The use of global modes to understand transition and perform flow control

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KTH Mechanics

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Aim of the investigations

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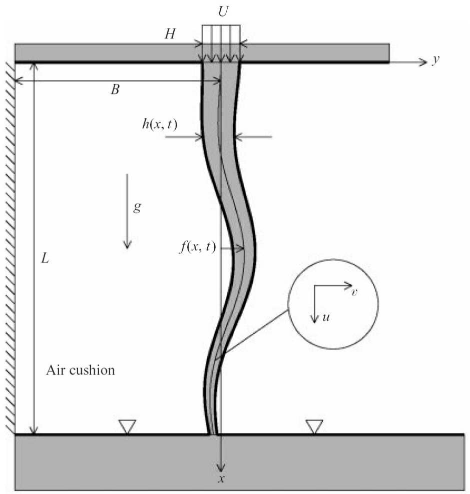
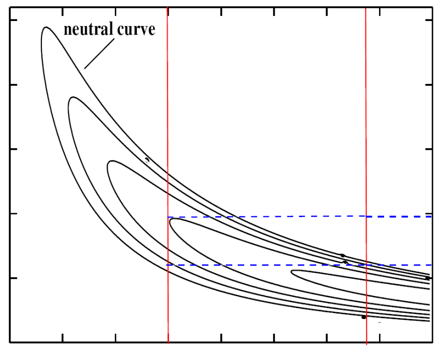
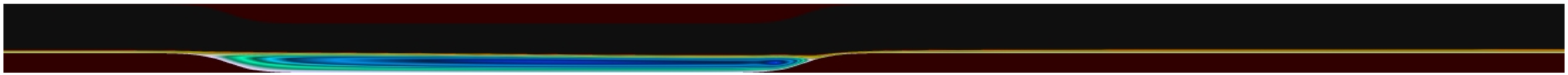
- Understanding instabilities and growth using superposition of global modes

- Design a feedback controller, using system of global modes

- Flow cases:
  Waterfall problem
  Blasius boundary layer
  Long shallow cavity







Background

- Streamwise non-normality and global modes

  Ginzburg-Landau, Cossu & Chomaz (1997)

  Waterfall problem, Schmid & Henningson (2002)

  Blasius boundary layer, Ehrenstein & Gallaire (2005)

- LQG feedback control applied to transition and turbulence

  Channel/BL turbulence, Kim *et al*

  Channel/BL transition, Bewley *et al*, Henningson, *et al*

  Many cavity control investigations …

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The water fall problem

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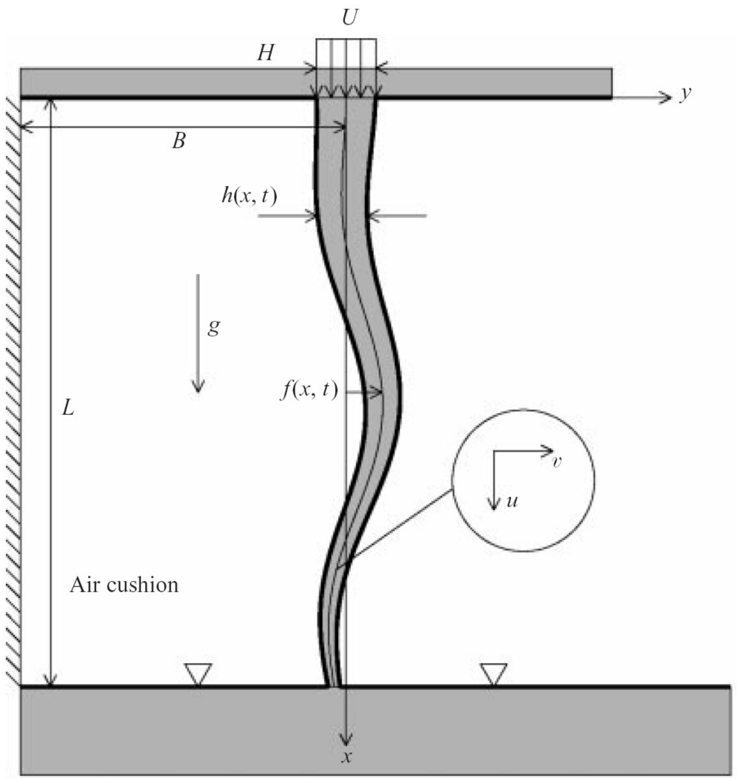


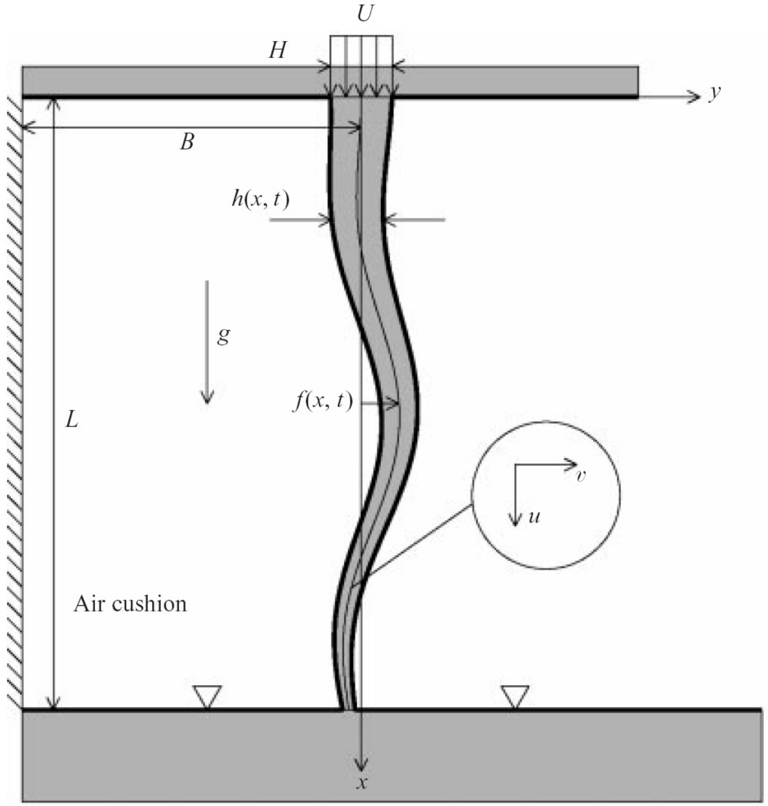
Formulation of waterfall problem

- Basic flow

$$\bar{u}(x) = \sqrt{U^2 + 2x}, \quad \bar{v}(x) = 0$$
$$\bar{p}(x,t) = 0, \quad \bar{h}(x) = \frac{U}{\sqrt{U^2+2x}}$$

- Disturbance equations

$$\underbrace{\begin{pmatrix} -\bar{u}(x)\mathcal{D} & -k\dfrac{\bar{u}(x)}{U}\displaystyle\int_0^1 dx \\ \mathcal{I} & -\bar{u}(x)\mathcal{D} \end{pmatrix}}_{\mathcal{L}} \begin{pmatrix} V \\ F \end{pmatrix} = i\omega \begin{pmatrix} V \\ F \end{pmatrix}$$



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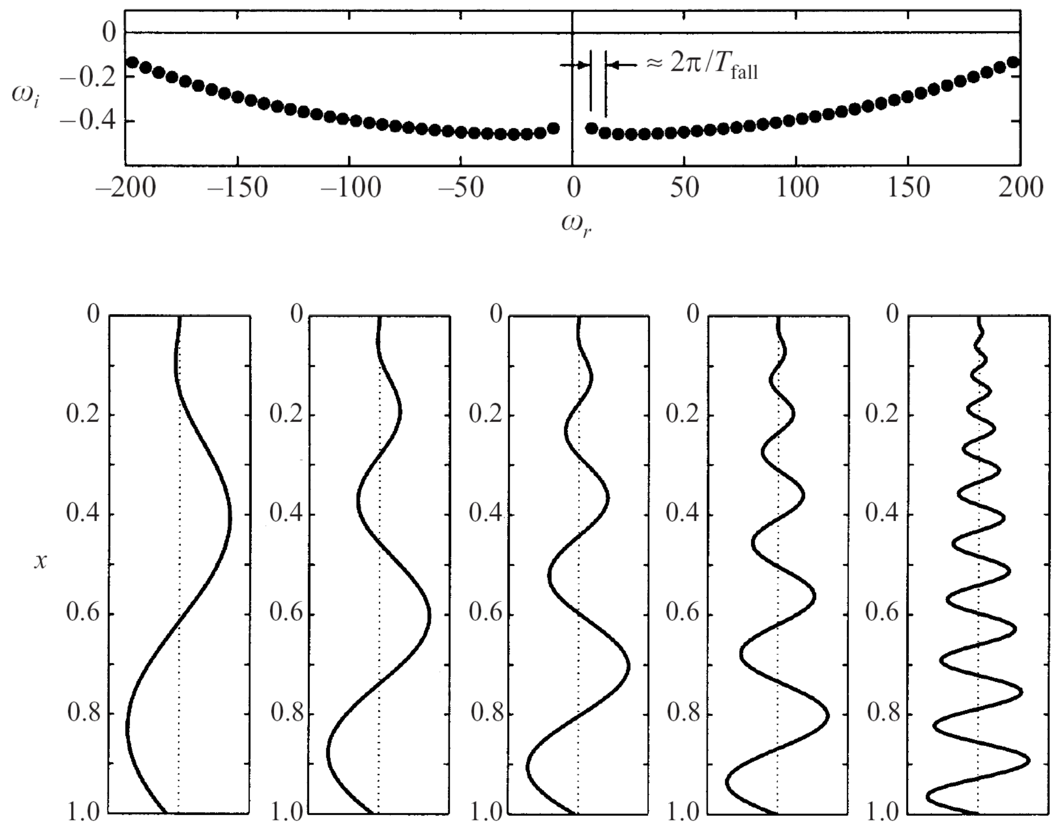
Spectrum and eigenmodes



