



# Fluid Simulation in the Movies

NAVIER AND STOKES MUST BE CIRCULATING IN THEIR GRAVES



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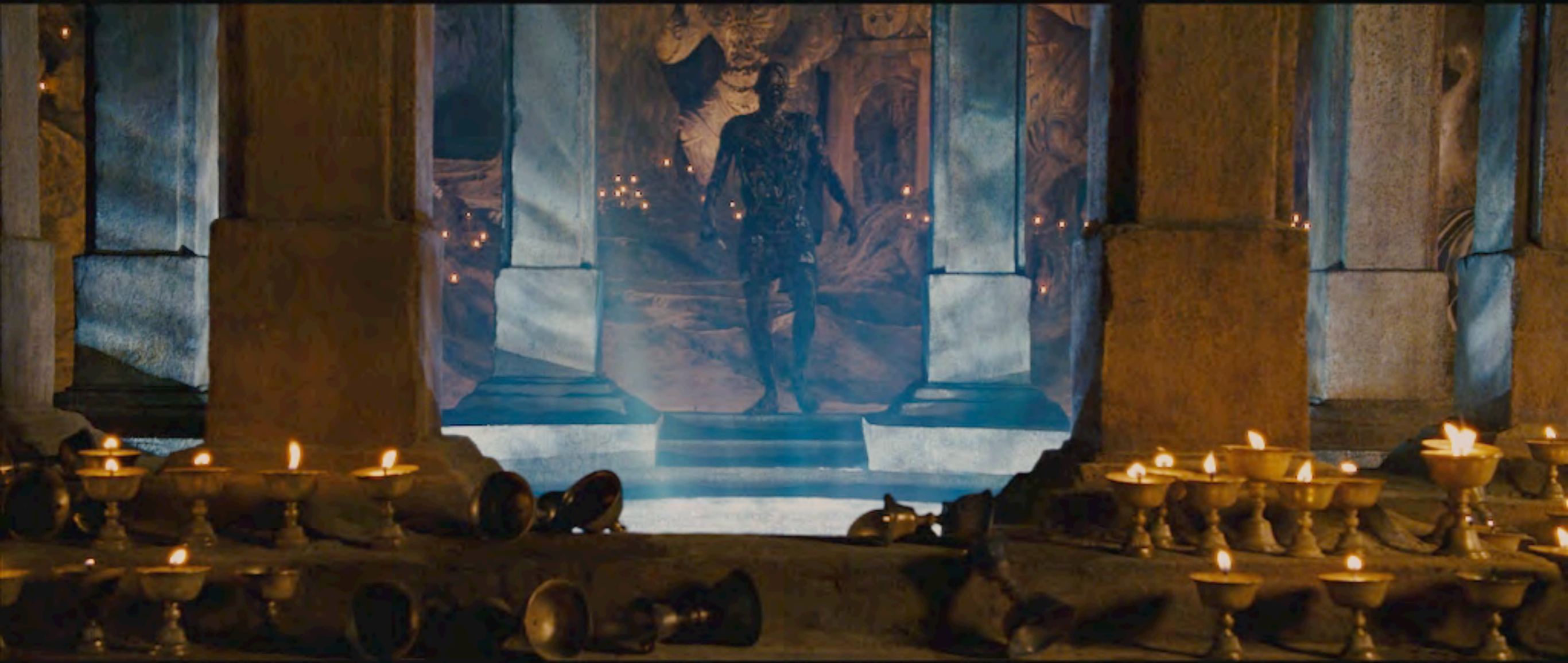
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Jeroen Molemaker  
Michael Kowalski  
Liliya Kharevych  
Taeyong Kim

THE MUMMY : TOMB OF THE DRAGON EMPEROR

0013

1066702 : patel:zc.100:FxPoolSim.AhabEmperorMult010Ren-0009 - 11:24 Jun 11

ZC100\_v0144



**THE MUMMY : TOMB OF THE DRAGON EMPEROR**



**THE A-TEAM**



The wolfman





# Production Issues

- ✦ CFD and Production workflows do not fit together well
- ✦ The language of visual effects is imprecise





