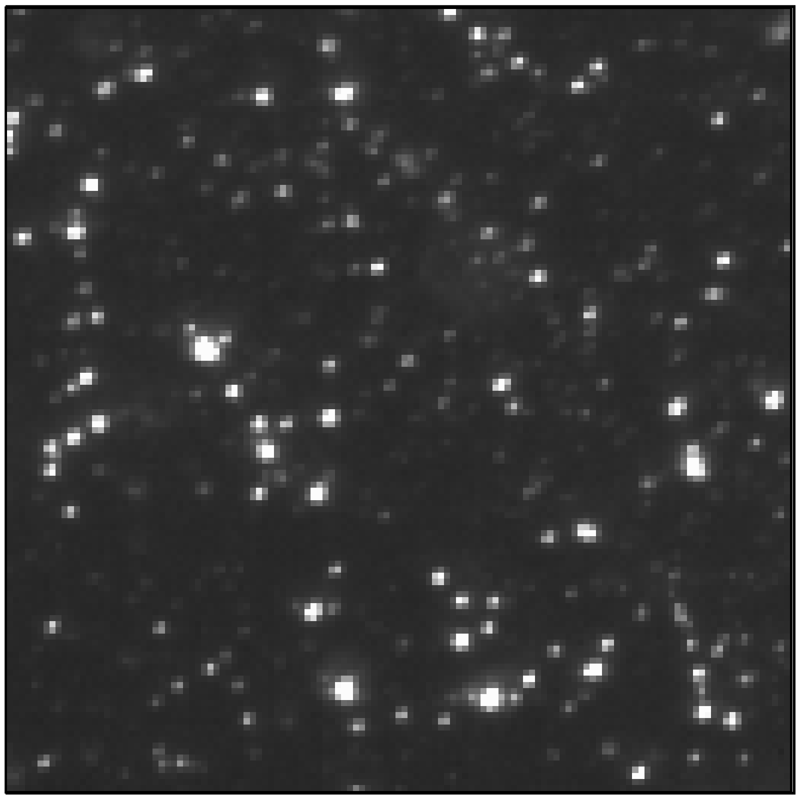
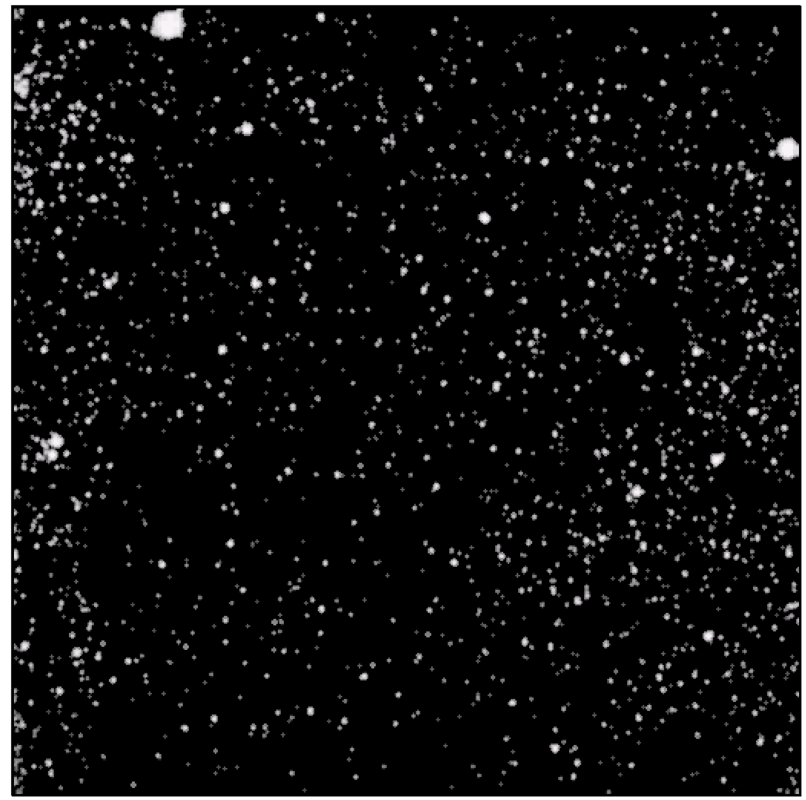
Quantum Mechanics meets Fluid Dynamics: Visualization of Vortex Reconnection in Superfluid Helium



Vortex Reconnection



Quantum Turbulence

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University of Maryland at College Park
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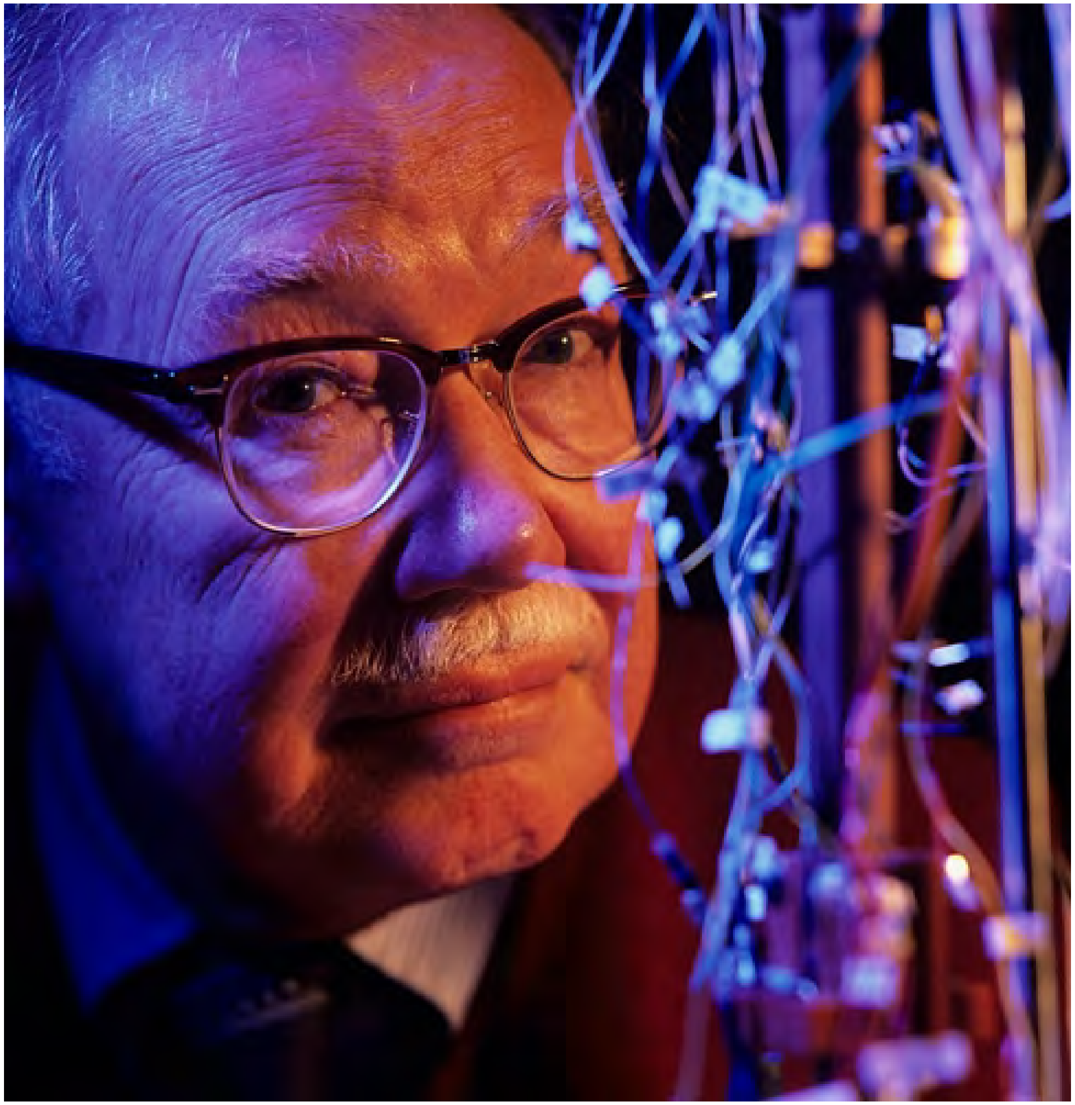
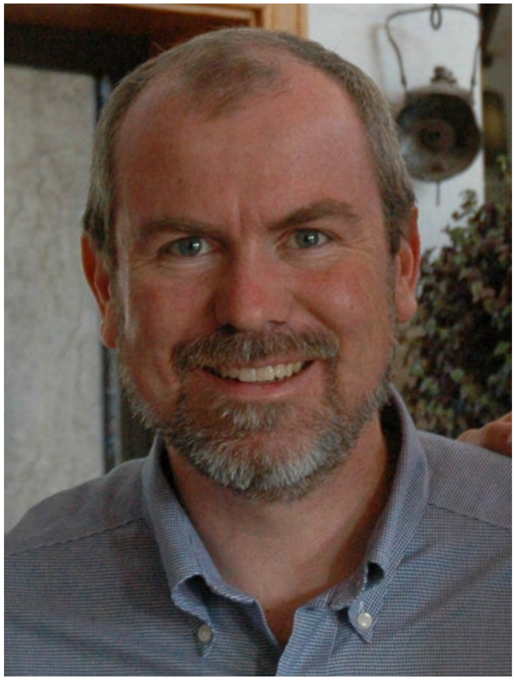
Happy (2nd) 40th birthday Russ Donnelly!



Acknowledgements



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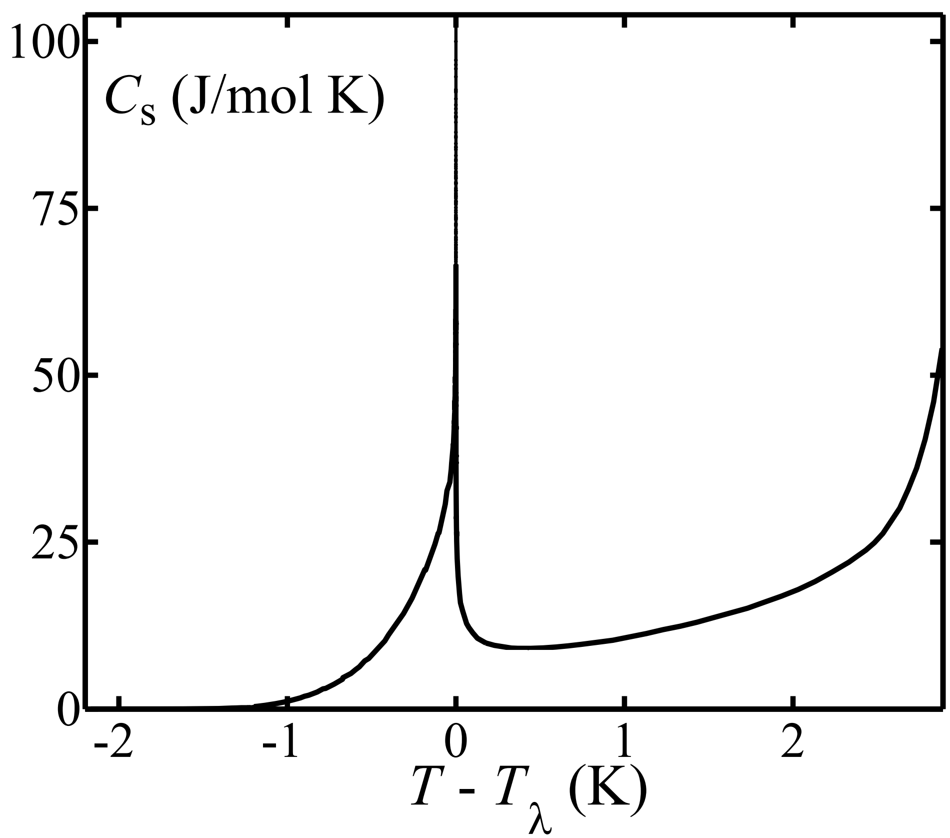
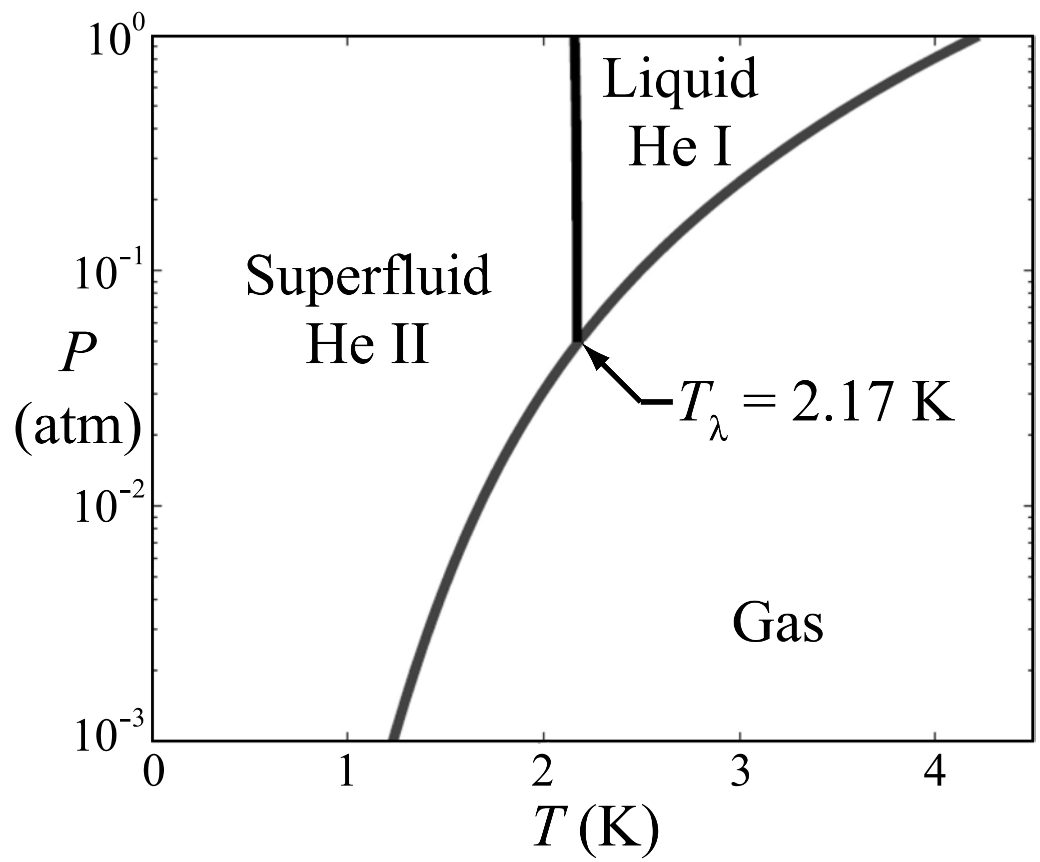
Superfluid Helium (He II)

$\lambda-$transition characterized by:

- Anomalous heat capacity (Keesom *et al.* 1928)
- Flow through very thin tubes with immeasurably small resistance (Keesom *et al.* 1930)

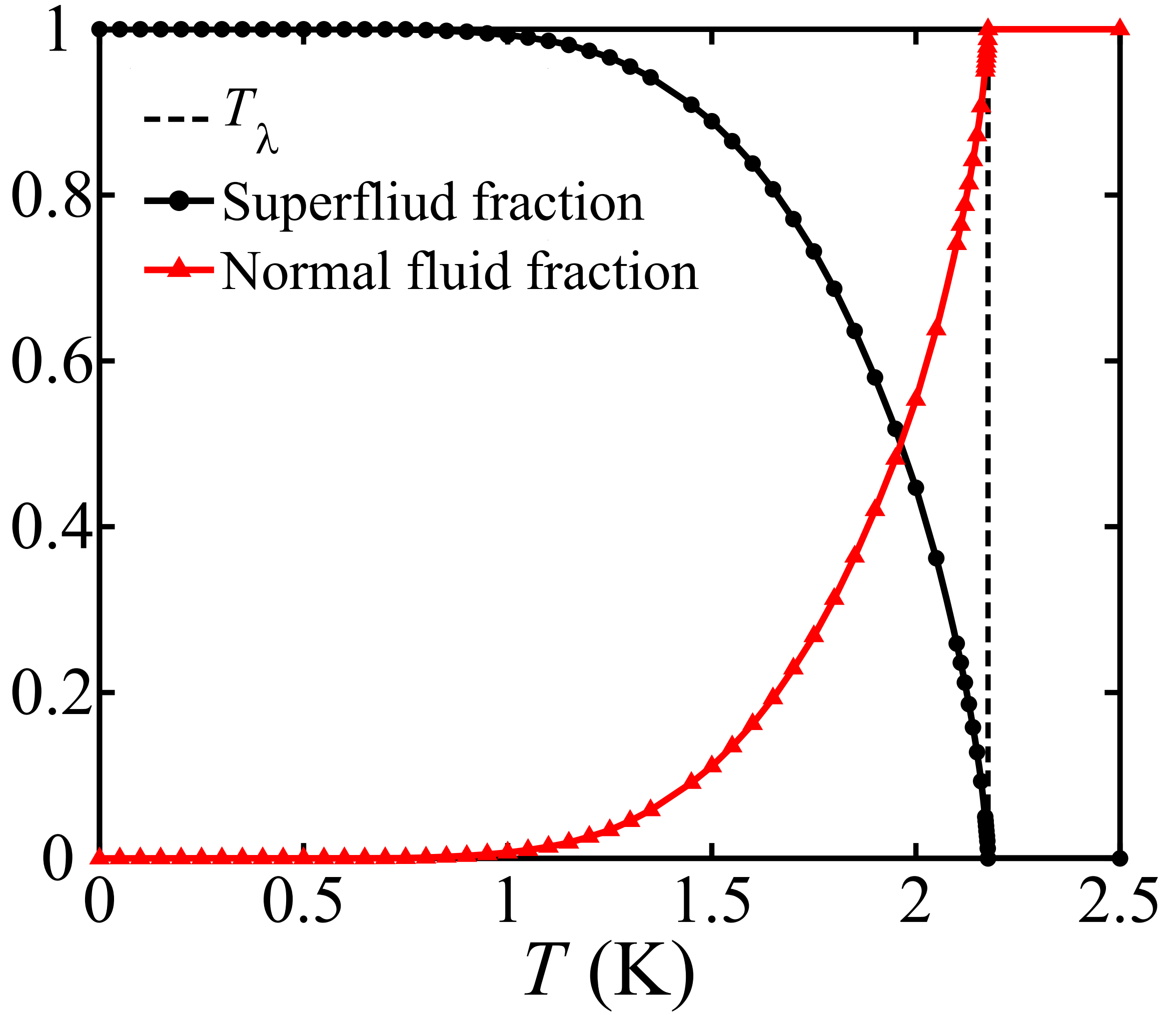




Data from Donnelly *et al.*, *J. Phys. Chem. Ref. Data* (1998)

Two-Fluid Model of Tisza and Landau

- He II behaves as a mixture of a viscous "normal" fluid and an inviscid "superfluid"
- Components interpenetrate and have distinct densities $\{\rho_n, \rho_s\}$ and velocity fields $\{\mathbf{v}_n, \mathbf{v}_s\}$



Data from Donnelly et al., *J. Phys. Chem. Ref. Data* (1998)

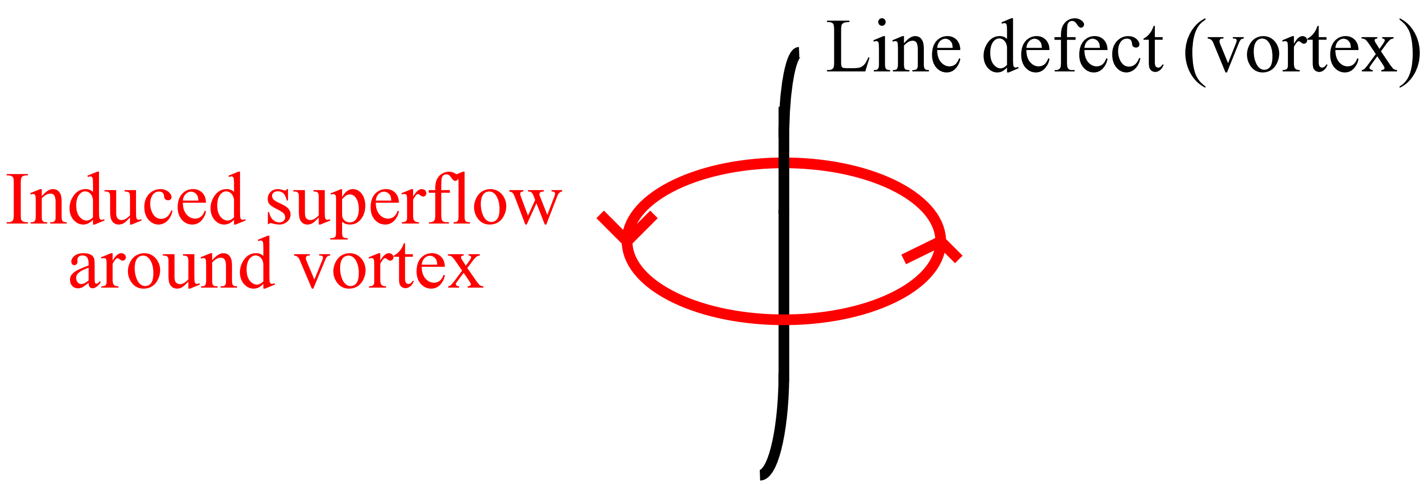
Quantized Vortices

Quantum mechanics restricts vorticity in superfluid to atomically-thin vortices with quantized circulation

