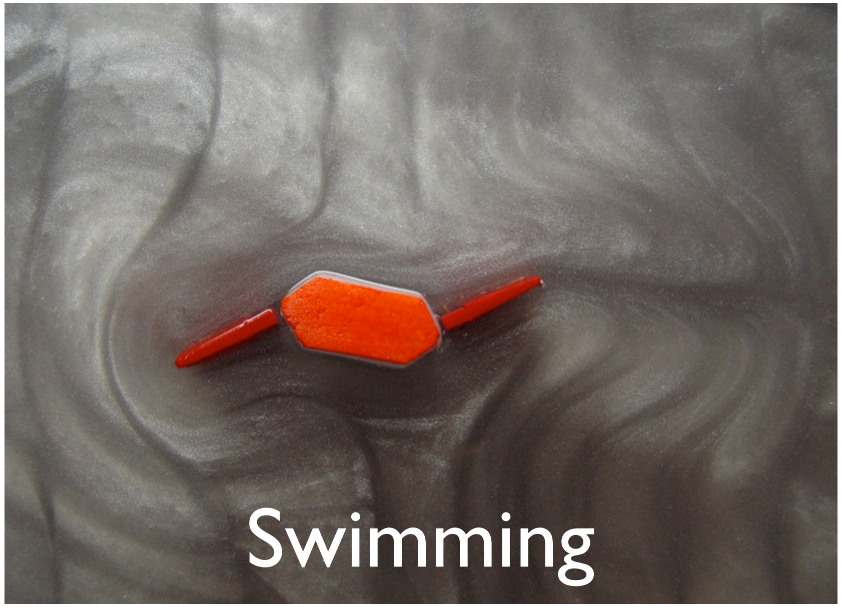
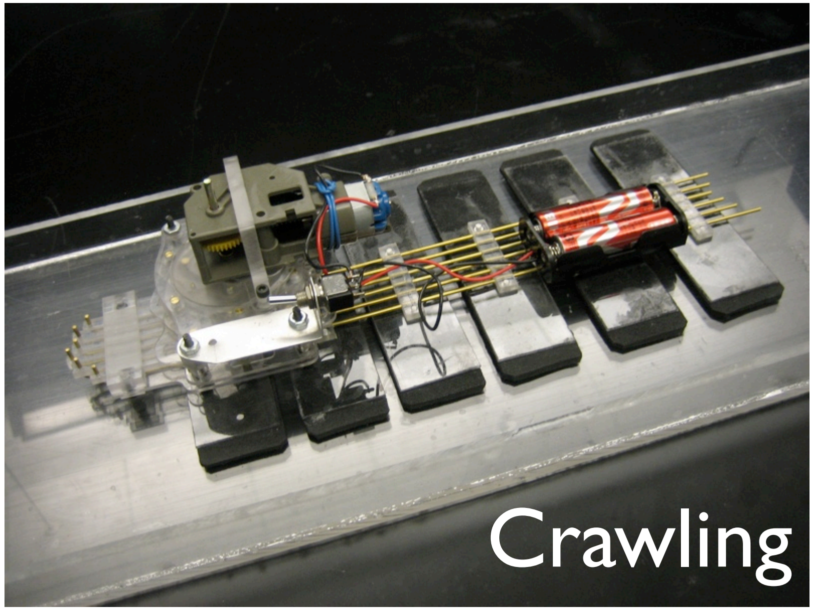
Optimizing Low Reynolds Number Locomotion

Anette (Peko) Hosoi
Hatsopoulos Microfluids Laboratory, MIT

What's in This Talk?

1. Optimal stroke patterns for 3-link swimmers
2. Building a better snail





Tiny Swimmers

Life at low Reynolds number

E. M. Purcell
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(Received 12 June 1976)

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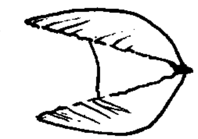
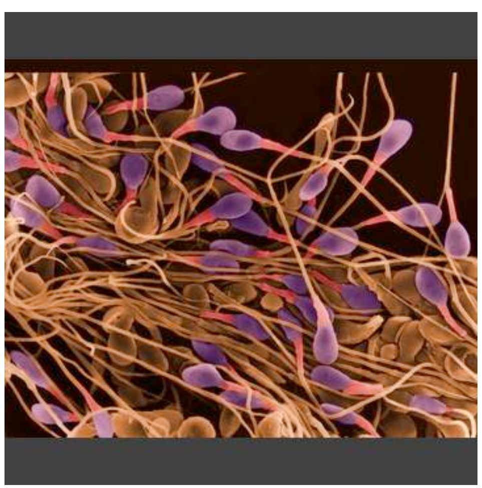
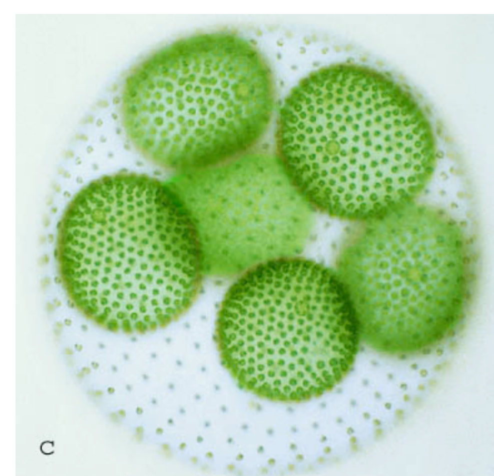
The Scallop Theorem



$$\boxed{\begin{aligned} -\nabla p + \mu \nabla^2 \mathbf{u} &= \mathbf{0} \\ \nabla \cdot \mathbf{u} &= 0 \end{aligned}}$$





First two images from http://www.astrographics.com/ - Dennis Kunkel
http://www.btinternet.com/~stephen.durr/volvoxtwo.html - Stephen Durr

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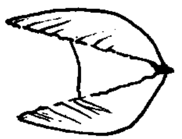

$$-\nabla p + \mu \nabla^2 \mathbf{u} = \mathbf{0}$$
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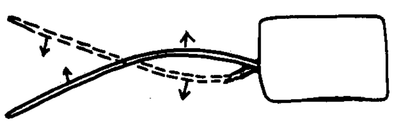
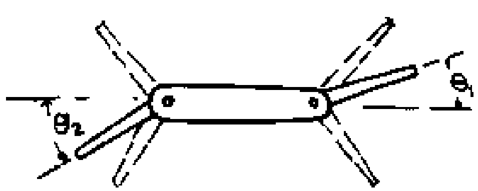
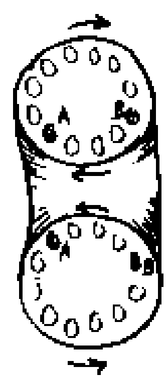
The flexible oar

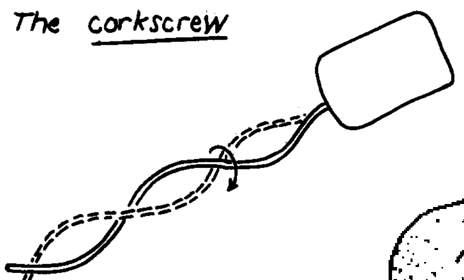


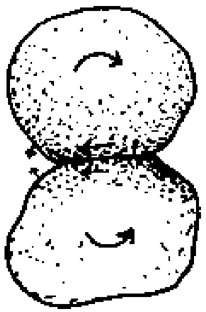


The 3-link swimmer



The corkscrew





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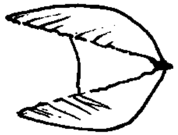

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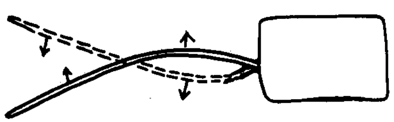

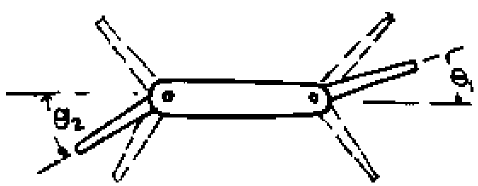

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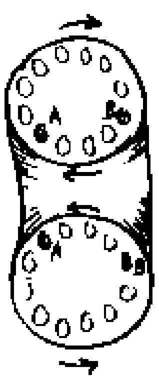

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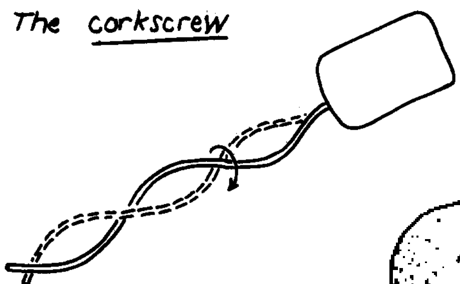

