Turbulence in Strongly-Stratified Flows

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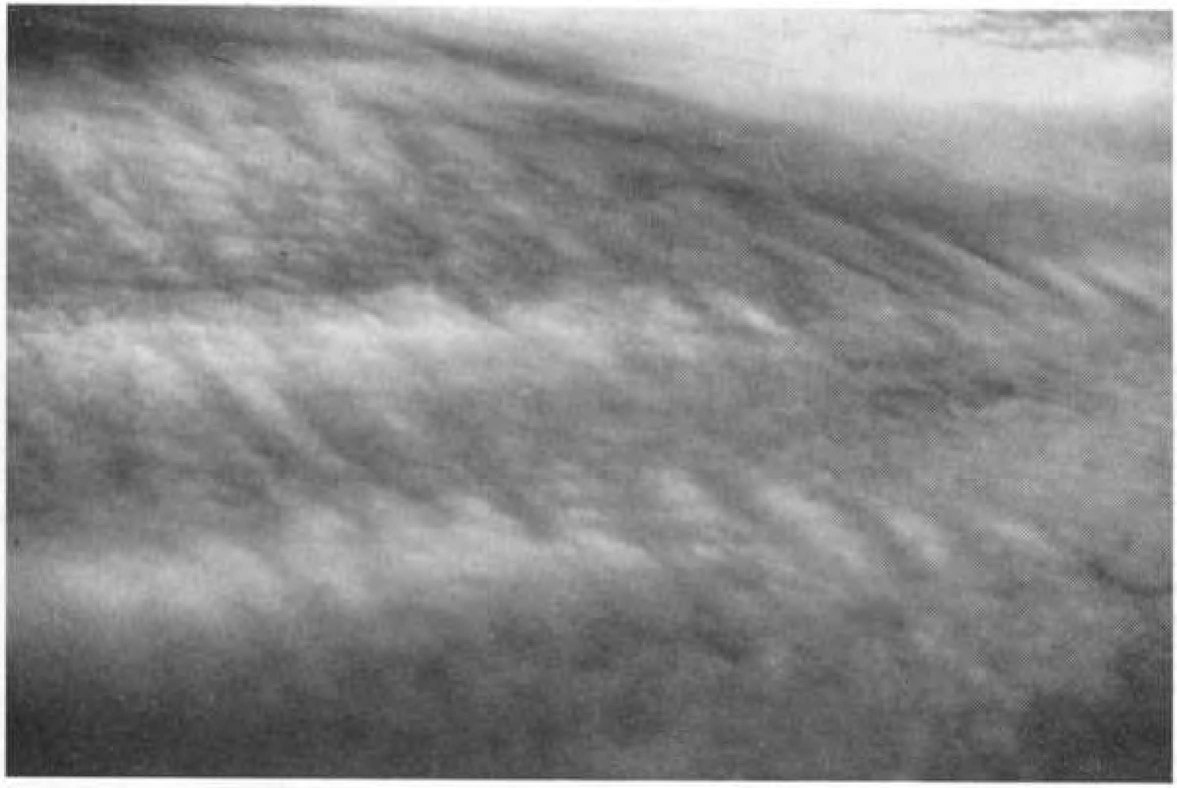
Examples of Instabilities – Atmosphere

Denver, Colorado, 1953 (photo by Paul E. Branstine)

