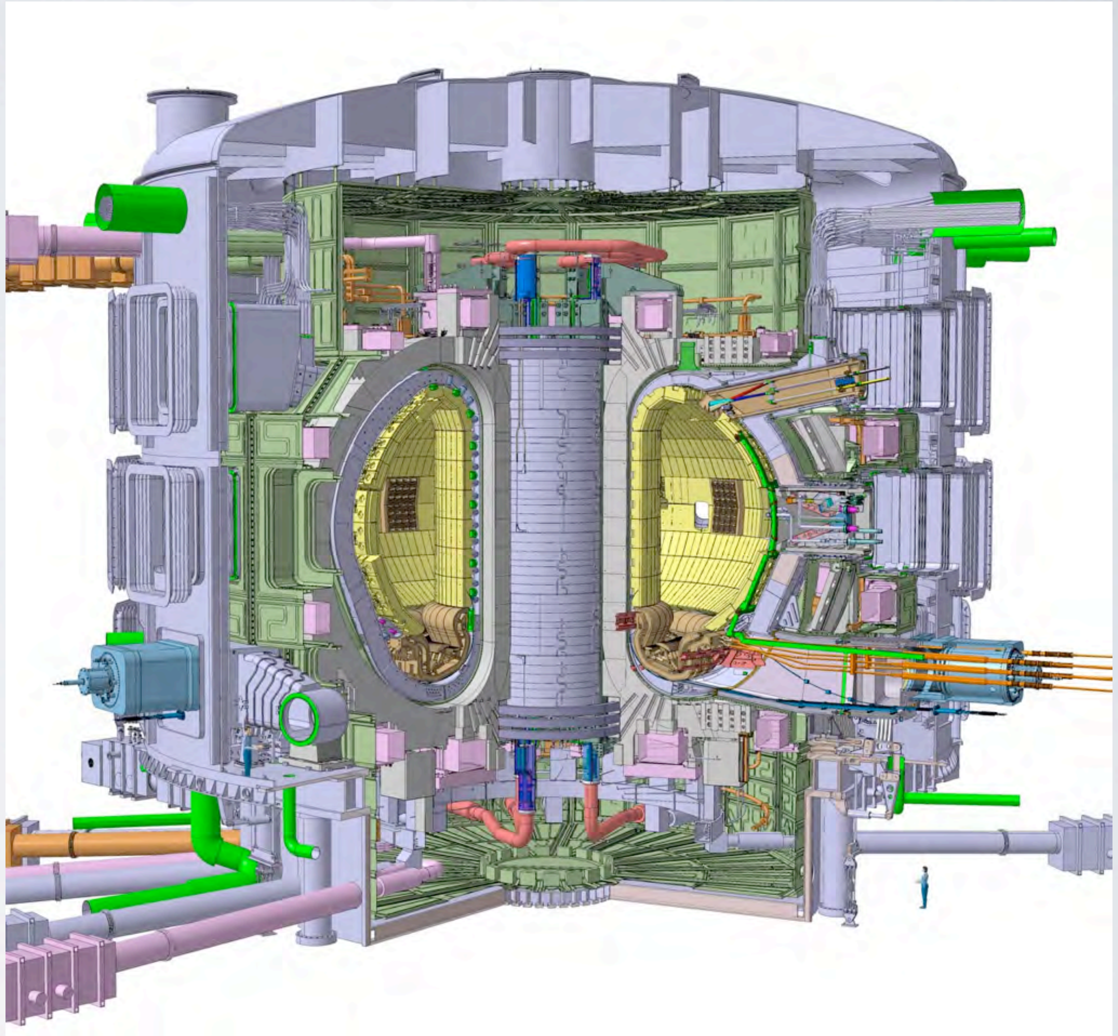
http://www.iter.org

Like in combustion problems, the **fluid species** may be important. Even in absence of reaction, different species may interact differently with the electromagnetic field.

Multi-fluid MHD is often needed to describe the fluid regime of **fusion plasmas**.

The ITER experiment will be a very complex system in which controlling the MHD instabilities will be essential.

- Edge localized modes
- Tearing modes
- Resistive ballooning modes
- ...



http://www.iter.org

ITER is an international collaboration between China, European Union, India, Japan, Korea, Russia and USA.

Currently under construction in the south of France, it aims to demonstrate that fusion is an energy source of the future.

Part 3

Numerical results

- The number of numerical results that could be presented is enormous

    - Astrophysics (solar wind, accretion disk, solar flares, ...)
    - Earth's magnetic field
    - Liquid metal flows (steel & aluminum industries, blanket in tokamaks, ...)
    - Plasma physics (fusion plasmas, magnetic control in arcs,...)

- We have chosen to limit the presentation to two sub-domains

    - Quasi-static approximation of liquid metal flows (low magnetic Reynolds numbers) **B. Knaepen & R. Moreau, Annu. Rev. Fluid Mech. (2008)**

    - Fully developed MHD turbulence (high magnetic Reynolds numbers) **P. Mininni, Annu. Rev. Fluid Mech. (2010 online, 2011 paper?)**

QS MHD flows (DNS)