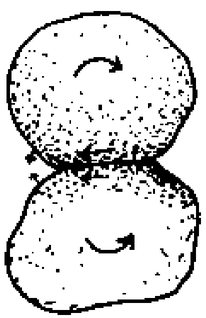

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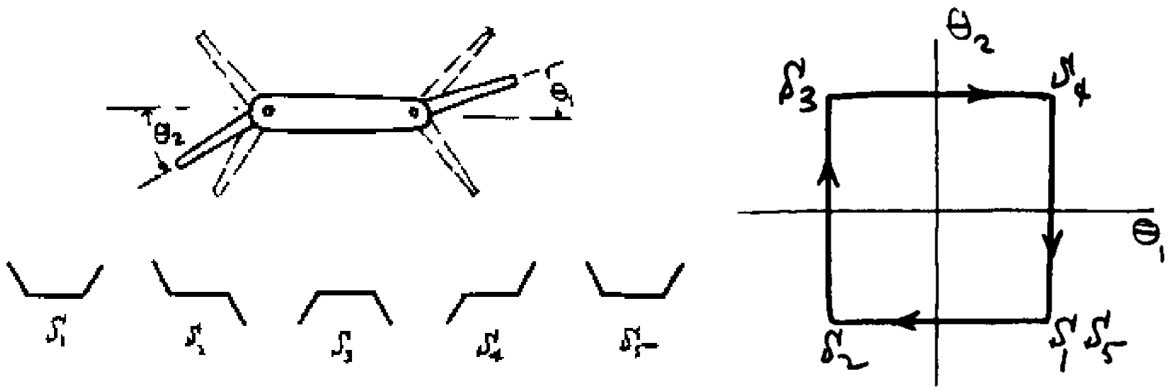

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- Becker, Koehler and Stone (2003): optimized geometry (arm length/body length and stroke angle)

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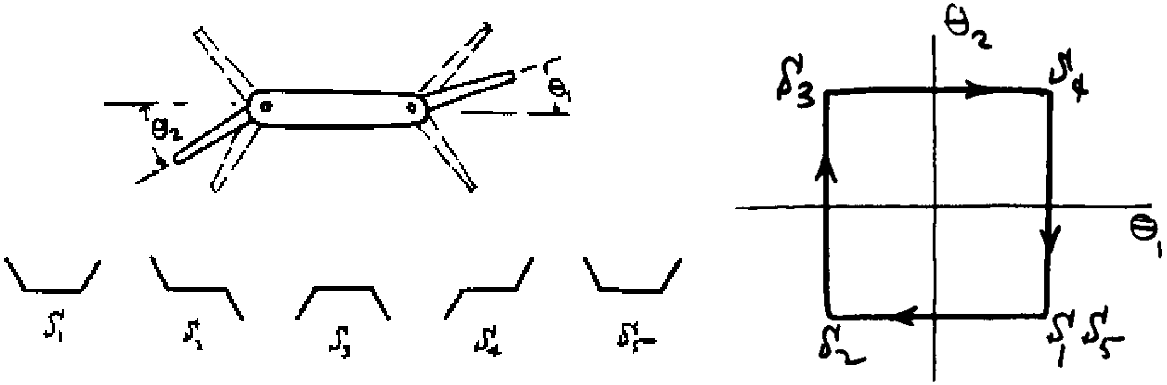

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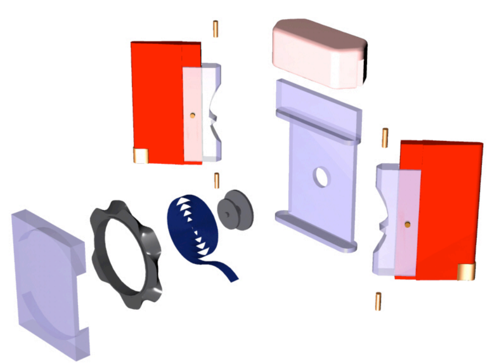

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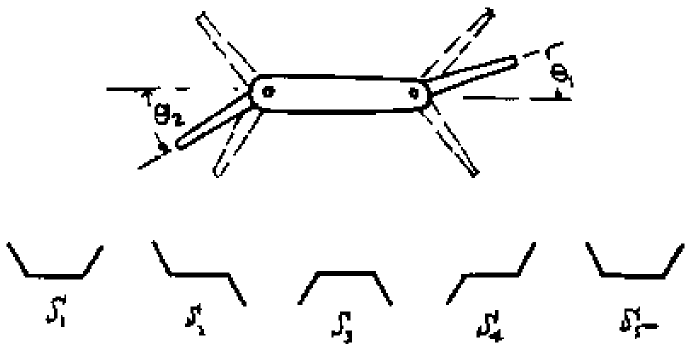

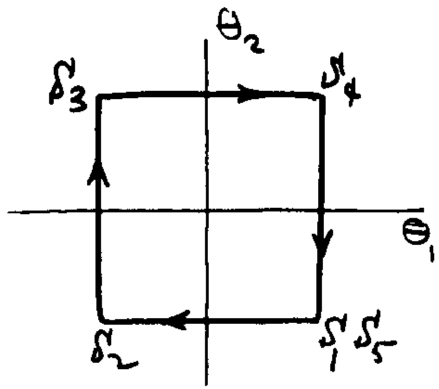

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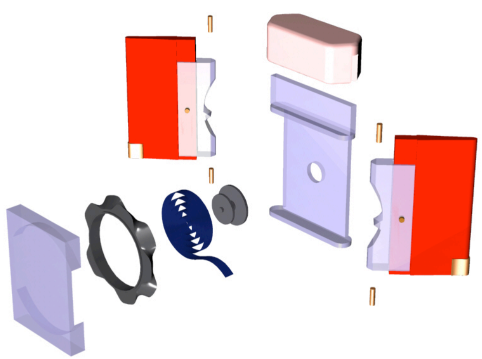

Can we do better?

Optimizing Kinematics

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Fixed geometry

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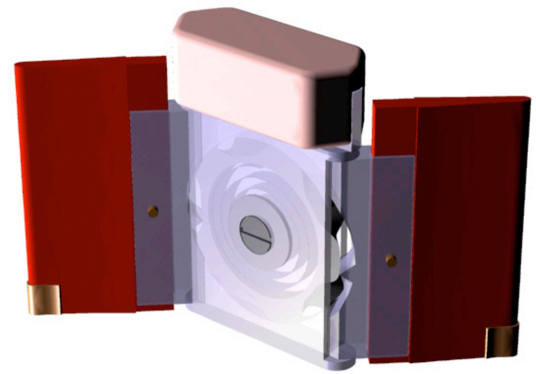
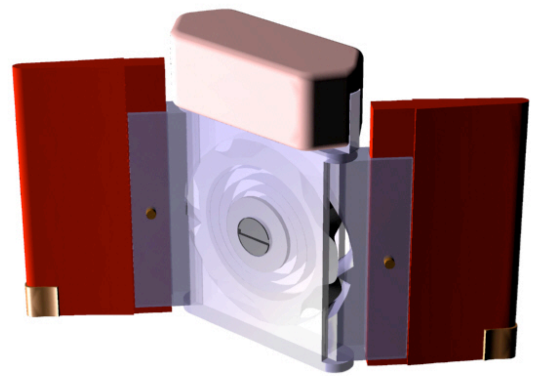
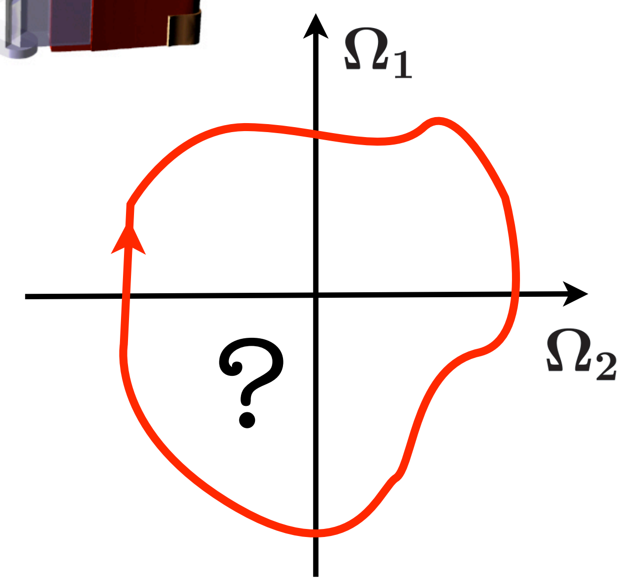