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- Damped global eigenmodes with frequencies not corresponding to experimentally observed values

Optimal sum of eigenmodes

$$G(t) = \max_{u_0 \neq 0} \frac{\|u(t)\|^2}{\|u_0\|^2} \qquad u \equiv \begin{pmatrix} v \\ f \end{pmatrix} = \sum_n \kappa_n \begin{pmatrix} V \\ F \end{pmatrix}_n e^{\lambda_n t}$$

$$= \|F e^{\Lambda t} F^{-1}\|_2^2$$

$$\leq \|F\|_2^2 \; \|F^{-1}\|_2^2 \; e^{2\Re\{\lambda_{max}\}t}$$

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$$A = F^H F, \qquad A_{mn} = \int_0^1 V_m^* V_n + k F_m^* F_n dV$$

$\|F\|_2 \|F^{-1}\|_2 = 1$ normal operator ($\mathcal{L}^*\mathcal{L} = \mathcal{L}\mathcal{L}^*$), orthogonal eigenfunctions

$\|F\|_2 \|F^{-1}\|_2 >> 1$ non-normal operator, non-orthogonal eigenfunctions

Transient growth of a wavepacket

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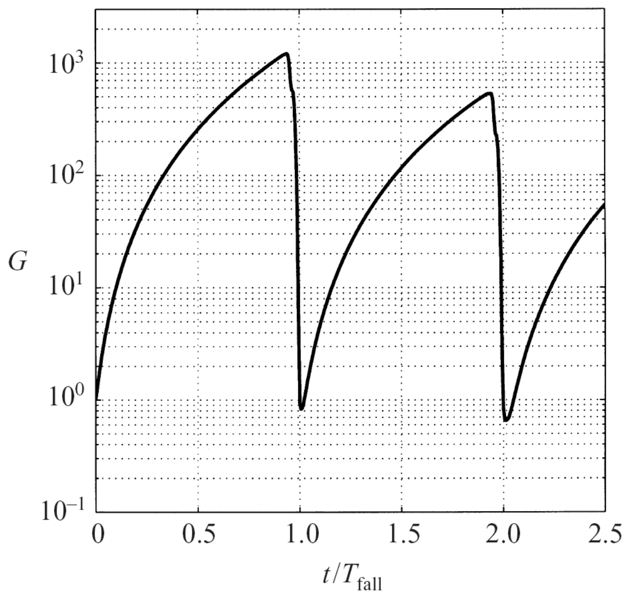
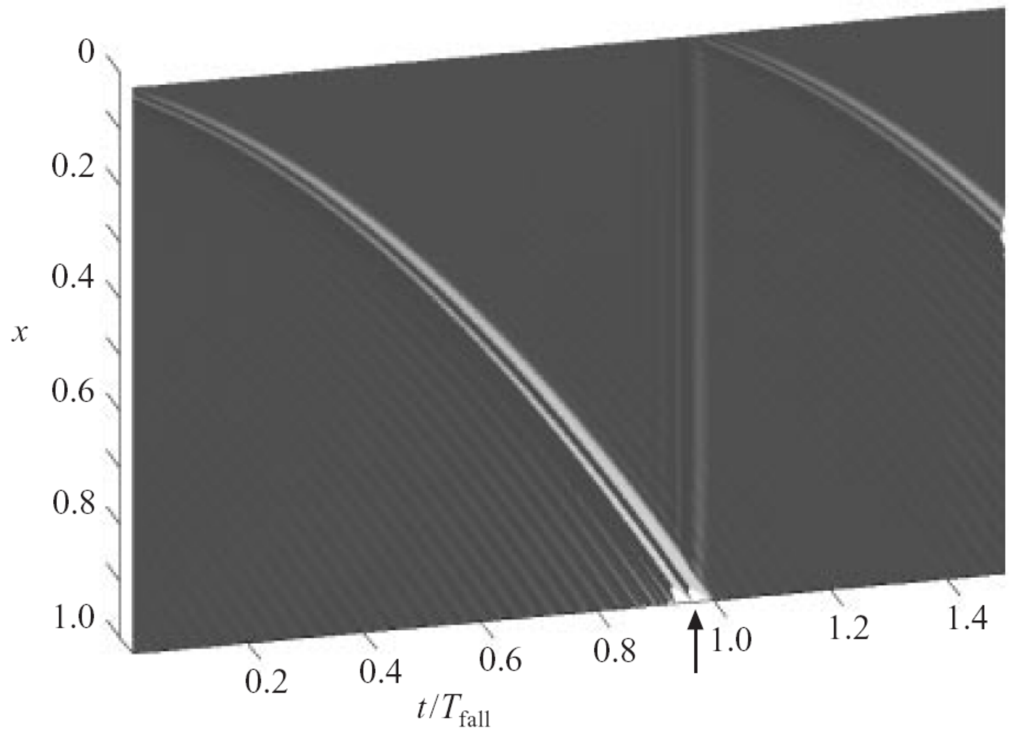
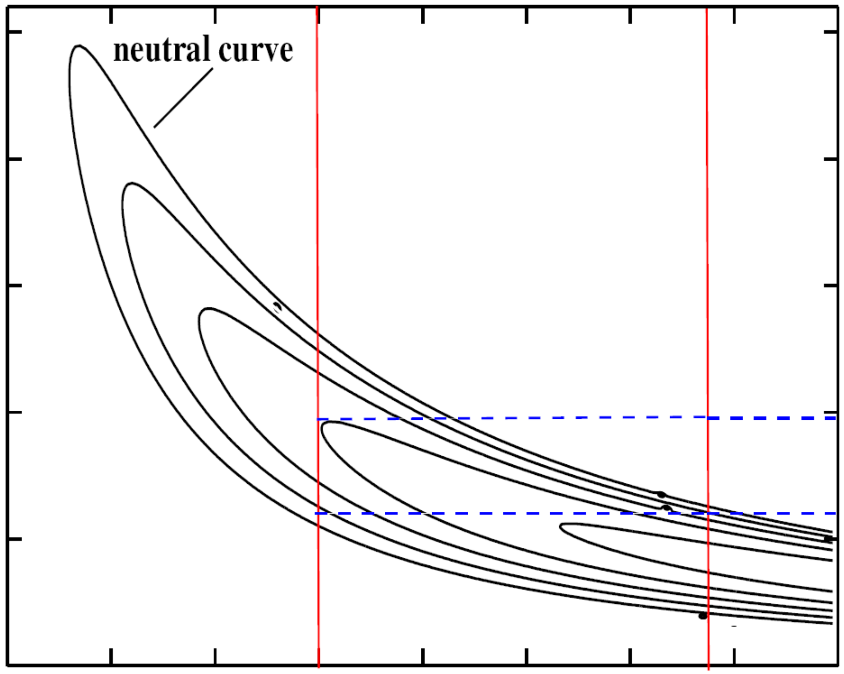




- Optimal disturbance describes a wavepacket which propagates down the liquid curtain
- Pressure pulse at downstream end triggrers a new disturbance in a global cycle
- Frequency of wavepackets matches experiments with period equal to fall time

The Blasius boundary layer



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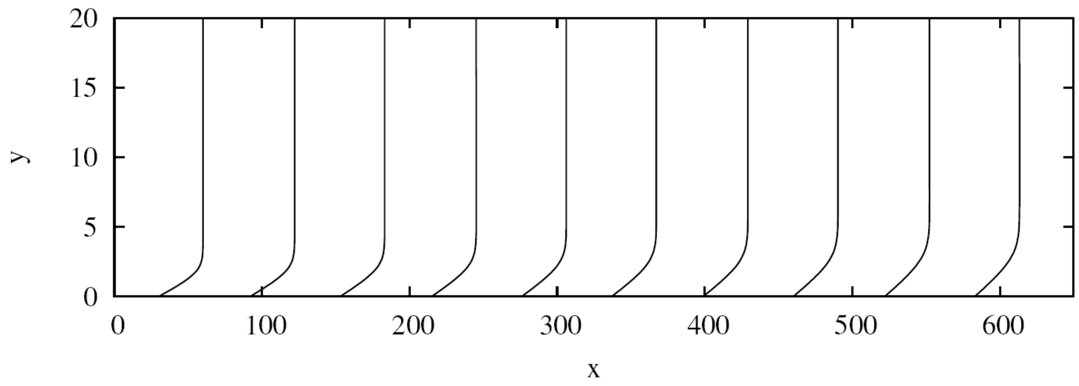
Blasius boundary layer