Induces a superflow around the line-vortex:

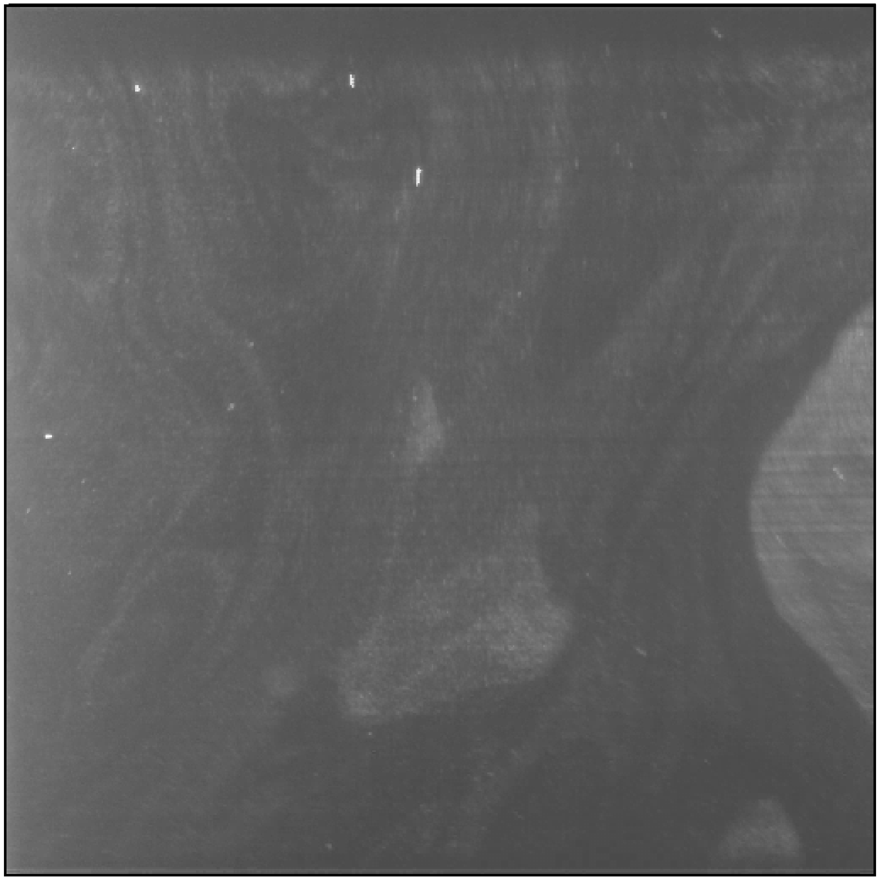
$$\mathbf{v}_s = \frac{\kappa}{2\pi s}\hat{\phi}$$

κ is quantum of circulation



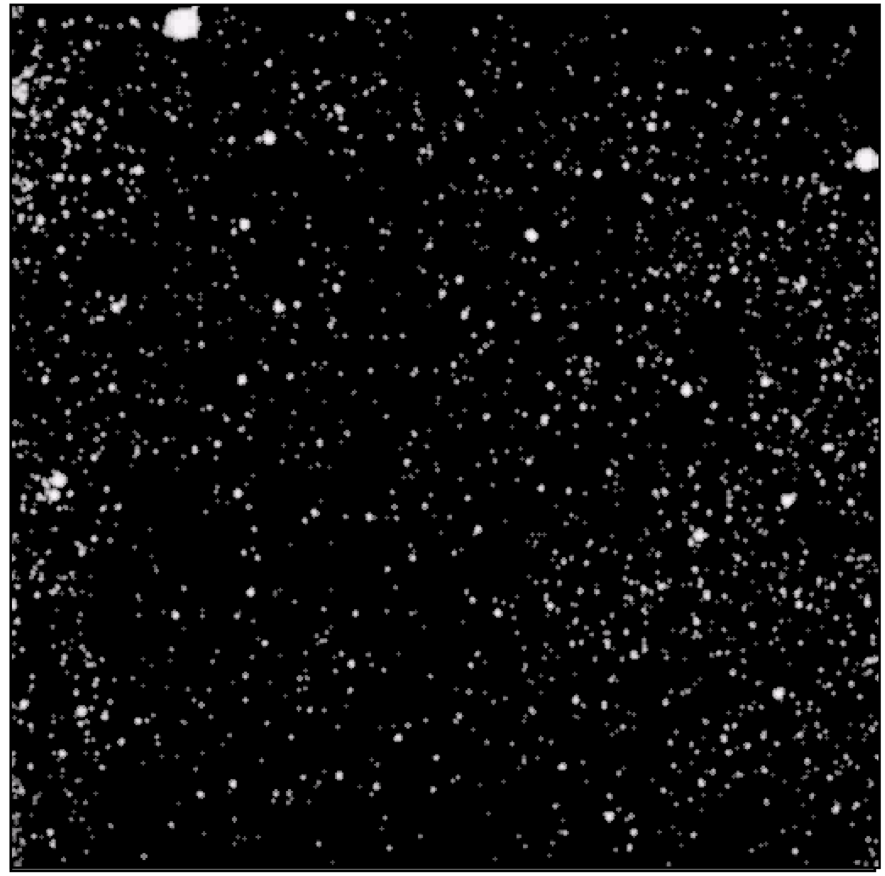
Quantum turbulence composed of a complex tangle of quantized vortices

Classical vs. Quantum Turbulence



Classical Turbulence

Velocity smoothed by viscosity
Vorticity also diffuses
Interactions spanning many length- and time-scales



Quantum Turbulence

Two-fluid nature
Vorticity topologically confined
Circulation quantized
Erratic velocity field

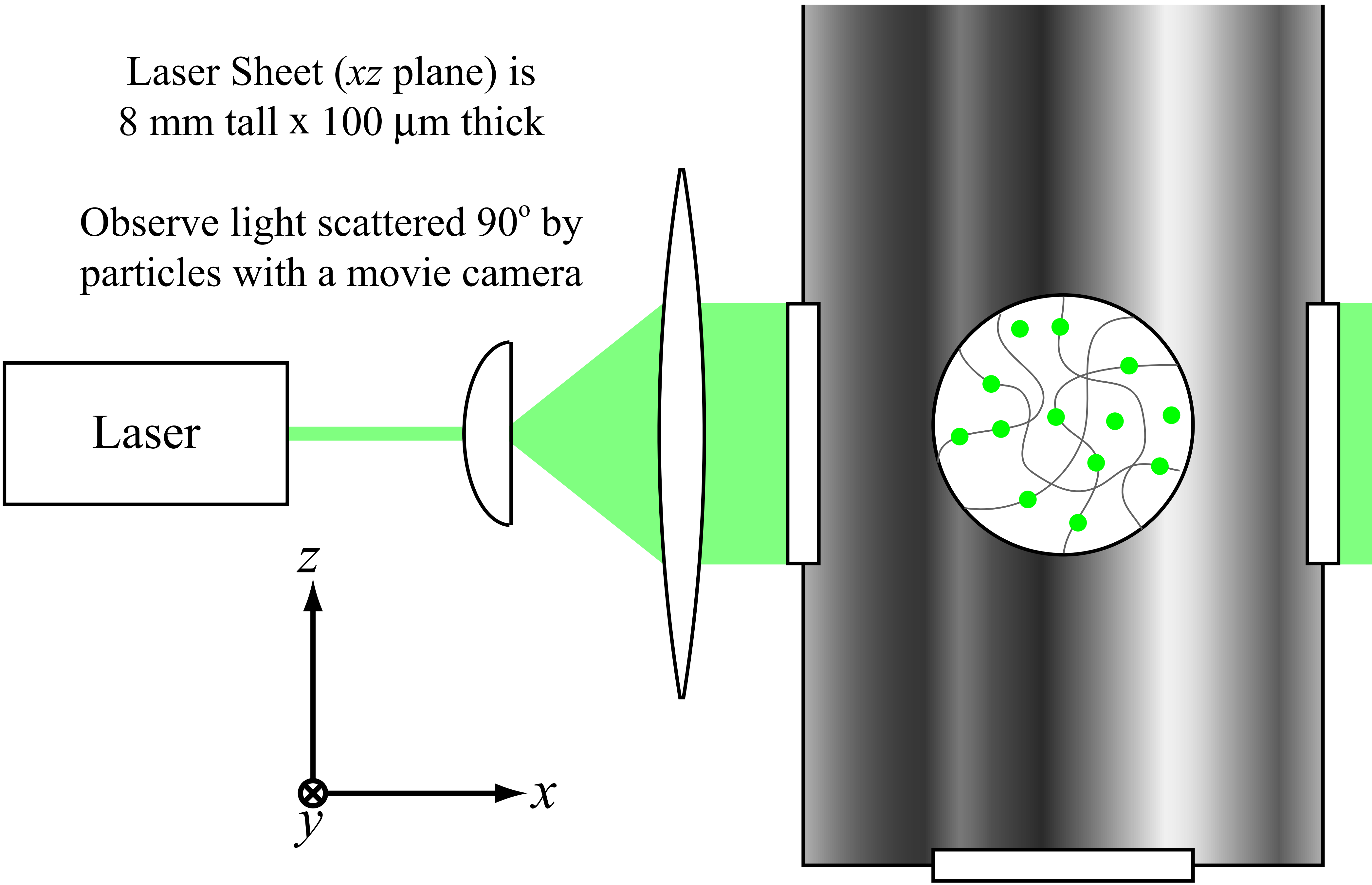
Objective

Examine the role and dynamics of quantized vortices in quantum turbulence by direct visualization

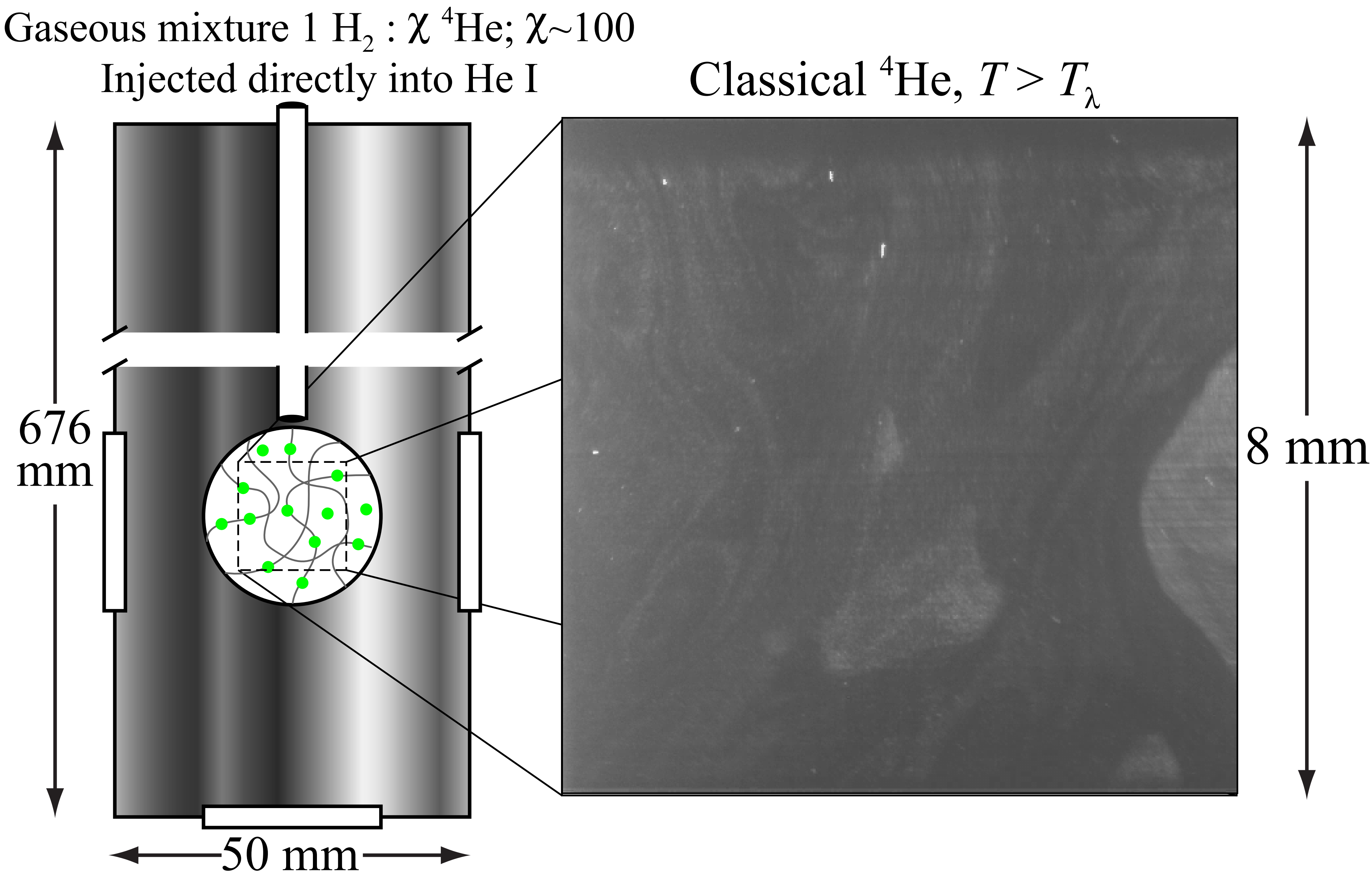
Visualization Technique

Laser Sheet (xz plane) is
8 mm tall x 100 μm thick

Observe light scattered 90° by
particles with a movie camera



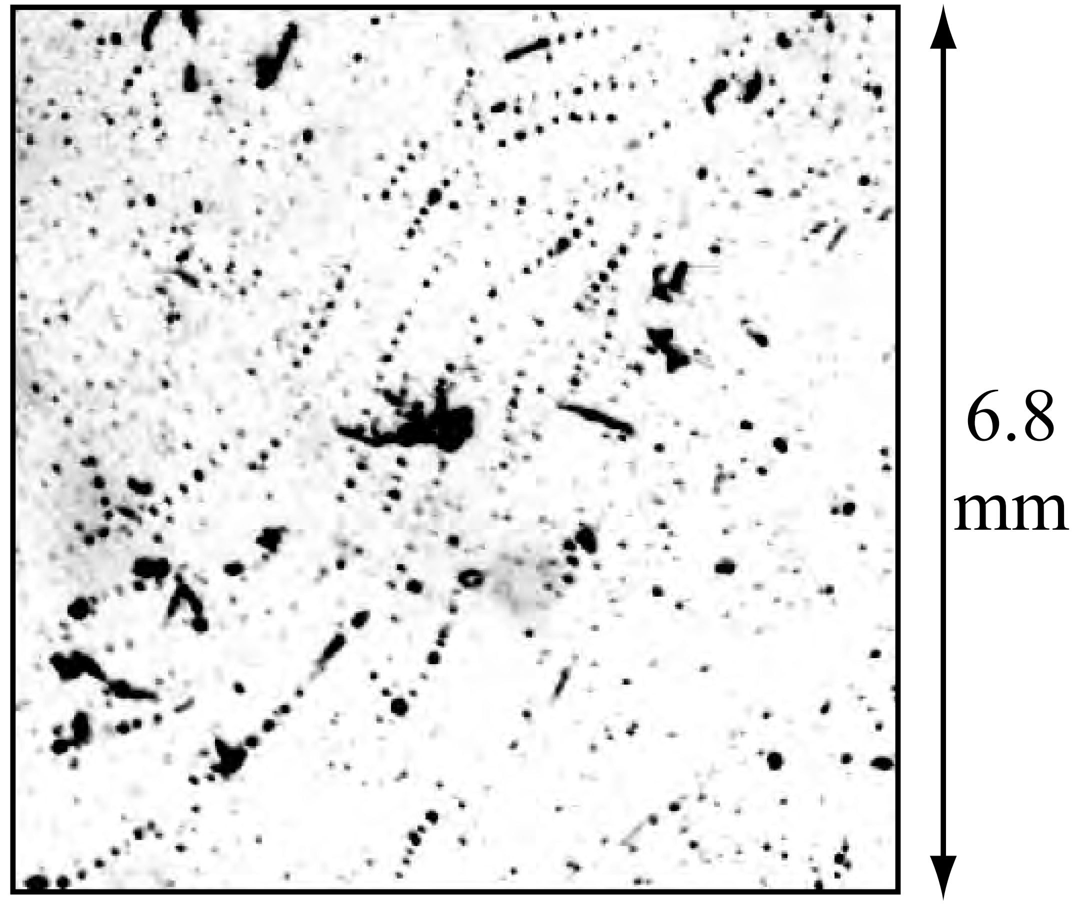
Hydrogen Particle Production



Visualizing Superfluid Vortices in He II

Below T_λ hydrogen particles collect onto filaments

Previous work has shown these filaments are particles trapped on the superfluid vortices (Bewley, *et al.*, *Nature* 2006)

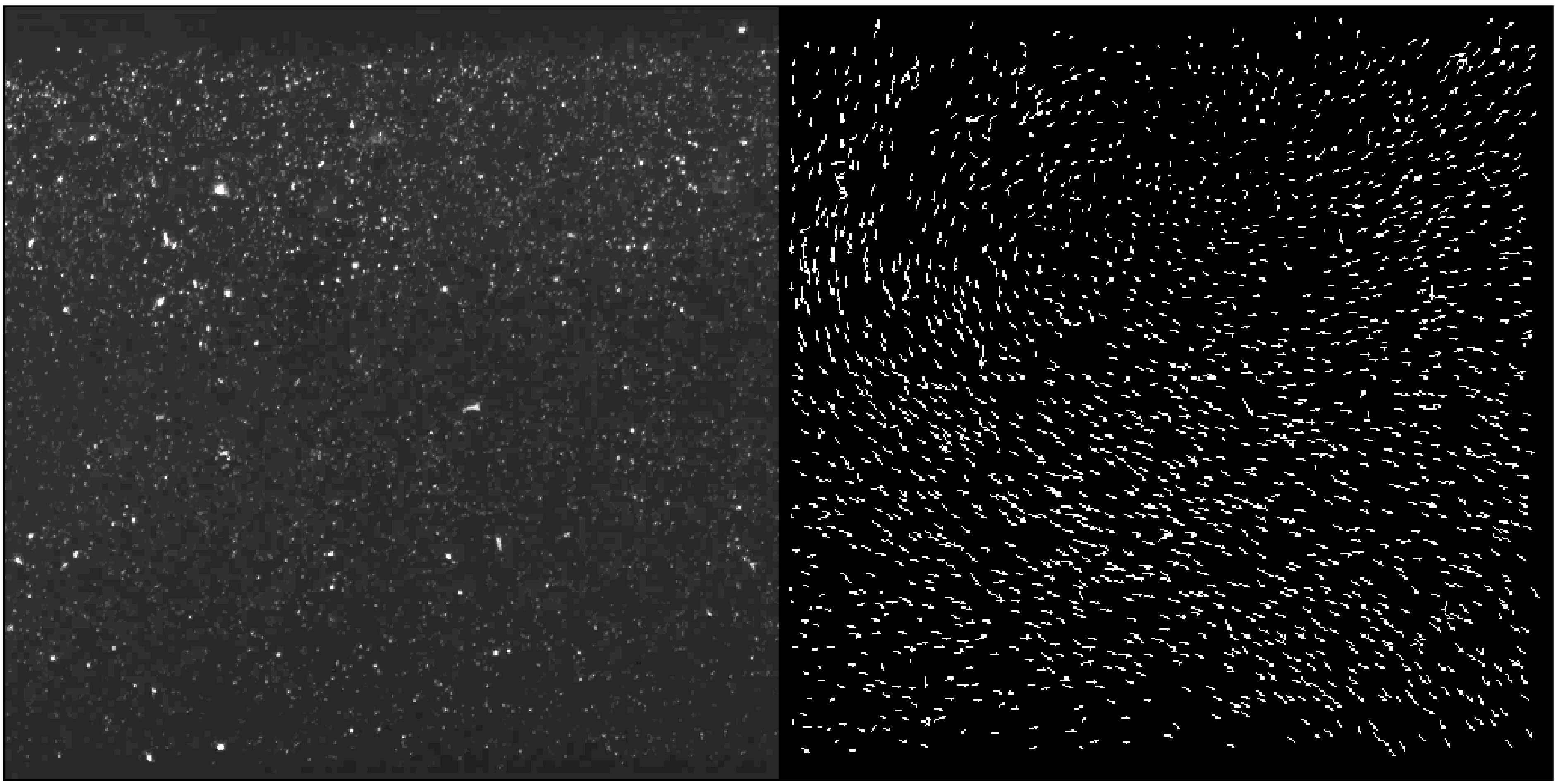


This allows for observations of both the normal fluid and superfluid for first time