Optimizing Kinematics

Fixed geometry





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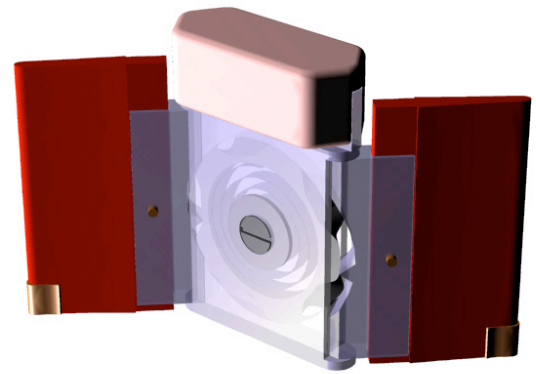
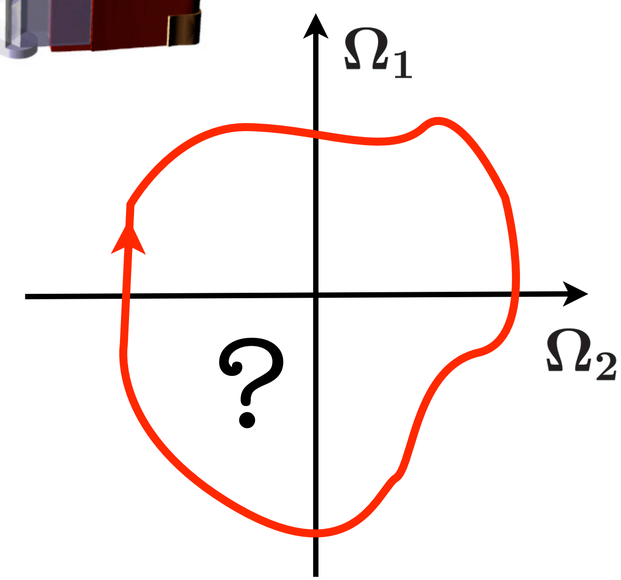
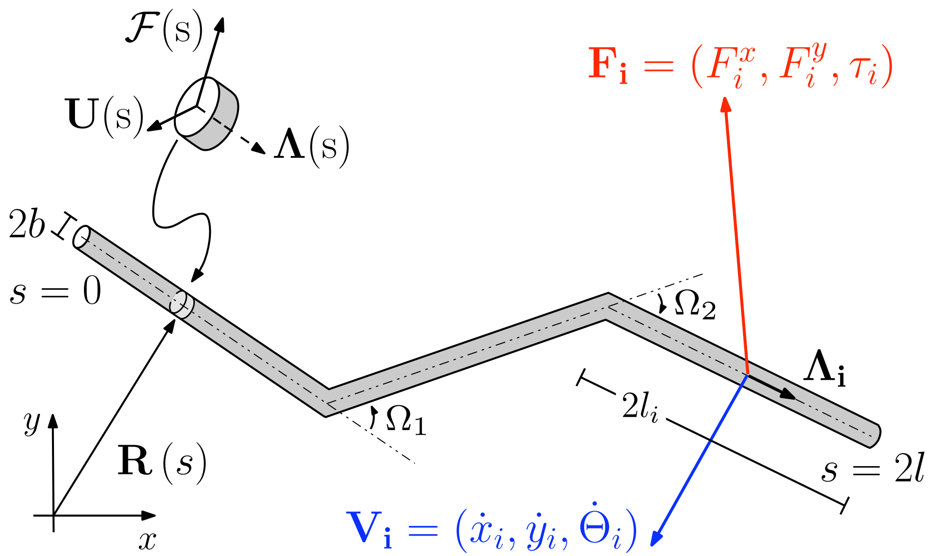
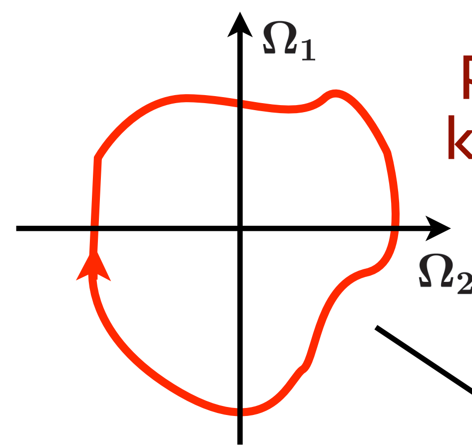




Kanso and Marsden (2005) - 3-link fish
Berman and Wang (2006) - insect flight

Model Swimmer





prescribe kinematics

force balance

Solve linear system

$$\sum_{i=1}^{3} \mathbf{F_i} = \sum_{j=1}^{3} \left(\sum_{i=1}^{3} \mathbf{A_i^j} \right) \dot{\mathbf{X}}_\mathbf{j} = \mathbf{0}$$

Constraint: links are attached

Swimming velocities and efficiencies

- Lowest order: resistive force theory
- Next order: can incorporate effects of slenderness and interactions between links

R. Cox, Journal of fluid mechanics **44 (4)**, 791 (1970).

Optimized Stroke Patterns

- Two cost functions
  - Efficiency
    - [useful work]/[energy dissipated]
    - Unique parametrization that optimizes efficiency for a given curve
  - Speed
- Symmetry axes

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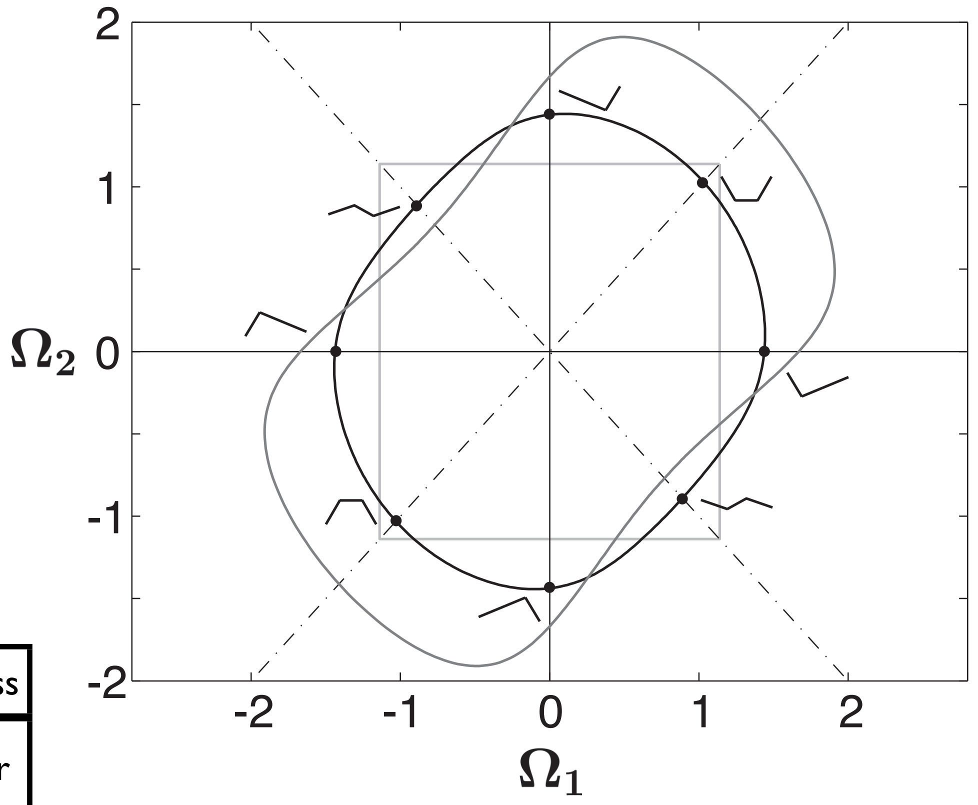

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Purcell	0.483	0.0077	0.809	As slender as possible
Optimal	0.623	0.013	0.933	

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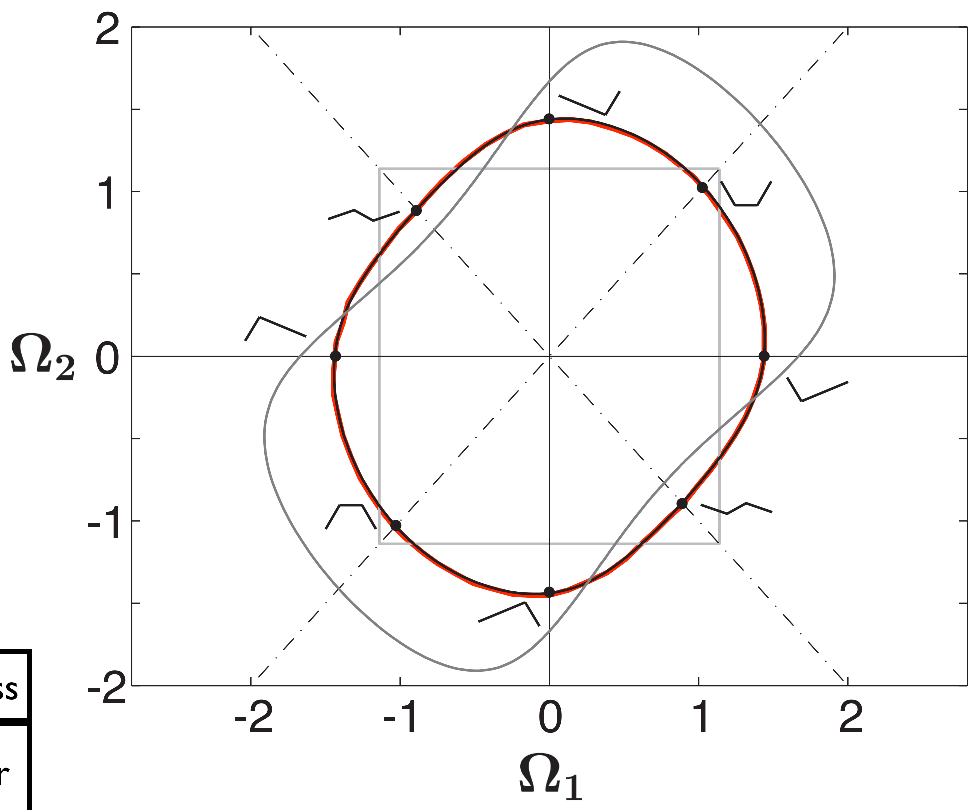