P. G. Drazin and W. H. Reid, *Hydrodynamic Stability*, 1981

Examples of Instabilities – Atmosphere

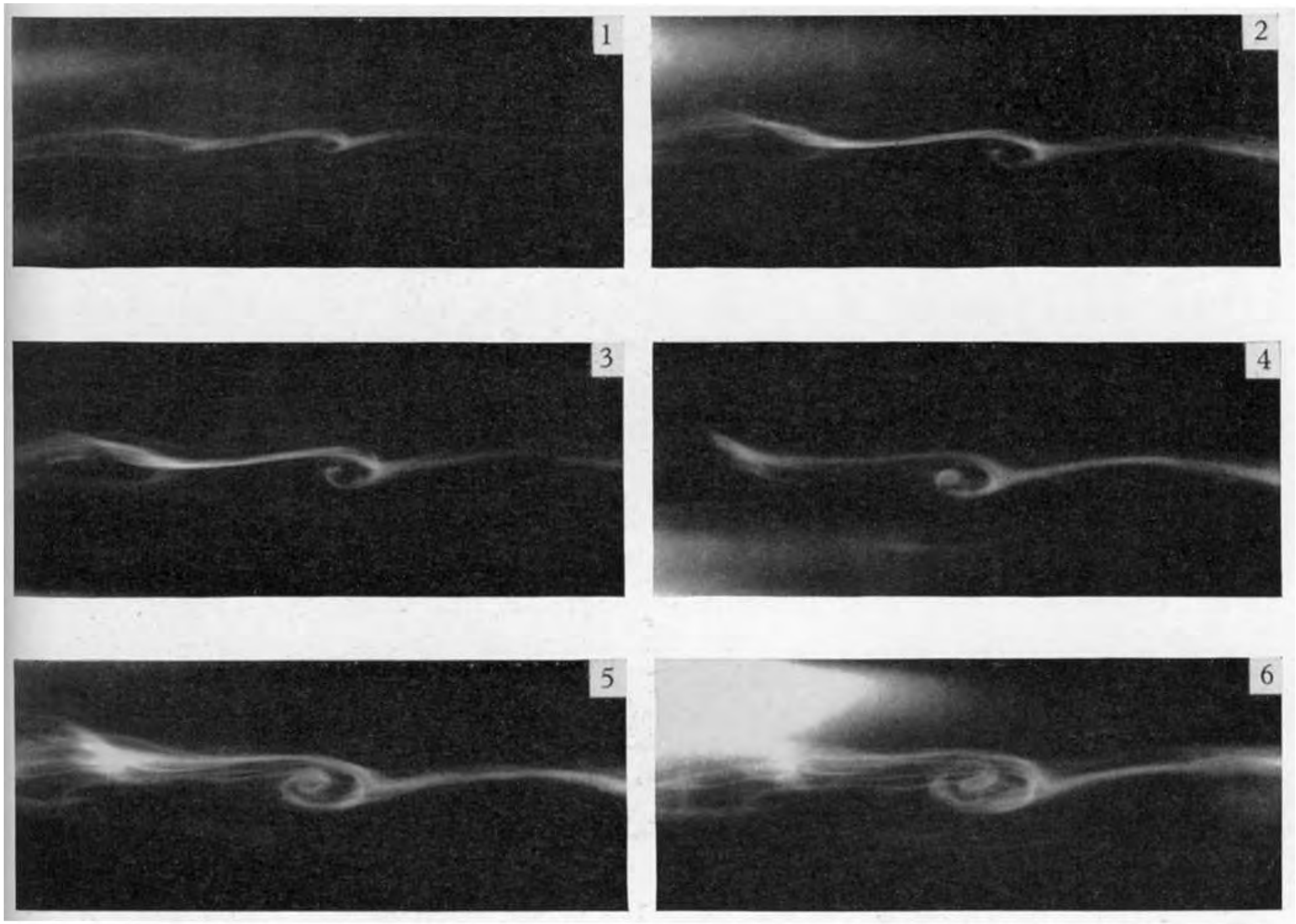


P. Atsavapranee and M. Gharib, *J. Fluid Mech.*, **342**, 1997

Examples of Instabilities – Ocean

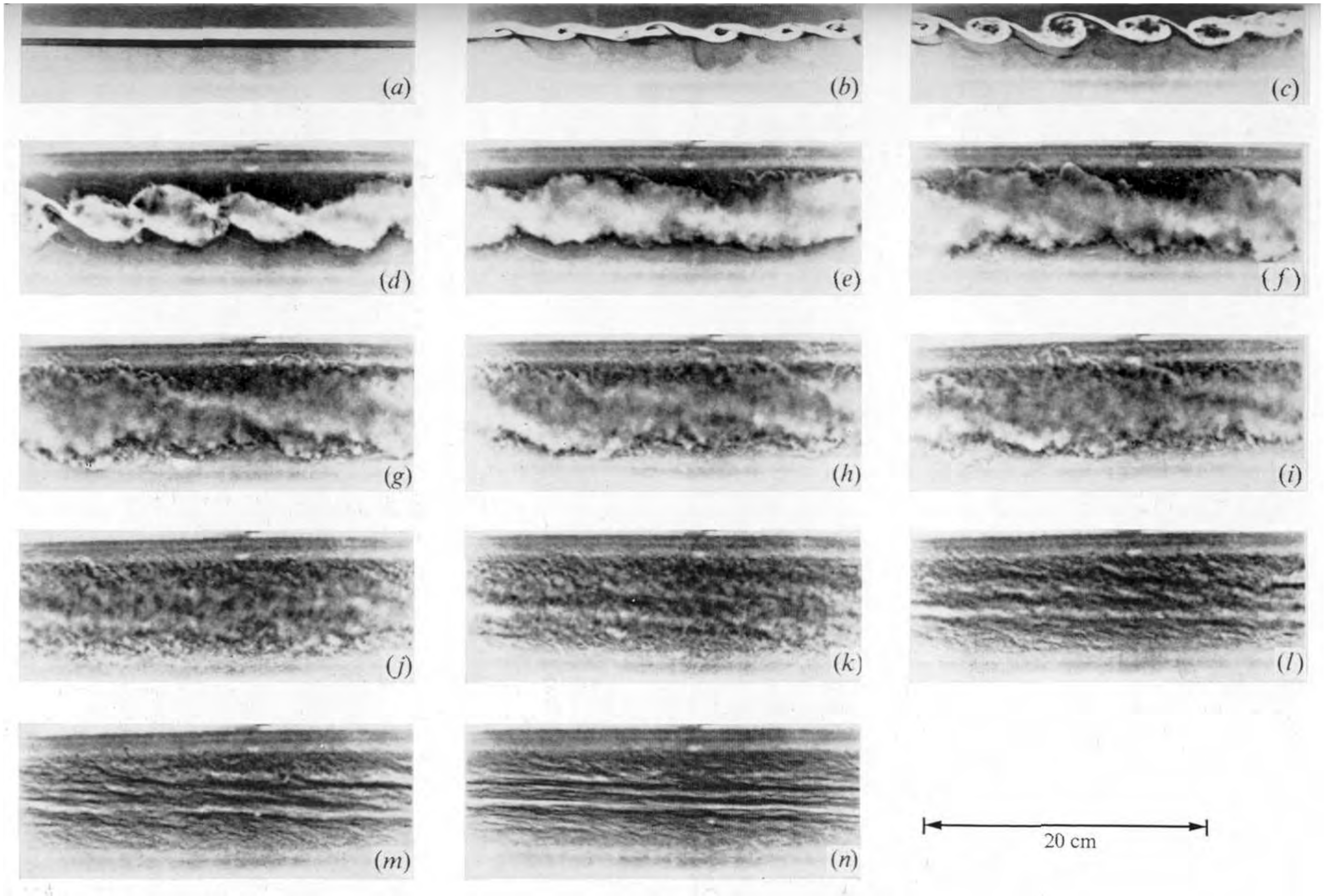
J. D. Woods, *J. Fluid Mech.*, **32**, 1968

Examples of Instabilities – Ocean



J. D. Woods, *J. Fluid Mech.*, **32**, 1968

Examples of Instabilities – Laboratory



S. A. Thorpe, *J. Fluid Mech.*, **46**, 1971

Interest – Sources of the Turbulence

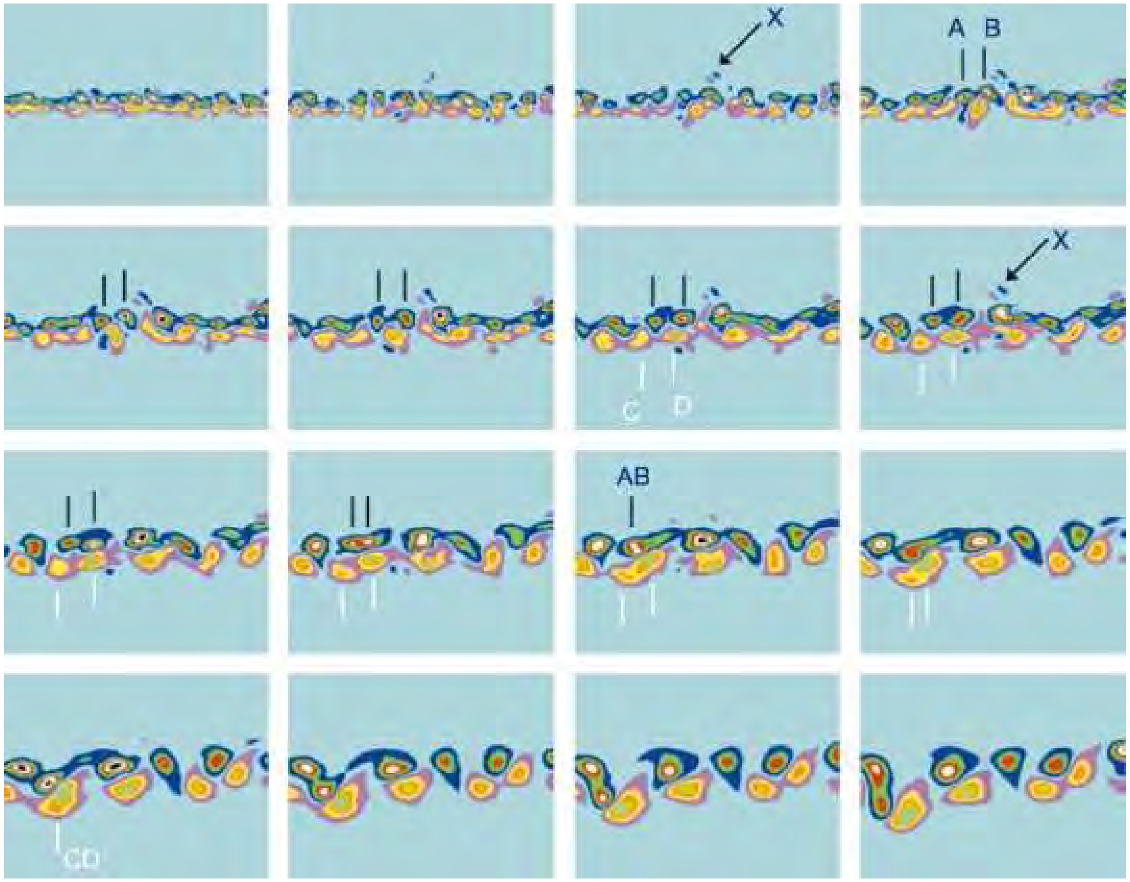
- Larger-scale motions, dynamics which lead to 3-D turbulence in a strongly, stably-stratified environment
  - control the energy transfer to smaller scales
  - overall dissipation rate, mixing rate, dispersion
  - often need to be parameterized in even larger-scale models