# Golden Compass



0021

#875022 : FxDeath.FxCmp-0009 10:23 Oct 11



0021

#875022 : FxDeath.FxCmp-0009 10:23 Oct 11

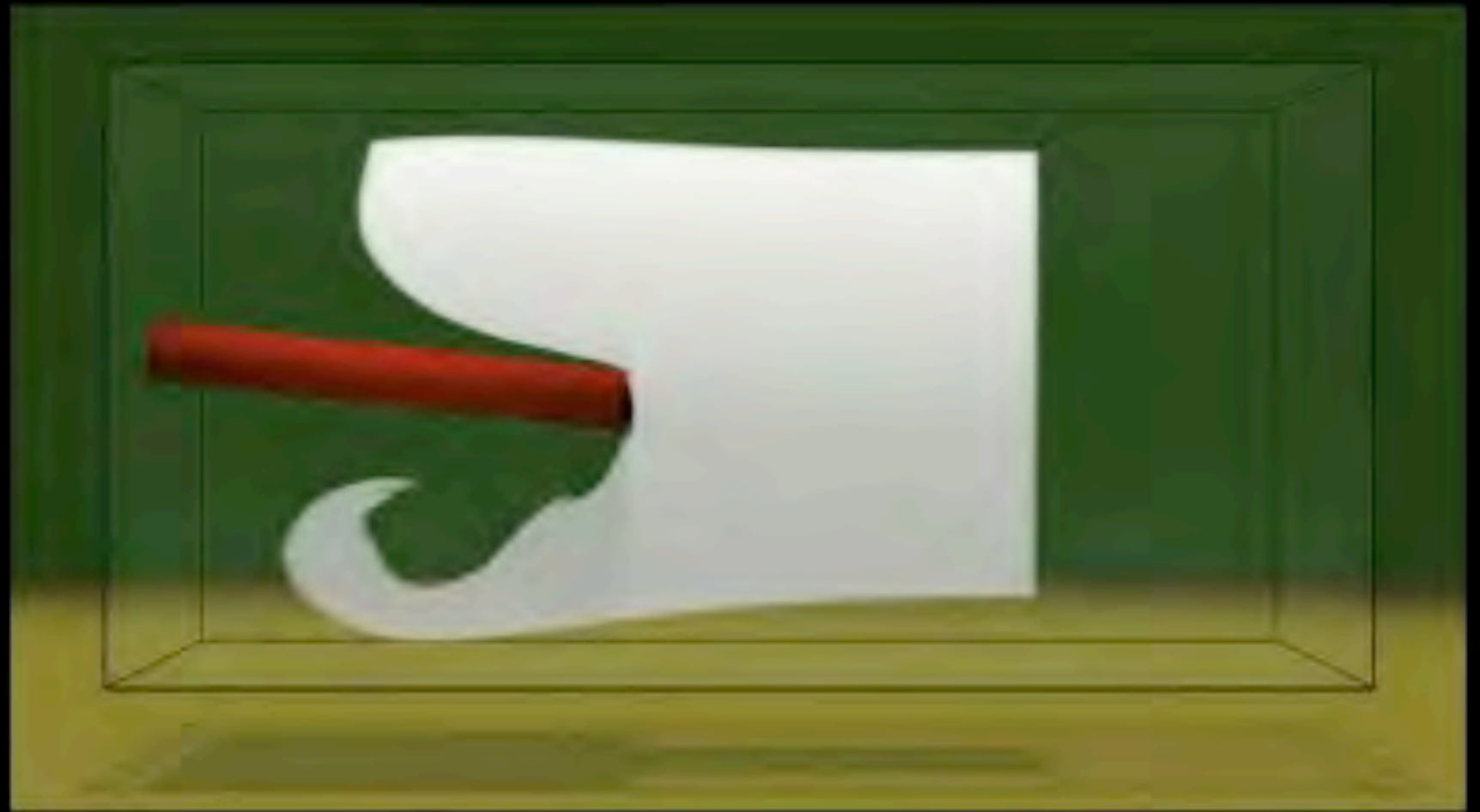
# 3D Gas Solver

- \* Incompressible Navier-Stokes
- \* Fixed rectangular Eulerian grid
- \* Multiple Advection Schemes
  - \* QUICK (space) + 2nd order Adams-Bashforth (time)
  - \* FLIP
  - \* Semi-Lagrangian
- \* Pressure Projection
  - \* Multigrid
  - \* Iterated Orthogonal Projections for complex boundaries
- \* Recently parallelized with MPI