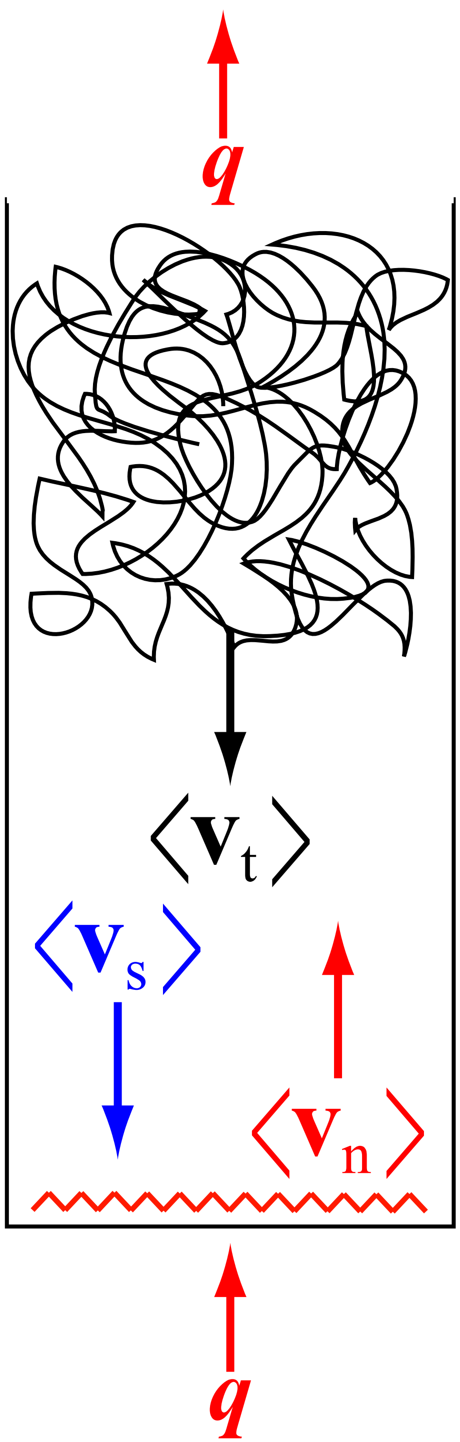
Particle Tracking

Particle-tracking allows us to analyze the particle dynamics without assuming smooth velocity fields (as in PIV)

Algorithm adapted from Eric Weeks and John Crocker



Thermal Counterflow



Reproducibly drive turbulence by applying a heat flux ***q*** to the bottom of the channel

Entropy only carried by normal fluid - entropy gradient drives normal fluid upward (not buoyancy)

$$\mathbf{v}_{\mathrm{n}} = v_{\mathrm{n}}\hat{z} = \frac{q}{\rho S T}\hat{z}$$

Superfluid moves downward to conserve mass

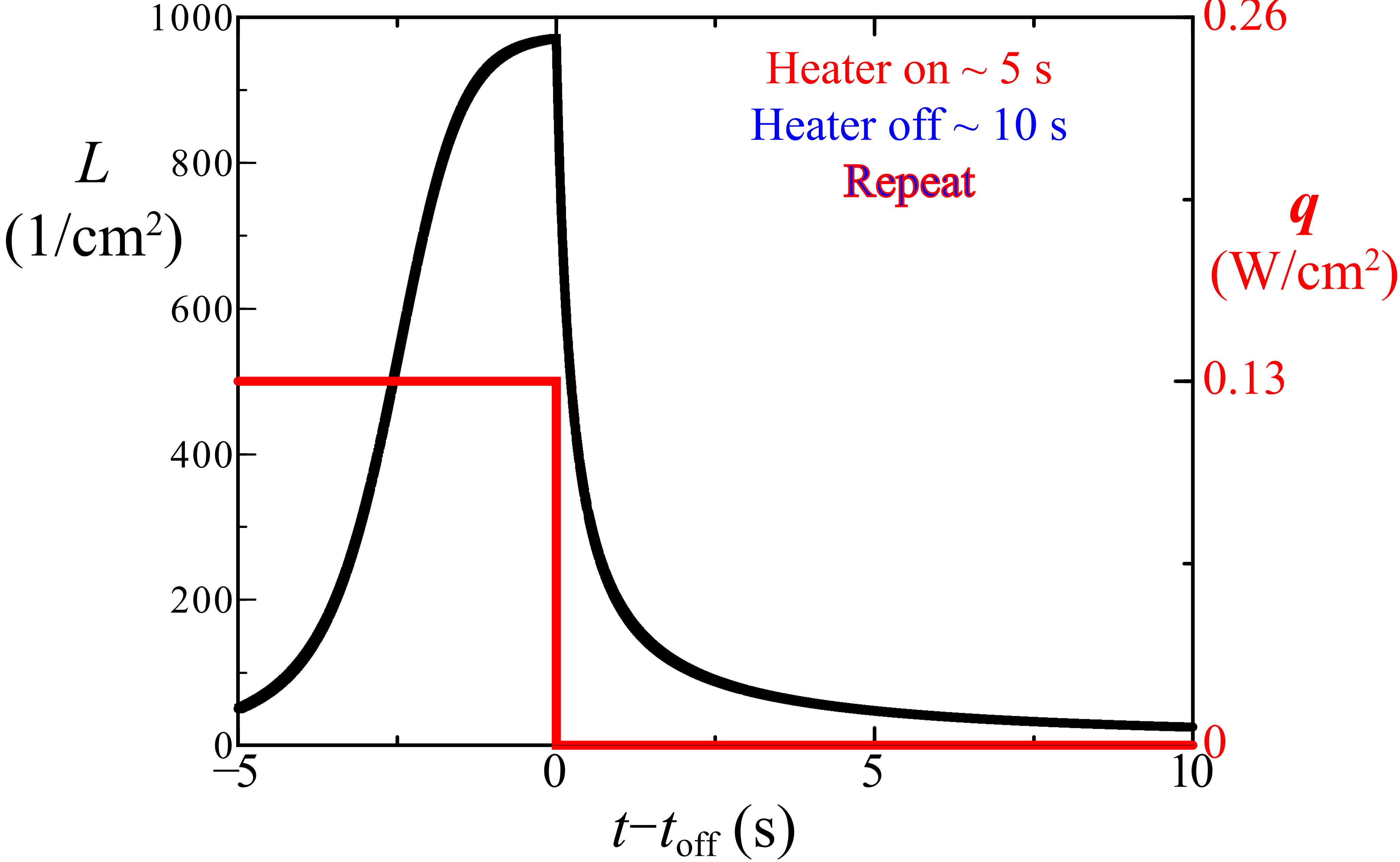
$$\mathbf{v}_{\mathrm{s}} = v_{\mathrm{s}}\hat{z} = -\frac{\rho_{\mathrm{n}}}{\rho_{\mathrm{s}}}v_{\mathrm{n}}\hat{z}$$

L. D. Landau, *Phys. Rev.* **60**, 356 (1941)

MSP, Fiorito, Sreenivasan, and Lathrop, *JPSJ* **77**, 111007 (2008)

Decaying Counterflow Turbulence

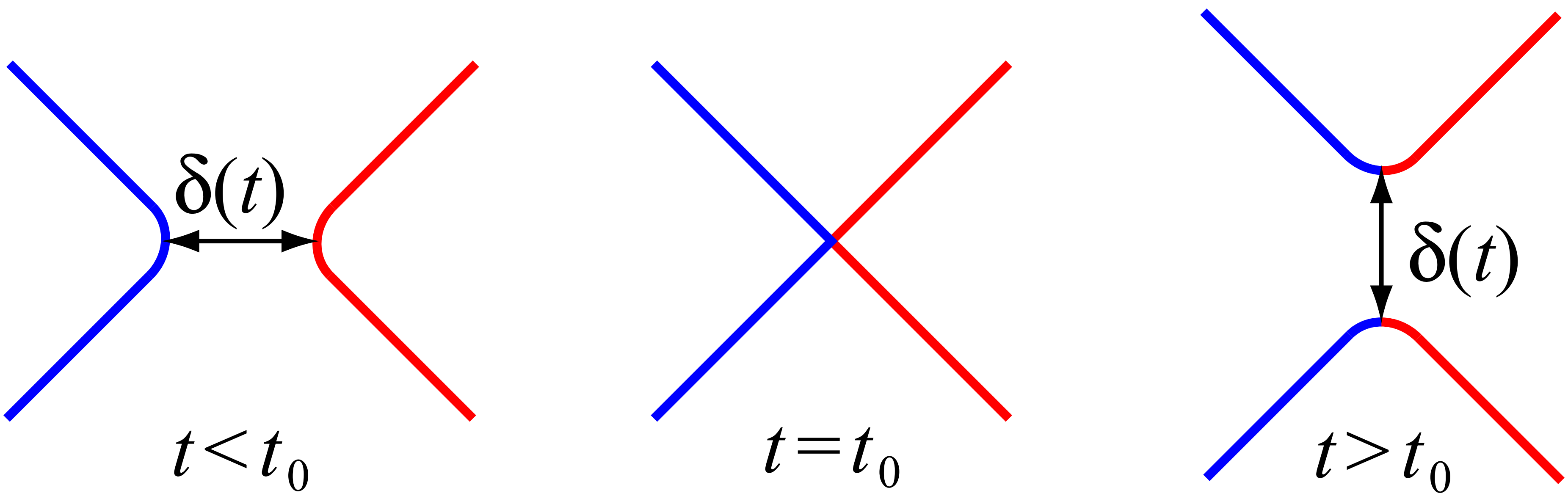
$$\frac{dL}{dt} = \alpha \left| \mathbf{v}_{ns} \right| L^{3/2} - \beta \kappa L^2$$



Superfluild Vortex Reconnection

Feynman, Prog. LTP (1955)
Schwarz, PRB (1985)
Schwarz, PRB (1988)
Tsubota and Maekawa, JPSJ (1992)
Koplik and Levine, PRL (1993)
de Waele and Aarts, PRL (1994)
Lipniacki, EJ Mech. B-Fluids (2000)
Nazarenko and West, JLTP (2003)

Previous theoretical studies predict that when two vortices cross they reconnect and that the dynamics are (nearly) time-reversible



$$\delta(t) = A[\kappa(t_0 - t)]^{1/2}$$

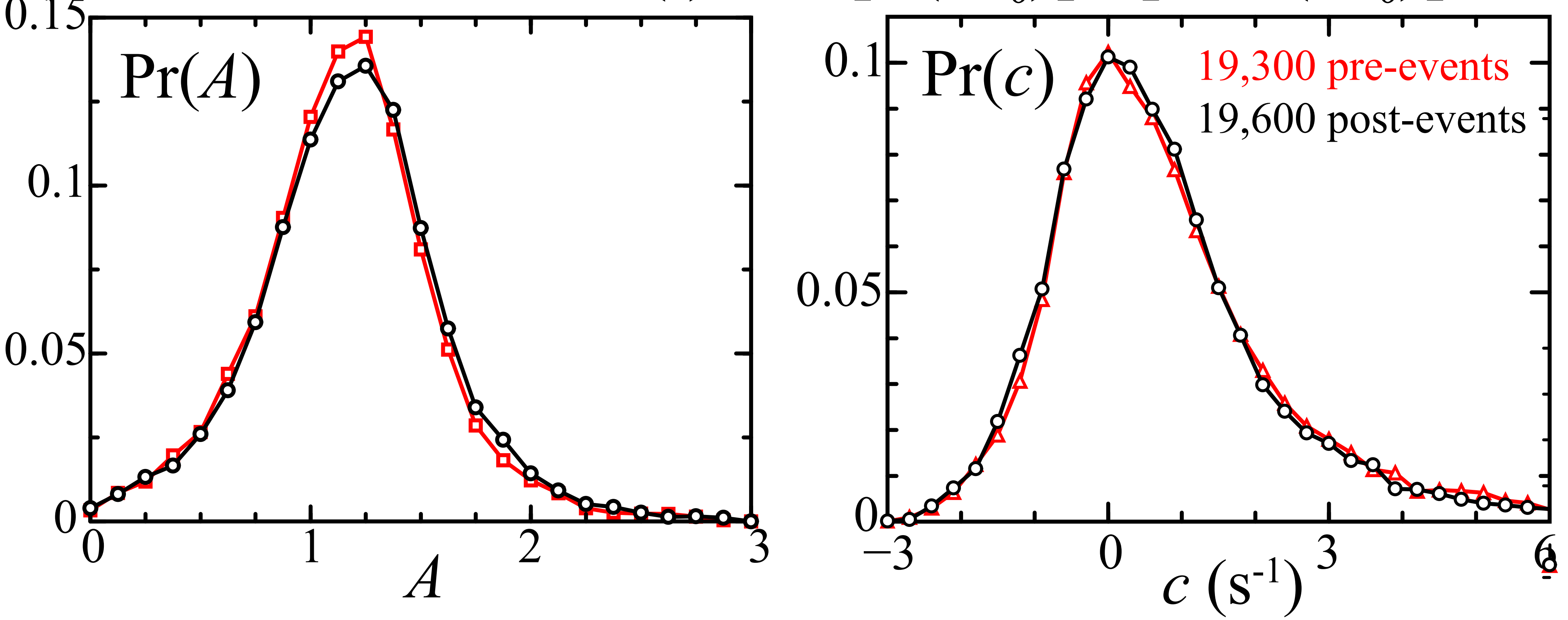
$$\delta(t) = A[\kappa(t - t_0)]^{1/2}$$

Bewley, MSP, Sreenivasan and Lathrop, *PNAS* **105**, 13707 (2008)

Correction-Factor Expression

Pre-reconnection: $\delta(t) = A[\kappa(t_0-t)]^{1/2}[1+c(t_0-t)]$

Post-reconnection: $\delta(t) = A[\kappa(t-t_0)]^{1/2}[1+c(t-t_0)]$

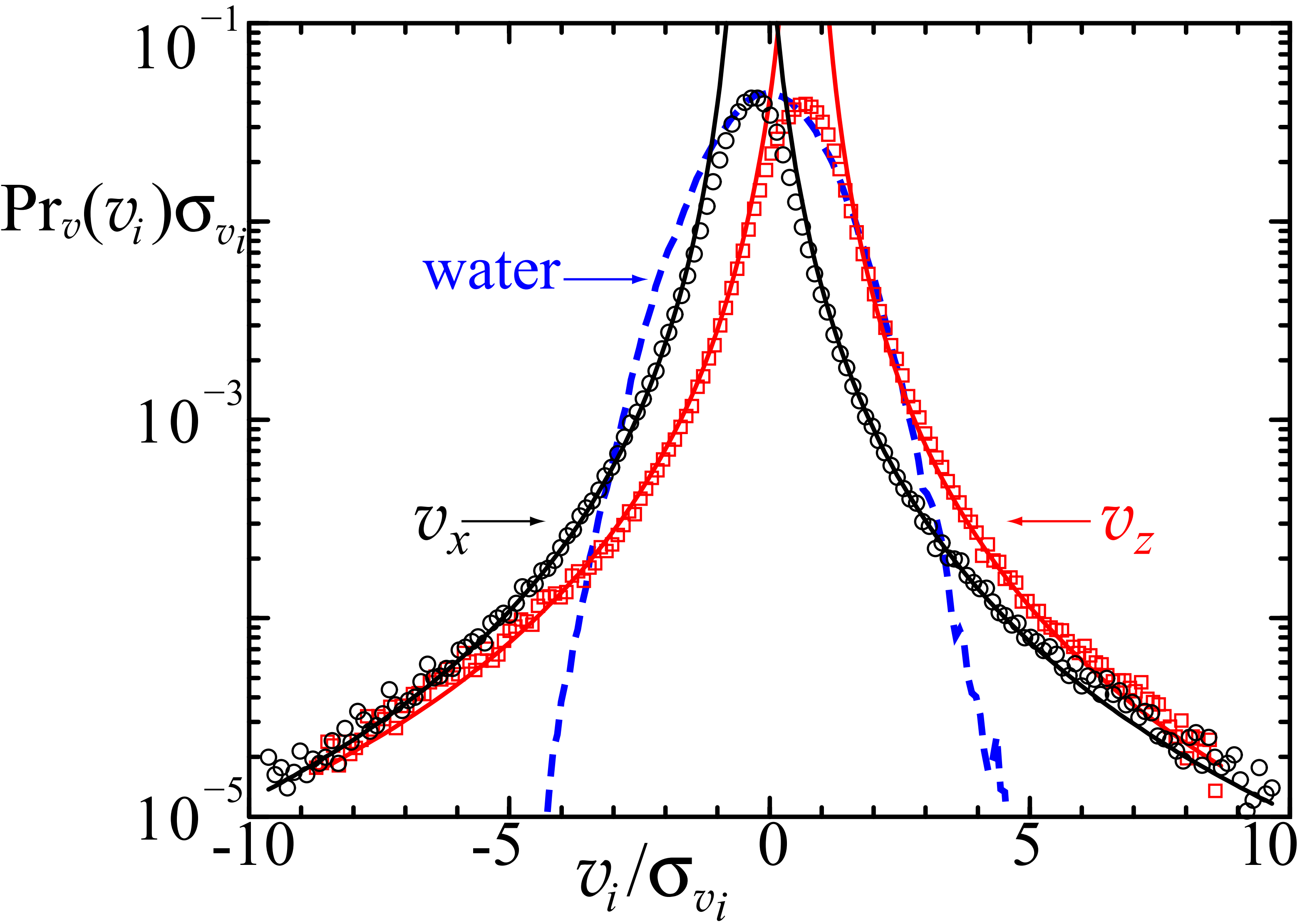


All measured statistics appear time-reversal invariant

MSP, M. E. Fisher and D. P. Lathrop, *Physica D* **239**, 1367 (2010)

Velocity Statistics

Velocity pdfs computed from *all* particle trajectories (no caveats)



$l(t) \sim \kappa^{1/2}|t-t_0|^{1/2}$

$v(t) \sim \kappa^{1/2}|t-t_0|^{-1/2}$

$\Pr(v)dv = \Pr(t)dt$

$\Pr(v) \sim |dt/dv|$

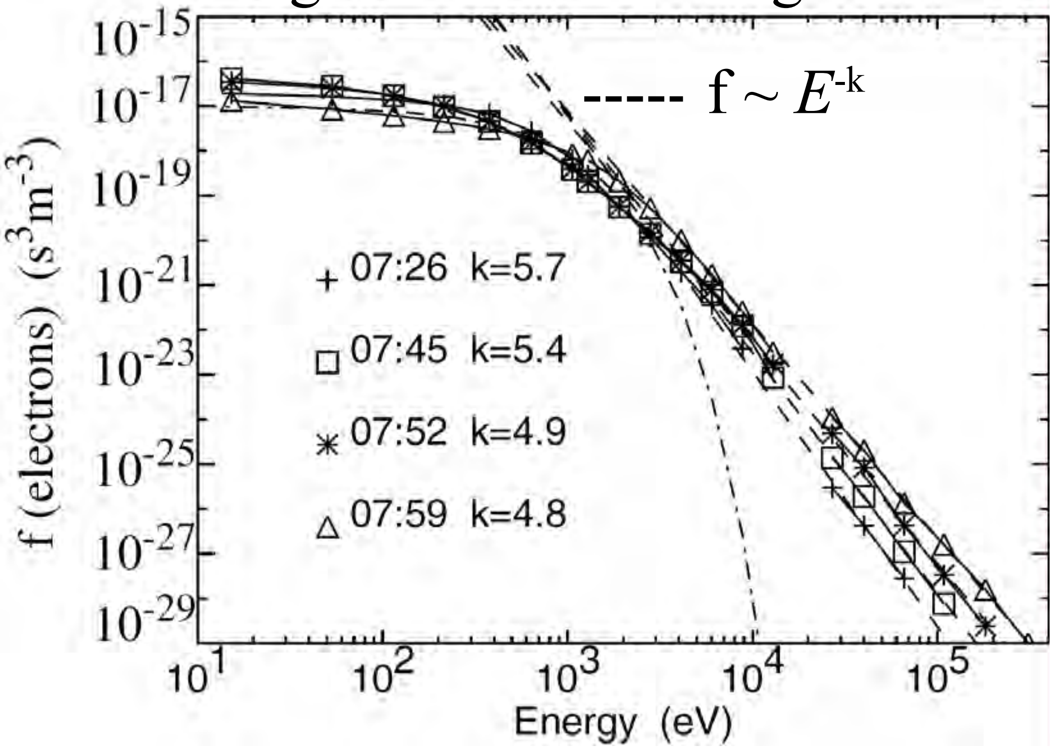
$\Pr(v) \sim v^{-3}$

MSP, Fisher, Sreenivasan, and Lathrop, *PRL* **101**, 154501 (2008)