- lower bound is approximately the Ozmidov scale, $\ell_O = \left(\frac{\epsilon}{N^3}\right)^{1/2}$ the maximum horizontal scale that can overturn
  - typically, in the ocean, $\ell_O \sim 1\,\mathrm{m}$
  - in strongly stable atmosphere boundary layers, $\ell_O \sim 1\,\mathrm{m}$
  - in the upper troposphere, stratosphere, $\ell_O \sim 10$'s m

Sources of Turbulence in Stably-Stratified Flows

- Breakdown of internal waves
  - sources of internal waves
    - wind, through surface waves, weather pressure disturbances
    - topographic generation by islands, ridges, seamounts, etc.
      - strong tidal influence
      - localized sources
    - spontaneous generation, e.g., by larger-scale motions
    - energy cascade from larger-scale waves
    - others sources (?)

Laboratory Results Indicate Other Motions



Plan view of wake at various times (Spedding, *Phys. Fl.*, 2002)

Sources of turbulence in stratified flows (cont'd)

- Quasi-horizontal vortices – 'Stratified Turbulence' $\Leftarrow$ discuss today
  Lilly (1983)
  - forward energy cascade from larger-scale motions (McWilliams, 2009)
    - $*$ loss of balance, frontogenesis, instabilities
  - topographic generation (McCabe, MacCready, and Pawlak, 2006)
  - by anticyclonic eddies – Meddies (Krahmann et al., 2008; Ménesguen et al., 2009)
  - Subantarctic Front (Sheen et al., 2009)
  - stable boundary layers (Frehlich and Sharman, 2010)
  - other sources (?)

Characteristics of Motions on These Scales

- Controlling parameters
  - Reynolds number: $R_\ell = u'\ell_H/\nu$
    - u' – characteristic rms velocity
    - ℓ_H – horizontal scale of energy-containing motions
  - Froude number: $F_\ell = u'/N\ell_H \sim \ell_B/\ell_H$
    - N – buoyancy frequency
    - $\ell_B = u'/N$ – buoyancy scale
  - Gradient Richardson number: $Ri = N^2 \Big/ \left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right]$
  - Rossby number: $Ro = u'/f\ell_H$
    - f – Coriolis frequency

- For atmospheric/oceanic motions at these horizontal scales (ℓ_H),
  - $(\ell_O/\ell_H)^{2/3} \sim F_\ell \ll 1$, $R_\ell \gg 1$, $Ro = u'/f\ell_H \gg 1$

Outline

- Describe strongly-stratified flow – ‘stratified turbulence’
  - provides ‘pathways’ to ‘classical’ 3-D turbulence
- Some results from numerical simulations of stratified turbulence
- Scaling arguments
  - possible ‘stratified turbulence’ inertial range
- Field data from oceans, atmosphere
- Some conclusions

Scaling Arguments

- Turbulent flows are expected to exist, locally, when

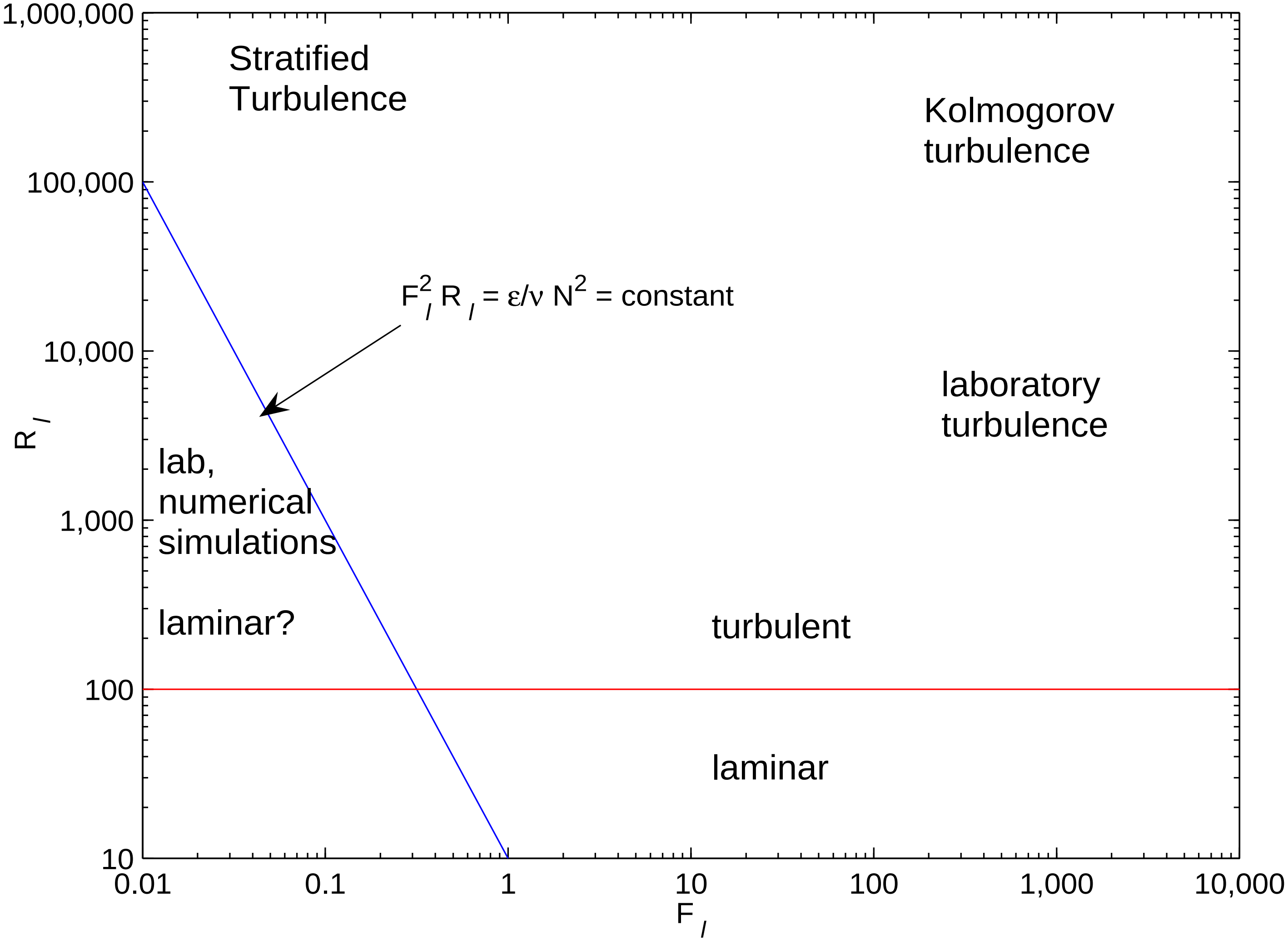
$$Ri = N^2 \Big/ \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right] < \mathcal{O}(1)$$