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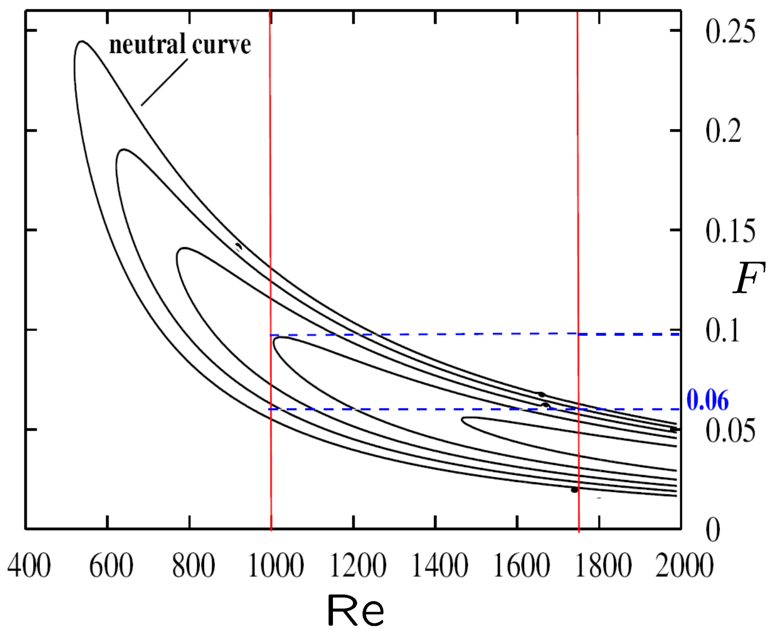
- Krylov method for global eigenmodes
- Inflow/outflow boundary conditions based on local dispersion relation

$$\frac{\partial u}{\partial x} = i\alpha u$$
$$\alpha = \alpha_0 + \frac{\partial \alpha}{\partial \omega}(\omega_0)(\omega - \omega_0)$$



Ehrenstein & Gallaire (2005)

Krylov method for global eigenvalue problem

- Discretized 2D linearized Navier-Stokes gives generalized eigenvalue problem, solved using Krylov subspace with Arnoldi algorithm

$$\begin{cases} \dfrac{\partial u}{\partial t} & = & -(\mathbf{U}\cdot\nabla)u - (\mathbf{u}\cdot\nabla)U - \dfrac{\partial p}{\partial x} + \dfrac{1}{\mathrm{Re}}\nabla^2 u \\ \dfrac{\partial v}{\partial t} & = & -(\mathbf{U}\cdot\nabla)v - (\mathbf{u}\cdot\nabla)V - \dfrac{\partial p}{\partial y} + \dfrac{1}{\mathrm{Re}}\nabla^2 v \\ 0 & = & \nabla\cdot\mathbf{u} \end{cases}$$

$$\frac{du}{dt} = \underbrace{B^{-1}A}_{\bar{A}}\,u \qquad u = \hat{u}e^{i\lambda t}$$

$$\bar{A}^{-1}\hat{u} = \frac{i}{\lambda}\hat{u} \qquad \Rightarrow \quad \lambda_n, \quad q_n$$

Krylov subspace: $\{v_1, \bar{A}^{-1}v_1, (\bar{A}^{-1})^2 v_1, ...\}$

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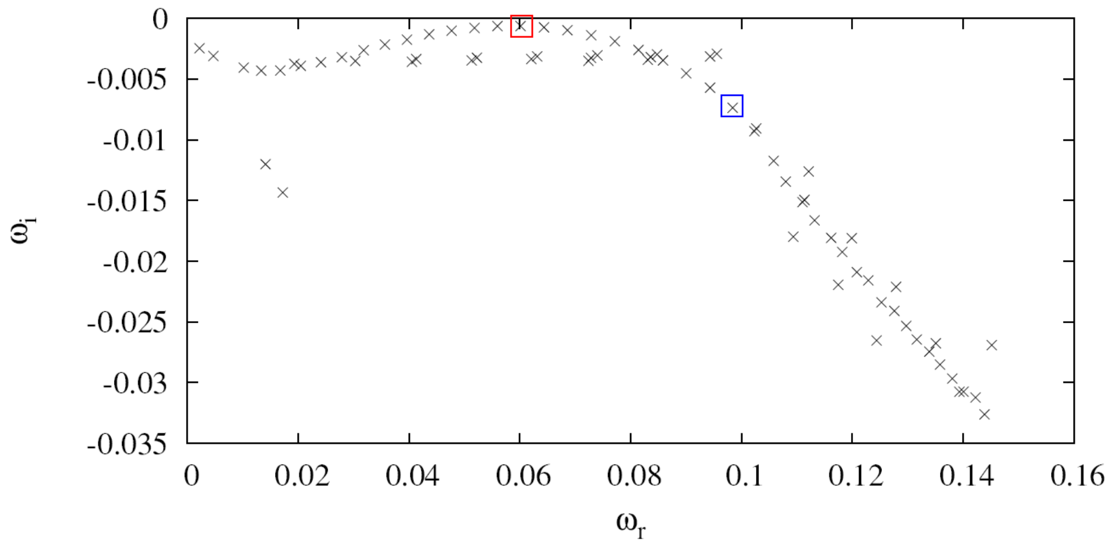
Spectrum associated with TS-instability



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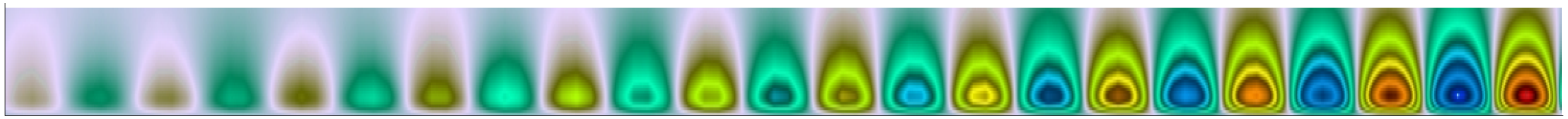
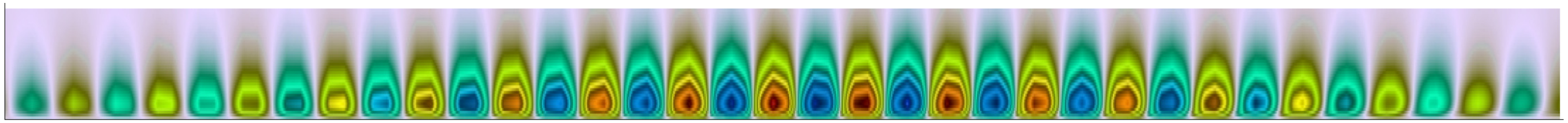
- Eigenmode with ω=0.06: normal velocity



- Eigenmode with ω=0.098: normal velocity



Evolution of optimal disturbance

- Total spectrum: blue
- TS-branch only: red

- Transient since disturbance propagates out of box

$$u = \sum_n \kappa_n(t) q_n$$

$$\max_{\kappa_n(0)} \frac{E(t)}{E(0)} = \|F e^{i\Lambda t} F^{-1}\|$$

$$\{F^H F\}_{ij} = \int\int q_i^H q_j \, dx dy$$

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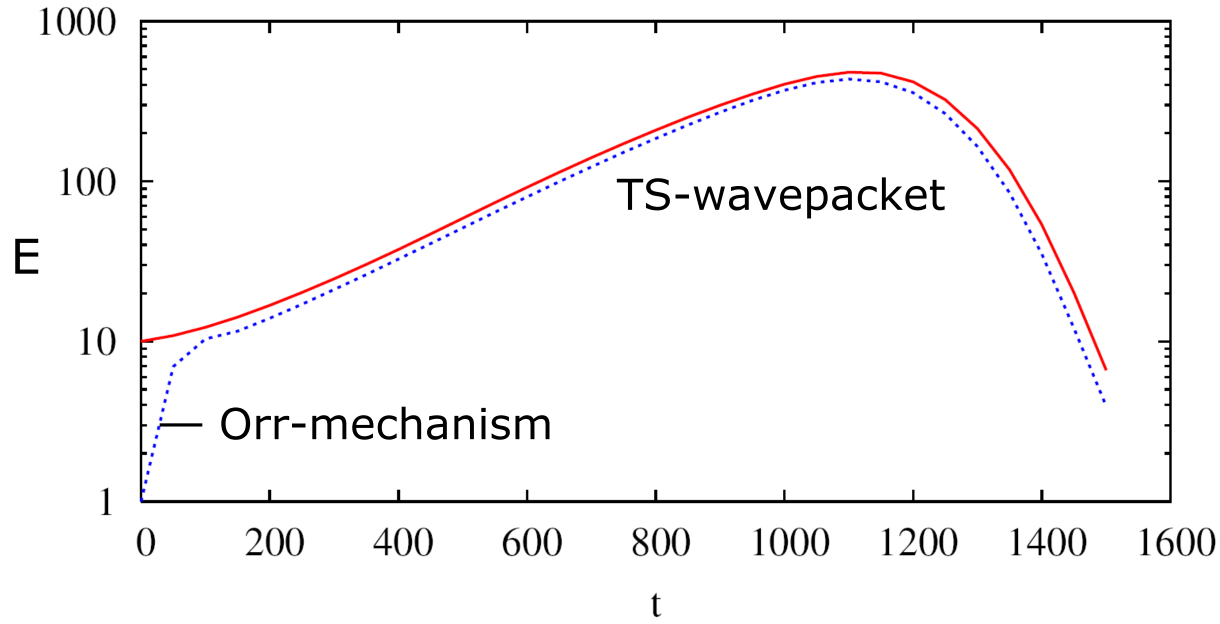