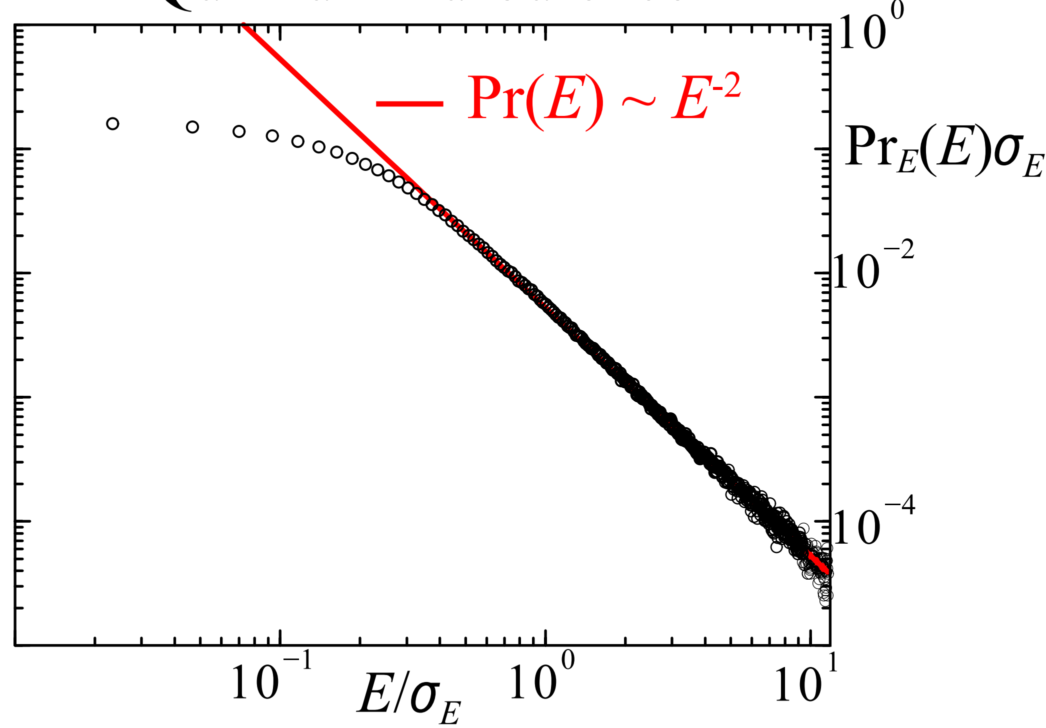
Analogies with MHD Turbulence

Motivated by J. F. Drake

Magnetic Reconnection Diffusion Region of Earth's Magnetotail



Reconnection-dominated Quantum Turbulence



Magnetic field lines in highly-magnetized plasmas reconnect and accelerate electrons with similar power-law distributions of energy

M. Oieroset *et al.*, *Phys. Rev. Lett.* **89**, 195001 (2002)

Conclusions

Visualize thermal counterflows and confirm Landau's theory

- MSP, Fiorito, Sreenivasan, and Lathrop, *J. Phys. Soc. Japan* **77**, 111007 (2008)

Visualize quantized vortex reconnection

- Bewley, MSP, Sreenivasan, and Lathrop, *Proc. Natl. Acad. Sci.* **105**, 13707 (2008)

Characterize dynamics of 20,000 reconnection events

- MSP, Fisher, and Lathrop, *Physica D* **239**, 1367 (2010)

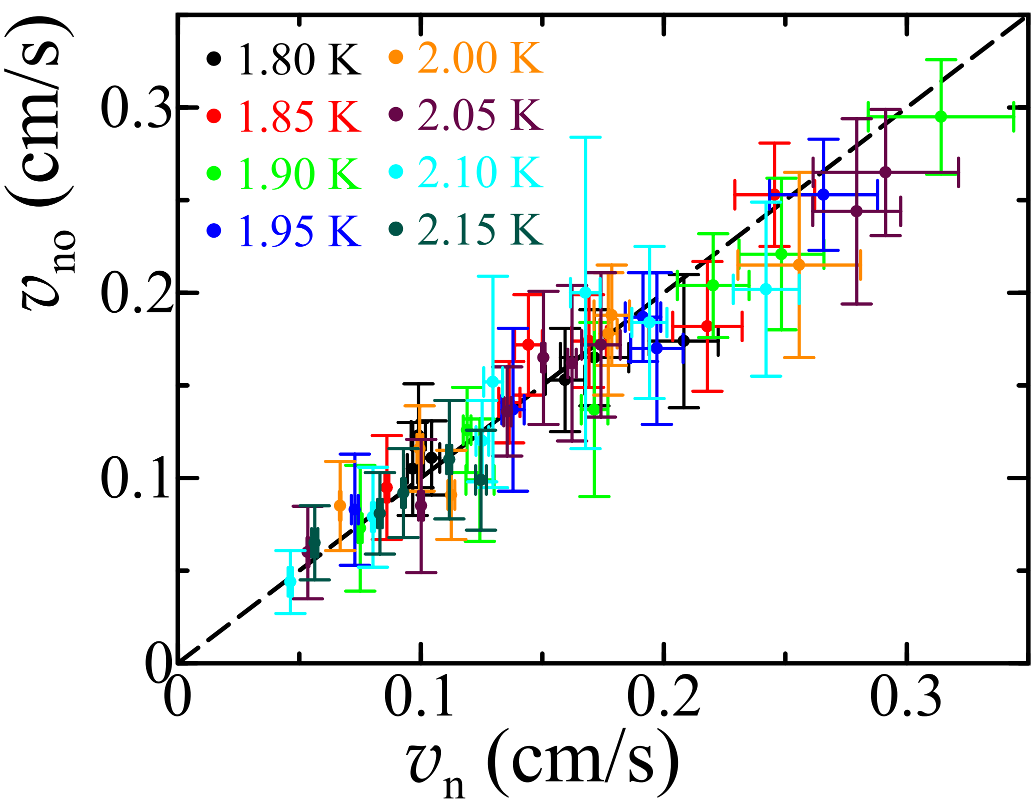
Velocity statistics of quantum turbulence are non-classical

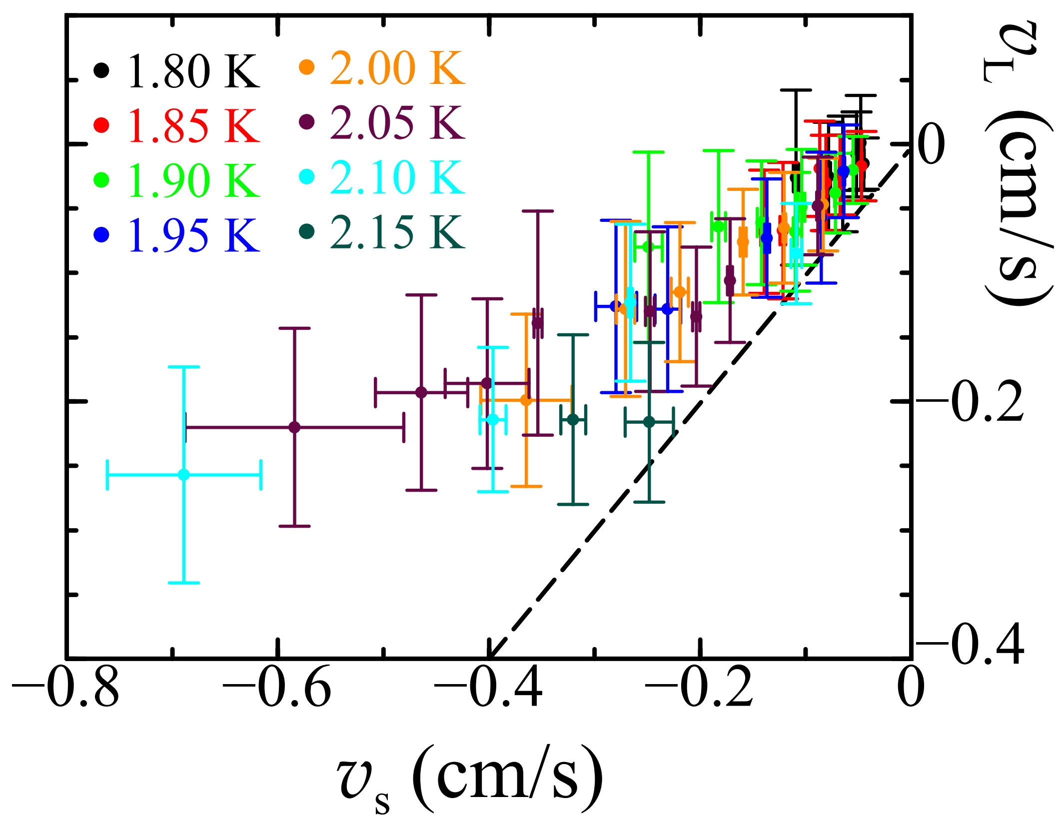
- MSP, Fisher, Sreenivasan, and Lathrop, *Phys. Rev. Lett.* **101**, 154501 (2008)

Quantum turbulence review

- MSP and Lathrop, *Ann. Rev. Cond. Matt. Phys.* in press

Thermal Counterflow Velocities





Observed normal fluid velocities match those predicted for all tempeartures and heat fluxes

Observed vortex line velocities are always below v_s, likely due to mutual friction

MSP, Fiorito, Sreenivasan, and Lathrop, *J. Phys. Soc. Japan* **77**, 111007 (2008)

Stokes Drag vs. Vortex Trapping

$$\mathbf{F}_{\mathrm{drag}} = -6\pi\eta a v_{\delta} \qquad \mathbf{F}_{\mathrm{trap}} = \rho_s \kappa^2 a^3 / 3\pi s^3$$

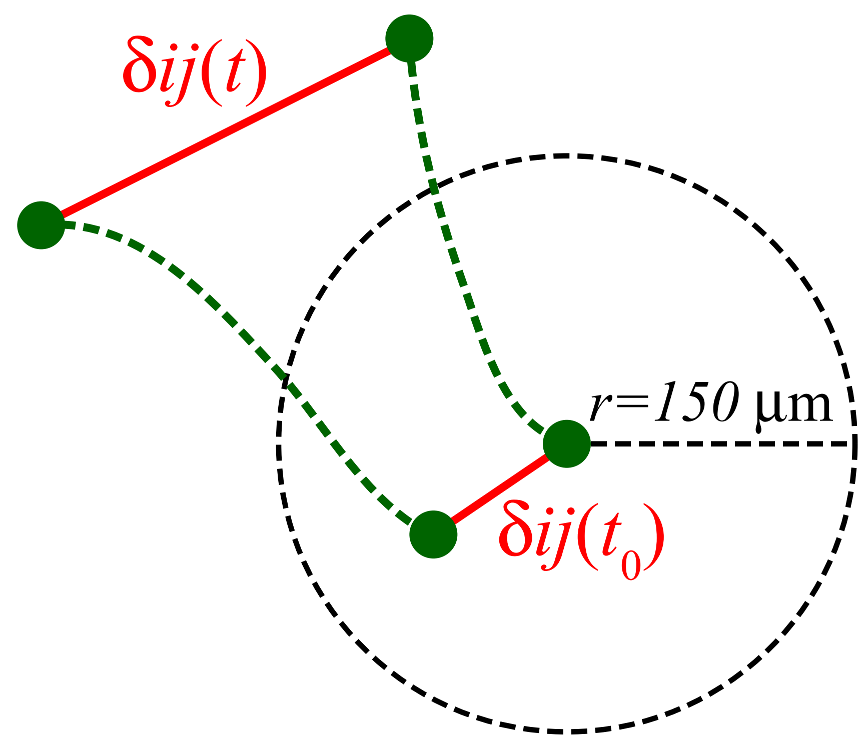
Stokes drag pulls particles away from vortices when they oppose $\mathbf{v}_n$

Particle trapping depends upon flow and particle properties and T

Characterize Stokes drag by separation of neighboring particles

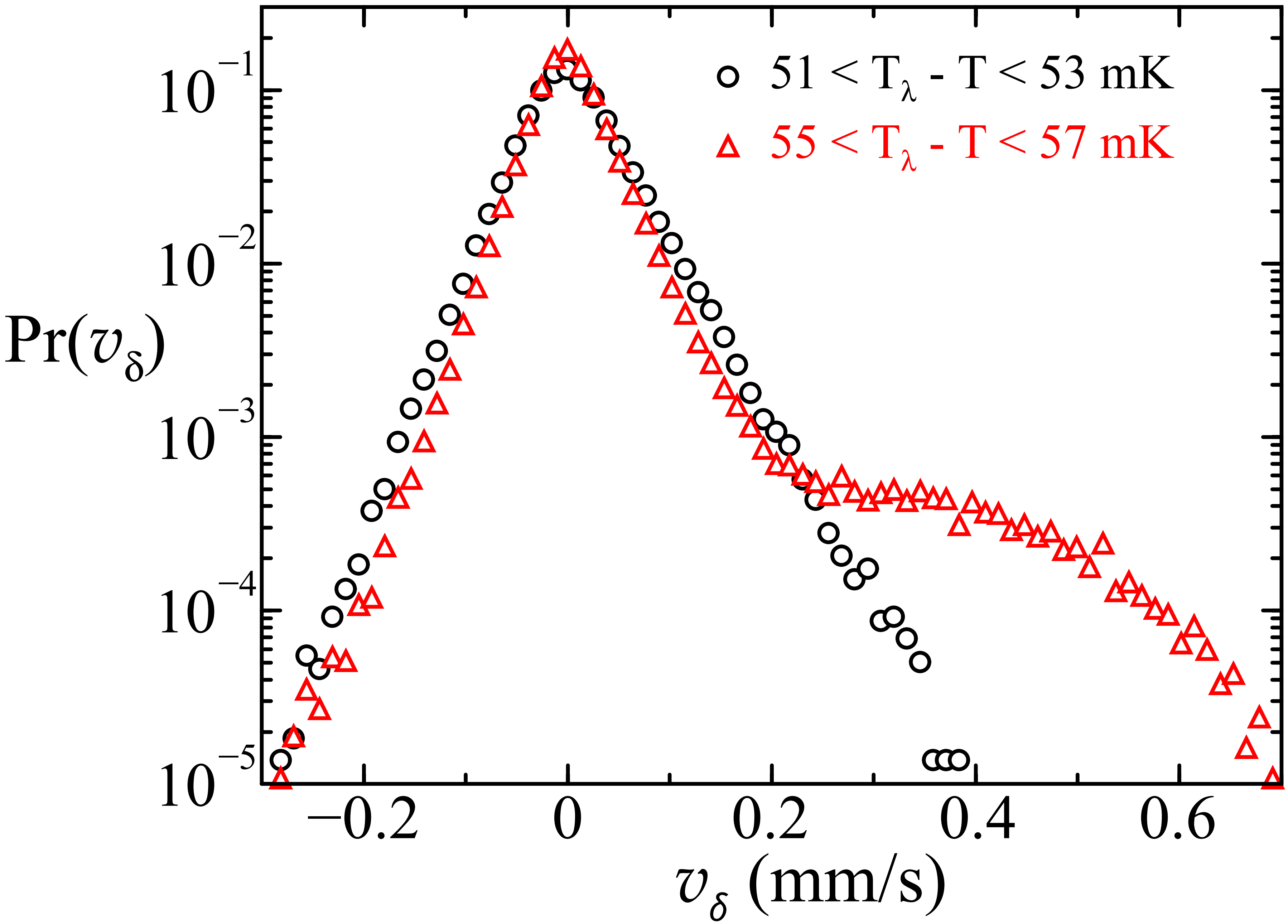
$$\delta_{ij}(t) = |\mathbf{r}_i(t) - \mathbf{r}_j(t)| = v_{\delta} t + \delta_{ij}(t_0)$$

Trapped particles characterized by large v_{δ} which oppose $\mathbf{v}_n$



Particle Trapping vs. Temperature

Data obtained while slowly cooling the system -152 μK/s with a 4 mW/cm^2 heat flux applied to drive gentle counterflow



Statistical Measure

Event defind as:

$$\frac{\delta_{mn}(t \pm 0.25\ s)}{\delta_{mn}(t)} > 4$$

\+ forward event

\- reversed event

Goodness-of-fit requirement

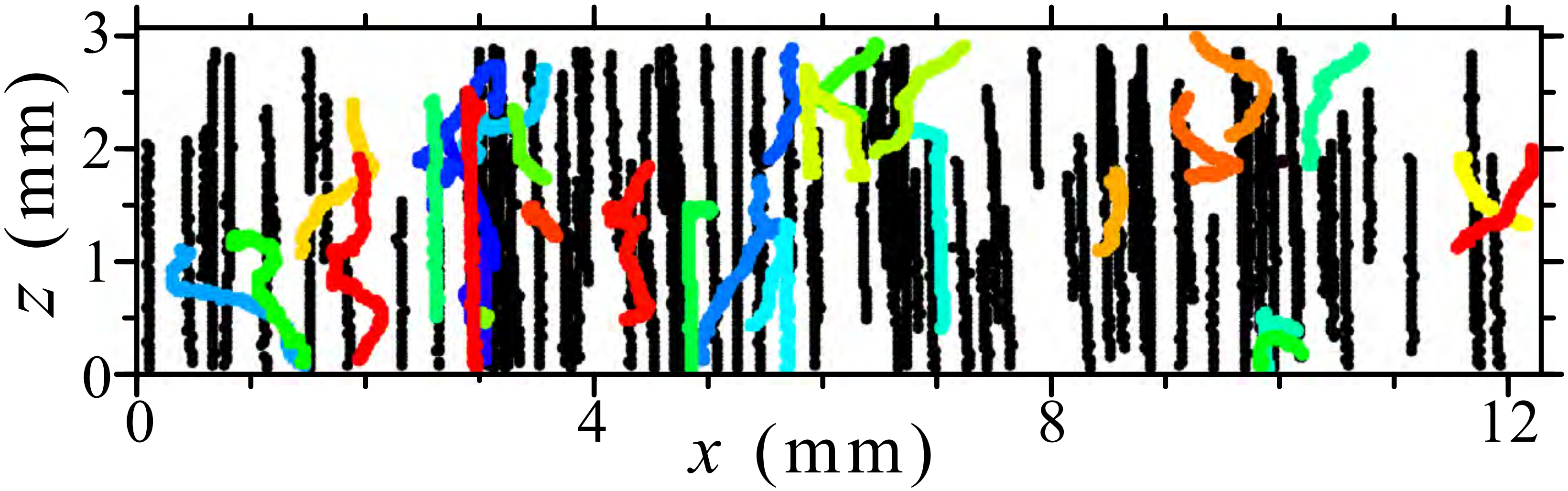
$$\chi^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\delta_{fit_i} - \delta_i}{\sigma}\right)^2 < 4$$

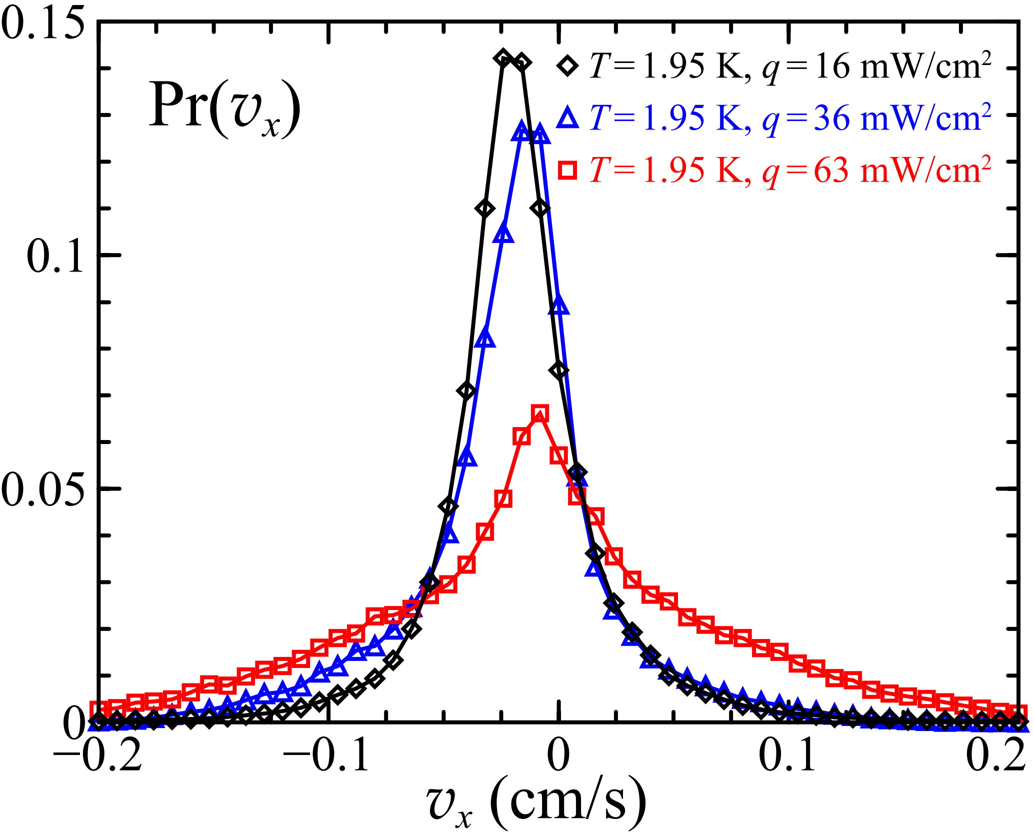
σ=4 μm (0.25 pixels)