- This can be shown to imply that, for turbulence to exist

$$\begin{aligned} R_b &= F_\ell^2 R_\ell > \mathcal{O}(1)\,, \quad \text{related to the ‘buoyancy Reynolds number’} \\ &\sim \epsilon/\nu N^2 \sim \left[\left(\frac{\epsilon}{N^3} \right)^{1/2} \Big/ \left(\frac{\nu^3}{\epsilon} \right)^{1/4} \right]^{4/3} = \left[\frac{\text{Ozmidov scale}}{\text{Kolmogorov scale}} \right]^{4/3} \end{aligned}$$

- from oceanographic measurements, laboratory data, numerical simulations
  - turbulence is found to exist when $R_b > \mathcal{O}(10)$

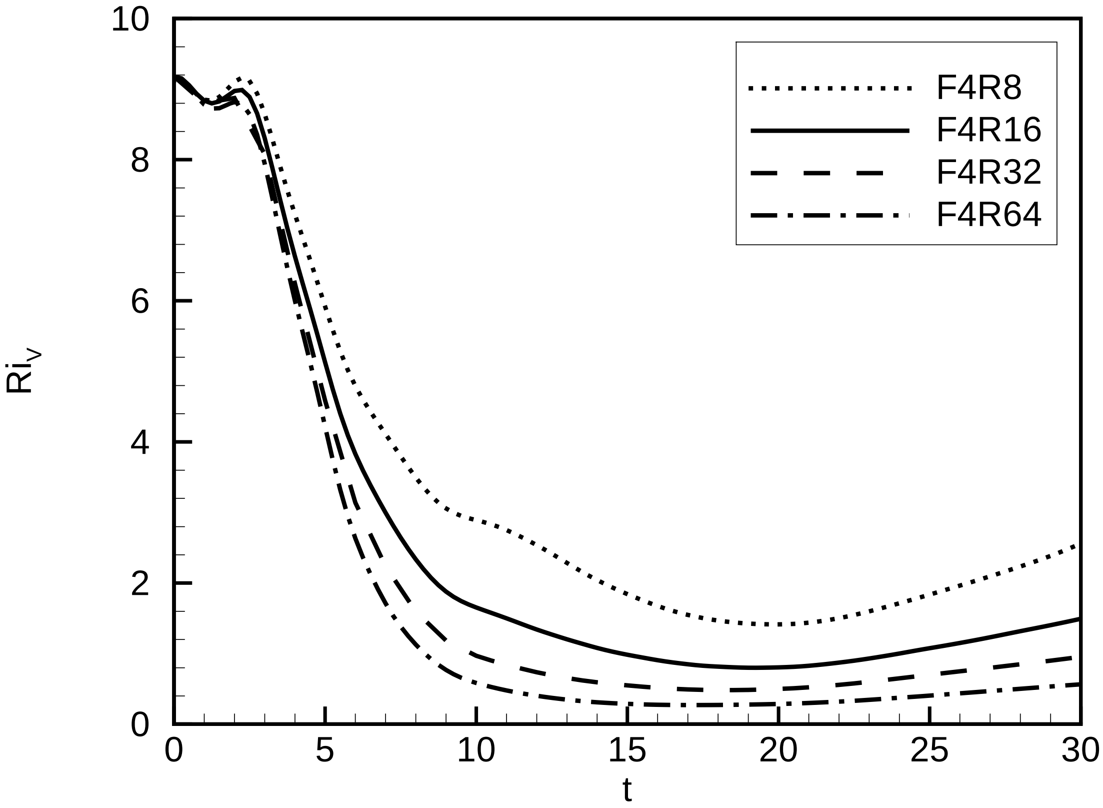
Stratified Turbulence (cont'd)



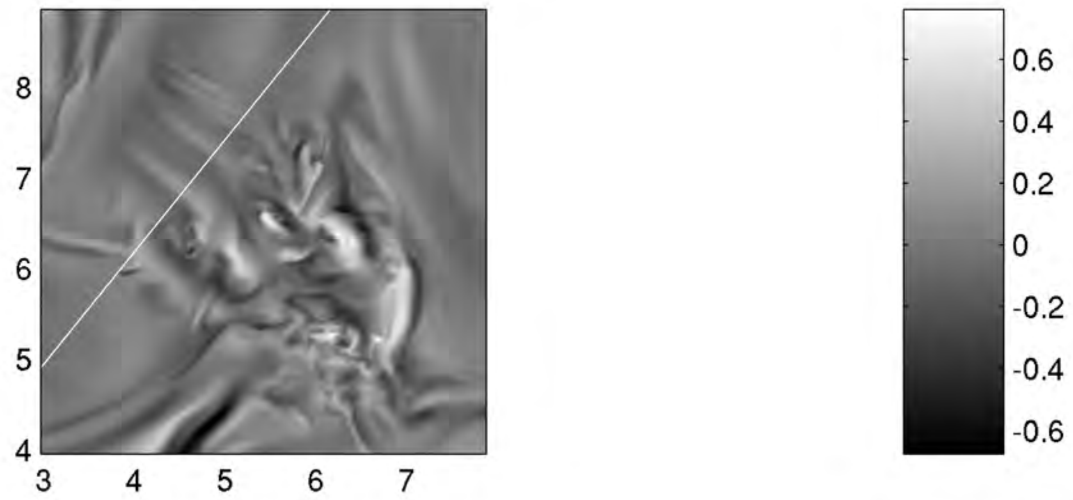
Theoretical Arguments – Stratified Turbulence