Snapshots of optimal disturbance evolution

- Orr-mechanism: initial disturbance leans against the shear raised up into TS-wavepacket
- Wavepacket propagates downstream, c.f. Ehrenstein & Gallaire (2005)

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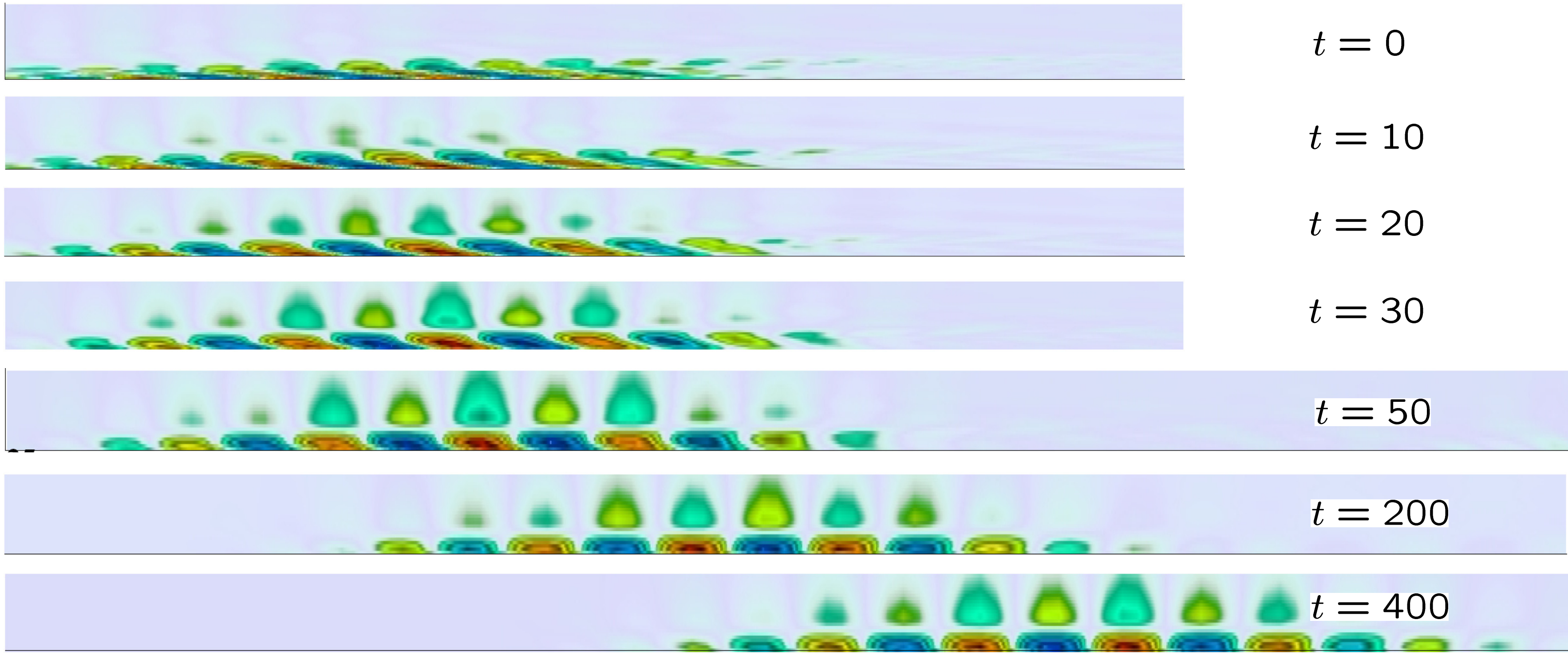

The long shallow cavity

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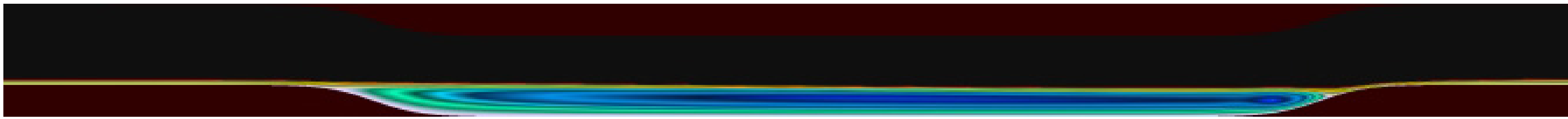

The long shallow cavity flow

- Basic flow from DNS with SFD:
  Åkervik et al., Phys. Fluids 18, 2006

- Strong shear layer at cavity top and recirculation at the downstream end of the cavity

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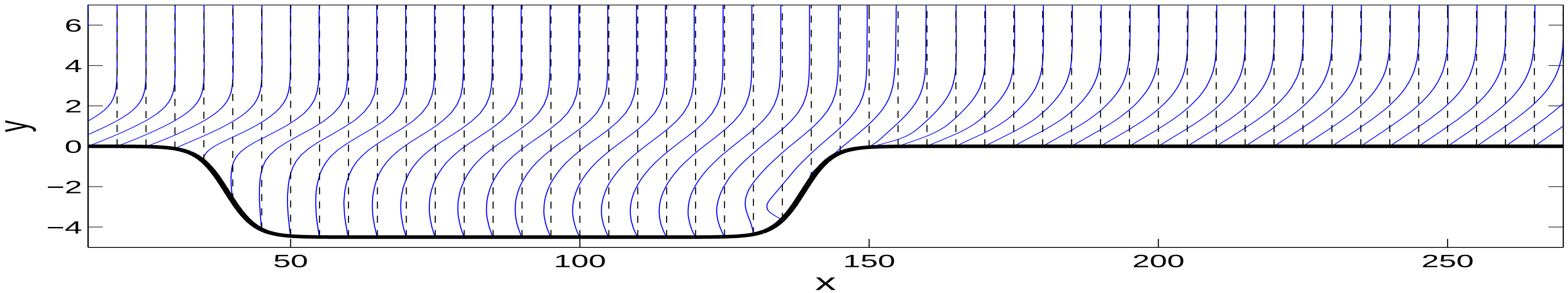

Global spectra

- Global eigenmodes found using Arnoldi method
- About 150 eigenvalues converged and 2 unstable