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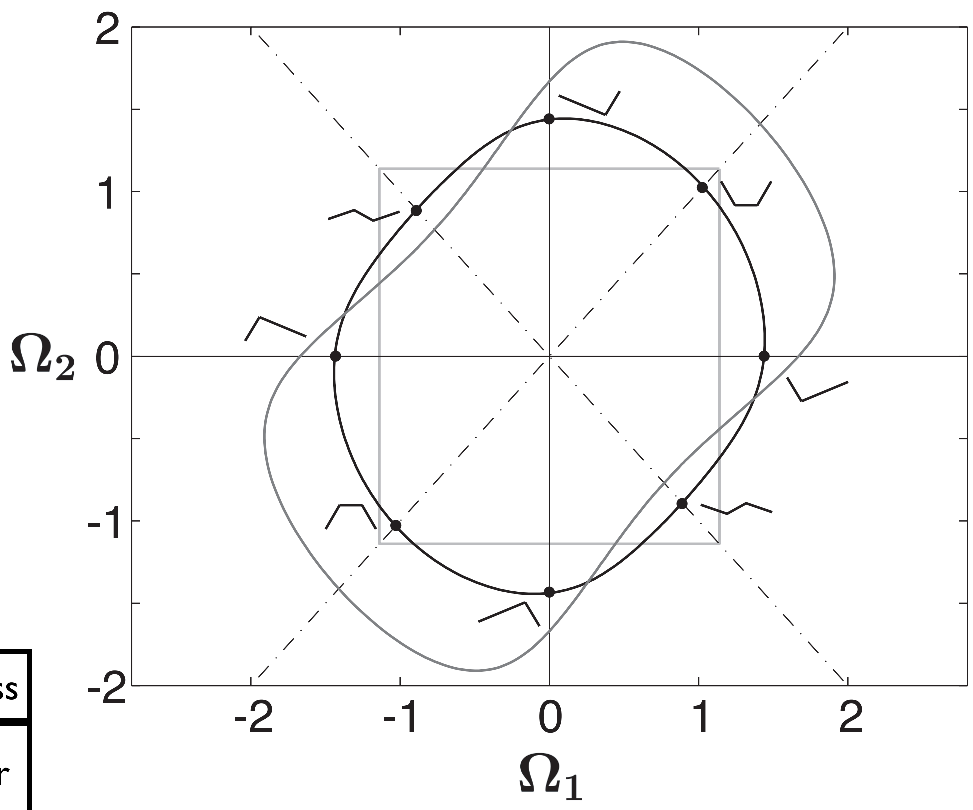

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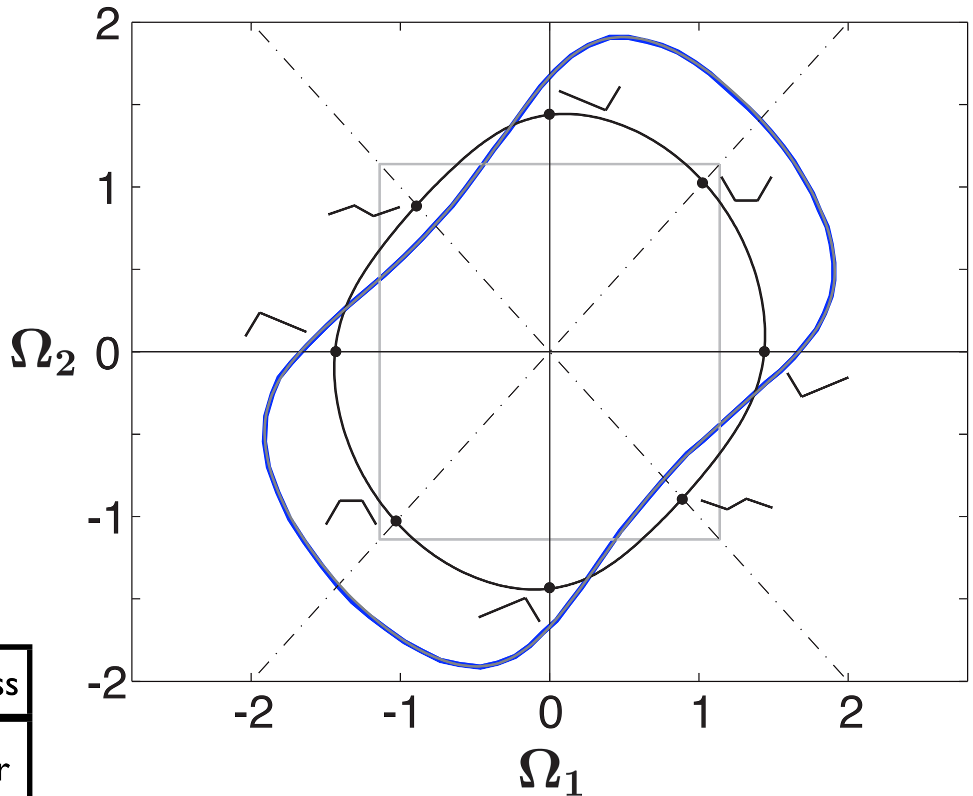

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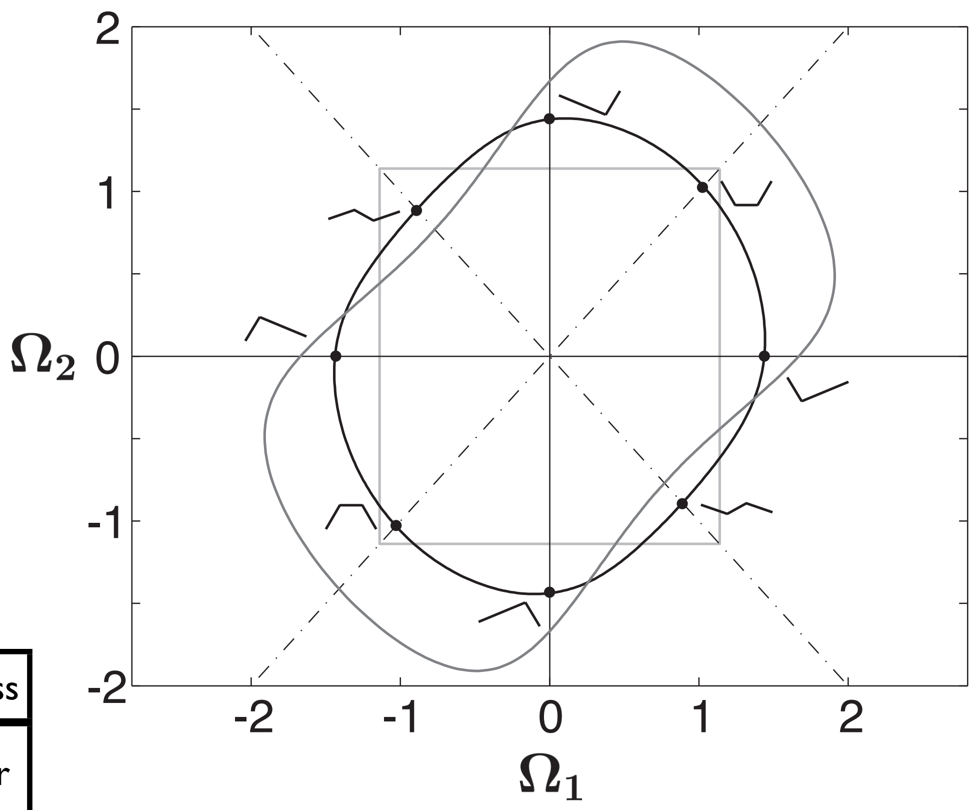

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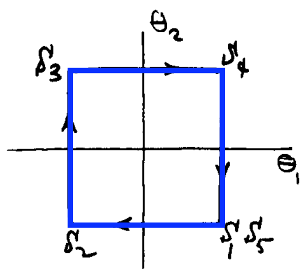
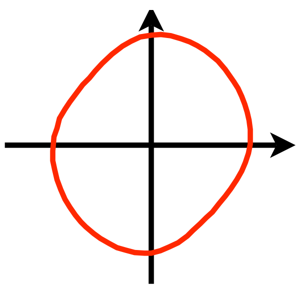
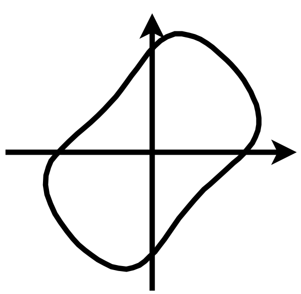
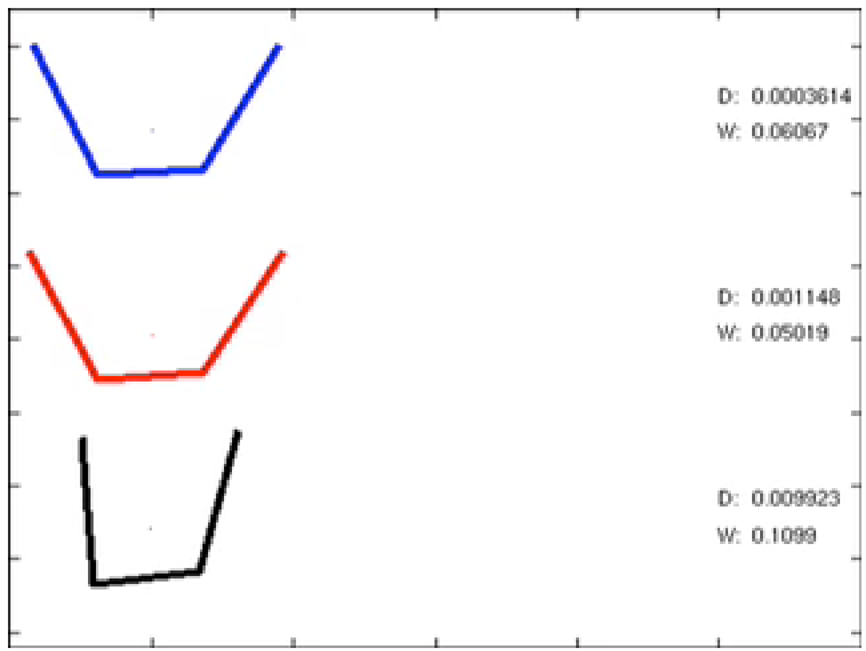
3-Link Race