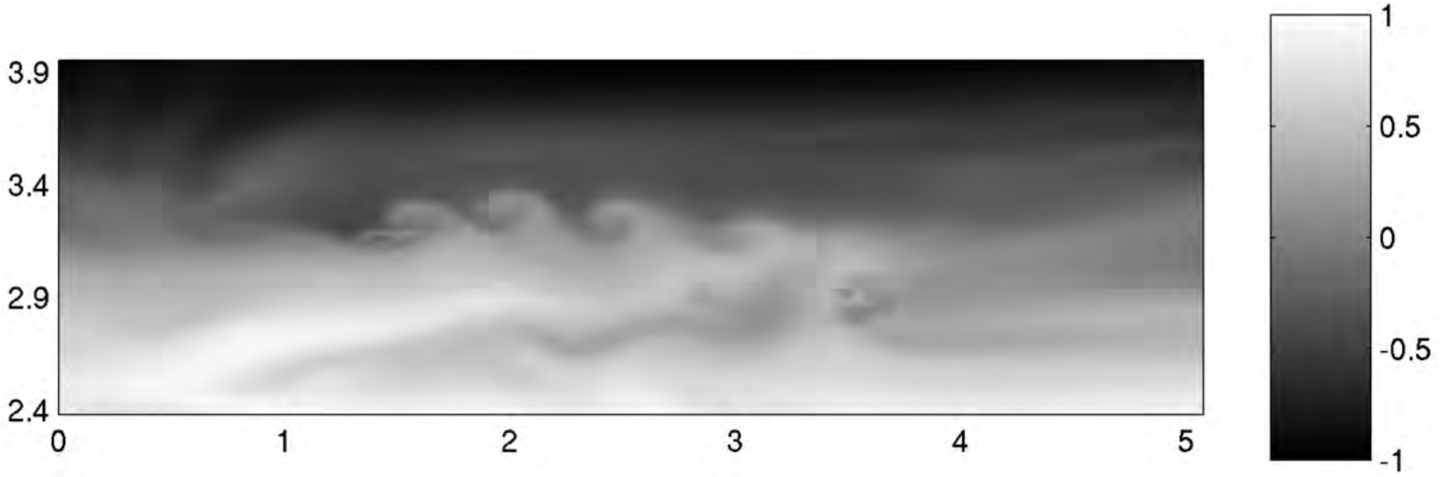
- Lilly (1983) used scaling arguments to suggest, for $F_\ell \ll 1$:
  - flows in ‘adjacent’ horizontal layers are somewhat decoupled
  - leads to increasing vertical shearing of horizontal flow
  - and to continually decreasing Richardson numbers
- Billant and Chomaz (2000)
  - induced velocities lead to large vertical inhomogeneities and layering
  - and to continually decreasing Richardson numbers
- Even though strong, stable stratification,
  at high Reynolds numbers, both mechanisms lead to
  - smaller vertical scales continually developing
  - local instabilities, ‘classical’ 3-D turbulence intermittently occurring

Volume-Averaged Gradient Richardson Number versus t

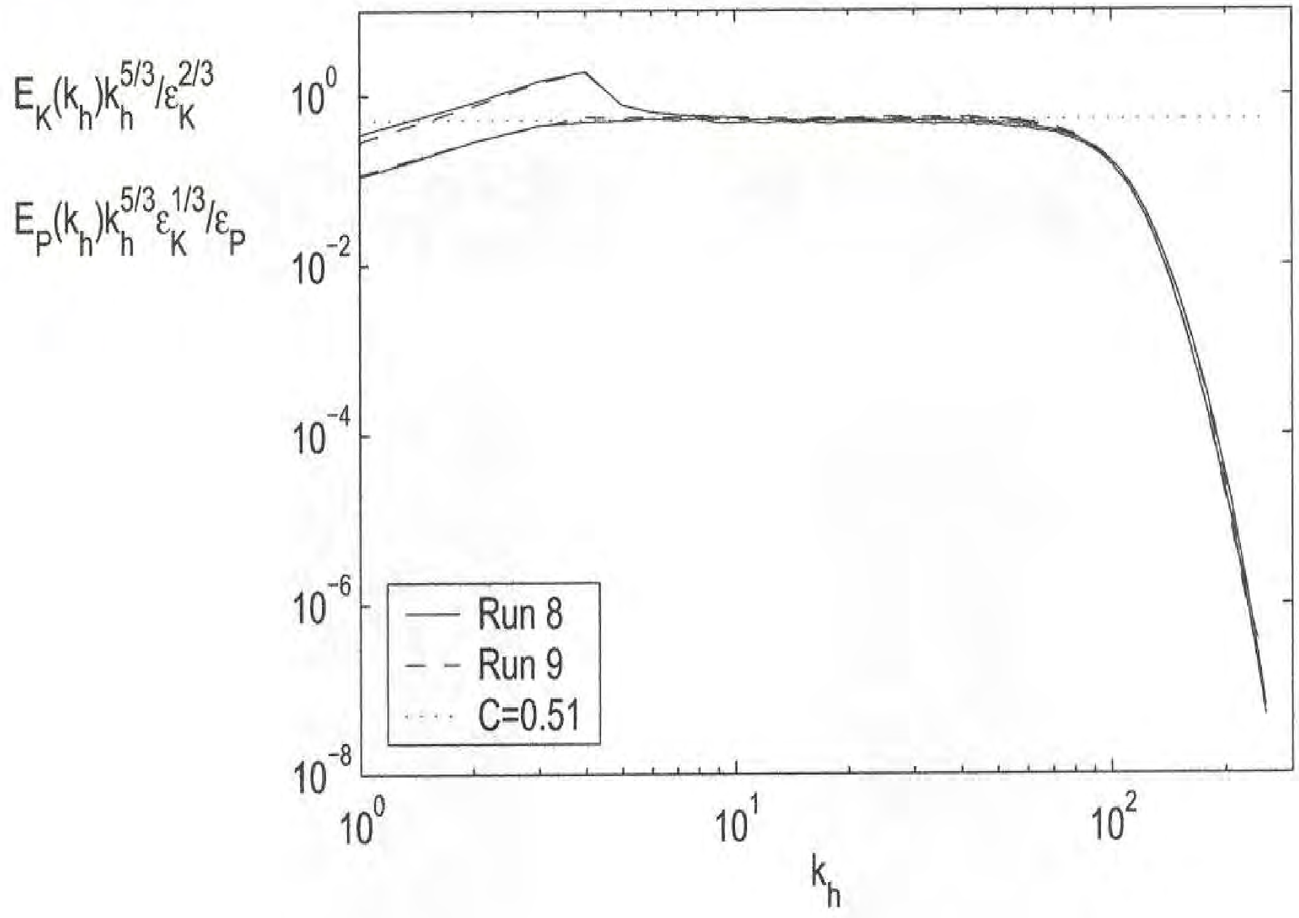


$$Ri_V = \frac{N^2}{\left\langle \left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2 \right\rangle}$$