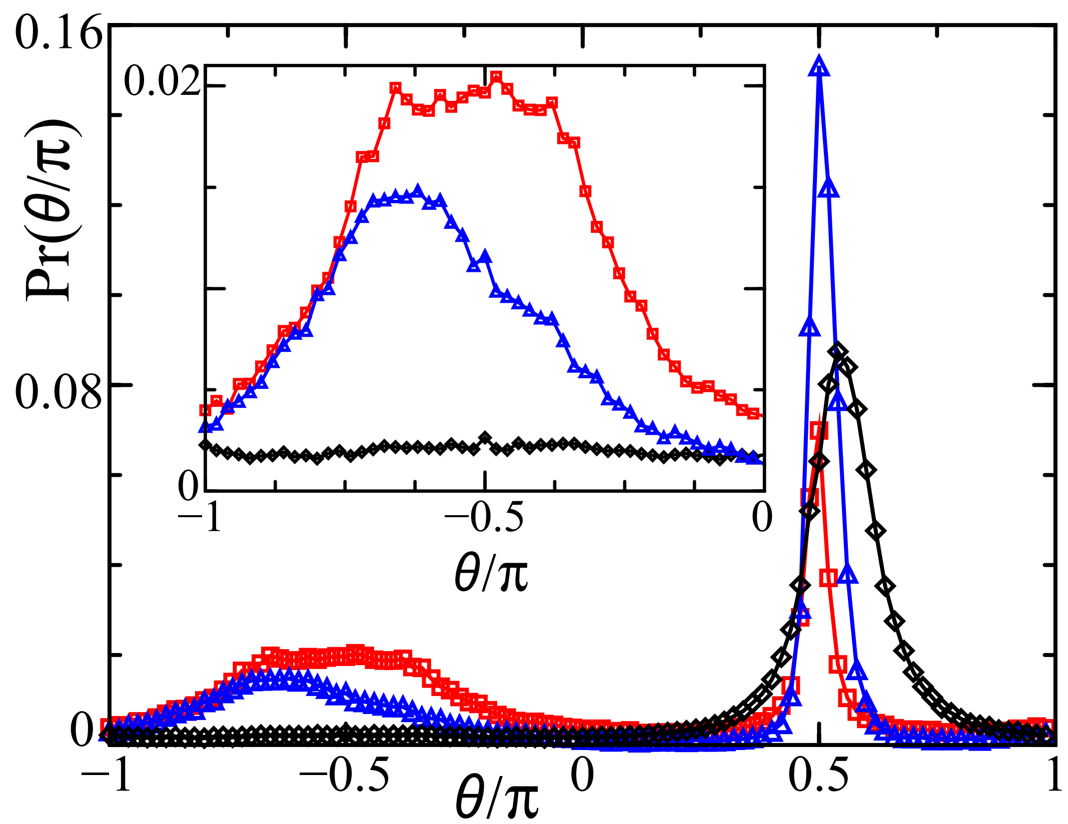
$$\delta_{fit} = B(t - t_0)^{\alpha}$$

$$\delta_{fit} = B(t_0 - t)^{\alpha}$$

Thermal Counterflow Velocity Statistics



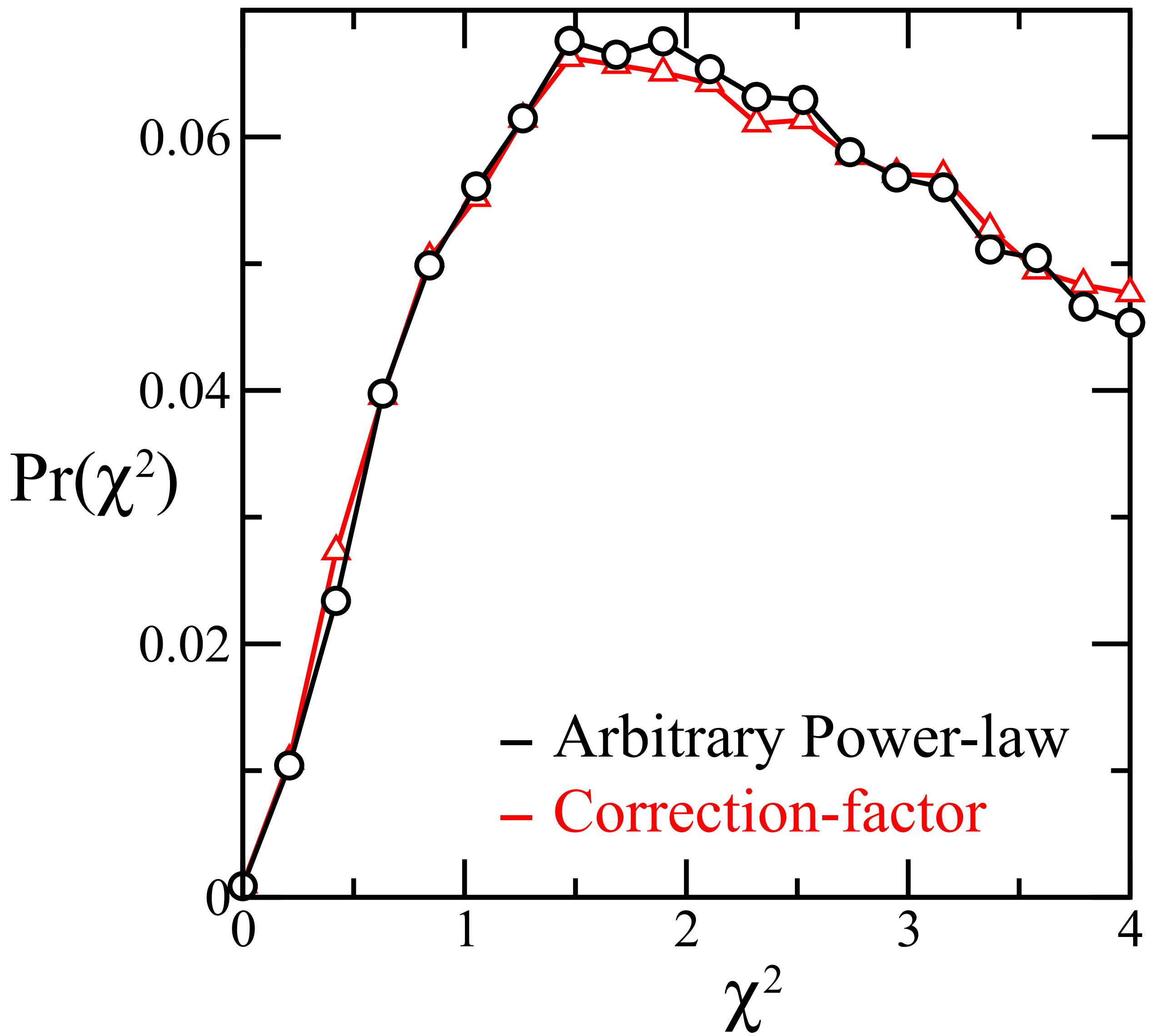




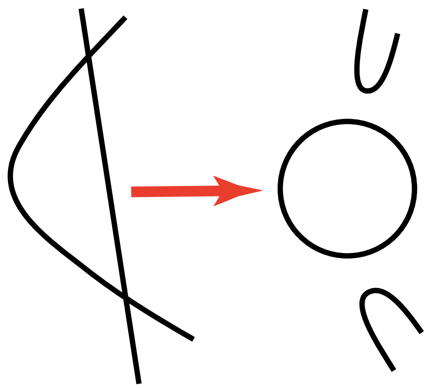
χ^2 Comparision

$$\chi^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\delta_{fit_i} - \delta_i}{\sigma}\right)^2$$

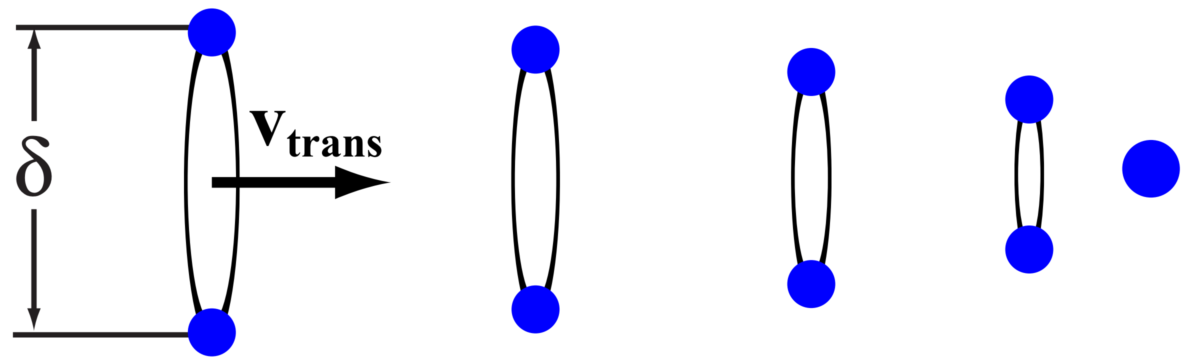
n = 15, 20, 25 for data take at 60, 80, 100 fps



Quantized Vortex Rings

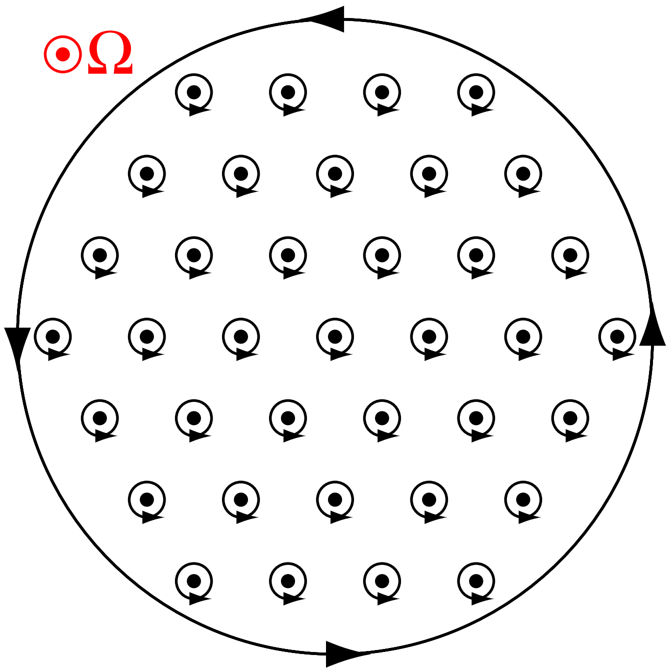


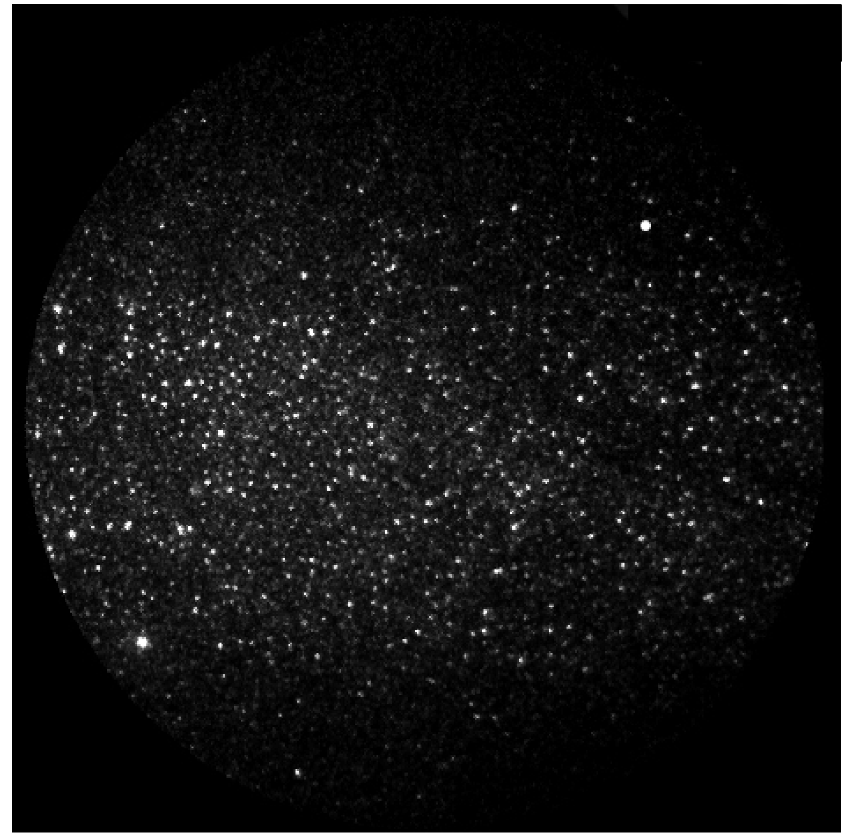
Reconnection
can produce
vortex rings

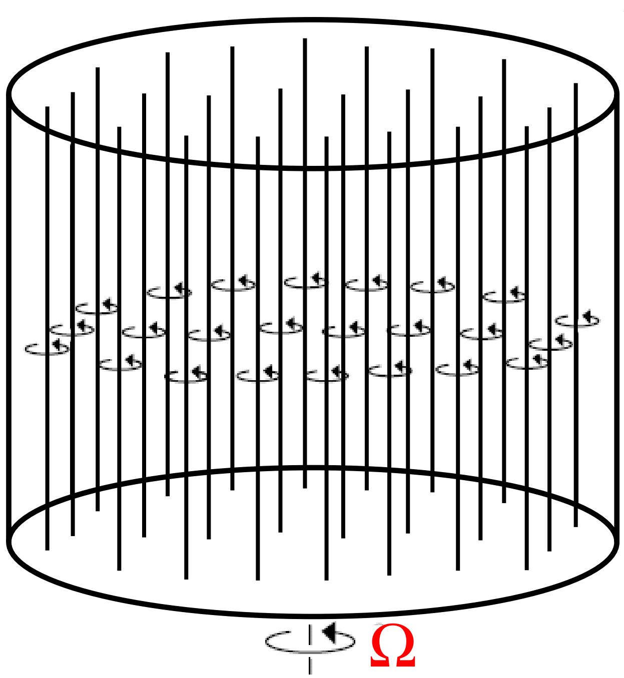


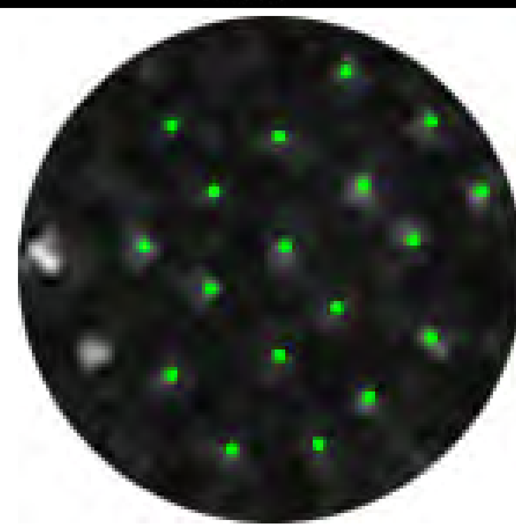
Pair of particles on collapsing rings
look like reconnection backwards in
time with additional transverse velocity