D = distance
W = Total "work"
(viscous dissipation)

Thin line = path of center of mass

Optimize geometry

Optimize geometry AND kinematics

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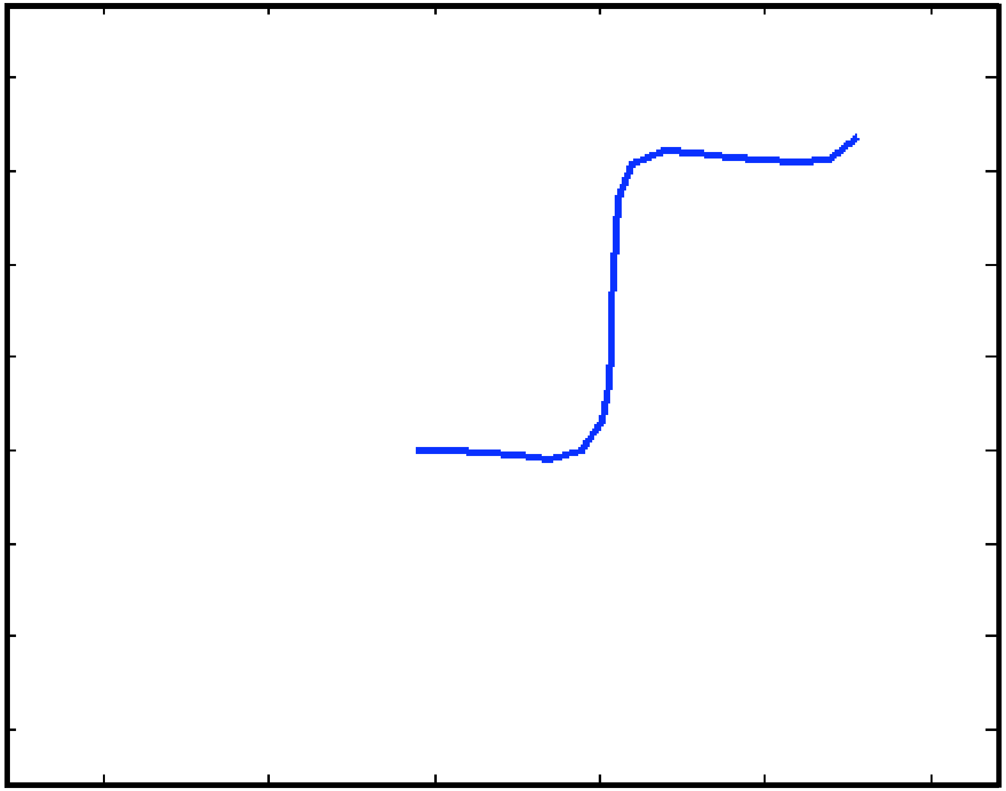
Multiple Links



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- Analytic solution by Lighthill (in *Mathematical Biofluiddynamics*)
    - 41 degree angle

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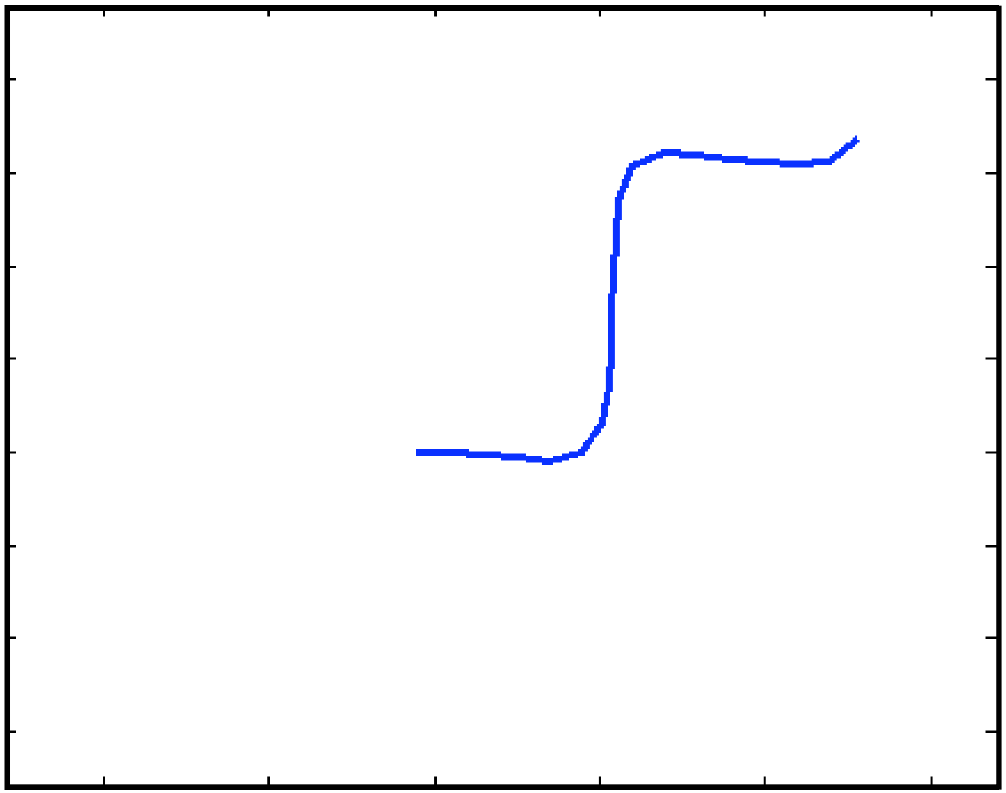

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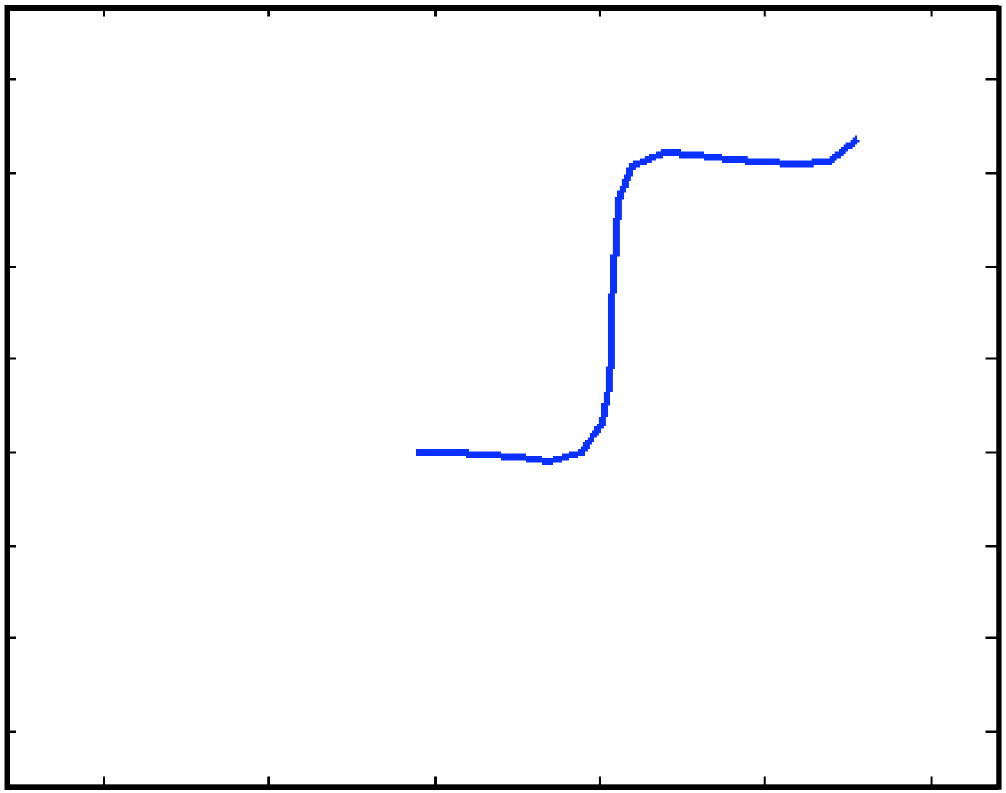