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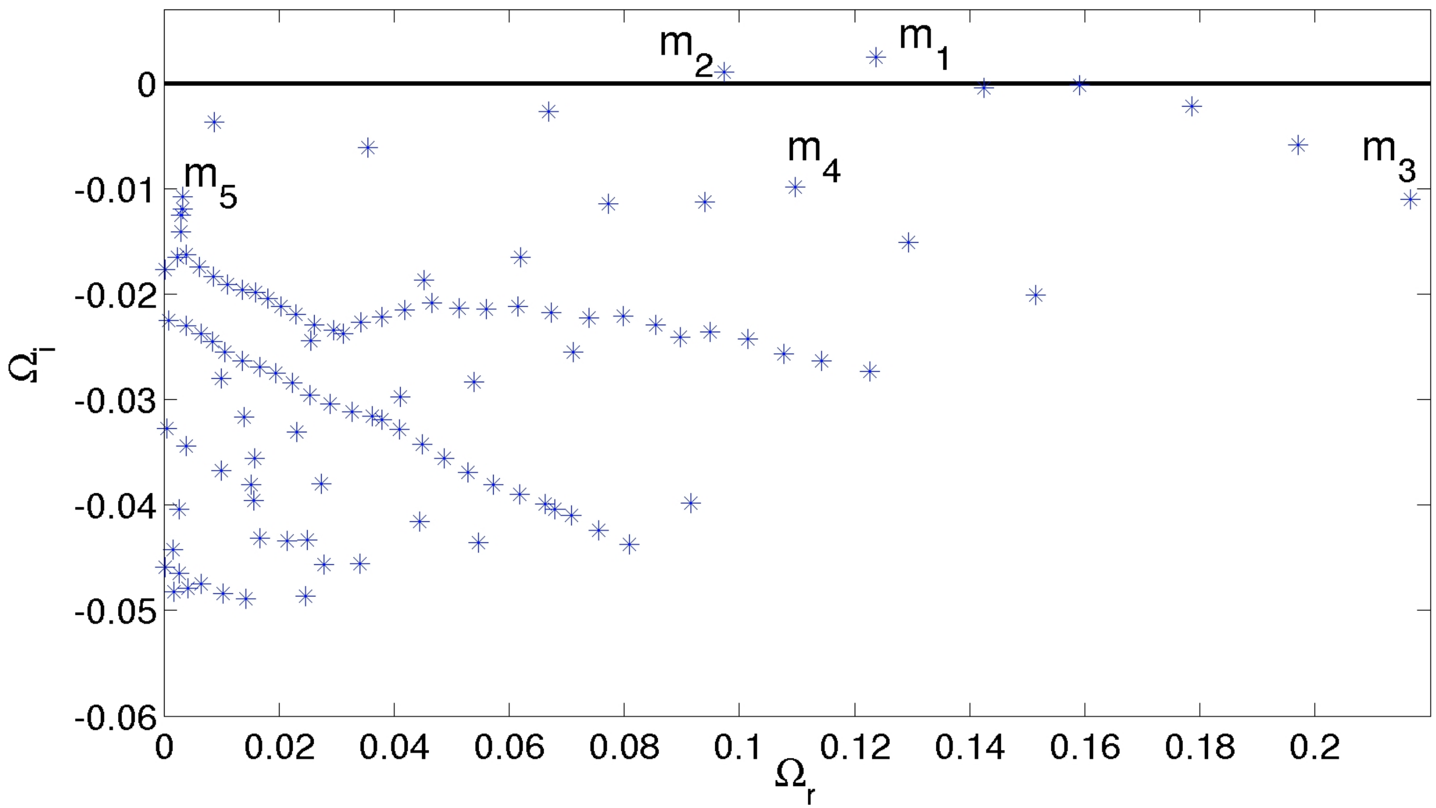
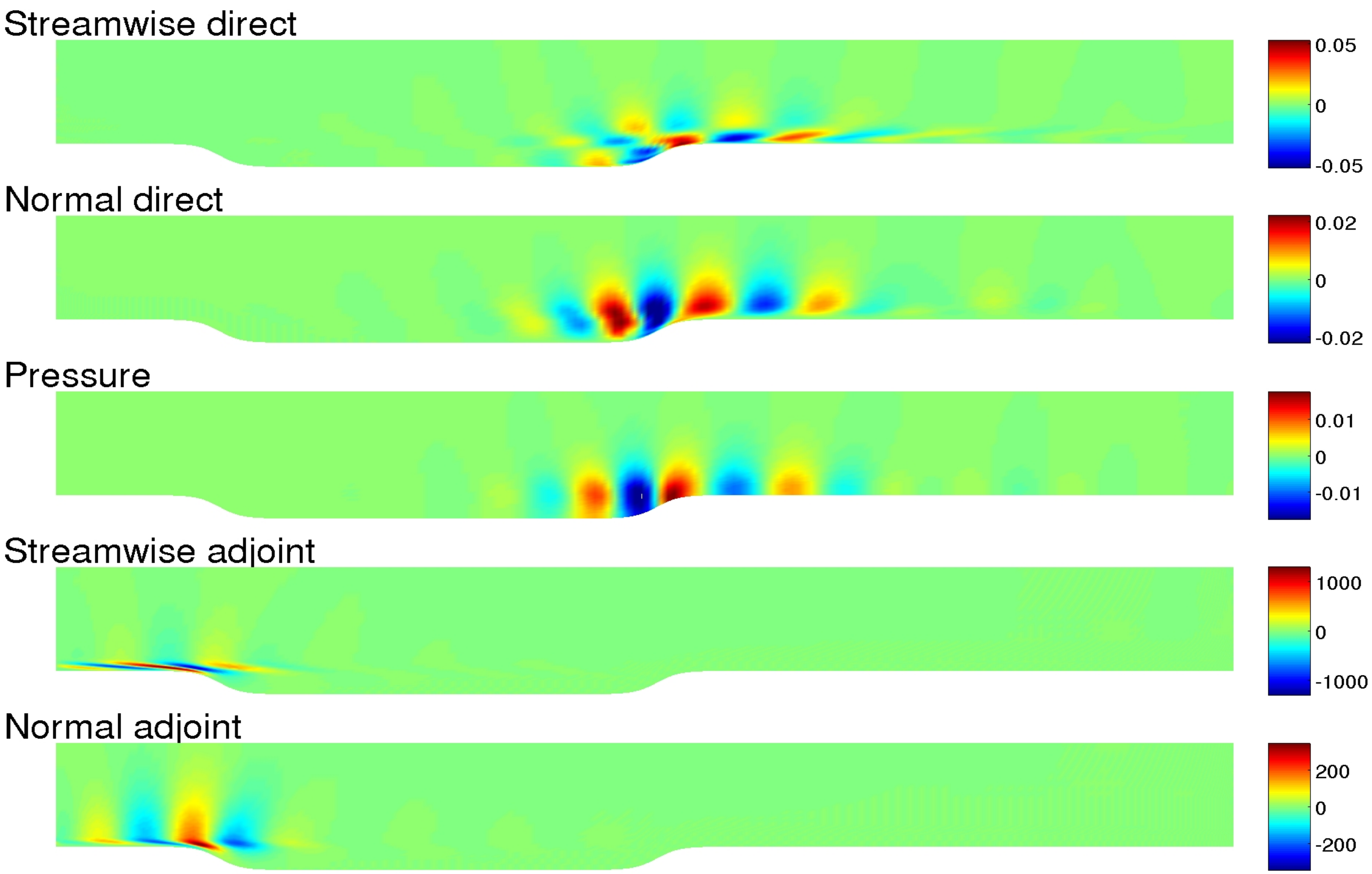


Most unstable mode



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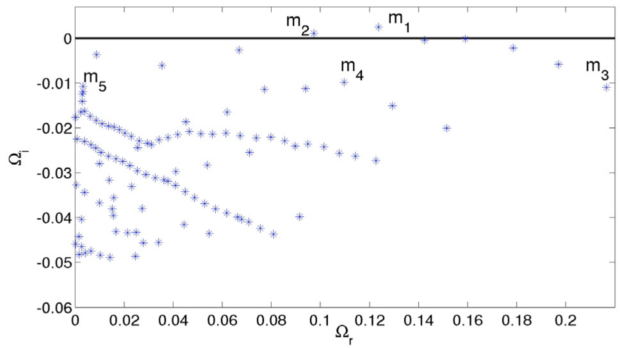
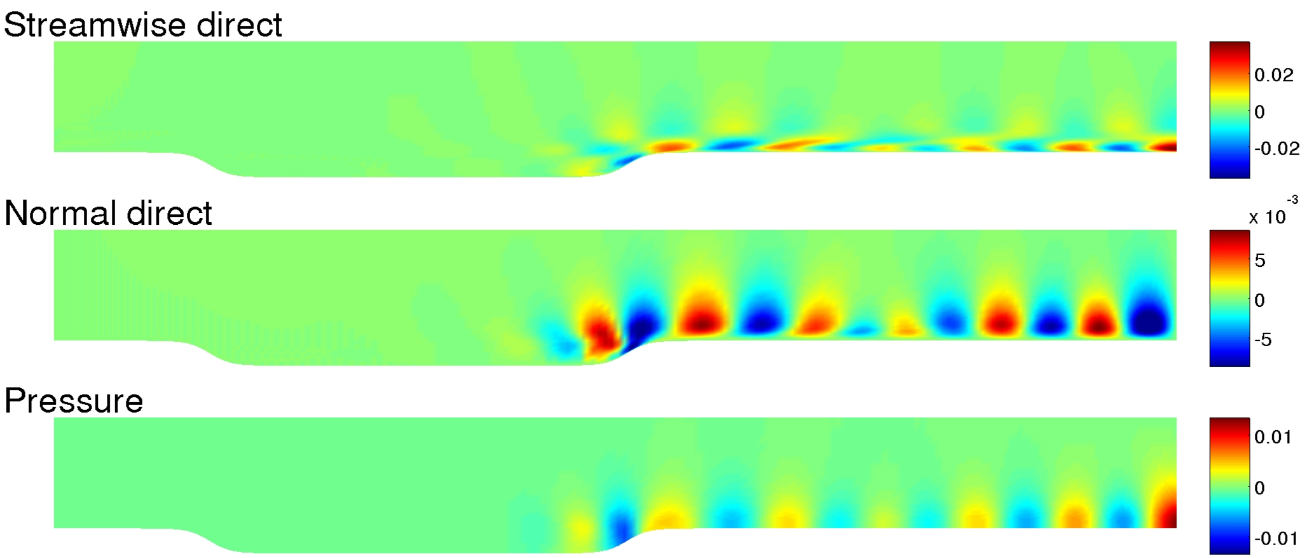
- Forward and adjoint mode located in different regions implies non-orthogonal eigenfunctions/non-normal operator
- Flow is sensitive where adjoint is large

Propagating mode m4





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- Needed in eigenfunction expansions describing spatially propagating disturbances