Low numerical  
viscosity

(QUICK)



High numerical  
viscosity

(SEMI-LAGRANGIAN)



# X2 - XMen United

swirly smokey puffy





# Routine Simulation Workflow

- ✦ Technical Directors (artists) run simulation software
- ✦ Many iterations per day, daily feedback
- ✦ Single machine (2-4 cores, 2-32 GB RAM)
- ✦ Low-res sims for quick approval, hi-res maybe
- ✦ “Sweetening” via gridless advection



# Advection via Characteristic

$$\frac{\partial C(\mathbf{x}, t)}{\partial t} + \mathbf{u}(\mathbf{x}, t) \cdot \nabla C(\mathbf{x}, t) = S(\mathbf{x}, t)$$

- Integral solution in terms of a characteristic function  $\mathbf{X}$

$$\mathbf{X}(\mathbf{x}, t') = \mathbf{x} - \int_0^{t'} dt'' \mathbf{u}(\mathbf{X}(\mathbf{x}, t' - t''), t'')$$

$$C(\mathbf{x}, t) = C(\mathbf{X}(\mathbf{x}, t)) + \int_0^t dt' S(\mathbf{X}(\mathbf{x}, t - t'), t')$$

# Multiple advection steps

$$C(\mathbf{x}, n\Delta t) = C_0(\mathbf{X}_n(\mathbf{x}))$$

$$\mathbf{X}_n(\mathbf{x}) = \mathbf{X}_{n-1}(\mathbf{x} - \mathbf{u}(\mathbf{x}, t + (n-1)\Delta t)\Delta t)$$

- ✦ Map generated post simulation
- ✦ Evaluating at render time is very slow for  $n > 4$
- ✦ Artistic tool for “sharpening” density and color fields
- ✦ “Gridless advection”