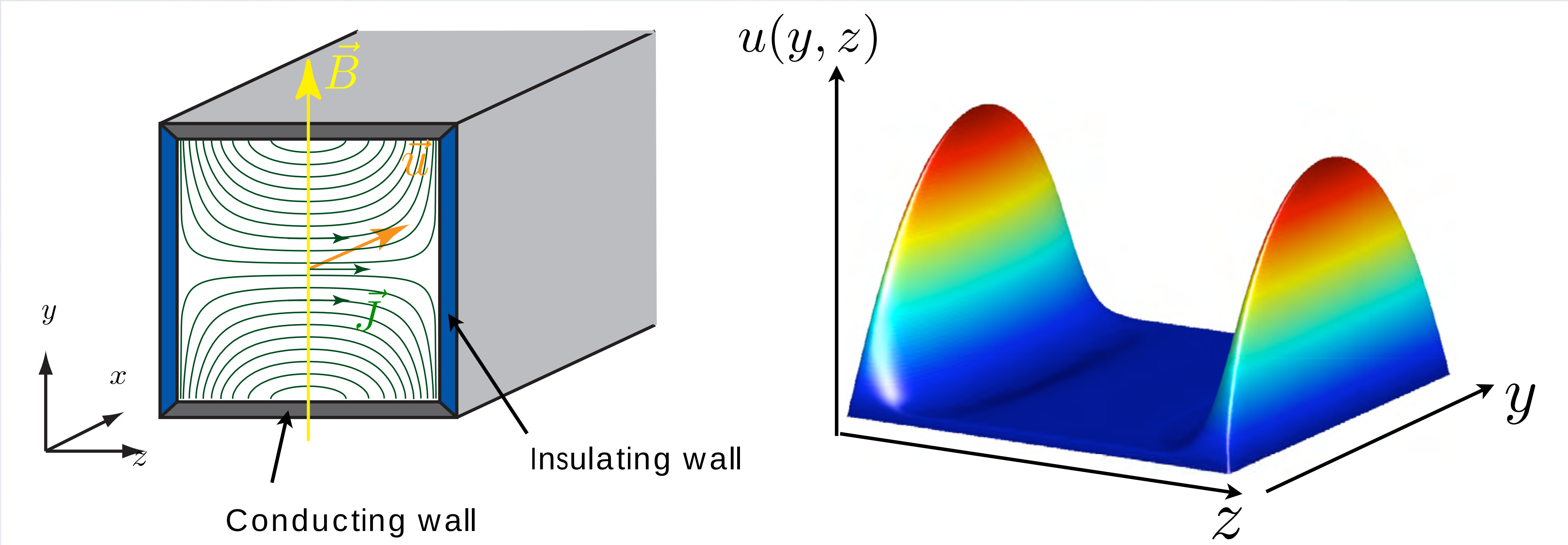
$$\frac{\partial \vec{u}}{\partial t} = -\vec{u} \cdot \nabla \vec{u} + \vec{b_0} \cdot \nabla \vec{b}' - \nabla p + \nu \nabla^2 \vec{u}$$

$$\kappa \nabla^2 \vec{b}' = -\vec{b}_0 \cdot \nabla \vec{u} \qquad \nabla \cdot \vec{u} = 0$$

- Main issue is usually very practical: Predict the flow / accurate simulations

QS MHD flows (DNS)

Laminar flows at small Reynolds numbers

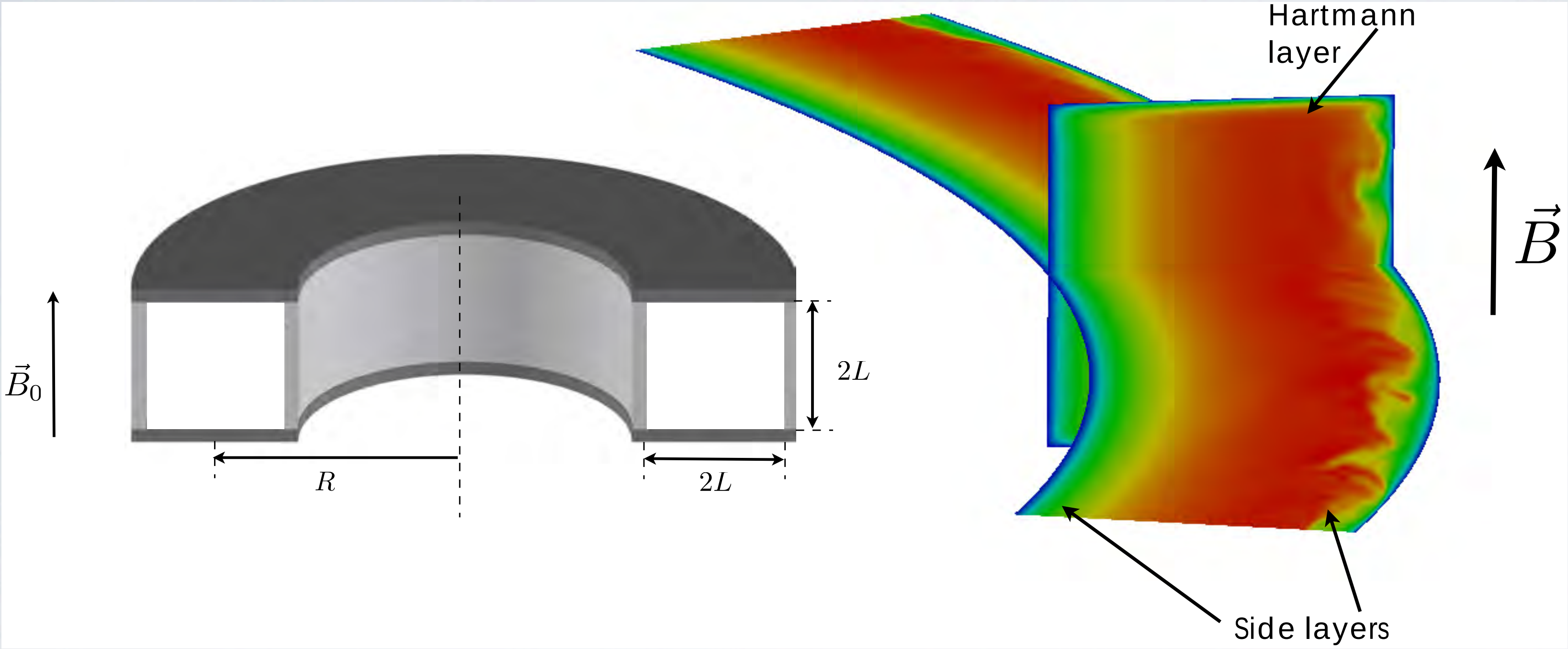


Quasi-static simulation of a rectilinear square duct.

U. Mühler & L. Bühler, *Magnetofluiddynamics in channels and containers*. Springer, 2001.
S. Smolentsev, Theor. Comput. Fluid Dyn. 2009.
...