Maximum energy growth

- Eigenfunction expansion in selected modes
- Optimization of energy output

$$u = \sum_n \kappa_n(t) q_n$$

$$\max_{\kappa_n(0)} \frac{E(t)}{E(0)} = \|F e^{i\Lambda t} F^{-1}\|$$

$$\{F^H F\}_{ij} = \int\int q_i^H q_j dx dy$$

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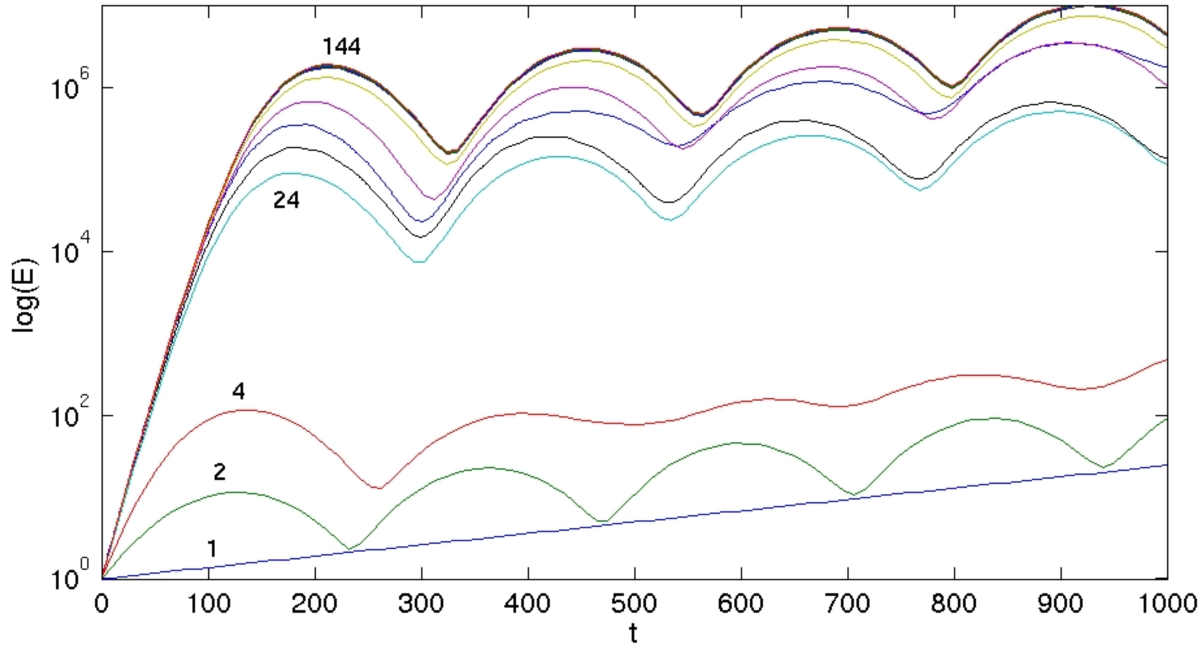

Initial development of wavepacket

- Integrated streamwise velocity from eigenfunction expansion
- Wavepacket grows exponentially in space as it propagates downstream
- Transient growth associated with superposition of non-normal global modes describe convectively growing wavepacket

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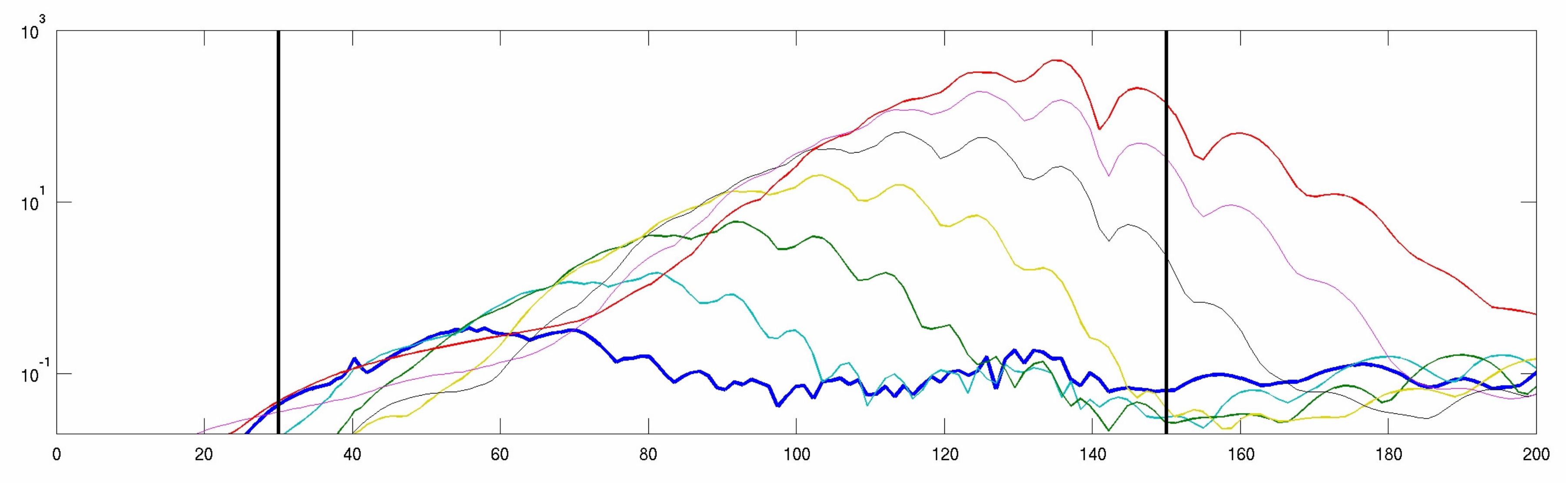
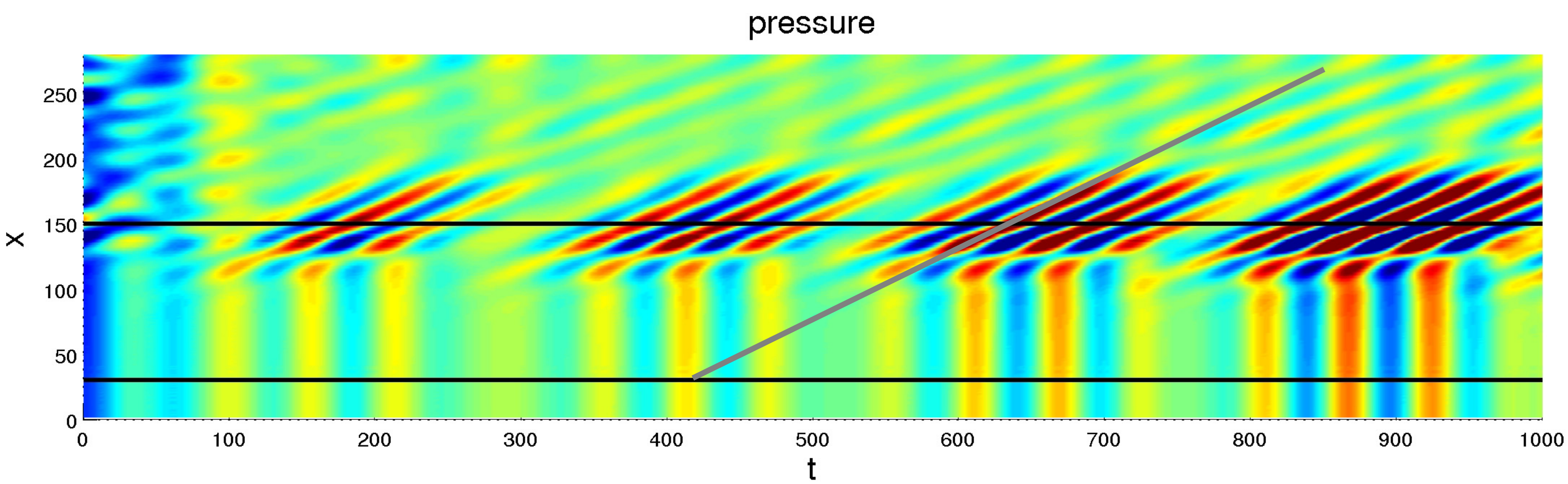


Development of wavepacket

- x-t diagrams of pressure at y=10 using eigenmode expansion
- Wavepacket generates pressure pulse when reaching downstream lip
- Pressure pulse triggers another wavepacket at upstream lip



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Evolution of wavepacket in DNS

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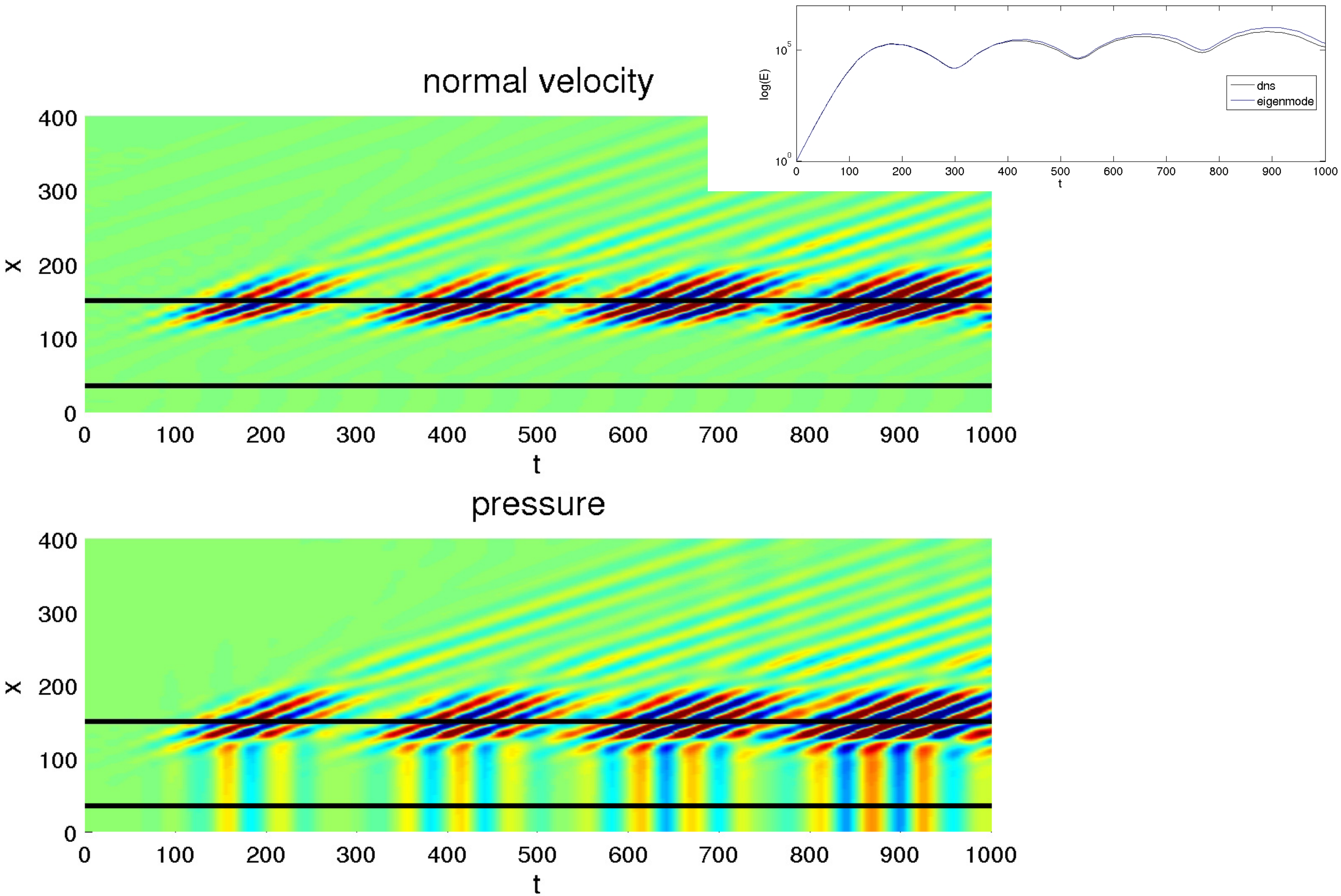