# The Wolfman: fire without Gridless Advection

# The Wolfman: fire with Gridless Advection



# A-Team

## Foreground cloud gridless advection

Without gridless advection →



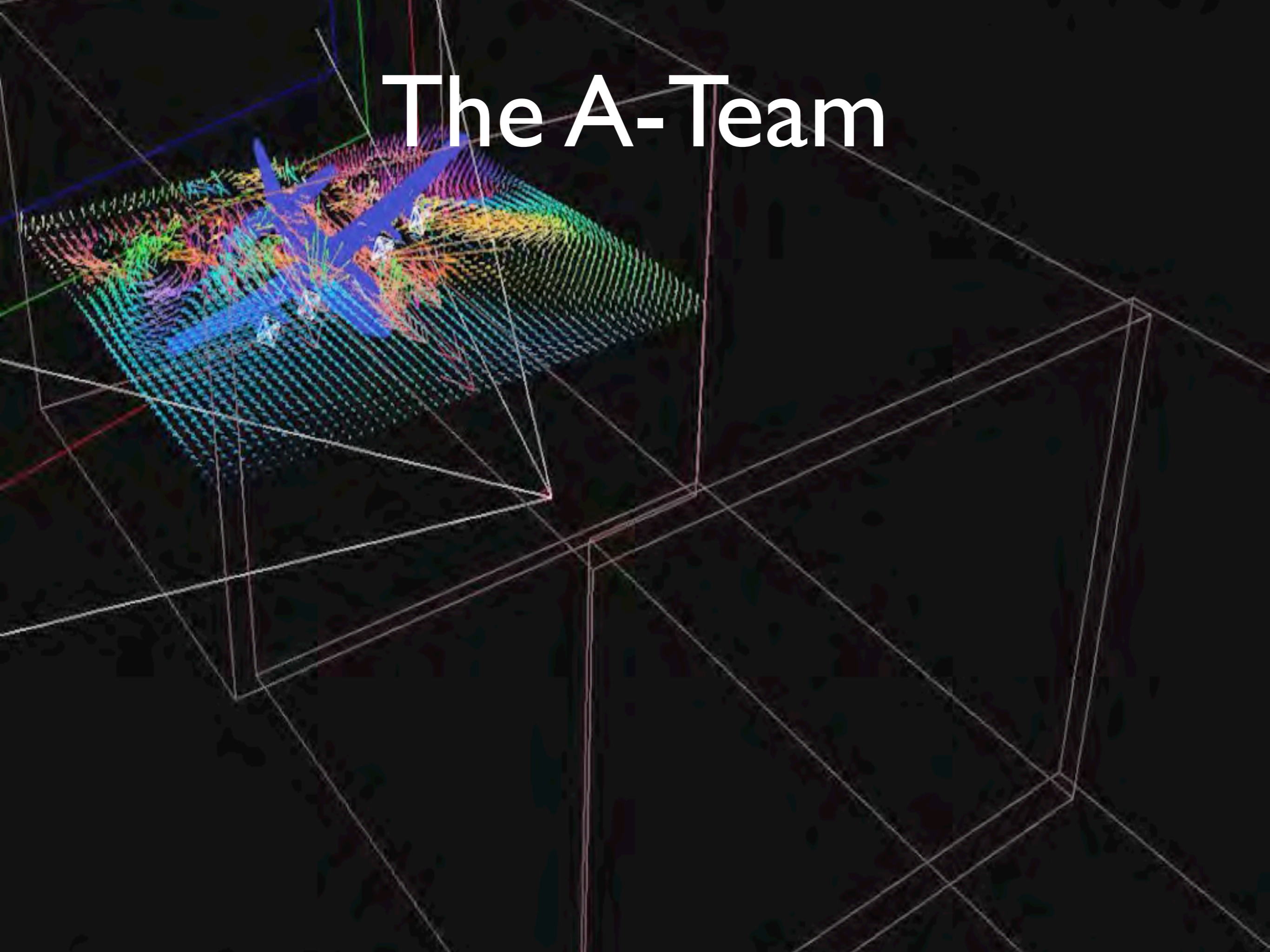
With two steps of gridless advection →



# The A-Team



# The A-Team



# Semi-Lagrangian Mapping (SELMA)

- Characteristic mapping function is the central object of advection

$$\mathbf{X}_n(\mathbf{x}) = \mathbf{X}_{n-1}(\mathbf{x} - \mathbf{u}_n(\mathbf{x})\Delta t)$$

- Applying this for more than 4-5 steps takes a lot of cpu time and memory
- Sampling the mapping function onto a grid each step reduces both resource issues

$$\mathbf{X}_n(\mathbf{x}_i) = \sum_j \omega_j \mathbf{X}_{n-1}(\mathbf{x}_j)$$

$$\mathbf{x}_j \in \text{Neighborhood}(\mathbf{x}_i - \mathbf{u}_n(\mathbf{x}_i)\Delta t)$$

- Preserves detail
- Causes numerical dissipation of the mapping function, but not the density



# The A-Team



# The A-Team



# The A-Team



# The A-Team

# The A-Team



# 3D Liquid Solver



- ✱ Incompressible Navier-Stokes
- ✱ Fixed rectangular Eulerian grid
- ✱ Advection via QUICK
  - ✱ 3rd order upwind advection
  - ✱ 2nd order Adams-Bashforth in time
- ✱ Pressure projection via conjugate gradient
- ✱ Free-surface tracking via levelset (gridded signed distance function)
  - ✱ Smooth Particle Hydrodynamics initiated at surface erosion
- ✱ One-phase only (liquid and vacuum)