QS MHD flows (DNS)

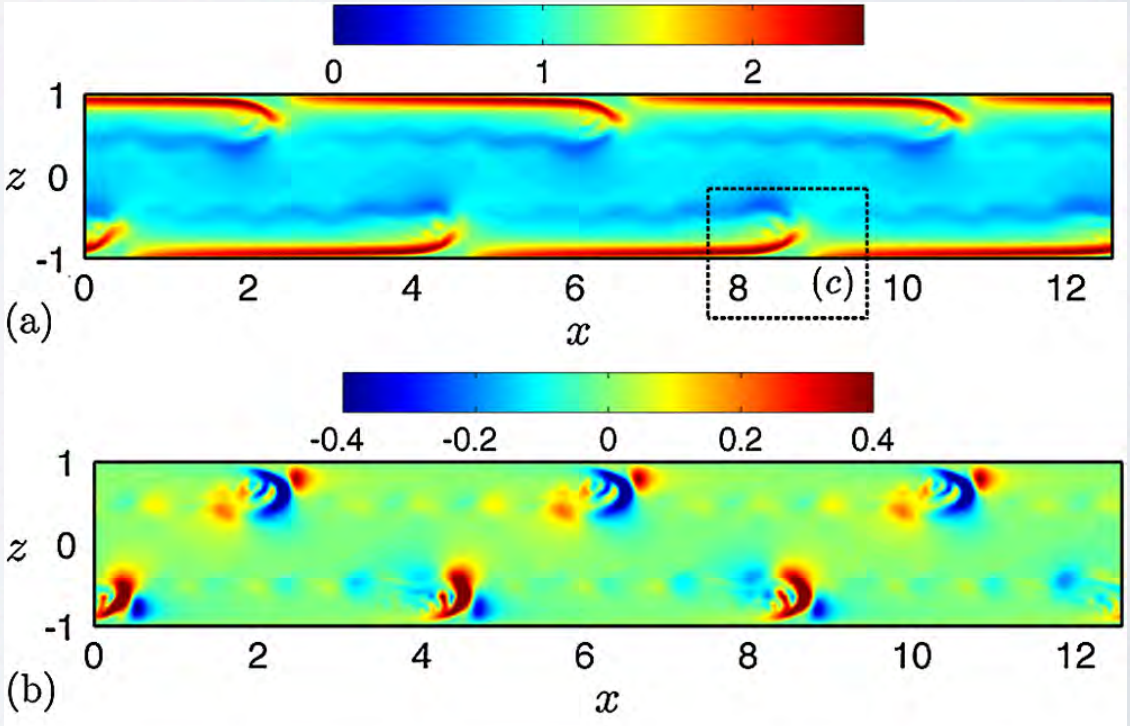
In some cases, **transitional** regimes can be observed



Quasi-static simulation of a toroidal square duct

S. Vantieghem, Ph.D. 2011 (DNS)
P. Moresco & T. Alboussière, JFM, 2004 (EXP)

M. Kinet, B. Knaepen & S. Molokov, PRL 2009 (DNS)



Quasi-static simulation of a rectilinear square duct

See also
Session MY, Tuesday 8pm
"magnetohydrodynamics"

QS MHD flows (LES & DNS)

Large Eddy Simulation is an attempt to solve numerically the fluid or MHD equation on a coarse grid without capturing all the small scales that can develop in a turbulent flow.

Mathematically, this is achieved by filtering the equations.

BUT :

$$\overline{u}_i = \mathrm{Filter}[u_i]$$

$$\tau_{ij} = \overline{u_i\,u_j} - \overline{u}_i\,\overline{u}_j$$

$$\mathrm{Filter}[u_i\,u_j] \neq \mathrm{Filter}[u_i]\,\mathrm{Filter}[u_j]$$

The new terms are linear.

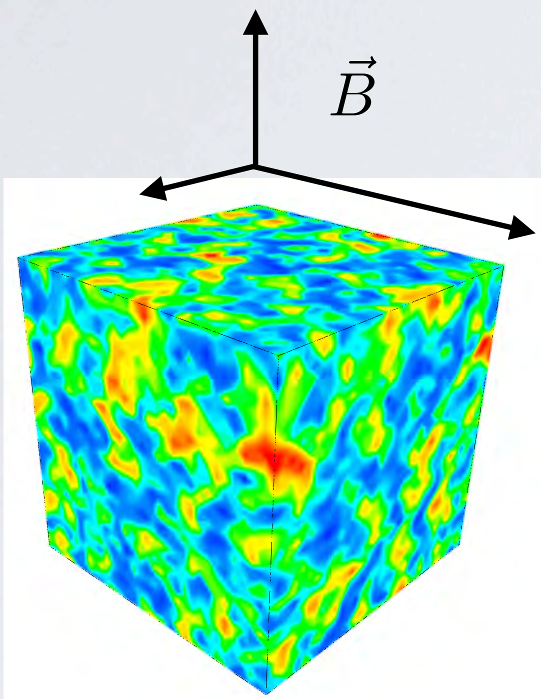
No additional modeling is required

$$\frac{\partial \overline{u}_i}{\partial t} = -\overline{u}_j \partial_j \overline{u}_i + \vec{b}_0 \cdot \nabla\, \overline{b}_i' - \nabla p + \nu \nabla^2\, \overline{u}_i - \partial_j \tau_{ij}$$

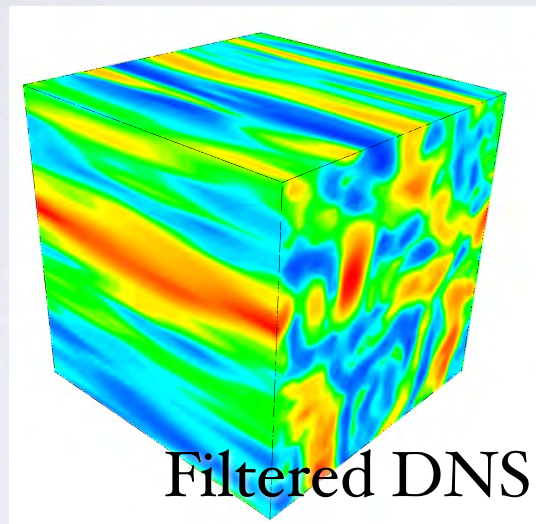
$$\kappa\, \nabla^2\, \overline{b}_i' = -\vec{b}_0 \cdot \nabla\, \overline{u}_i$$