8
7
6
5
4
3
4
5
6
7
0.6
0.4
0.2
0
-0.2
-0.4
-0.6


3.9
3.4
2.9
2.4
0
1
2
3
4
5
1
0.5
0
-0.5
-1

Compensated Horizontal Energy Spectra



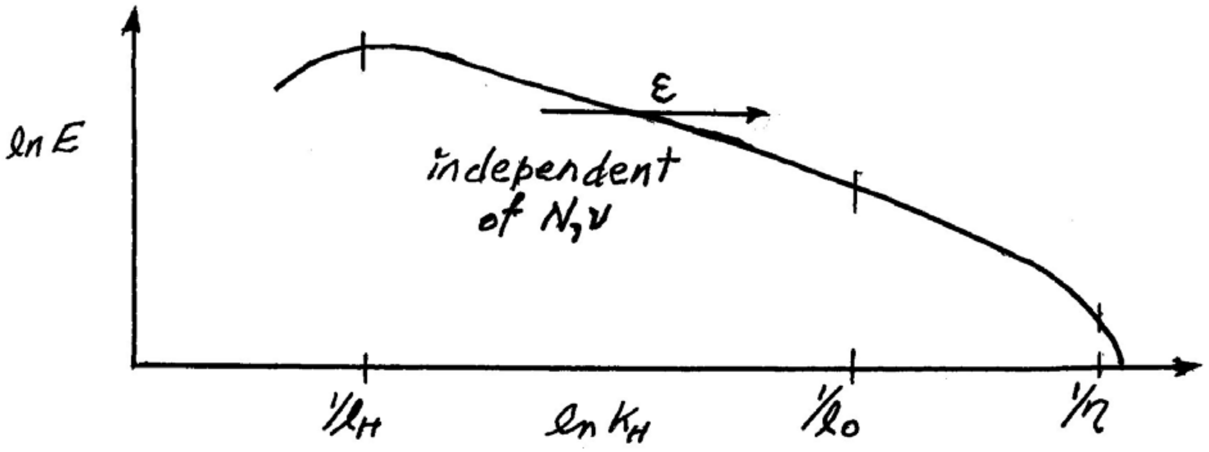
Compensated horizontal energy spectra, Lindborg (2005,2006).

Other Related Numerical Simulations

- Simulations showing approximate $k_H^{-5/3}$ behavior
  - Riley & deBruynKops (2003)
  - Waite & Bartello (2004, 2006); Waite (2010)
  - Lindborg (2005, 2006)
  - Brethouwer, Billant, Lindborg and Chomaz (2007)
  - Molemaker and McWilliams (2010)
  - deBruynKops (2010)
  - Diamessis, Spedding, and Domaradzki (2010)
- Simulations partially consistent with $k_H^{-5/3}$ behavior
  - Kimura and Herring (2010)

Scaling Arguments

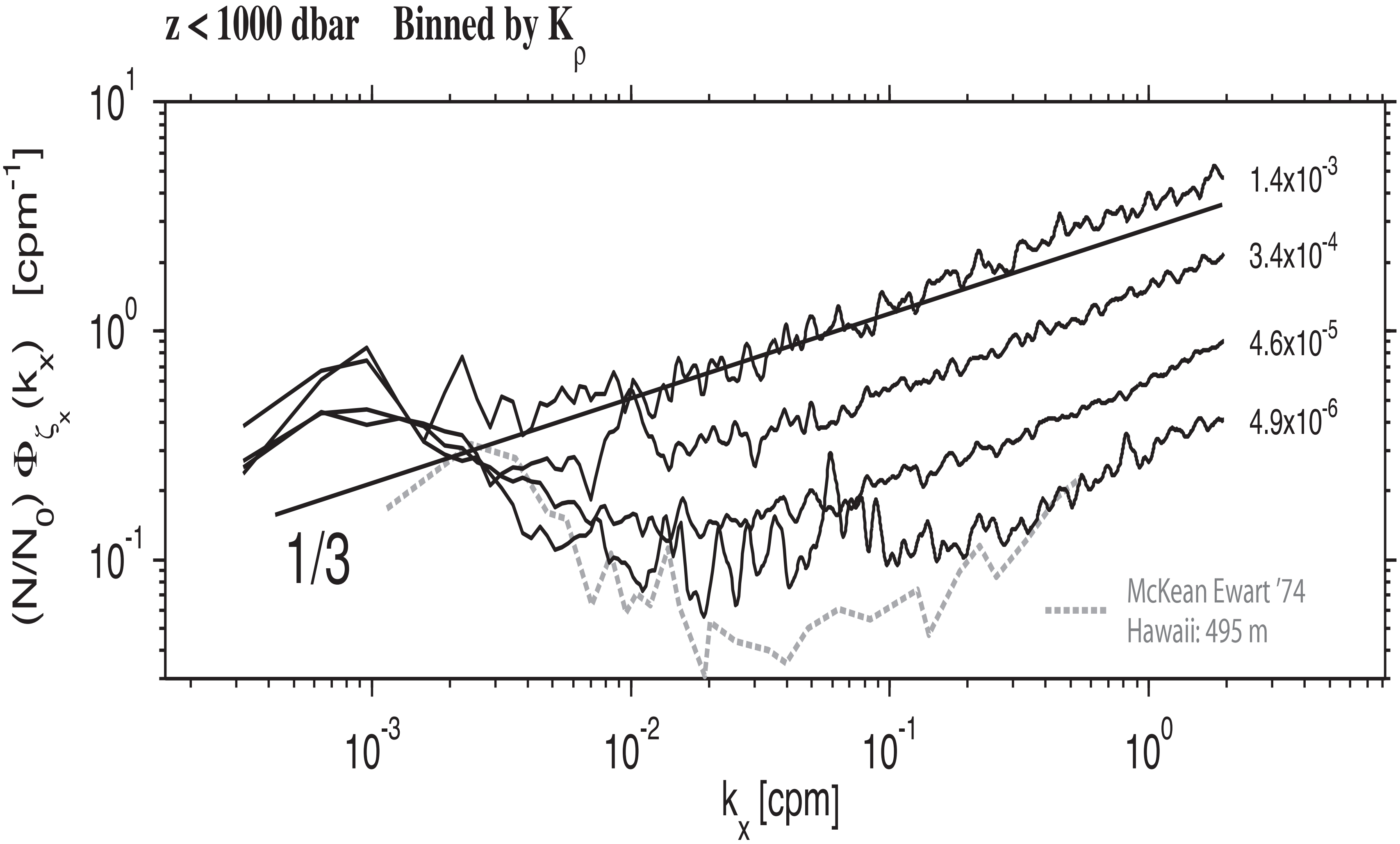
- Assume
  - $F_\ell \ll 1$ (strong stratification) — implies $\ell_O/\ell_H \sim {F_\ell}^{2/3} \ll 1$
  - $F_\ell^2 R_\ell > R_{B\text{crit}} \sim 20$ — implies $\eta/\ell_O < {R_{b\text{crit}}}^{-3/4} \simeq 0.1$
  - $\epsilon \sim u'^3/\ell_H$, independent of N, ν (or F_ℓ, R_ℓ)