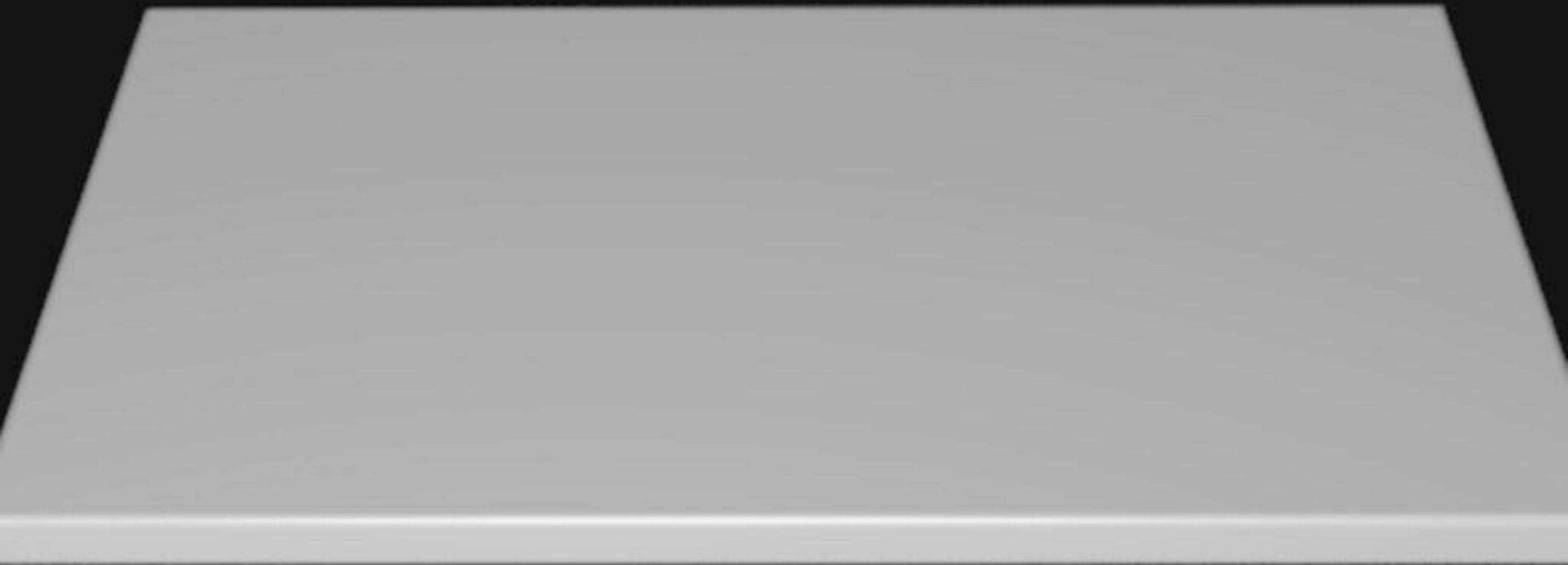
# Cat in the Hat



0002

#209161 : 32819

# Happy Feet



1250

#644022 : user:patel sc49.29:FxAhab.Out\_Cam1\_ConForce02-0001 - 17:12 Sep 16



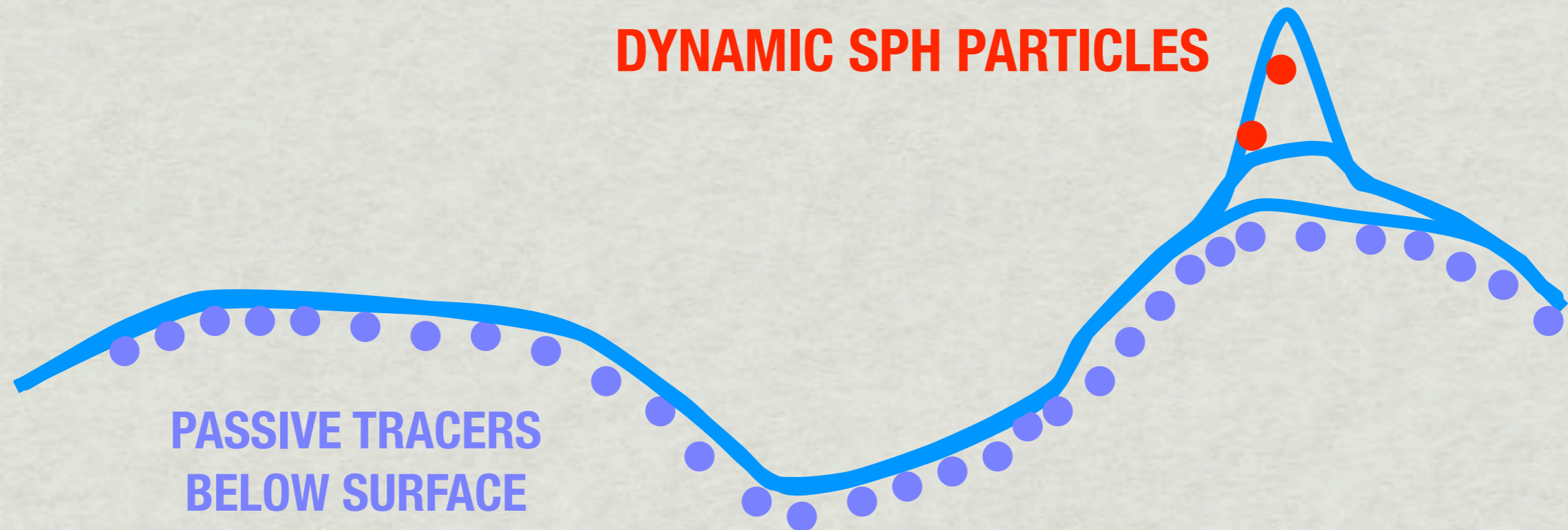
# iWave

Fast surface wave height simulator

Unphysical but visually nice boundary interactions



# Hybrid Eulerian Grid and SPH



- \* Levelset surface advected
- \* Passive tracers advected
- \* Levelset surface regrided - EROSION OCCURS
- \* Exposed tracers converted to SPH particles with momentum & mass from eroded volume

#1214294 : rd.mysteryCave:FxCmTests.FxShotCmp-0001 - dfh

09:00:37

**LOTL-PROPERTY OF UNIVERSAL**

0085

11 / 17 / 08

rd.mysteryCave.1214294

**Coupled Simulations**

**3D HYBRID + IWAVE SURFACE + SPLASHES & FOAM TEXTURES**

# Marmaduke



0001

#1506859 : rd.dfn:FxSurfboard.Surf2-0001 - 12:40 Jul 23

# Graphics Research on CFD

## Wavelet Turbulence for Fluid Simulation



Theodore Kim<sup>1</sup>, Nils Thürey<sup>2</sup>, Doug James<sup>1</sup>, and Markus Gross<sup>2</sup>



Cornell University<sup>1</sup>

**ETH**<sup>2</sup>

Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zürich

# New Directions

- \* Iterate on simulations without full re-simulation
  - \* iterate on a subregion of the full simulation
  - \* inject density/velocity changes
  - \* inject objects
  - \* move objects around
  - \* local changes in dynamics or resolution
  - \* more external forcing
- \* Focus on characteristic function in simulation
  - \* recast Navier-Stokes

# Iterative NS Simulation

- \* Assume pre-existing velocity, density, pressure

$$\mathbf{U}_0(\mathbf{x}, t)$$

$$\rho_0(\mathbf{x}, t)$$

$$p_0(\mathbf{x}, t)$$

$$\frac{\partial \mathbf{U}_0}{\partial t} + \mathbf{U}_0 \cdot \nabla \mathbf{U}_0 + \nabla p_0 = f_0$$