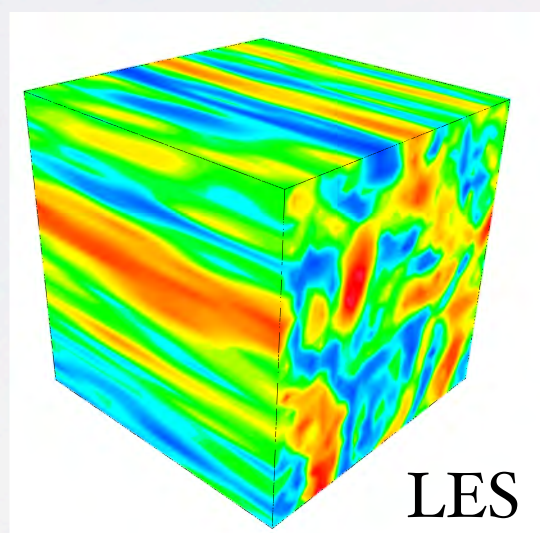
QS MHD flows (LES & DNS)

Fully developed turbulence



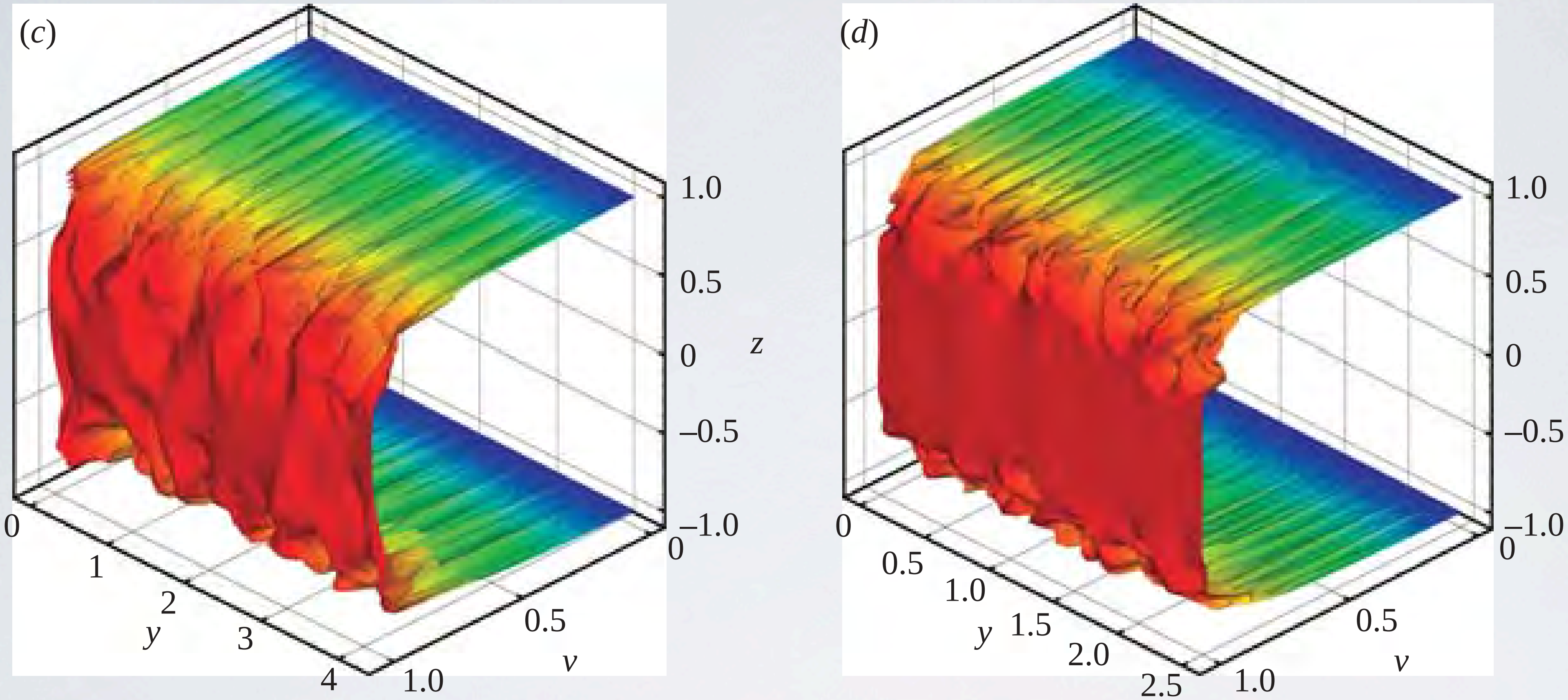
Quasi-static simulation of homogeneous, anisotropic turbulence





B. Knaepen & P. Moin, PoF 2004 (LES & DNS)
A. Vorobev, O. Zikanov, P.A. Davidson, B. Knaepen, PoF 2005 (LES & DNS)
...

QS MHD flows (LES & DNS)



T. Boeck, D. Krasnov & E. Zienicke, JFM 2007 (DNS)

Quasi-static simulation of a channel flow

H. Kobayashi, PoF 2006 (LES, Channel)
I. Sarris, S.C. Kassinos, DC, PoF 2007 (LES, Channel)
S. Satake, T. Kunugi, S. Smolentsev, J. of Turbulence (2004) (DNS, Pipe)
...

- LES with the "traditional" models performs very well for moderate Hartmann numbers.
- Hartmann layers remain an issue.

Full nonlinear MHD / DNS

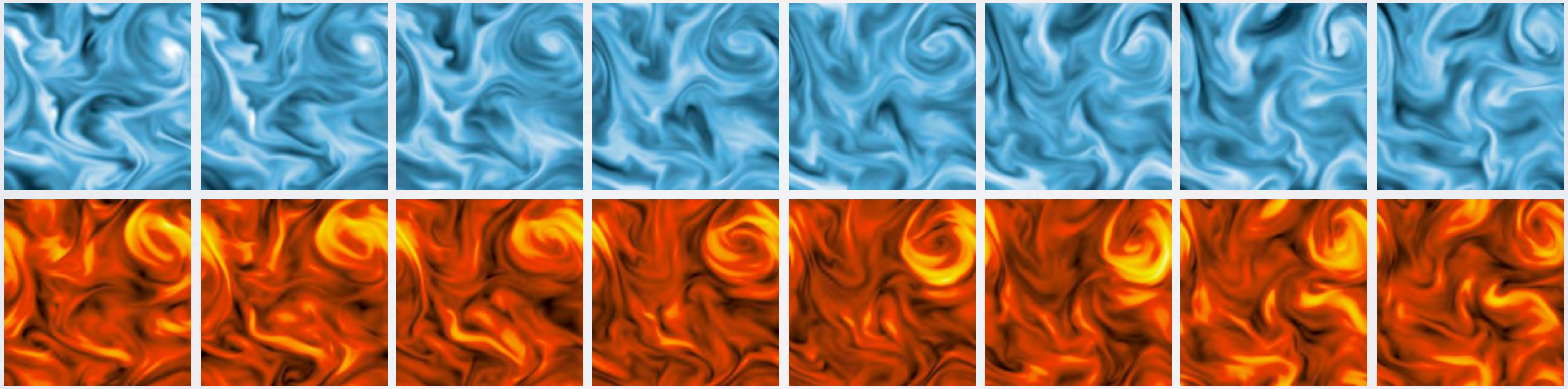
$$\frac{\partial \vec{u}}{\partial t} = -\vec{u} \cdot \nabla \vec{u} + \vec{b} \cdot \nabla \vec{b} - \nabla p' + \nu \nabla^2 \vec{u} \qquad \nabla \cdot \vec{u} = 0$$