Rotating Superfluids

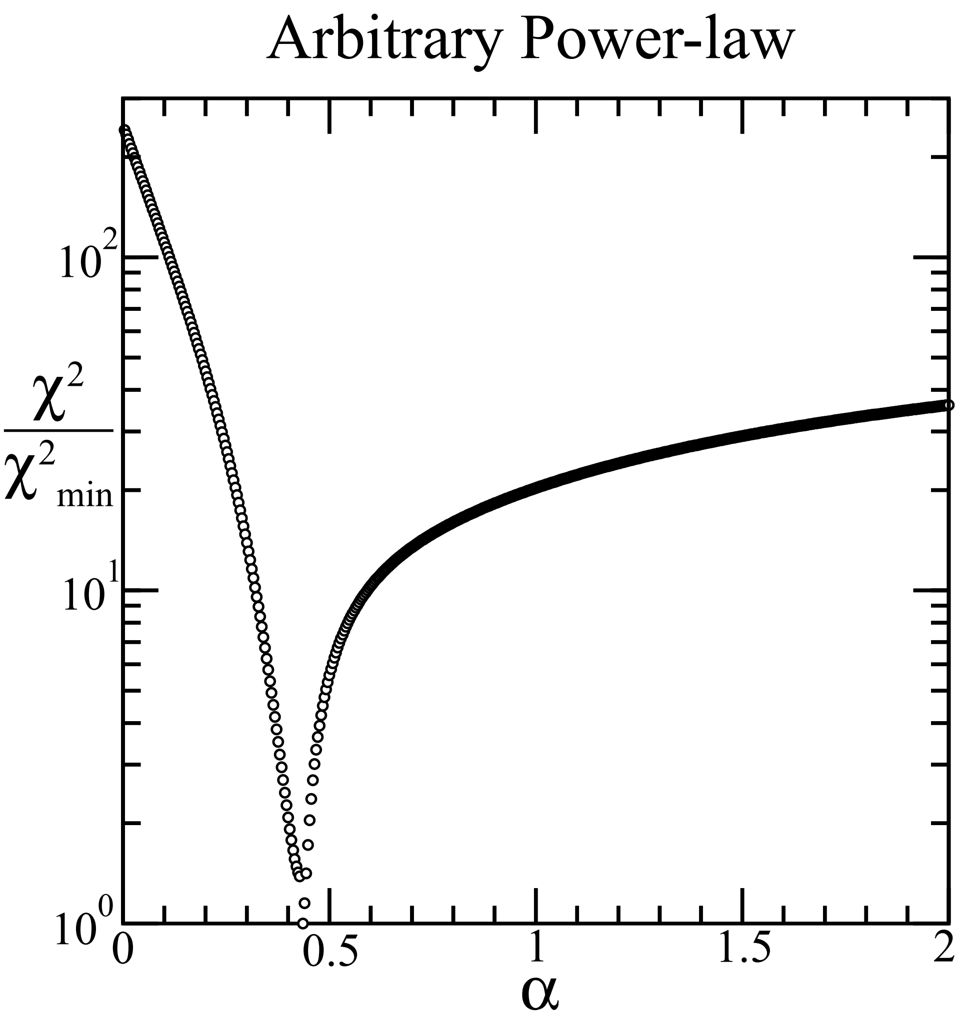


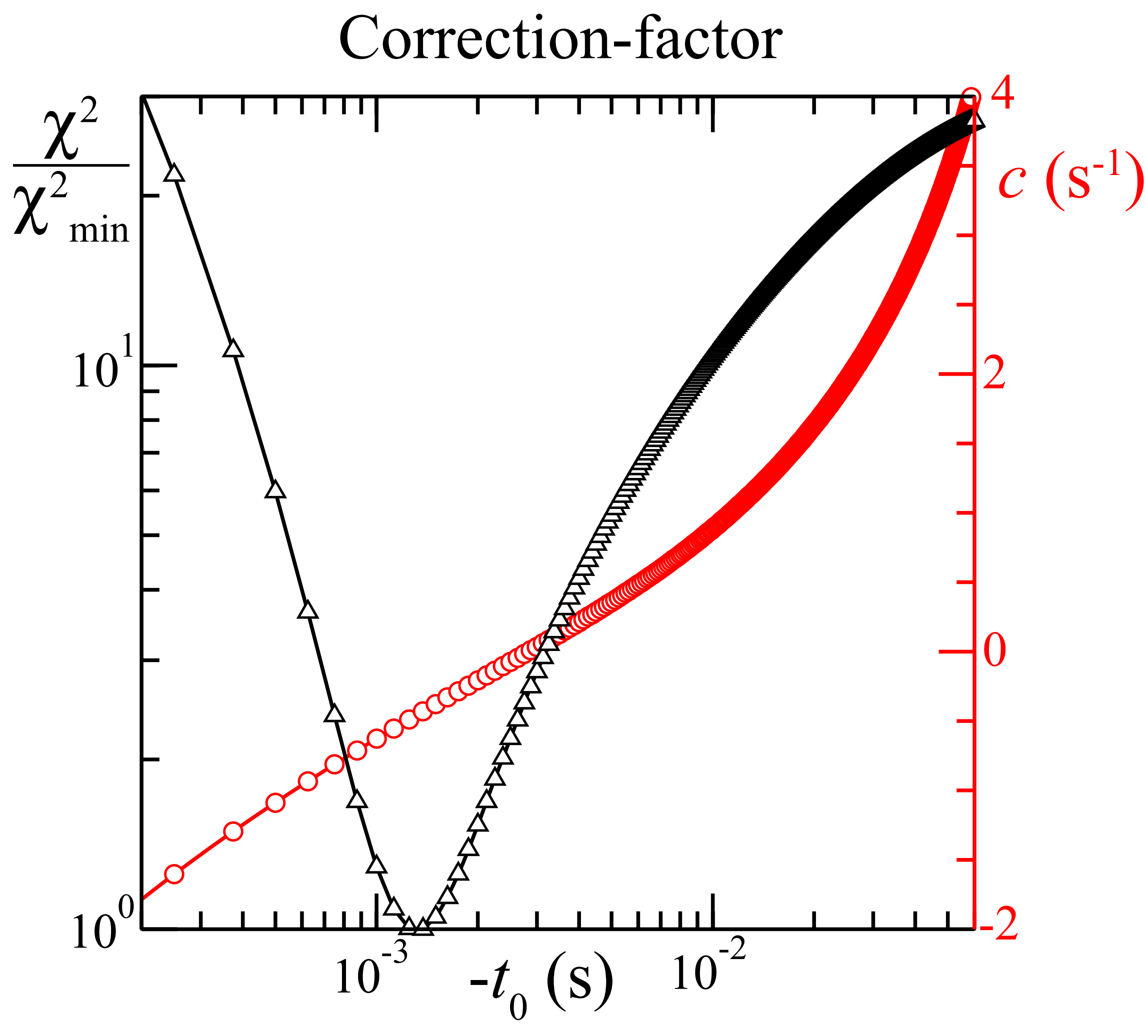






Selecting Fit Parameters



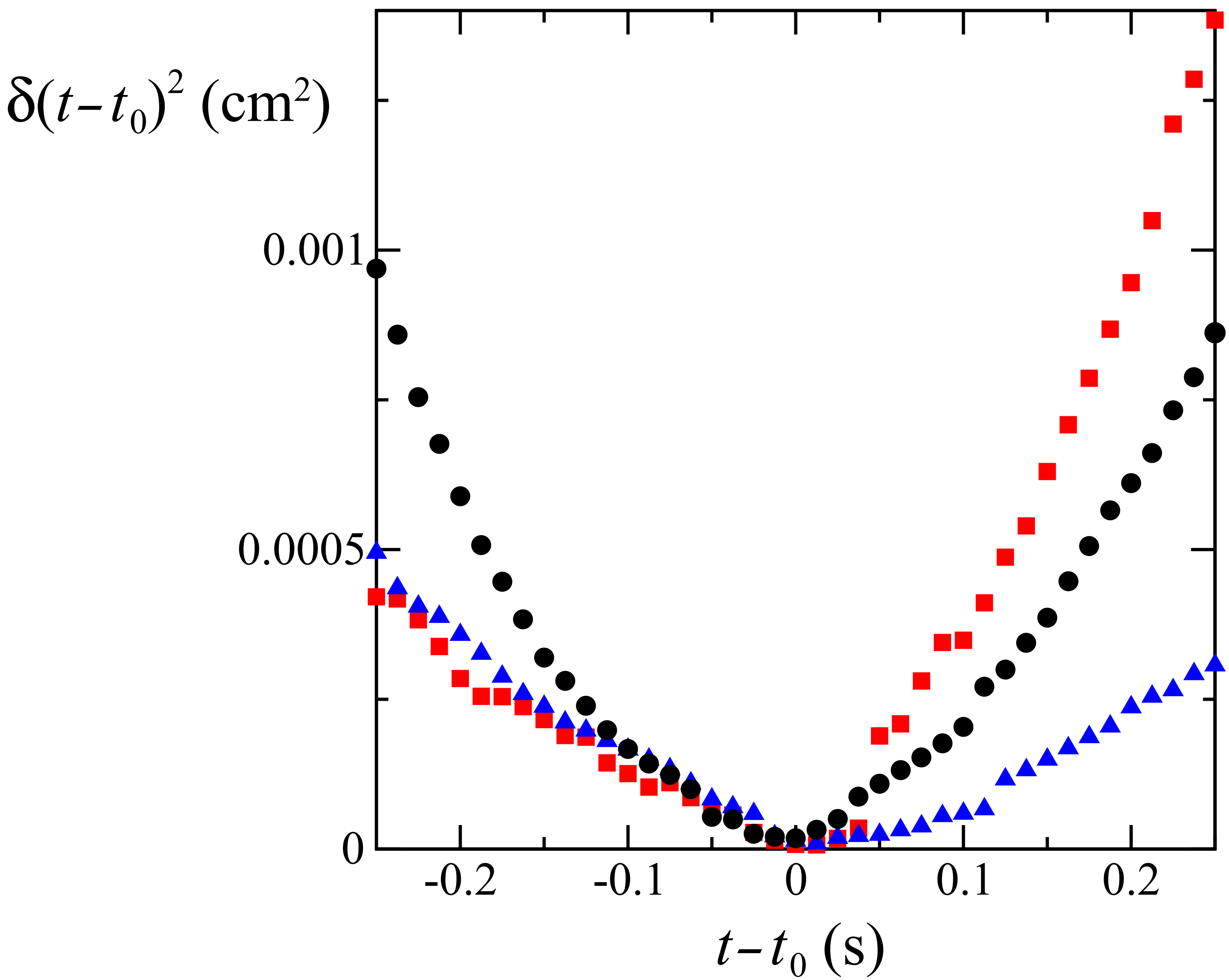


For each event, 500 values of α and t_0 are fit and the sets of $\{\alpha, B, t_0\}$ and $\{A, c, t_0\}$ that minimize χ^2 are chosen as the best fit

Time-Reversibility

Pre-reconnection: $\delta(t) = A\_[\kappa(t_0\text{-}t)]^{1/2}[1 + c\_(t_0\text{-}t)]$

Post-reconnection: $\delta(t) = A_+[\kappa(t\text{-}t_0)]^{1/2}[1 + c_+(t\text{-}t_0)]$



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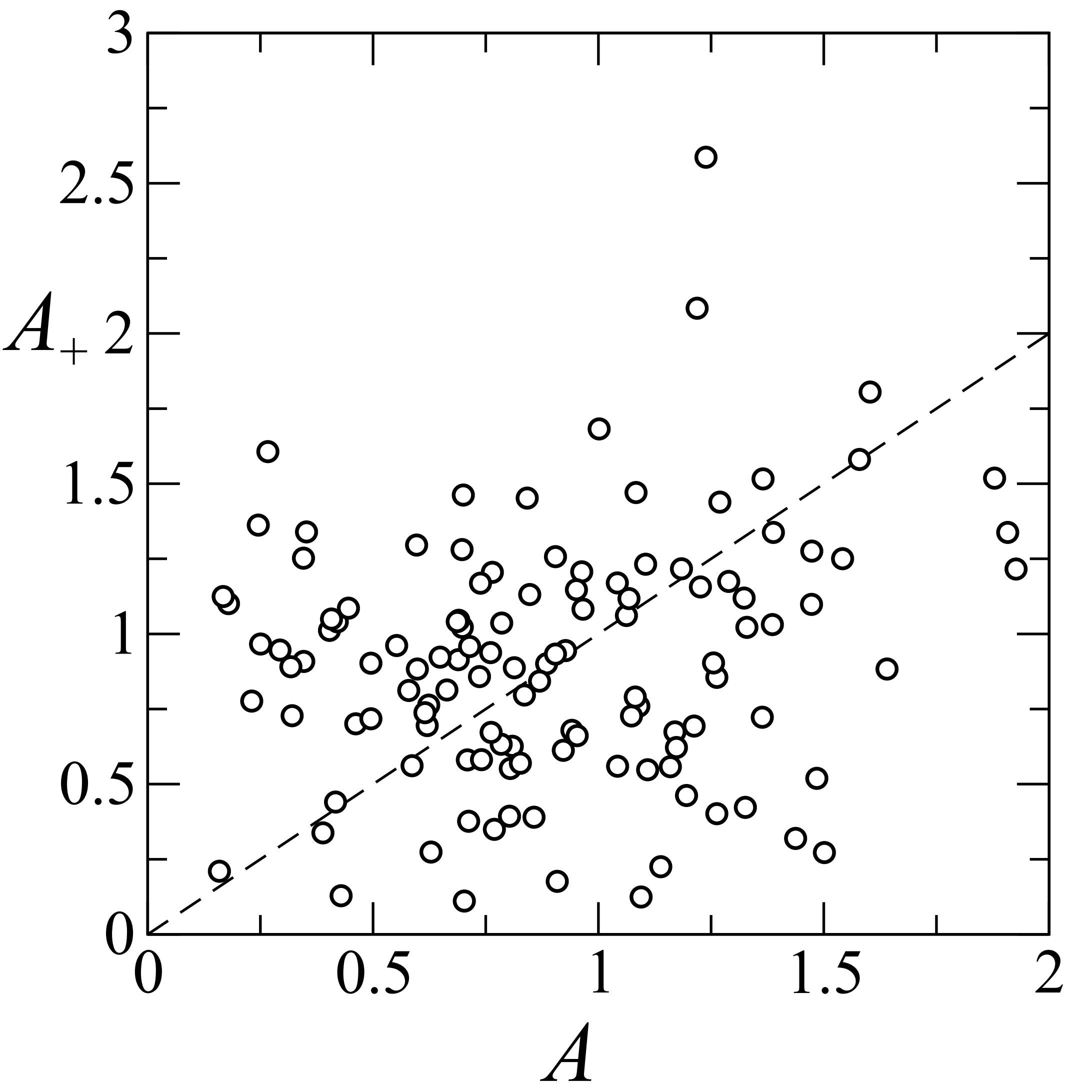
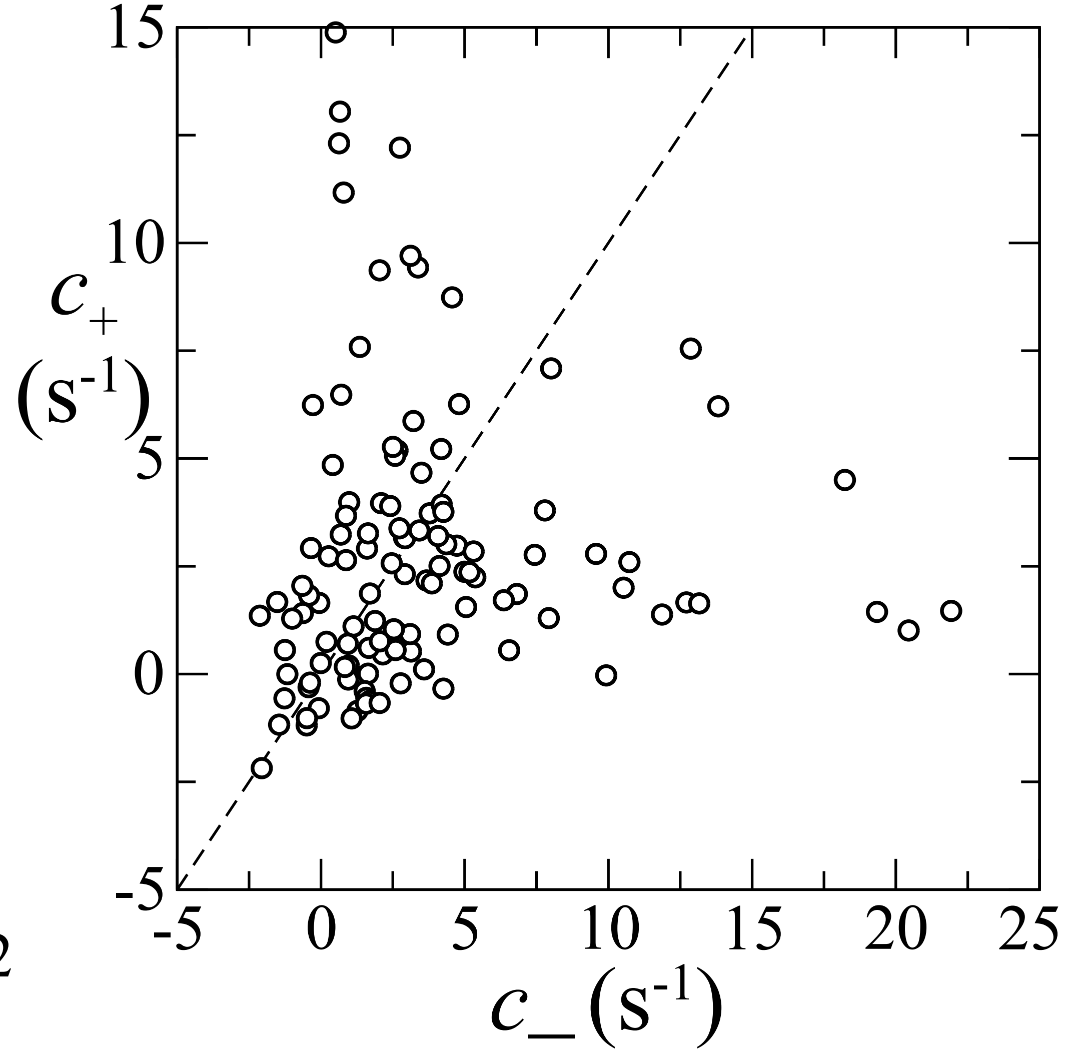
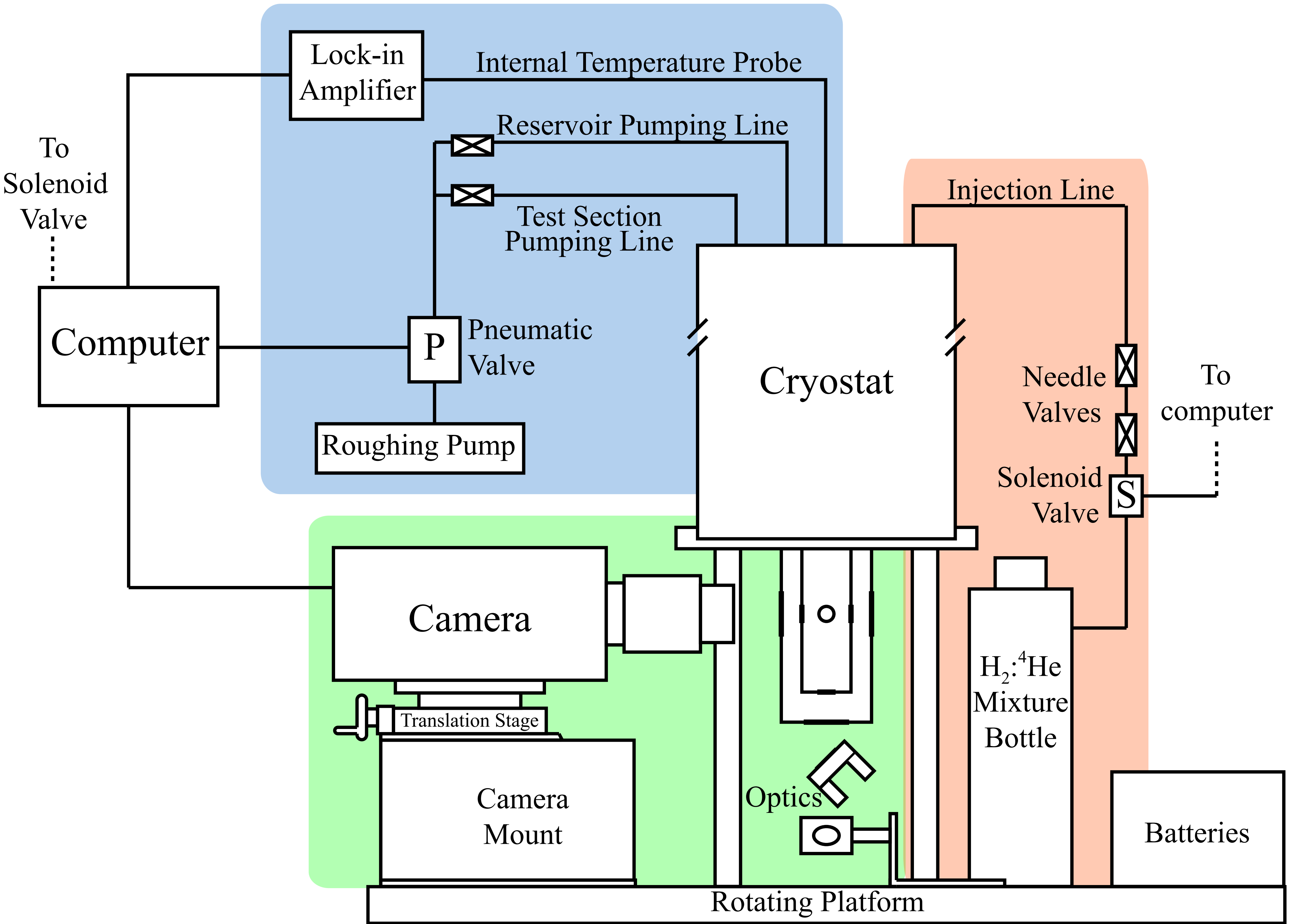




Apparatus

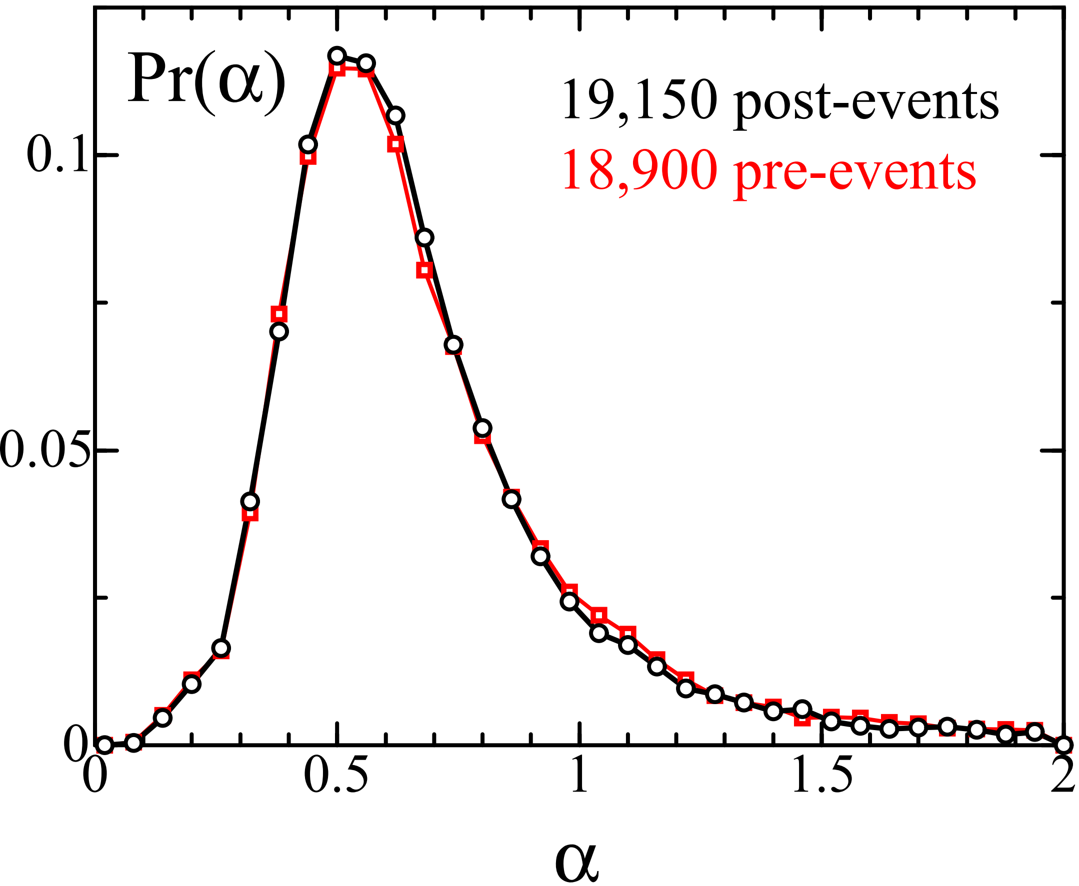


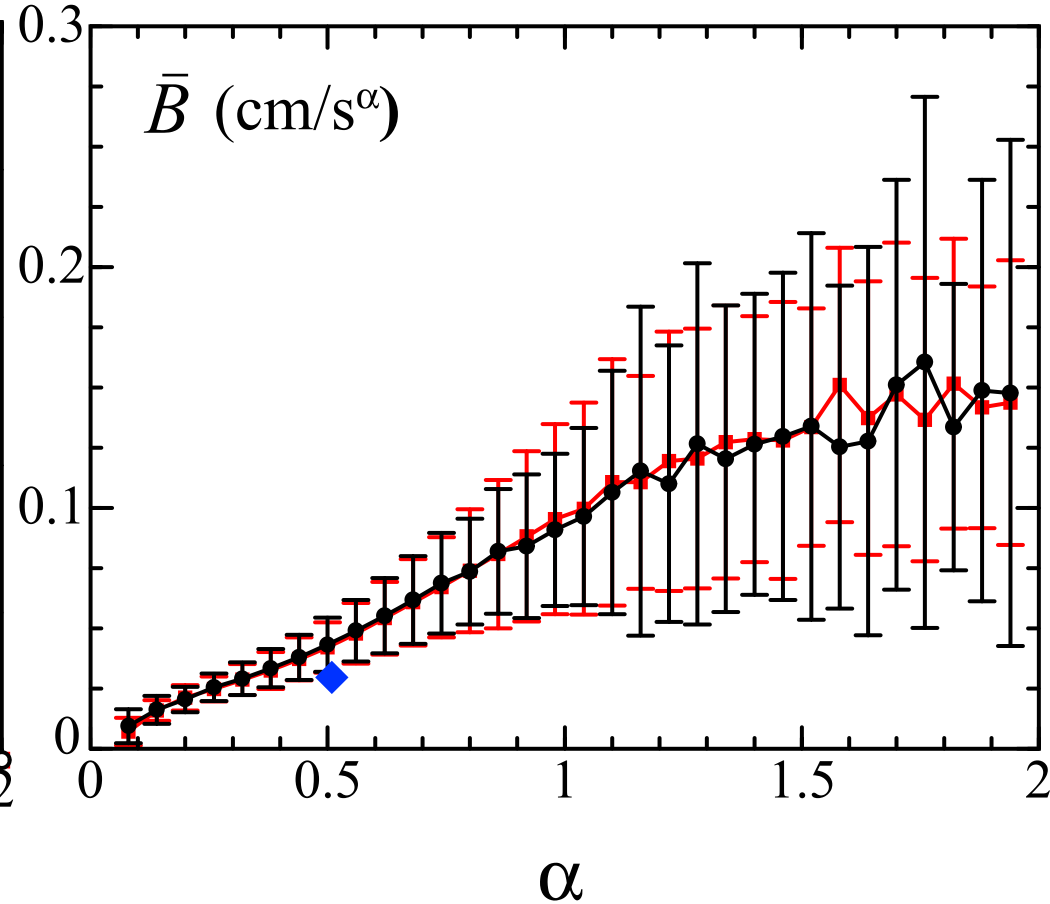
Lock-in Amplifier
Internal Temperature Probe
Reservoir Pumping Line
Test Section Pumping Line
To Solenoid Valve
Computer
P
Pneumatic Valve
Roughing Pump
Cryostat
Injection Line
Needle Valves
To computer
Solenoid Valve
S
Camera
Translation Stage
Camera Mount
Optics
H$_2$:^{4}He Mixture Bottle
Batteries
Rotating Platform