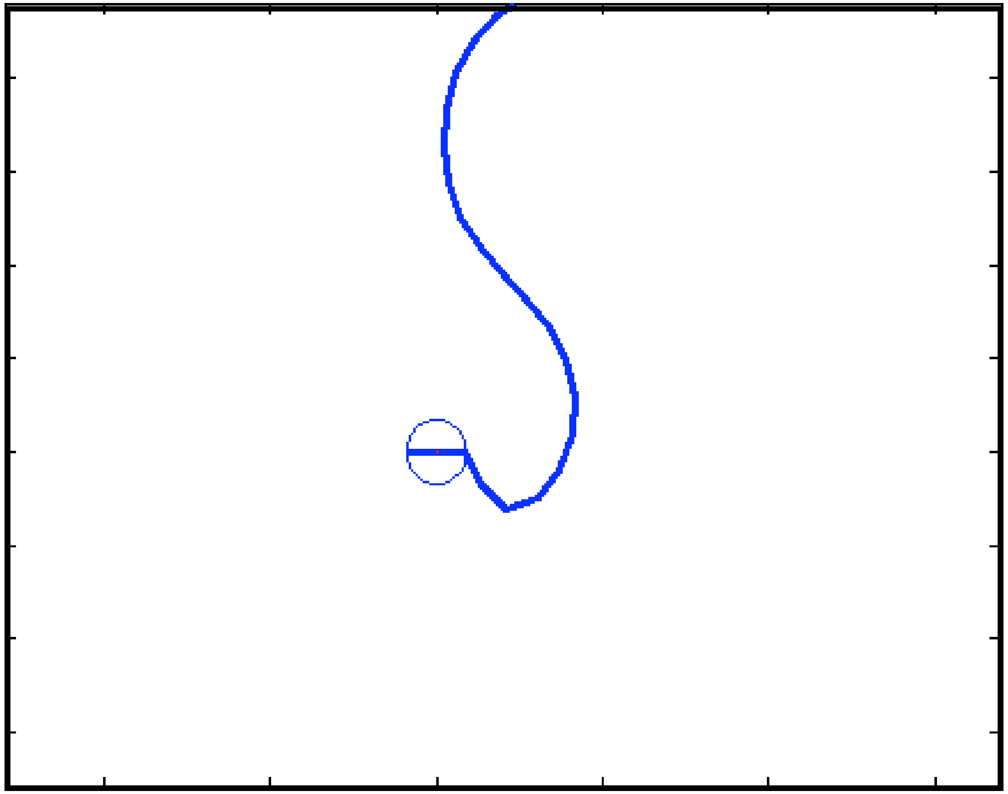

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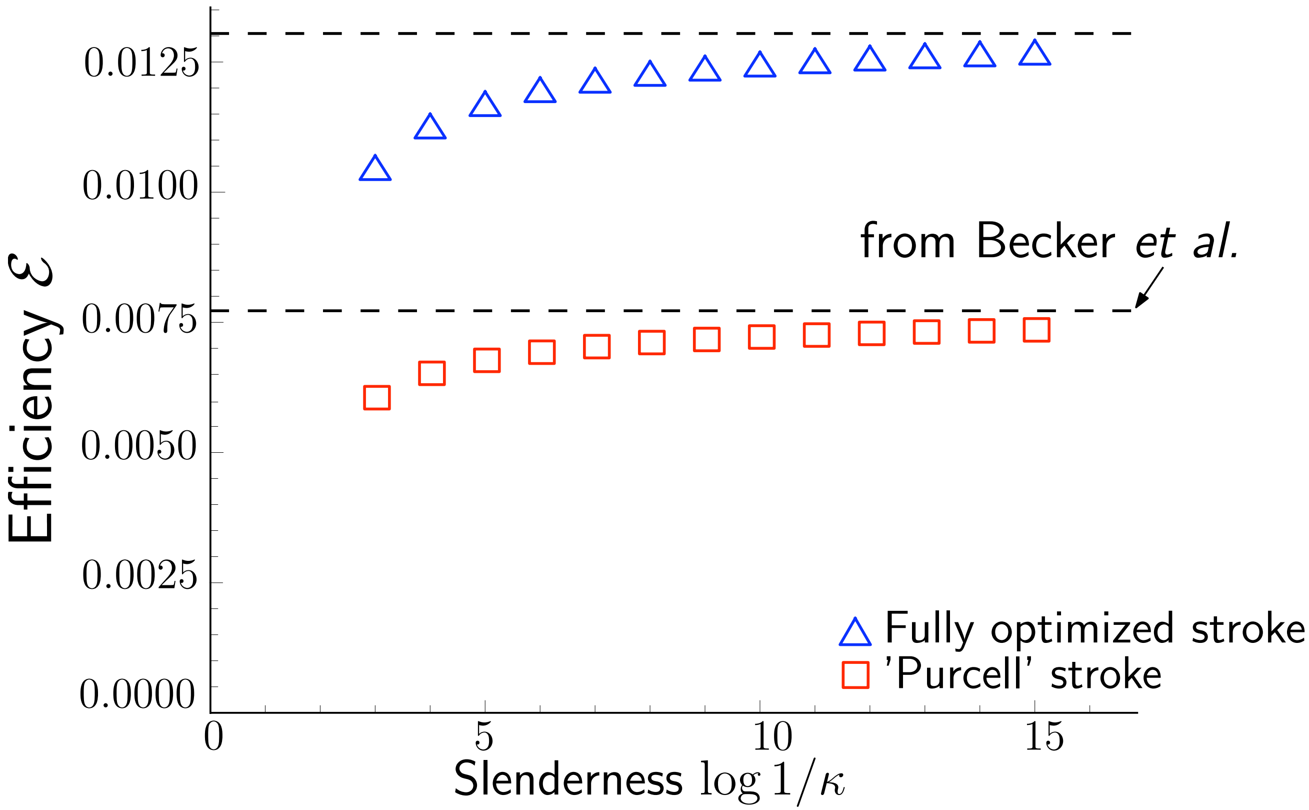
Effect of Slenderness



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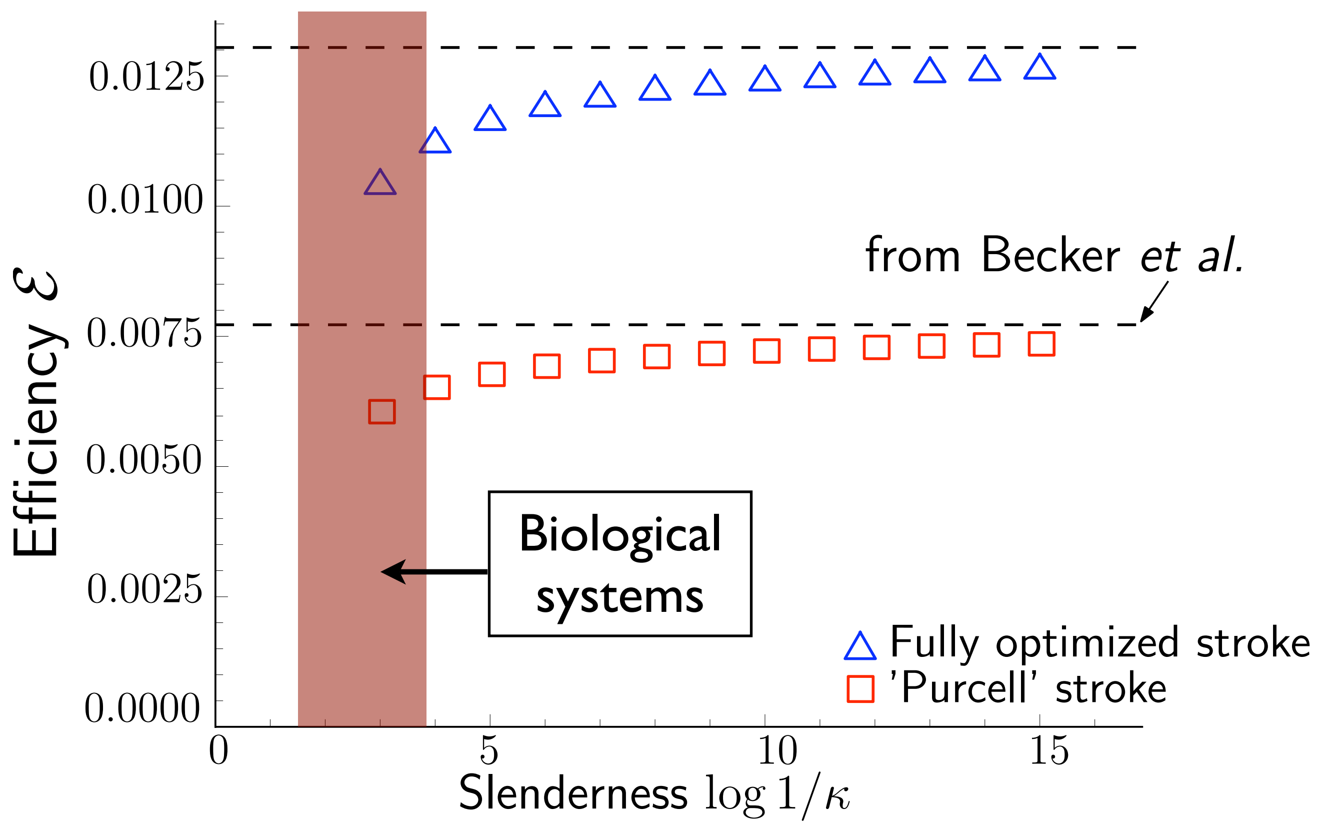
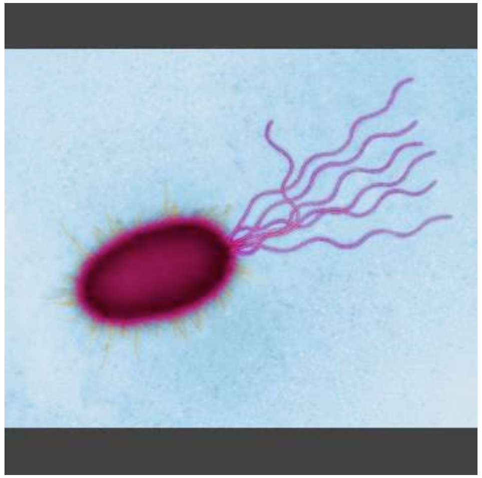
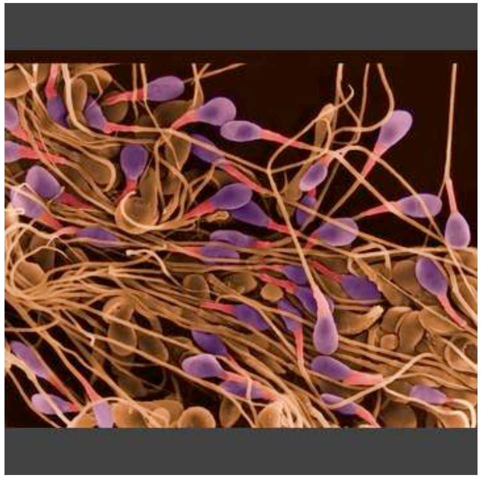
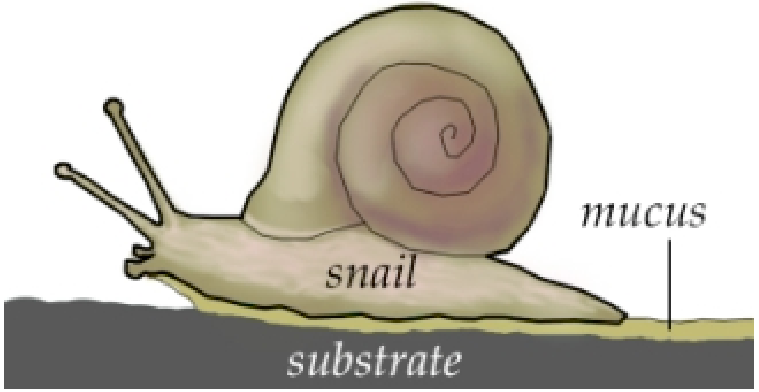