$$\frac{\partial \vec{b}}{\partial t} = -\vec{u} \cdot \nabla \vec{b} + \vec{b} \cdot \nabla \vec{u} + \kappa \nabla^2 \vec{b} \qquad \nabla \cdot \vec{b} = 0$$

- Main issues are more theoretical:
    - Understand the energy spectrum
    - Understand the role of the nonlinear invariants
    - Characterize the growth of the magnetic field (dynamo effect, not discussed here)
    - ...

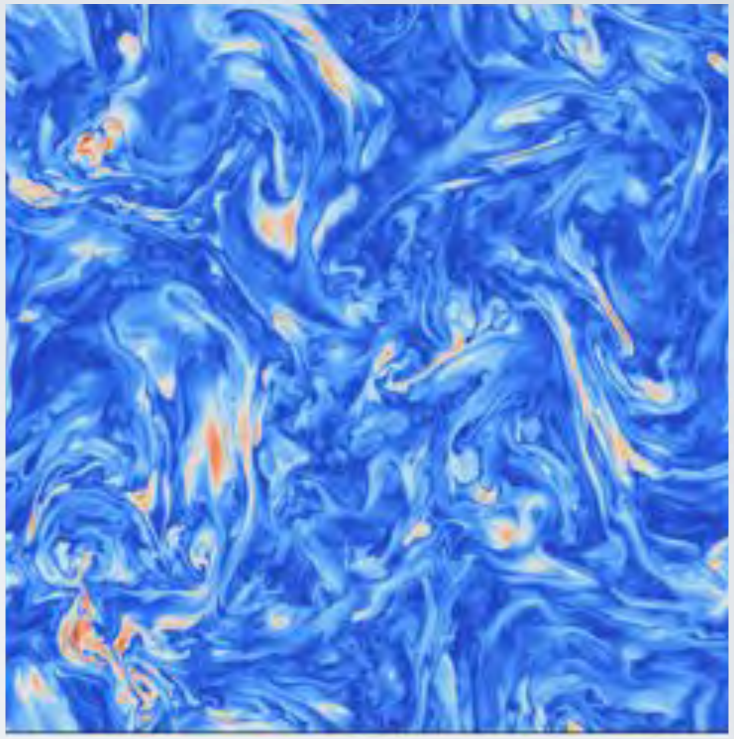
Full nonlinear MHD / DNS

D. Biskamp & W.C. Müller , PRL 1999, PoP 2000,
W.C. Müller , D. Biskamp & R. Grappin, PRE 2003
A. Alexakis, P. Mininni & A. Pouquet, PRL 2005, PRE 2005, Astoph. J. 2006,New J. Phys 2007
A . Brandenburg, K. Subramanian, Physics Report 2005
DC, O. Debliquy, B. Knaepen, B. Teaca & M. Verma, J. of Turbulence 2005
B. Teaca, M. Verma, B. Knaepen & DC, PRE 2009
...



Full nonlinear MHD / DNS

DNS 1024^3
Velocity field (left)
Magnetic field (right)