Arbitrary Power-Law Expression

Pre-reconnection: $\delta(t) = B(t_0 - t)^\alpha$

Post-reconnection: $\delta(t) = B(t - t_0)^\alpha$

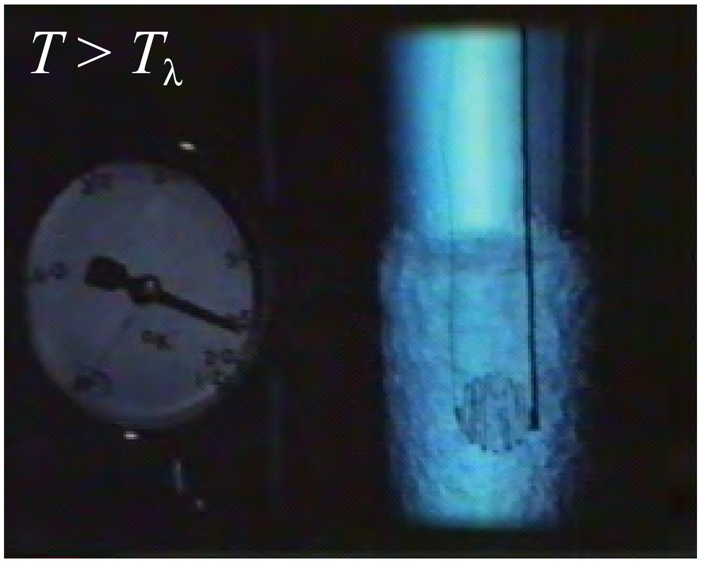


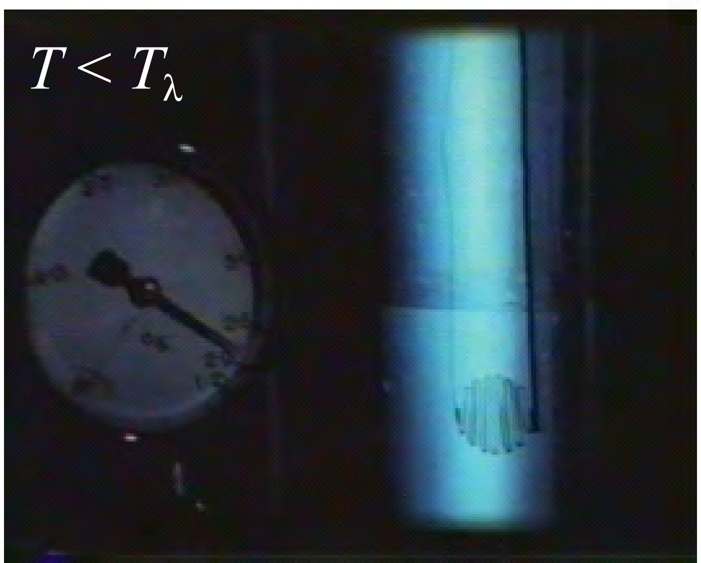


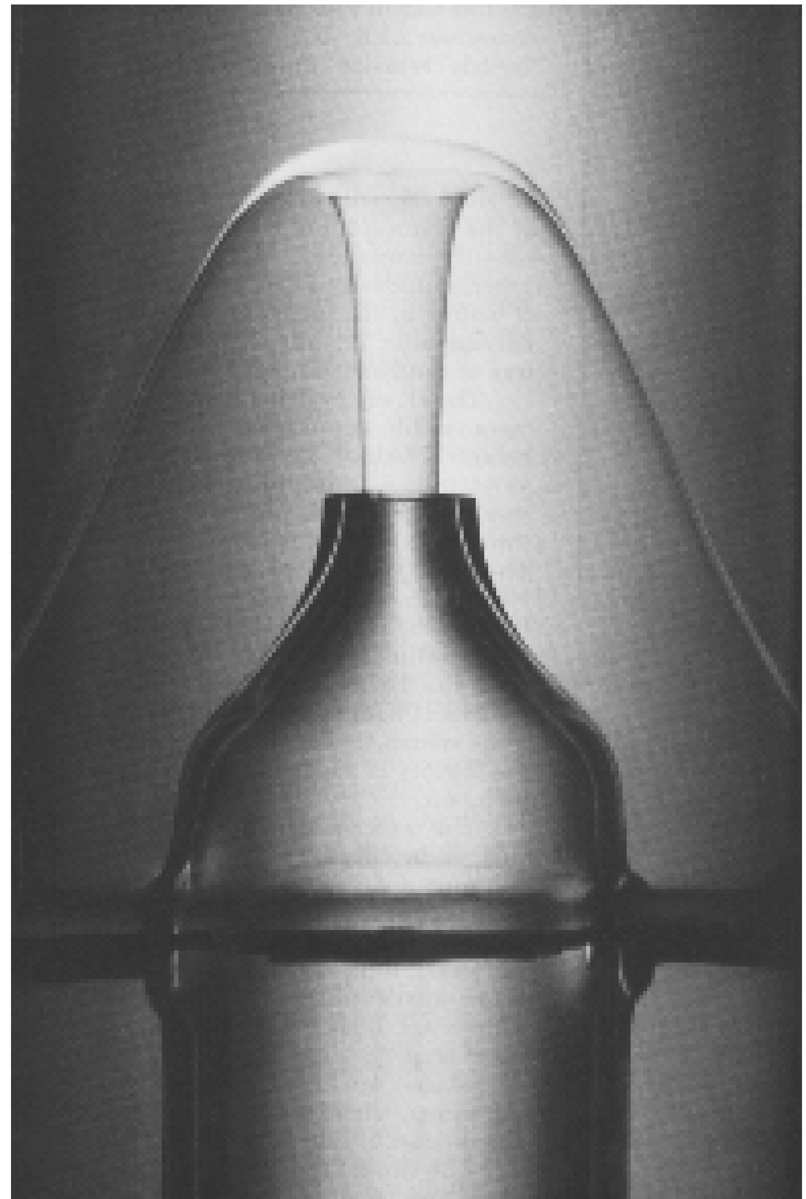
MSP, M. E. Fisher and D. P Lathrop, *Physica D* in press (2010)

Superfluid Helium (He II)

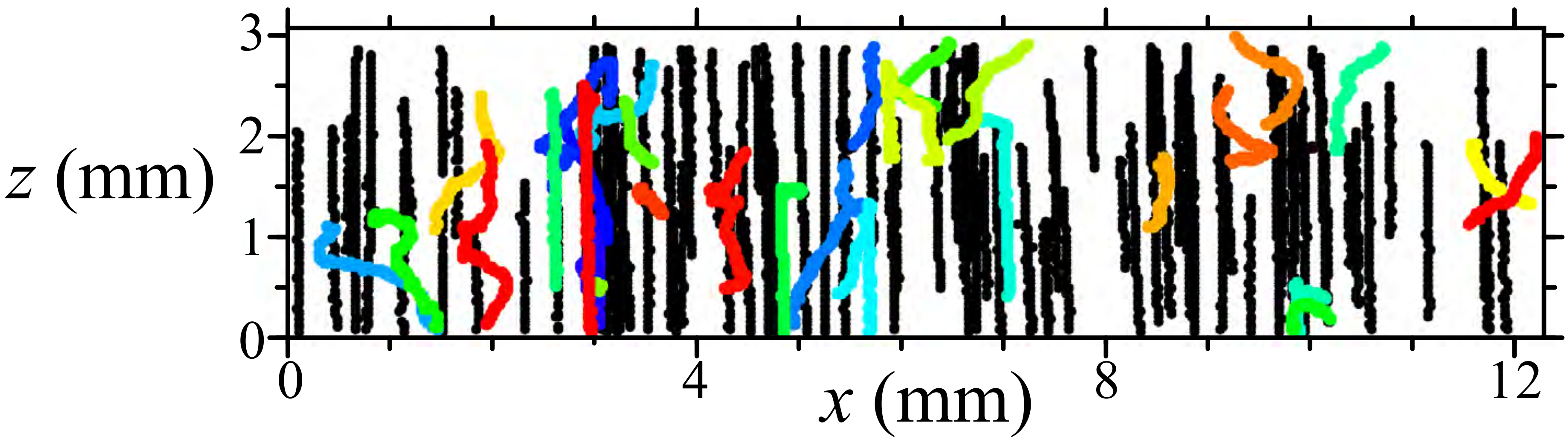
- Visible boiling and convection ceases (Mclennan *et al.* 1932)
- 10^7 fold increase in heat transport (Keesom 1936, Allen 1937)
- Flow produced by irradiation (Allen *et al.* 1938)

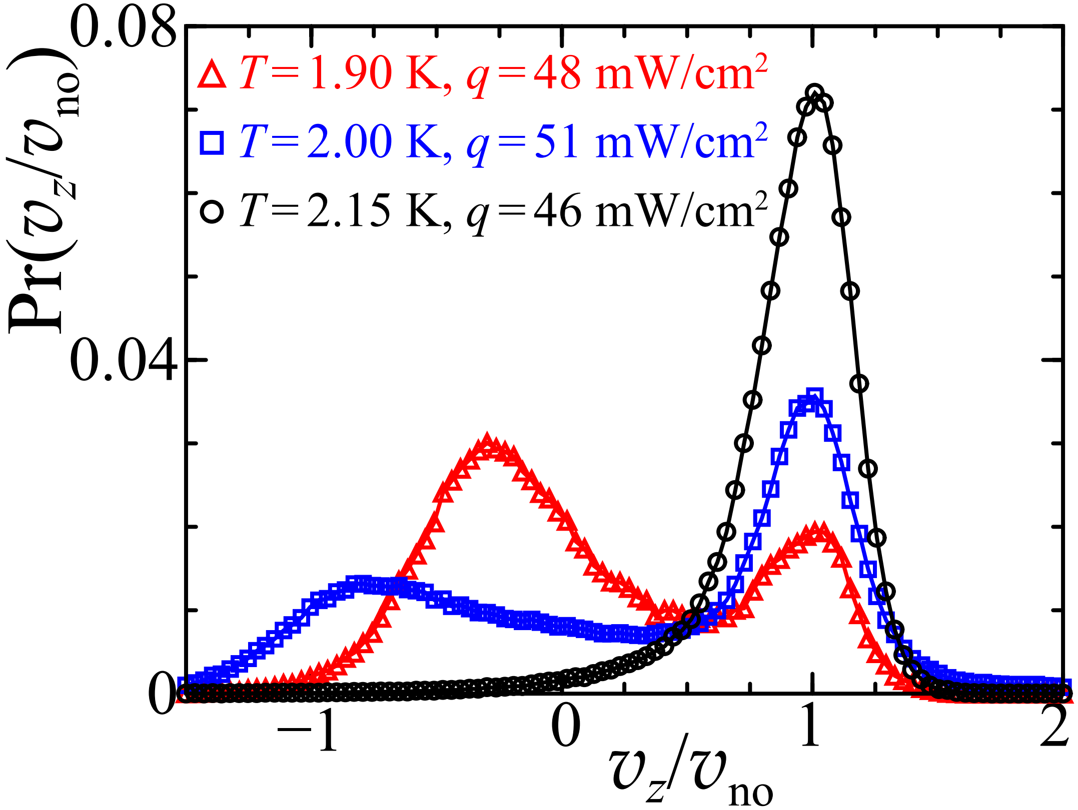


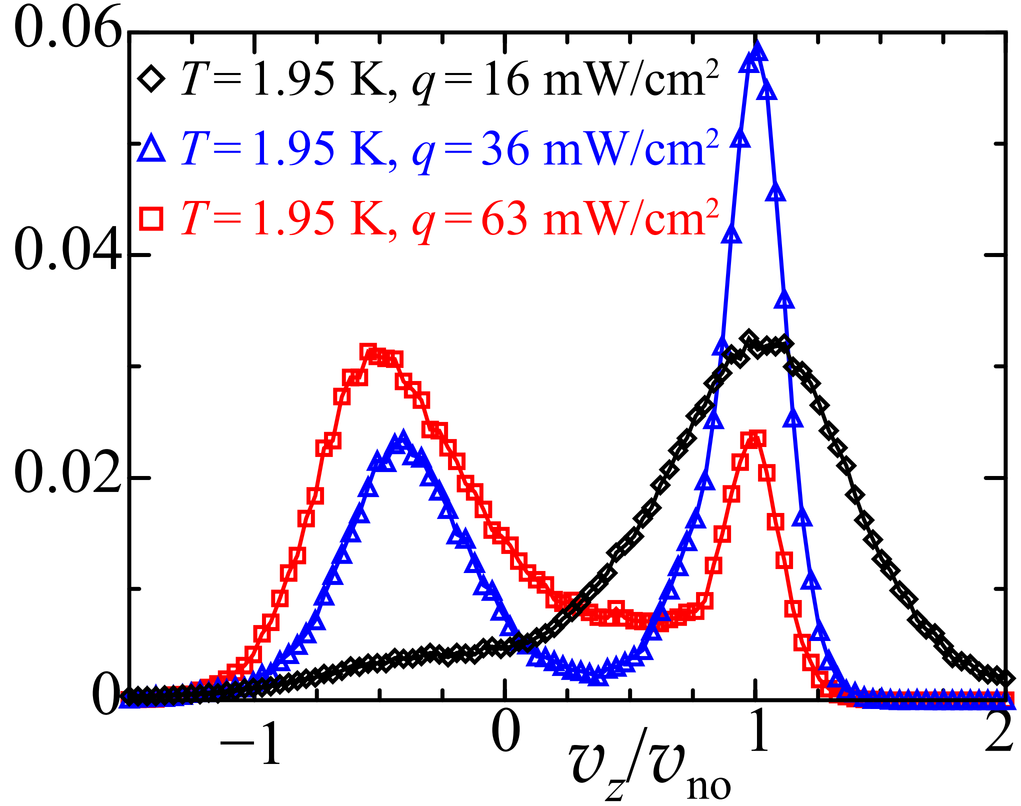




Thermal Counterflow Velocity Statistics



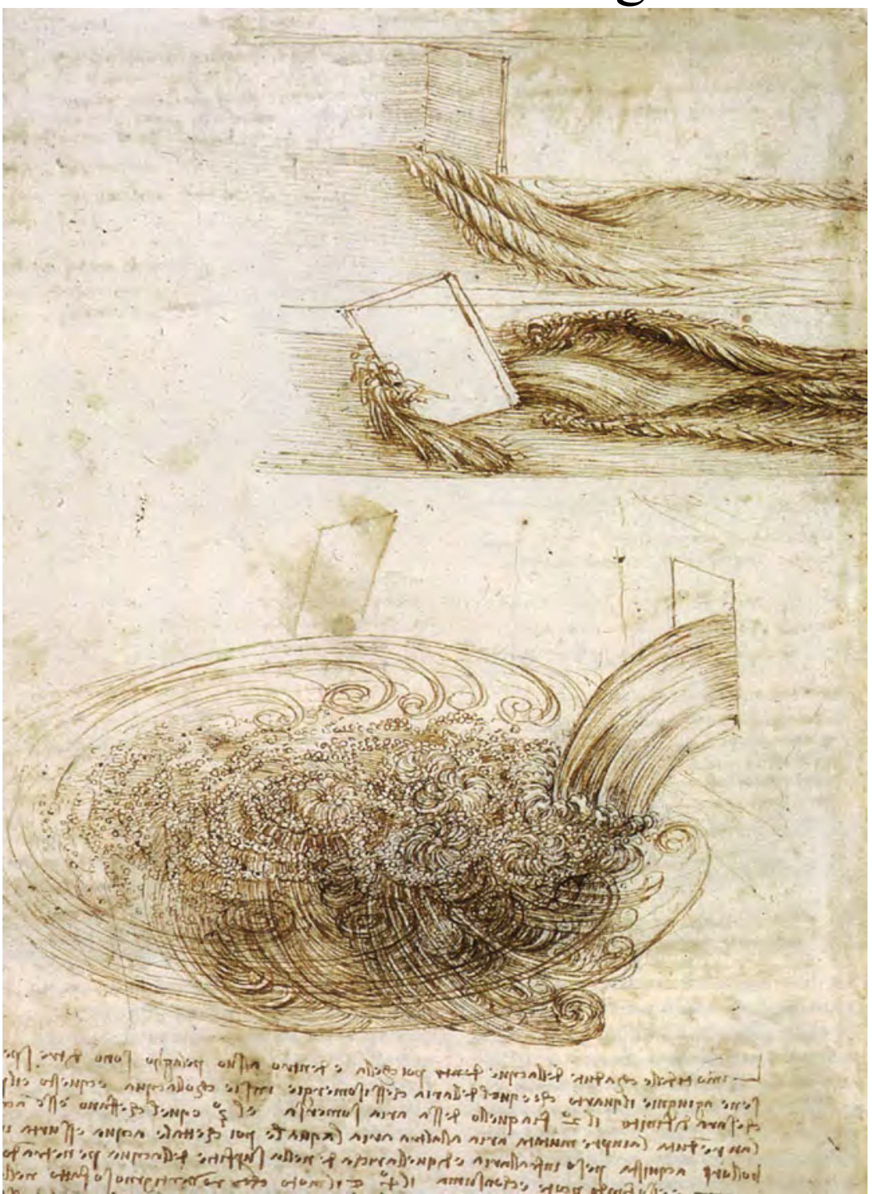




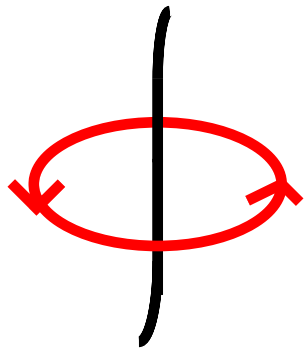
Fluid Turbulence

Classical

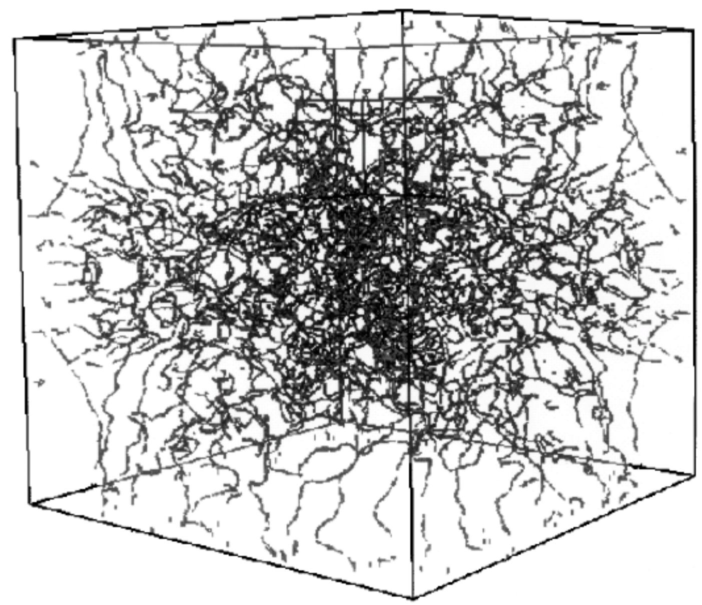
Vortices span a large range in size and strength