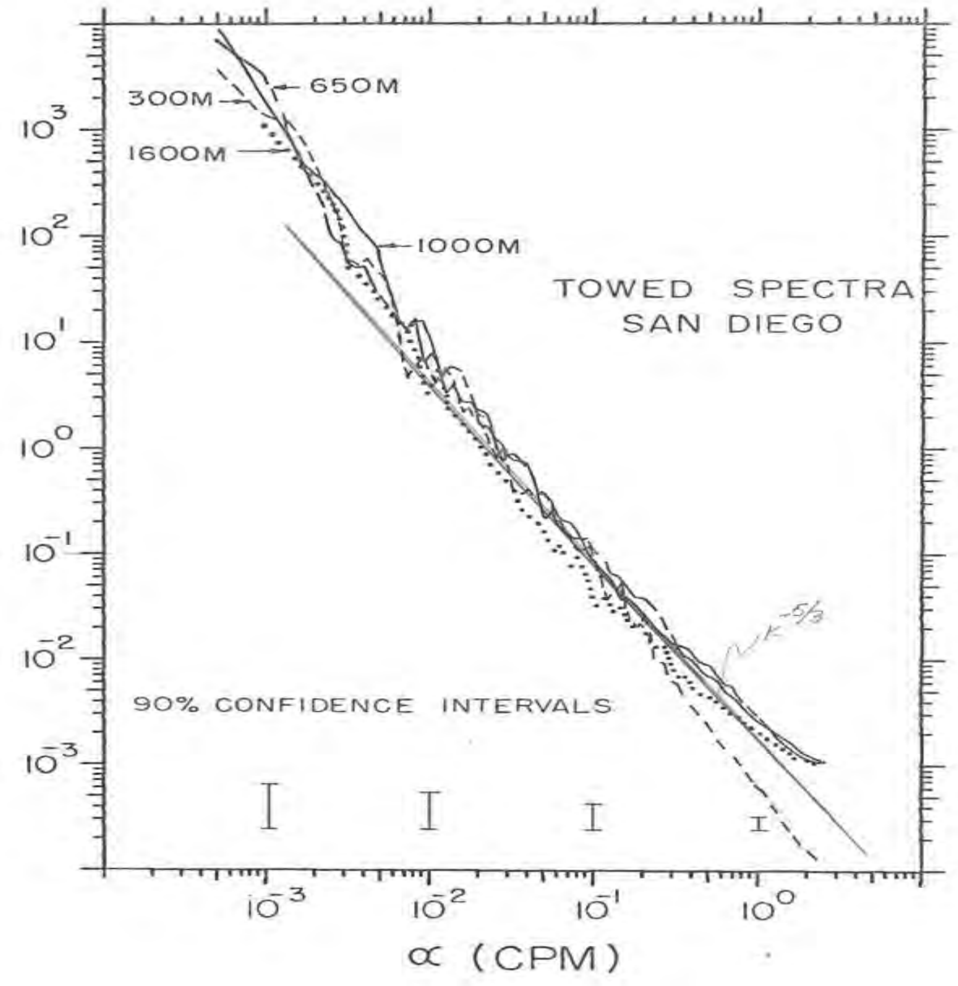
Implications

- Potential for stratified turbulence 'inertial cascade' for large R_ℓ (Riley and de Bruyn Kops, 2003; Lindborg, 2005, 2006)
  - expect to occur at scales $\ell_H \gg \ell \gg \ell_O$
  - if $F_\ell \ll 1$, $R_\ell \gg 1$, $R_b \geq \mathcal{O}(10)$
    - highly anisotropic 'inertial' subrange in the **horizontal**
      - spectral dependence only on ϵ, χ and k
    - $E_u(\kappa_H) = \mathcal{C}_u\, \epsilon^{2/3} \kappa_H^{-5/3}$
    - $E_\theta(\kappa_H) = \mathcal{C}_\theta \chi \epsilon^{-1/3} \kappa_H^{-5/3}$
    - $d_H^2(t) = \mathcal{C}_d\, \epsilon\, t^3$ (patch size; possibly)
- Replaces notion of 'buoyancy subrange' (Bolgiano, Ozmidov, Lumley)
- Results from the atmosphere, oceans?

Shear Spectra – Ocean (Klymak & Moum, 2007)

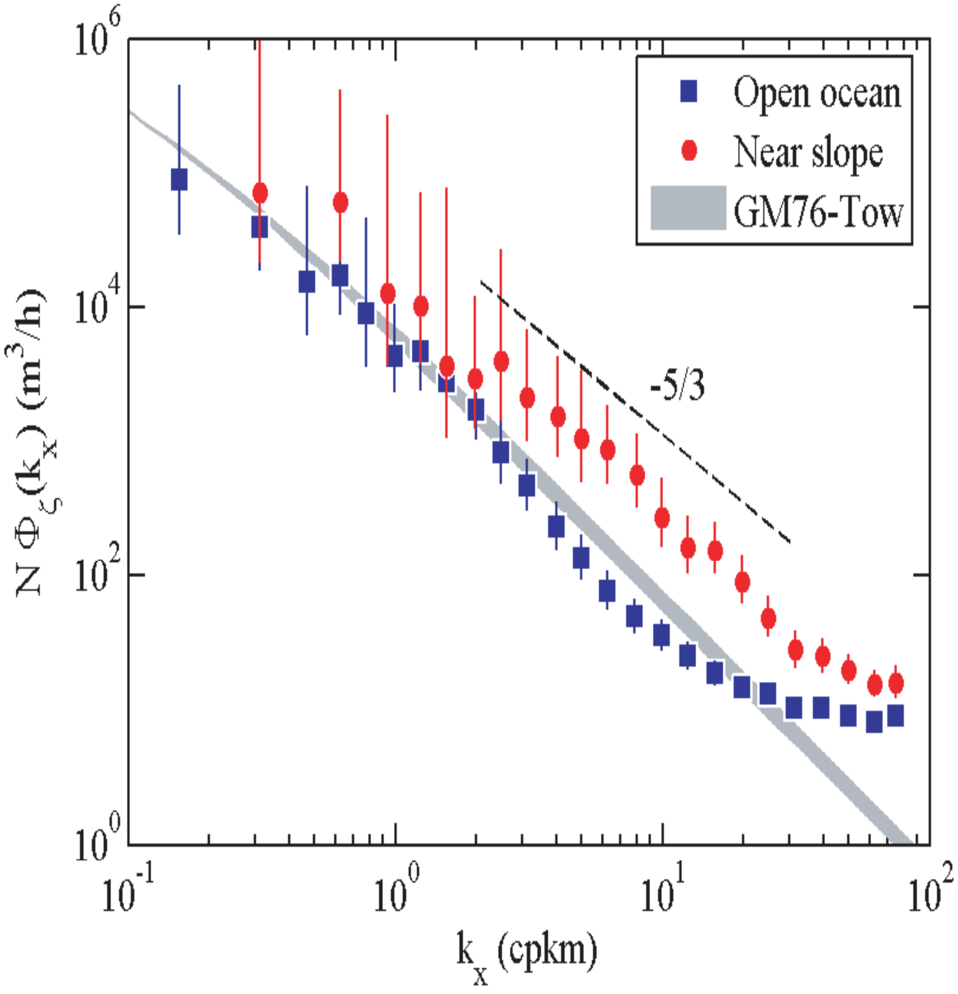


Temperature spectra – Ocean (Ewart, 1976)