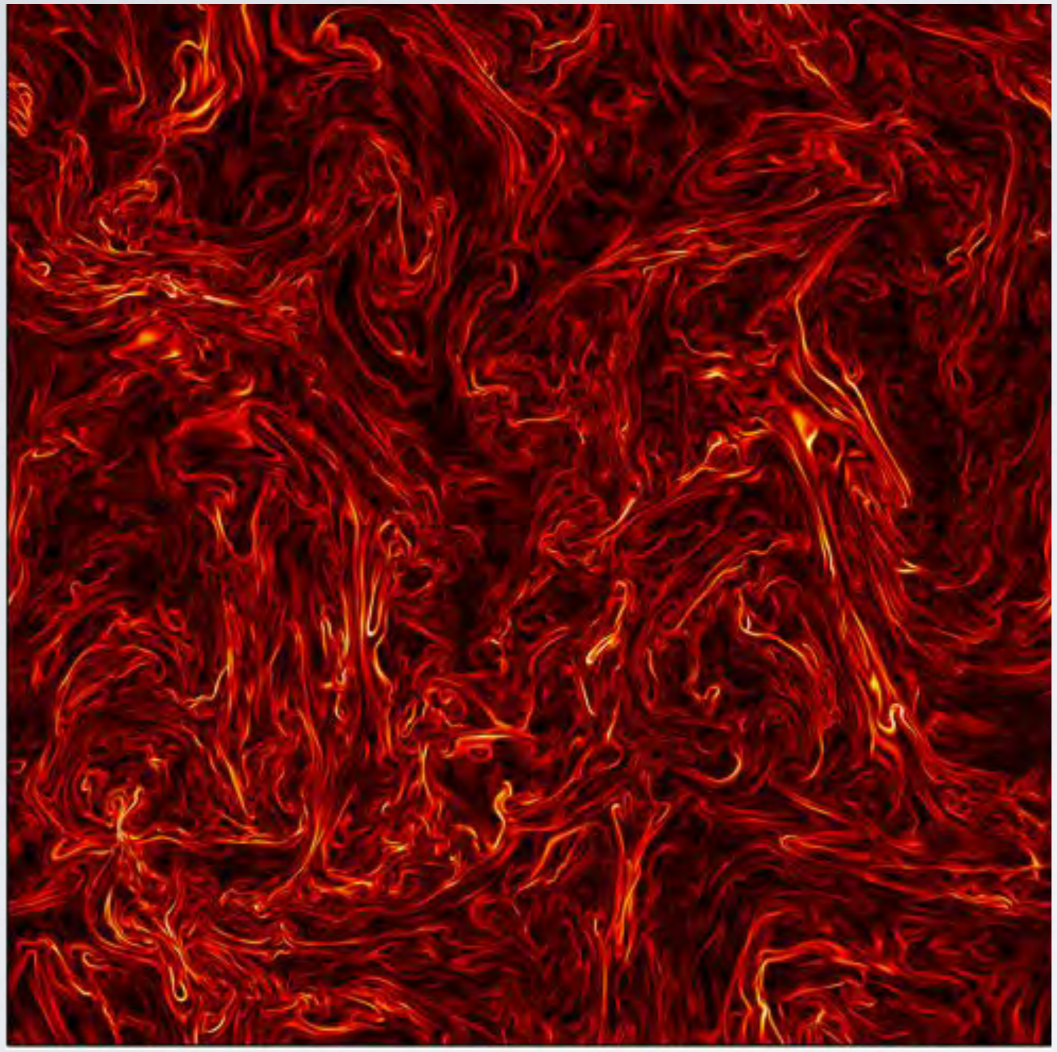
$$E(k) \propto k^{-3/2}$$

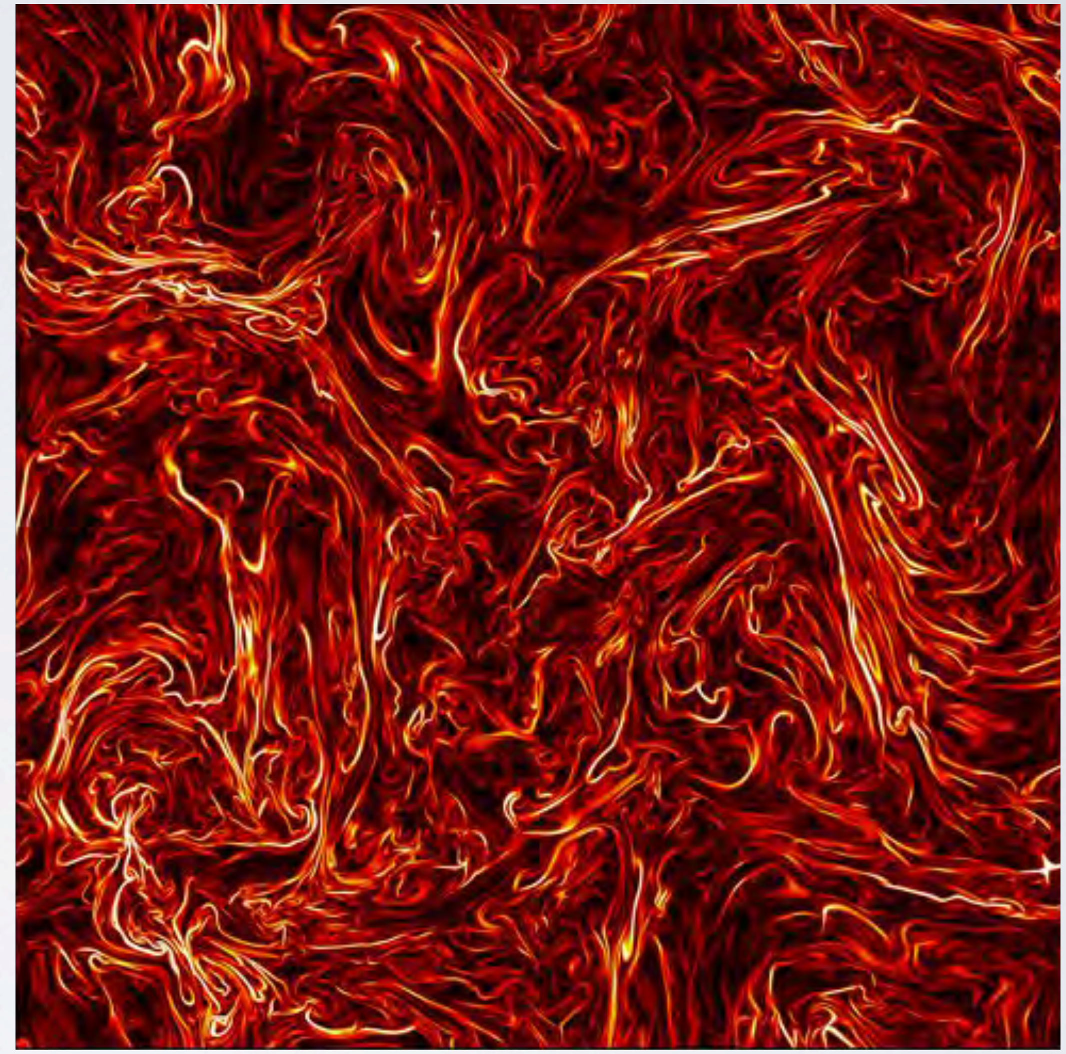
P. S. Iroshnikov, Astron. Zh. (1963)
R. H. Kraichnan, PoF (1965).

$$E(k) \propto k^{-5/3}$$

A.N. Kolmogorov, Sov. Phys. Dokl. (1941).

Full nonlinear MHD / DNS





$\vec{\omega} = \nabla \times \vec{u}$

$\vec{j} = \frac{1}{\mu_0} \nabla \times \vec{B}$

- Spectra are closer to Kolmogorov
- Filamentary structure of the current is even more marked than for the vorticity
- Current is highly correlated with the vorticity