Gastropod Locomotion

Locomotion is directly coupled to stresses in the thin fluid film



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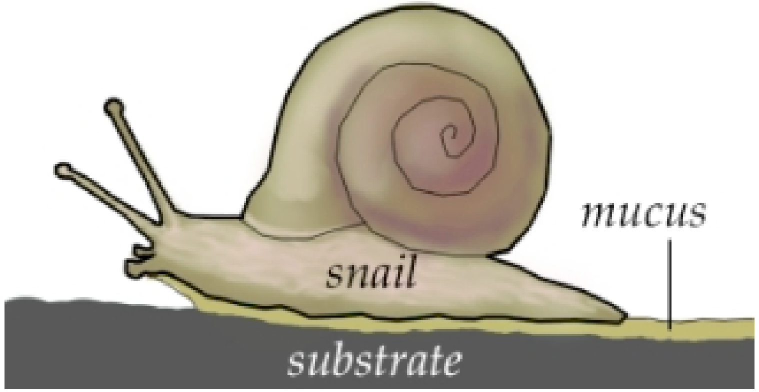

Gastropod Locomotion

Retrograde vs direct waves

F. Vles, C. R. Acad. Sci., Paris 145, 276 (1907)

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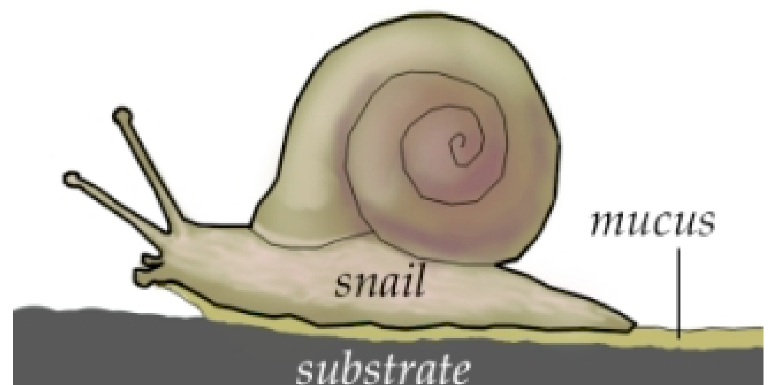
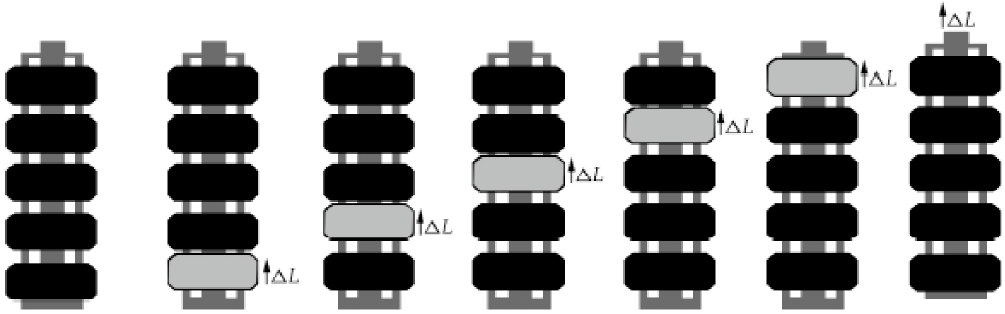
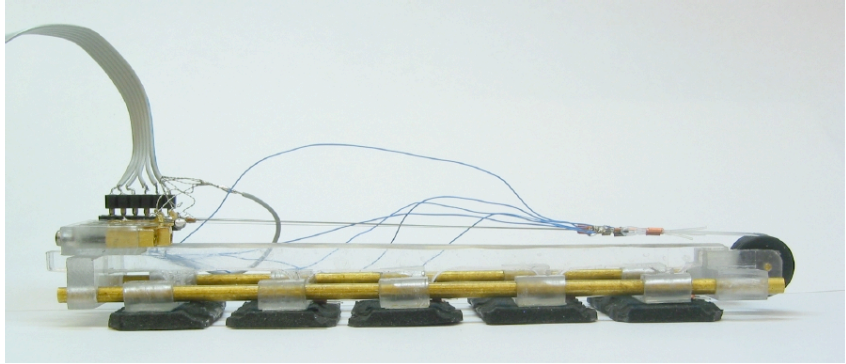


What is Required for Locomotion?





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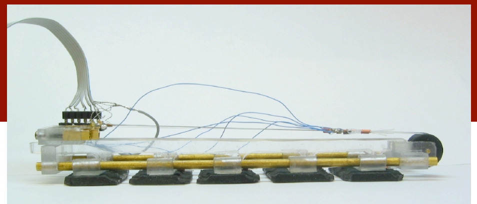

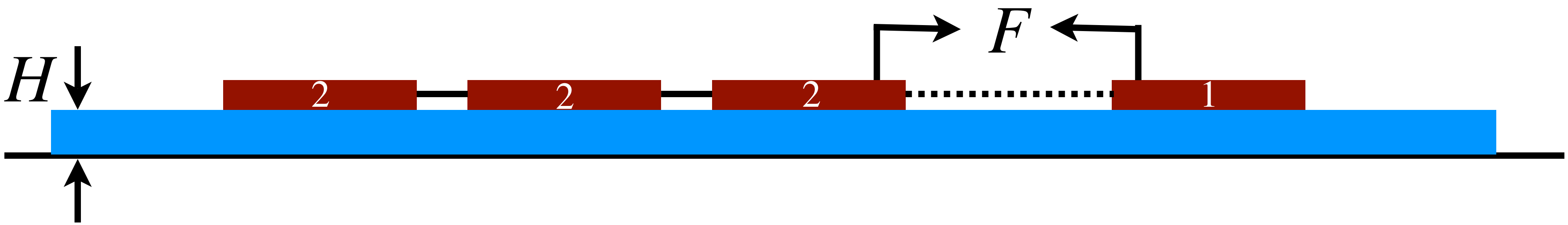

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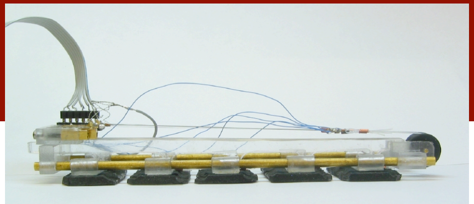