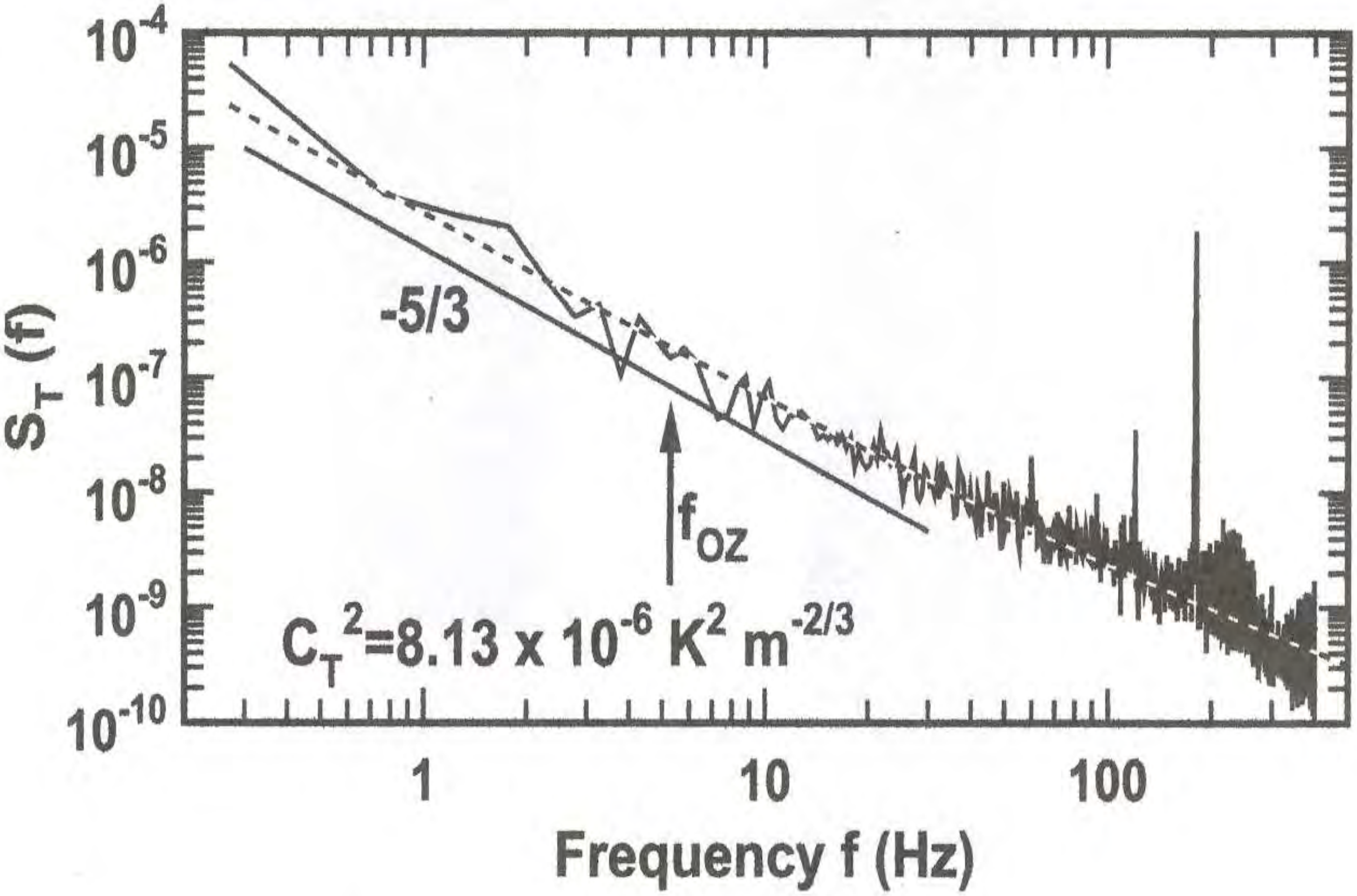
Power spectra of temperature off the coast of San Diego (30^oN, 124^oW).

Displacement spectra – Ocean (Hollbrook & Fer, 2005)



Vertical displacement spectra from open ocean (squares) and near slope (dots)

Temperature Spectrum – Atmosphere (Frehlich et al., 2008)



Temperature frequency (horizontal wave number) spectrum
in very stable atmospheric boundary layer

Other results from the Ocean

- Other results from the ocean consistent with $k_H^{-5/3}$ scaling
  - Sheen et al. (2009)
    - Subantarctic Front in South Atlantic Ocean
  - Ménesguen et al. (2009)
    - anticyclonic Meddies in Gulf of Cadiz
  - Krahmann et al. (2008)
    - same field campaign as Ménesguen
  - Bouruet-Aubertot, van Haren, and Lelong (2010)
    - Moorings in deep water on south-eastern slope of Rockall Channel

Summary of Field Results

- Field experiments
  - 3-D turbulence is very intermittent, sporadic
  - Often observe in the oceans at scales $\ell_O < \ell_H < 100$'s m
    - horizontal spectra in velocity, temperature consistent with $\kappa_H^{-5/3}$
    - vertical spectra more consistent with κ_V^{-3}
  - not classical Kolmogorov-Oboukov-Corrsin spectra
    - highly nonisotropic
    - scales are much too large
    - influence of stable density stratification
  - consistent with numerical simulations, scaling arguments

Conclusions

- Stratified turbulence $F_\ell \ll 1$, $R_\ell \gg 1$, $R_b \geq \mathcal{O}(10)$, a pathway to 3-D turbulence
  - e.g., at oceanic horizontal scales larger than a few meters
  - strong tendency for vertical shearing of horizontal motion
  - leads to intermittent, 3-D turbulence
  - spatial intermittency is part of a strong downscale transfer of energy
  - potential for stratified turbulence ‘inertial cascade’
    - highly nonisotropic inertial subrange
    - possible explanation of field data
    - replaces notion of a ‘buoyancy subrange’

Some Open Issues regarding Stratified Turbulence

- What are dynamics in 'inertial range'?
  - E.g., except in 3-D turbulent regions, vortex lines mainly horizontal.
- Role of internal waves
  - Are the density perturbations 'slaved' to velocity?
- Do laboratory results scale up to field?
  - Is there both upscale and downscale transfer of energy?
- What are the vertical spectra? E.g., $E_H(k_z) \propto k^{-3}$?
- The dynamical relationship between ℓ_B and ℓ_O?
- Does horizontal particle separation go as: $\langle D^2 \rangle = \mathcal{C}\epsilon t^3$?
  - Or does shear dispersion dominate?