Feedback control of cavity disturbances

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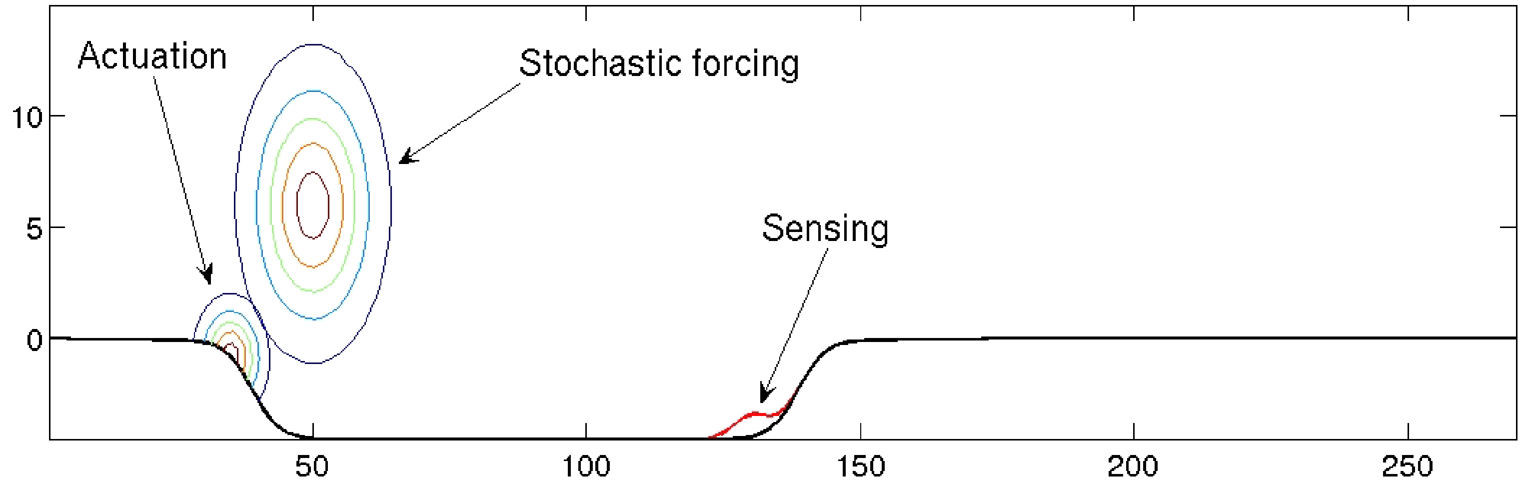

Feedback control using reduced model

- Project dynamics on least stable eigenmodes using adjoints
- Choose spatial location of *control* and *measurements*

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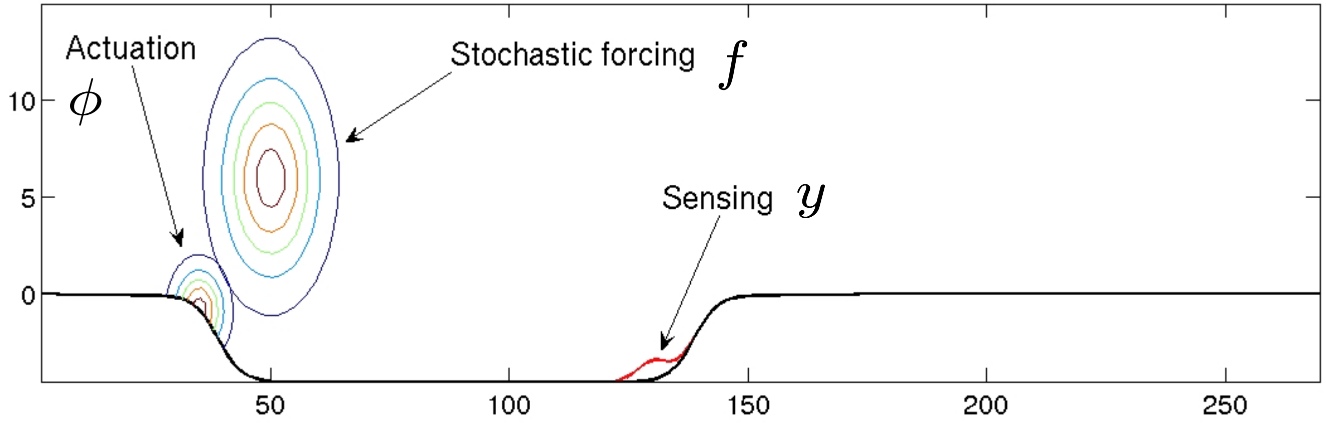
$$\frac{du}{dt} = \bar{A}u + \bar{B}\varphi \qquad u = \sum_n \kappa_n(t) q_n(x,y)$$

$$y = \bar{C}u \qquad \varphi = \sum_m \phi_m(t) \boxed{h_m(x,y)}$$

$$\frac{d\kappa}{dt} = A\kappa + B\phi \qquad \{B\}_{nm} = < \bar{B}h_m, q_n^+ >$$

$$y = C\kappa \qquad \{C\}_{in} = \boxed{r_i(x)}\ q_n'(x,0)$$

LQG feedback control



$$\frac{d\kappa}{dt} = A\kappa + B\phi + B_1 f$$

$$y = C\kappa + g$$

Model of real system/flow

$$\frac{d\kappa_e}{dt} = A\kappa_e + B\phi - L(y - y_e)$$

$$y_e = C\kappa_e$$

Estimator

$$\phi = K\kappa_e$$

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Riccati equations for control and estimation gains

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choose K to minimize disturbance energy

$$\min J = \frac{1}{2}\int_0^T \int_\Omega (\kappa^* Q \kappa + \phi^* M \phi)\, dt d\Omega$$

$$XA + A^* X - XBM^{-1}B^* X + Q = 0$$

$$\Rightarrow \quad K = -M^{-1}B^* X$$

choose L to minimize covariance of estimation error P

$$AP + PA^* - PC^* G^{-1} CP + B_1 R B_1^* = 0$$

$$\Rightarrow \quad L = -PC^* G^{-1}$$

Control gain K in physical space

- $\phi = \bar{K}u$
- Flow close to actuator influences control
- Actuator located where flow is sensitive (adjoint large)

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Estimation gain L in physical space

- Estimation forcing $\bar{L}(y - y_e)$ strongest close to sensor
- Sensor located where modes are large (easier to measure)

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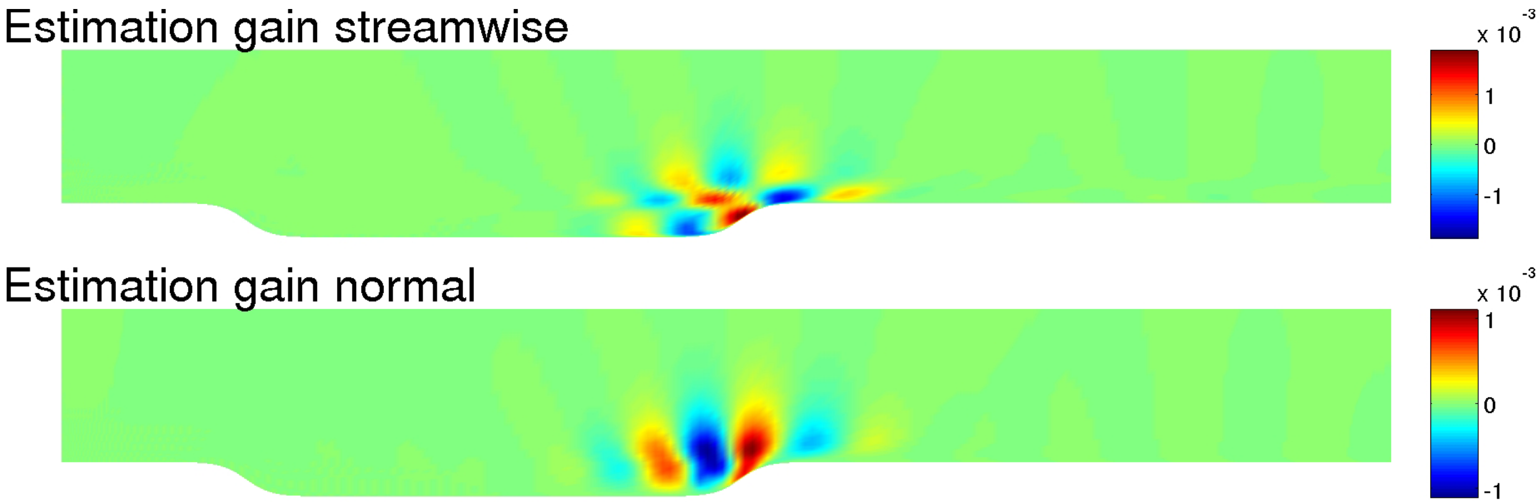