$$\nabla \cdot \mathbf{U}_0 = 0$$

$$\frac{\partial \rho_0}{\partial t} + \mathbf{U}_0 \cdot \nabla \rho_0 = 0$$

# Perturbing Simulation

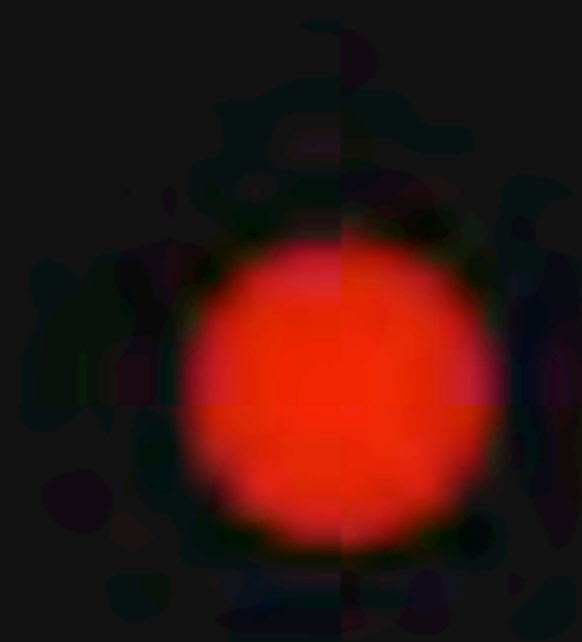
$$\begin{aligned}\mathbf{U}_0(\mathbf{x}, t) &+ \mathbf{U}'(\mathbf{x}, t) \\ \rho_0(\mathbf{x}, t) &+ \rho'(\mathbf{x}, t) \\ p_0(\mathbf{x}, t) &+ p'(\mathbf{x}, t)\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathbf{U}'}{\partial t} + (\mathbf{U}_0 + \mathbf{U}') \cdot \nabla \mathbf{U}' + \mathbf{U}' \cdot \nabla \mathbf{U}_0 + \nabla p' &= f' \\ \nabla \cdot \mathbf{U}' &= 0 \\ \frac{\partial \rho'}{\partial t} + (\mathbf{U}_0 + \mathbf{U}') \cdot \nabla \rho' + \mathbf{U}' \cdot \rho_0 &= 0\end{aligned}$$

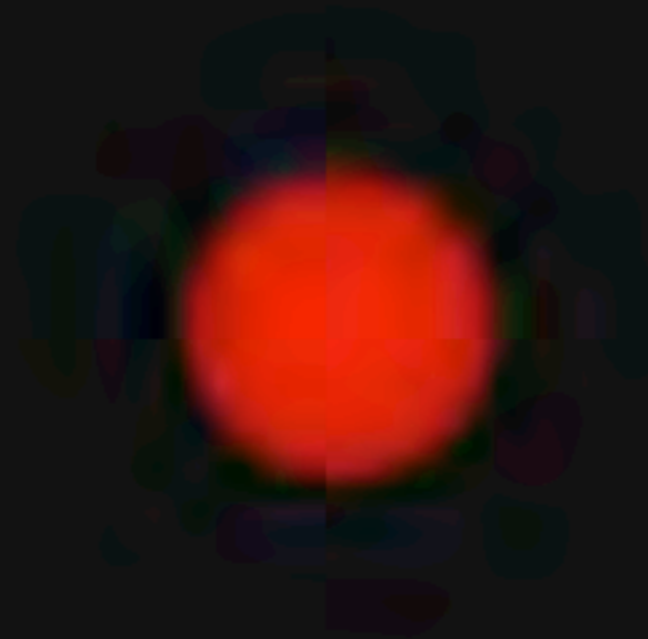


# Last night's Sim

Supervisor comment:  
“Make it veer to the right”



# Iterated Sim



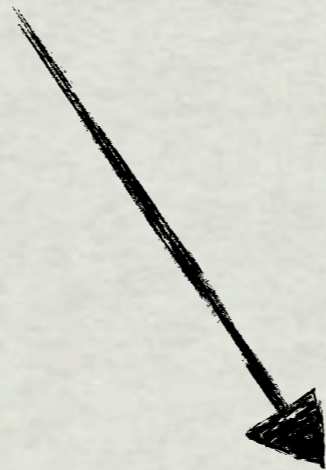
# Recast Navier-Stokes

- ✱ Characteristic function is the critically important feature
- ✱ Write NS equations to work directly with it
- ✱ Production benefits
  - conceptually similar to surface displacement
  - artistic modifications more controllable

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} + \nabla p = f$$

$$\nabla \cdot \mathbf{U} = 0$$

$$\frac{\partial \rho}{\partial t} + \mathbf{U} \cdot \nabla \rho = 0$$



$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} + \nabla p = f$$

$$\det(\nabla \mathbf{X}) = 1$$

$$\frac{\partial \mathbf{X}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{X} = 0$$

$$\mathbf{U}(\mathbf{x}, t + \Delta t) = \mathbf{X}(\mathbf{x}, t) - \mathbf{X}(\mathbf{x}, t + \Delta t)$$