Quantum



Vortices all atomically-thin with idential circulating flow



Turbulence: tangle of interacting quantized vortices

C. Nore, M Abid & M. E. Brachet *Phys. Rev. Lett* **78**, 3896 (1997)

Decaying Quantum Turbulence

Previous experimental studies:

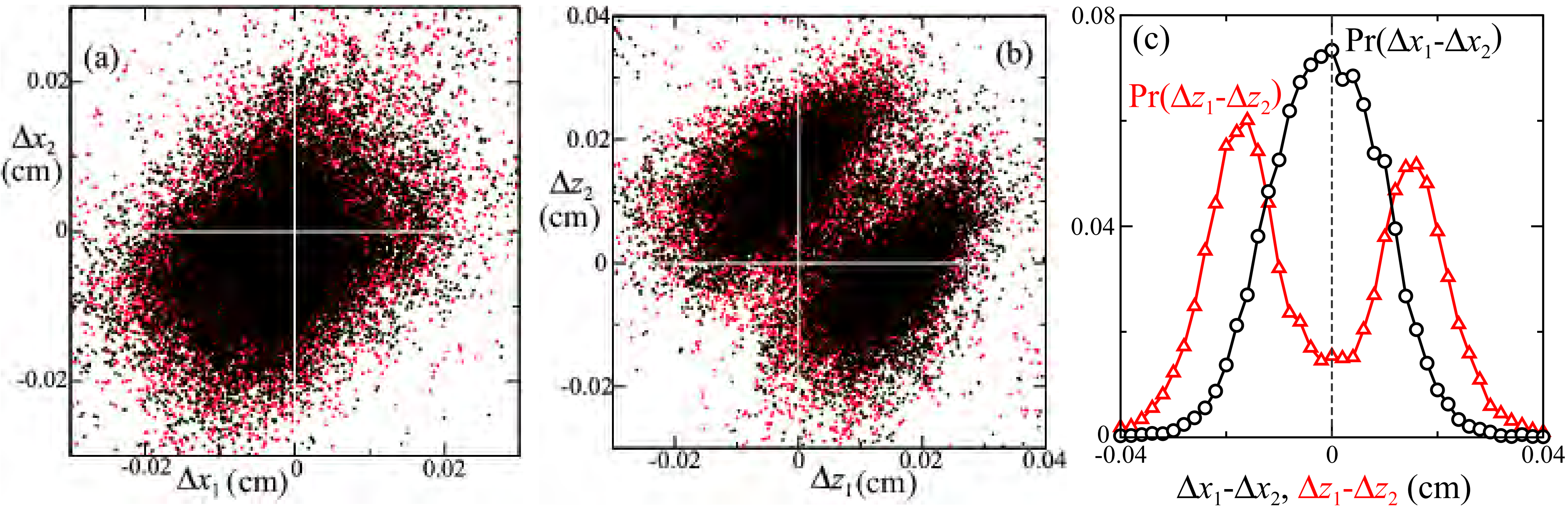
- Smith *et al.*, PRL 1993 (grid, second sound)
- Skrbek *et al.*, PRL 2000 (grid, second sound)
- Skrbek *et al.*, PRE 2003 (counterflow, second sound)
- Gordeev *et al.*, JLTP 2005 (counterflow, second sound)
- Niemela *et al.*, JLTP 2005 (grid, secound sound)
- Chagovets *et al.*, PRE 2007 (counterflow, second sound)
- Walmsley *et al.*, PRL 2007 (spin down, negative ions)
- Walmsley *et al.*, JLTP 2008 (spin down, negative ions)
- Walmsley *et al.*, PRL 2008 (ion jet, negative ions/CVRs)

Previous studies only measure spatially-averaged quantities over large volumes

Reconnection Displacement Vectors

Pre-reconnection: $\Delta \mathbf{r}_i = \mathbf{r}_i(t - 0.25\ \mathrm{s}) - \mathbf{r}_i(t)$

Post-reconnection: $\Delta \mathbf{r}_i = \mathbf{r}_i(t + 0.25\ \mathrm{s}) - \mathbf{r}_i(t)$



Displacements show anisotropy along direction of driving counterflow (z)

All measured *statistics* time-reversal invariant

Dissipation vs. Topological Defects

$$\frac{dL}{dt} = \alpha \left| \mathbf{v}_{ns} \right| L^{3/2} - \beta \kappa L^2$$

$$L = \frac{\text{defect line length}}{\text{volume}}, \ \mathbf{v}_{ns} = \mathbf{v}_n - \mathbf{v}_s$$

Defect generation term - drives the system away from equilibrium
increases line length through counterflow

Dissipation term - relaxes system toward equilibrium
reduces line length through dissipation

WF Vinen: Proc. R. Soc. London Ser. A **242**, 493 (1957)

Dissipation vs. Topological Defects

Dissipative processes relax systems toward equilibrium

Topological defects are structurally constrained, frustrating a system's ability to equilibrate

Ex: magnetic domains walls can prevent a ferromagnet from reaching its minimum energy state

How do topological defects interact as a system relaxes toward equilibrium?

Why $c \neq 0$?

Expect sub-dominant corrections for crossover between scales

Effects of local environment (Tsubota studying numerically)

Influence of neighboring vortices – convert c to a length l

$$c|t - t_0| \equiv \pm\kappa|t - t_0|/l^2$$

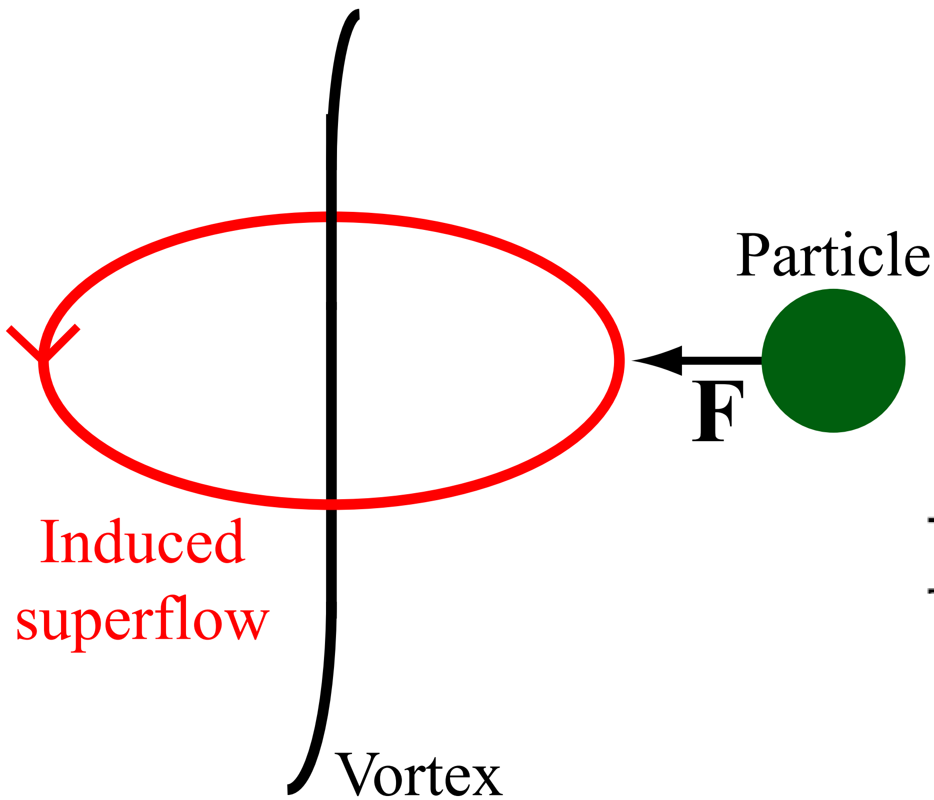
$$l_{\text{mean}} = 0.40 \text{ mm}$$

(typical intervortex spacing 0.1 – 1 mm)

Particle Trapping Mechanism

Pressure gradient acts to balance centrifugal force of circulating superfluid around vortex

Hydrogen particles do not circulate, only feel pressure gradient that traps them along quantized vortices



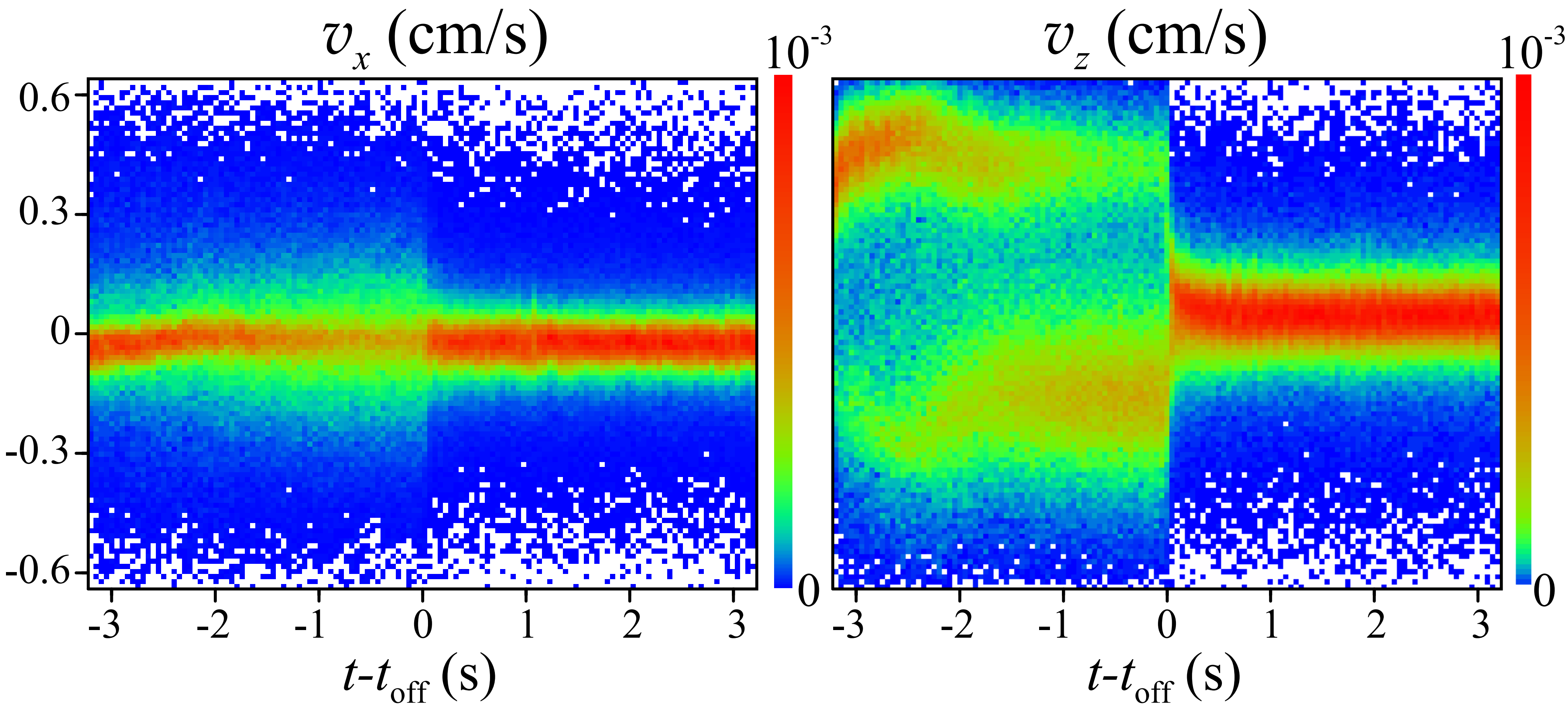
$$P = -\frac{\rho_{\mathrm{S}}\kappa^2}{8\pi^2 s^2}$$

$$-\nabla P = -\frac{\rho_{\mathrm{S}}\kappa^2}{4\pi^2 s^3}\hat{s}$$

$$\mathbf{F} = \mathbf{F}(T) = \oint_{\partial\Omega} P\hat{n}\,dA$$

P. E. Parks and R. J. Donnelly,
Phys. Rev. Lett. **16**, 45 – 48 (1966)

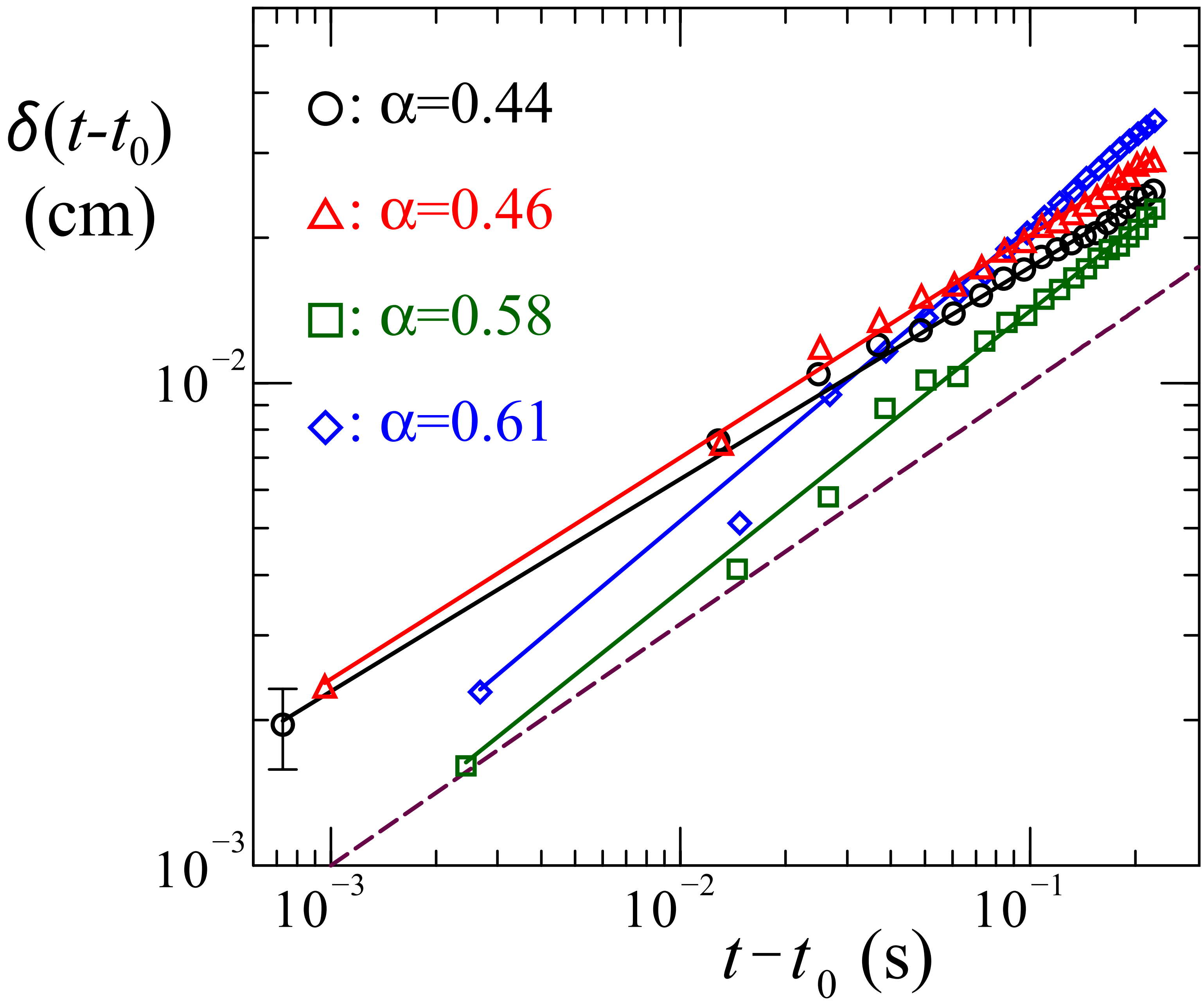
Pulsed Counterflow Velocities



What is the source of high velocity trajectories when heater is off?

Reconnection Dynamics

$$\delta(t) = B(t\text{-}t_0)^{\alpha}$$



Superfluid Order Parameter

Order parameter for superfluid helium is a complex field,

$$\Psi(\mathbf{x}) = f e^{i\phi}$$

f is amplitude, and ϕ is phase

Superfluid velocity given by:

$$\mathbf{v}_{\mathrm{S}} = \frac{\kappa}{2\pi}\nabla\phi, \;\; \kappa \equiv \frac{h}{m}$$

$h =$ Planck's constant

$m =$ mass of helium atom

Superfluid Topological Defects

By continuity ϕ must be 2π periodic, quantizes circulation

$$\Gamma = \oint_C \mathbf{v}_\mathrm{s} \cdot d\ell = n\kappa$$

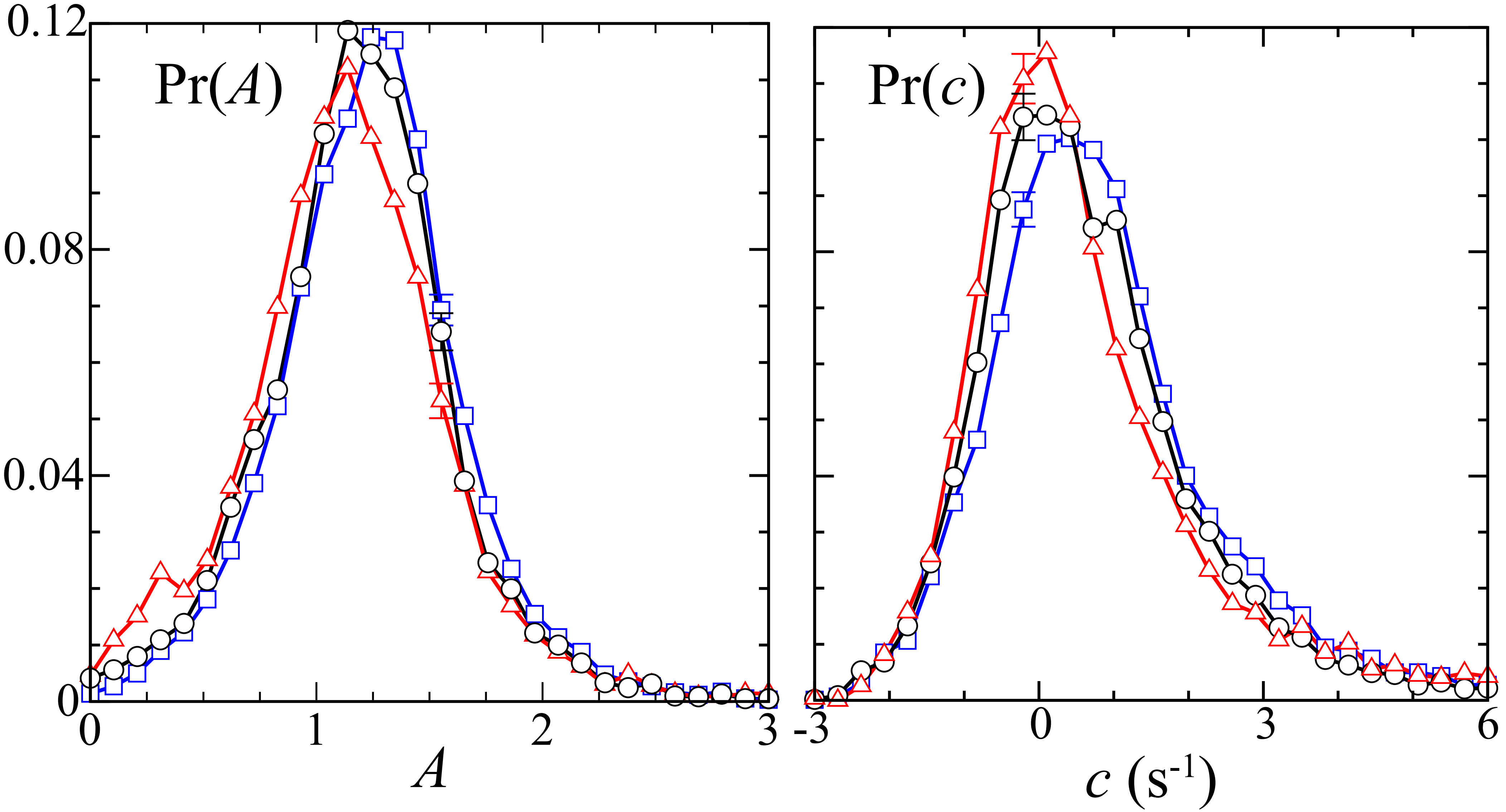
n is integer, κ is quantum of circulation

For nonzero n, the phase is ill-defined resulting in 1D topological defects with $f = 0$

These 1D defects are gradients in ϕ, inducing flow of superfluid

Temperature Dependence

1.70 K < *T* < 1.88 K 1.88 K < T < 1.96 K 1.96 K < T < 2.05 K



MSP, M. E. Fisher and D. P Lathrop, *Physica D* in press (2010)