LQG controller formulation with DNS

- Apply in Navier-Stokes simulation

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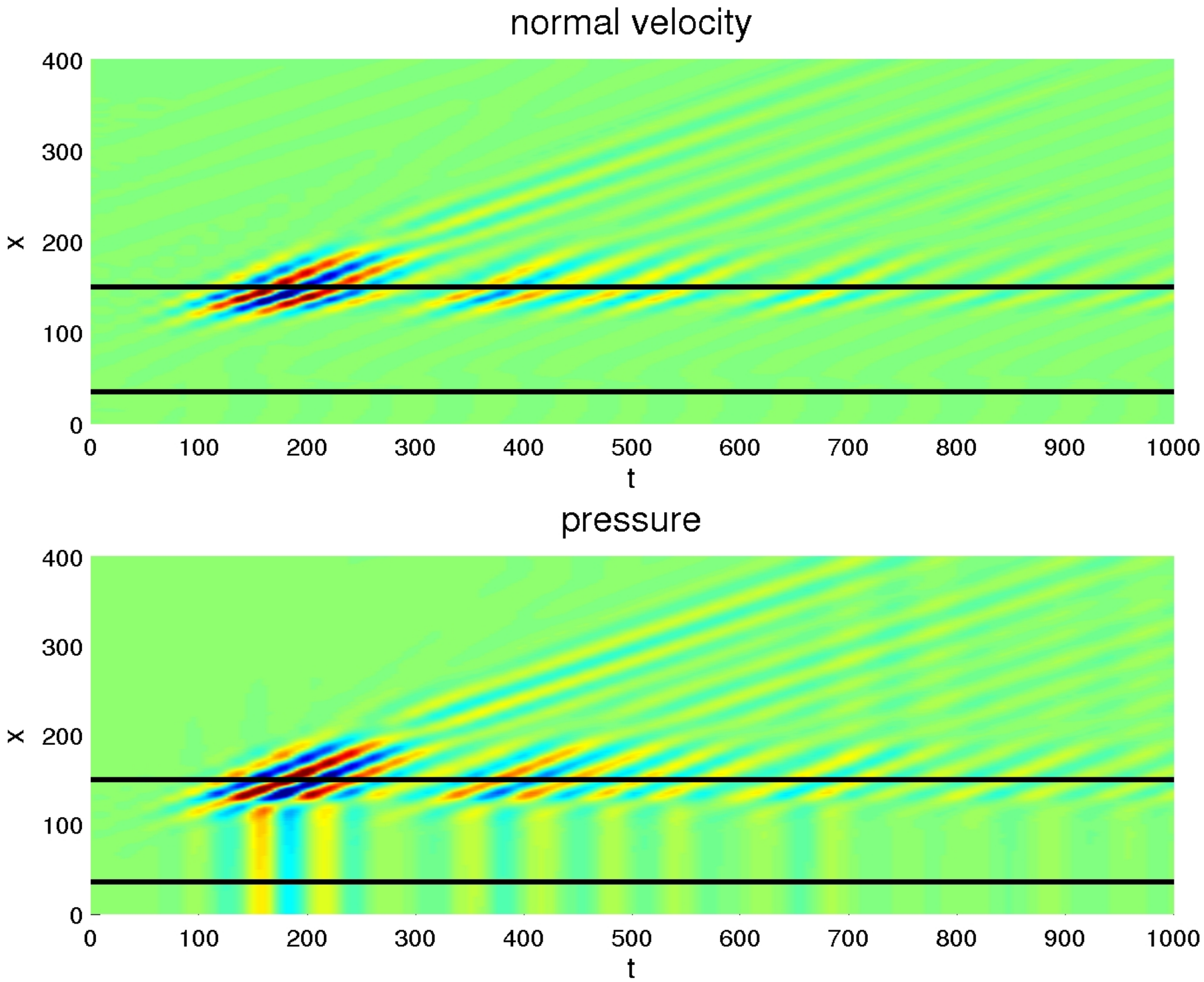
$$\begin{aligned} \dot{u} &= NS(u) + \bar{B}h(x,y)\phi(t) \\ y &= \bar{C}u \end{aligned}$$

$$\begin{cases} \dot{\kappa}_e &= (A + BK + LC)\kappa_e - Ly \\ \phi &= K\kappa_e \end{cases}$$

DNS of controlled flow

- Estimator based on 4 to 25 global modes



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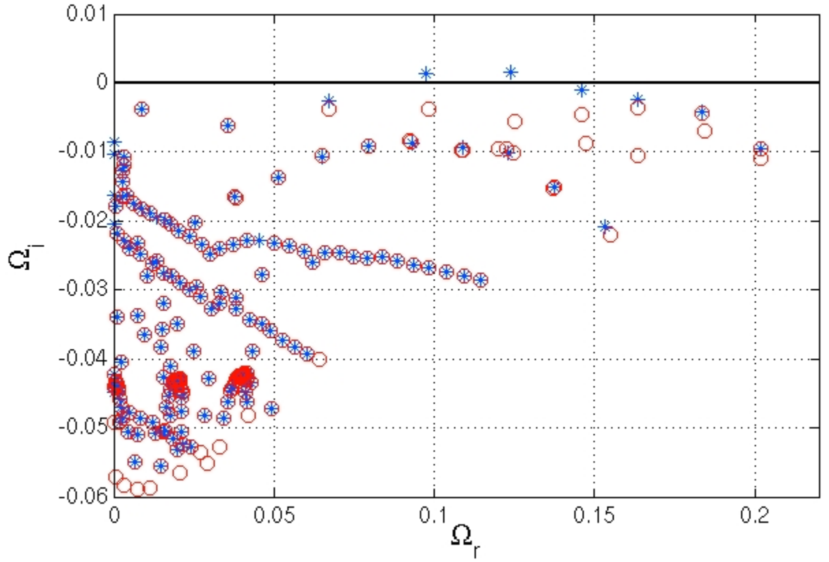
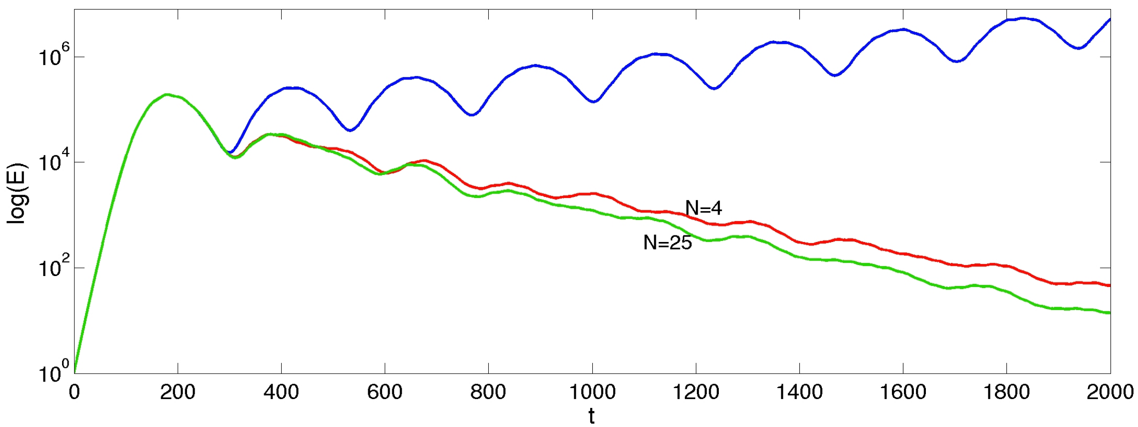
Controller performance

- Least stable eigenvalues are rearranged
- Exponential growth turned into exponential decay
- Good performance in DNS using only 4 global modes





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Conclusions

- Global eigenmodes from Krylov subspace with Arnoldi algorithm
- Transient growth associated with superposition of non-normal global modes describe convectively growing wavepackets in waterfall, Blasius and shallow cavity problem
- Optimal sum of global modes brought out pressure triggering mechanism and Orr-mechanism automatically
- Feedback control of the shallow cavity flow
  - LQG control design using reduced order model with global modes
  - Controller based on small number of modes works well in DNS
- Outlook:
  - Calculation of optimal growth directly in Krylov subspace or using adjoint iterations, to avoid sensitive non-orthogonal eigenmodes
  - Balanced truncation modes in control design

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