Full nonlinear MHD / LES

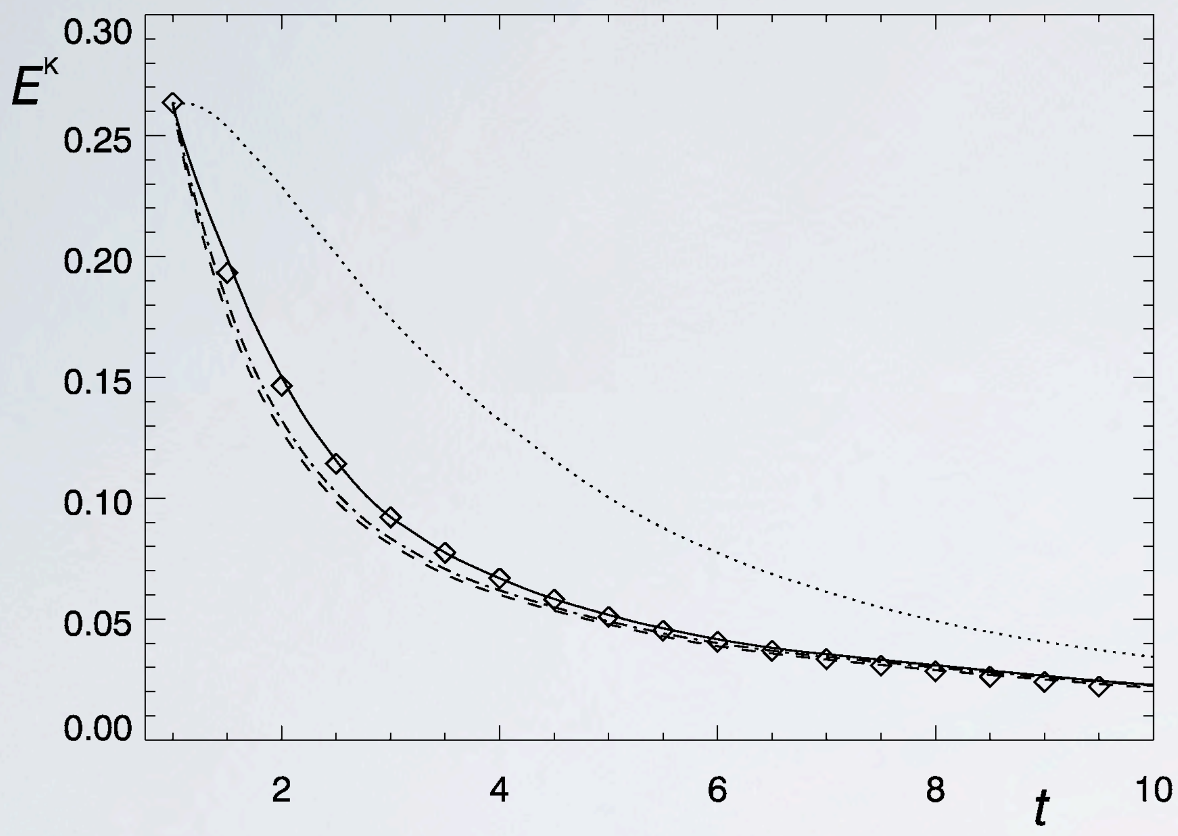
Large eddy simulations are more difficult.

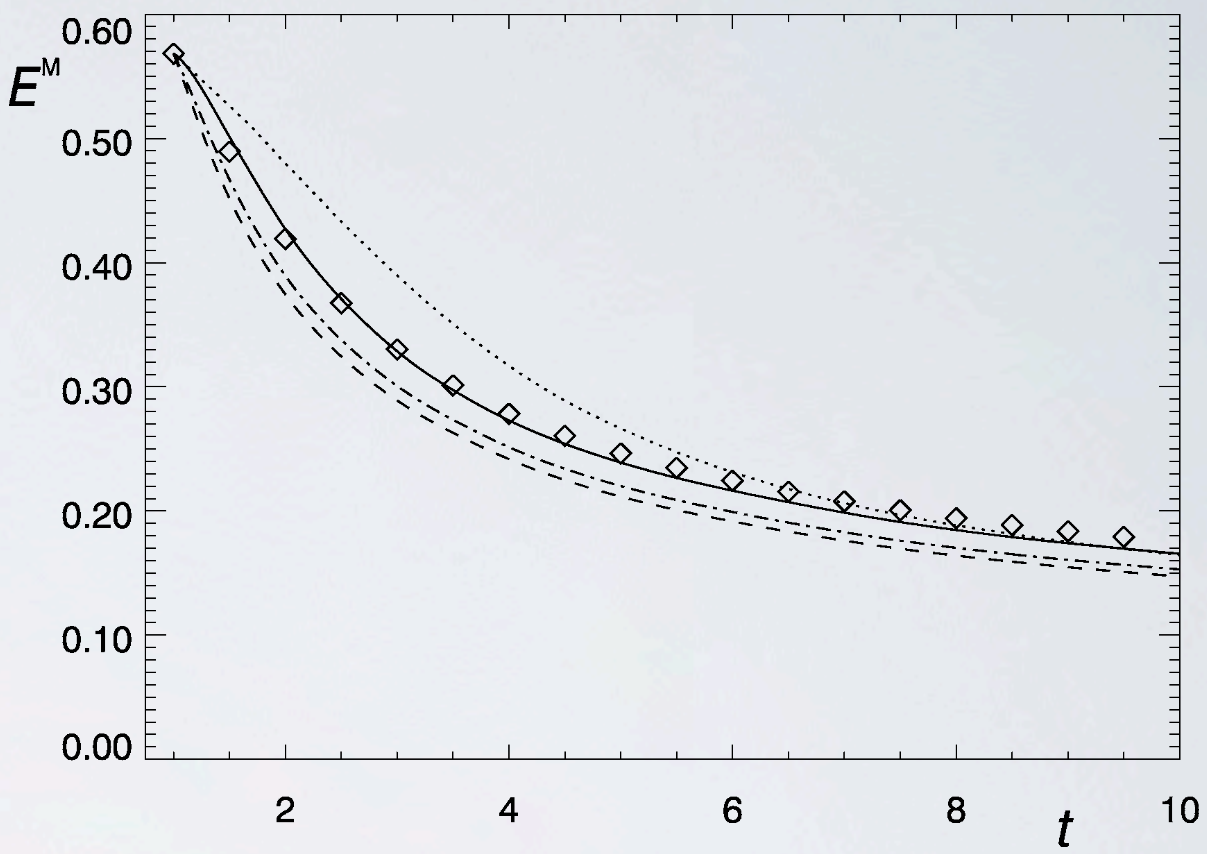
There are more nonlinear terms.

More terms to model if you filter...

A. Yoshizawa (PoF,1987)
T. Passot, H. Politano, A. Pouquet, & P. L. Sulem (Theor. Comput. Fluid Dyn, 1990)
Y. Zhou & G. Vahala (J. Plasma Phys.,1991)
M. L. Theobald, P. A. Fox & S. Sofia (PoP, 1994)
O. Agullo, W.-C. Müller, B. Knaepen & DC (PoP, 2001)
W.-C. Müller & DC (PoP, 2002)
B. Knaepen & P. Moin (PoF, 2004)
P. D. Mininni, D.C. Montgomery & A Pouquet (PoF, 2005 - PRE, 2005).
J. Pietarila Graham,P. D. Mininni & A. Pouquet (PRE 2009)

Full nonlinear MHD / LES





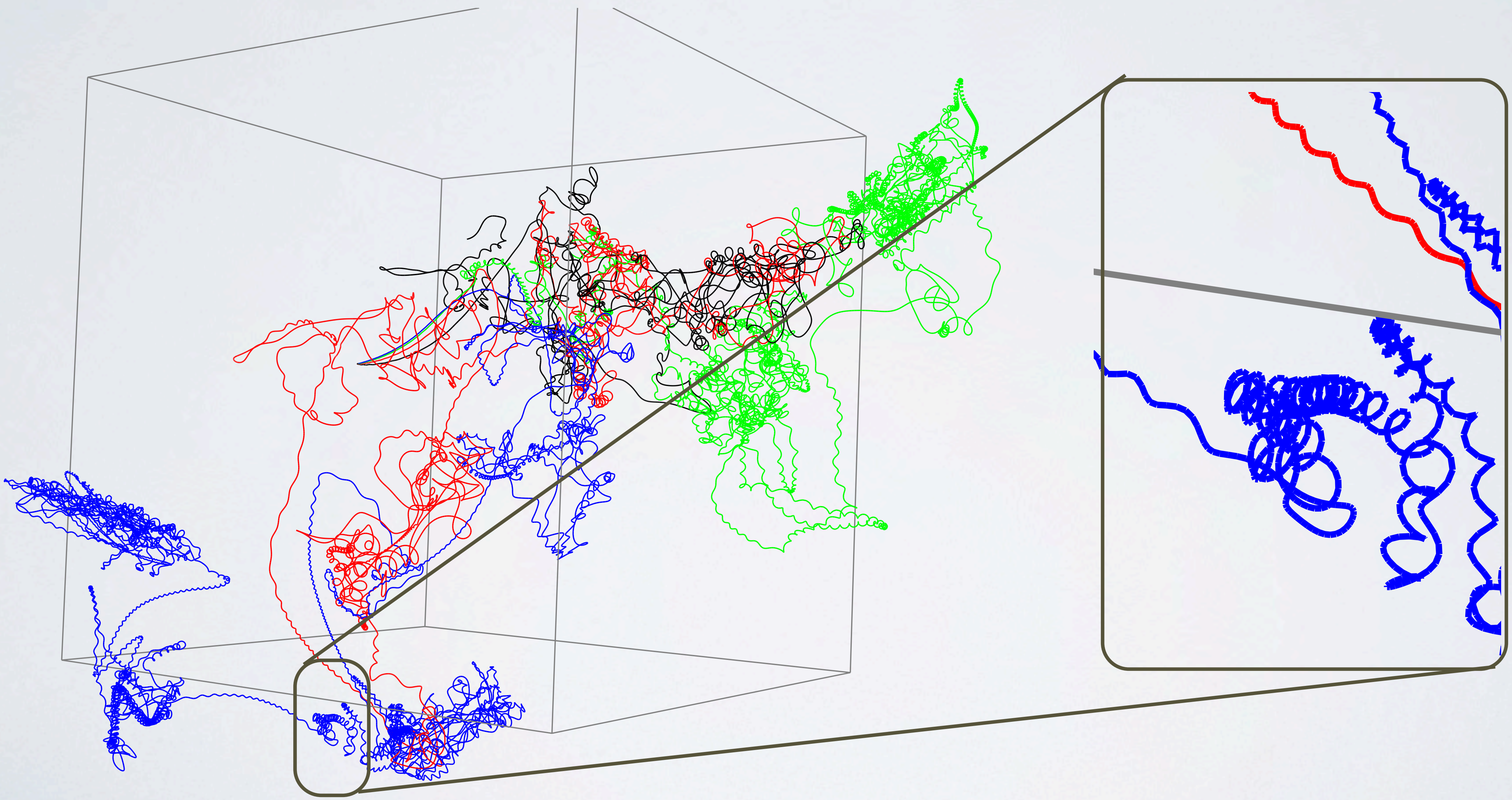
- Even for magnetic Prandtl number =1, magnetic energy is easier to capture
- Best performances are achieved for the dynamic Smagorinsky model in the velocity equation and a dynamic magnetic resistivity model in the magnetic field equation with

$$\kappa_m \propto \Delta^2 \sqrt{|\vec{j} \cdot \vec{\omega}}$$

W.C. Müller & DC, Comp. Phys. Com. 2002

Full nonlinear MHD / trajectory of charged particles

Influence of electric and magnetic field on test particles (important for fusion plasmas, since confinement is the main issue)



C. Lalescu, B. Teaca & DC, J. Comp. Phys. 2010

Full nonlinear MHD / Energy transfers

There is a also number of results on the energy transfers in the MHD cascade

- locality
- direct/inverse cascade
- scale invariance

A. Alexakis, P. Mininni & A. Pouquet, PRL 2005, PRE 2005
DC, O. Debliquy, B. Knaepen, B. Teaca & M. Verma, J. of Turbulence 2005
B. Teaca, M. Verma, B. Knaepen & DC, PRE 2009
J.A. Domaradzki, B.Teaca, DC, PoF 2010
P. Mininni, Annu. Rev. Fluid Mech. (2010 online, 2011 paper?)

but no time to discuss all these issues today...

Concluding remarks

- Electrically conducting fluids are very diverse
  - In size
  - In composition
  - In behavior

- In many systems, conducting fluids are turbulent
  - Their analysis can be very much inspired by fluid dynamics
  - However, the analogy with fluid dynamics has limitations

- Computational analysis of MHD flows is very rich
  - Even laminar flows are challenging
  - More nonlinear invariants
  - Large eddy simulation is quite successful

Concluding remarks

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- Computational analysis of MHD flows is very rich
  - Even laminar flows are challenging
  - More nonlinear invariants
  - Large eddy simulation is quite successful

There is certainly room for another famous physicist or engineer and I would be glad to add the picture of one of you in my next talk

???

(19.. - 2...)