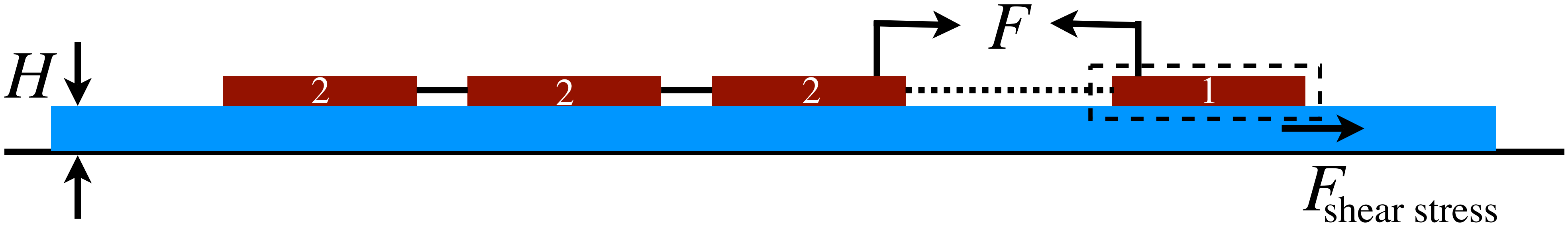

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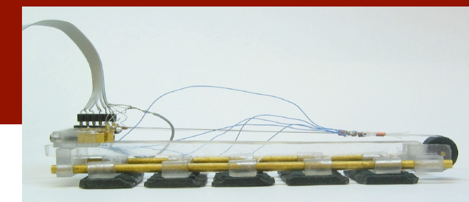

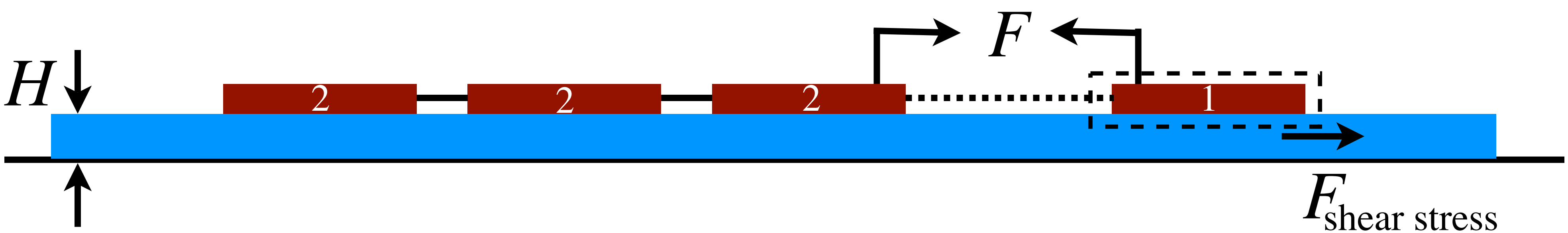

$$F = \tau_1 A = \mu(\tau_1) V_1 A / H$$

Couette flow in small gap

$$V_{cm} = \left[V_2(N-1) + V_1 \right] / N$$

Each pad carries equal mass

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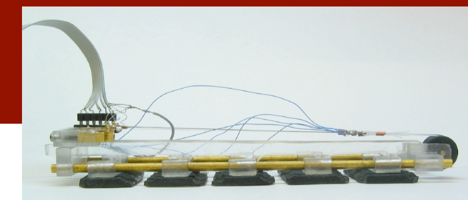

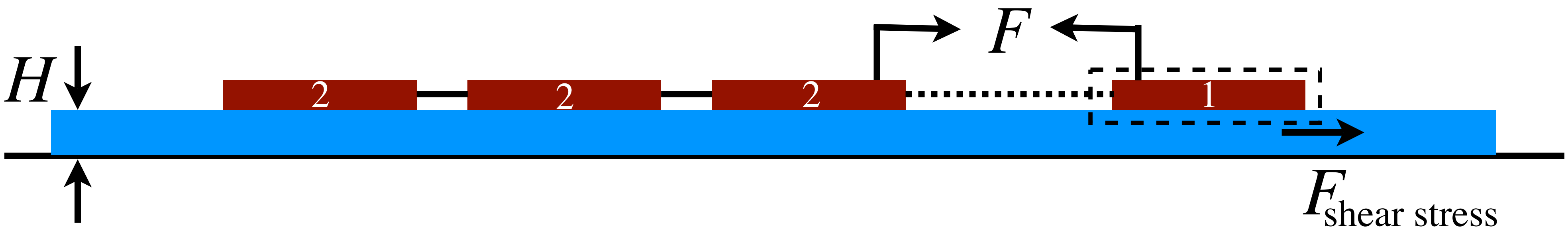

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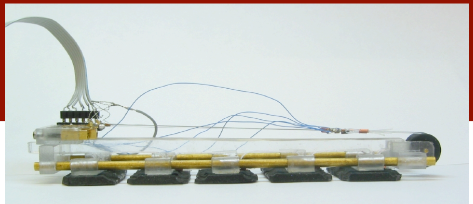

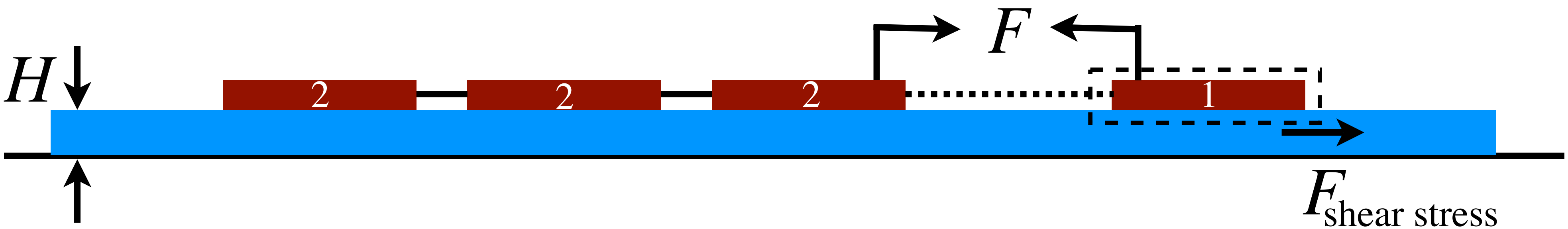

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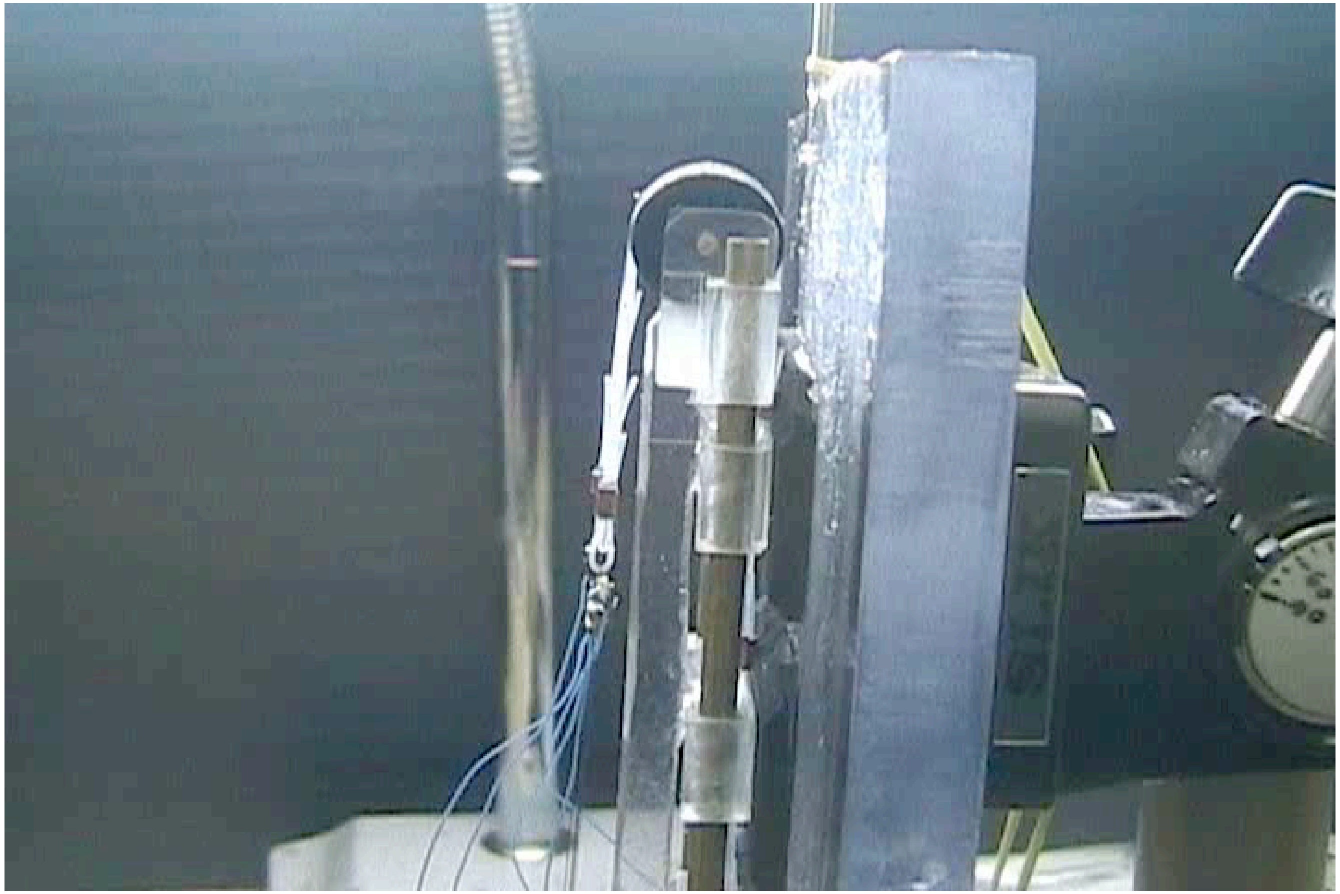
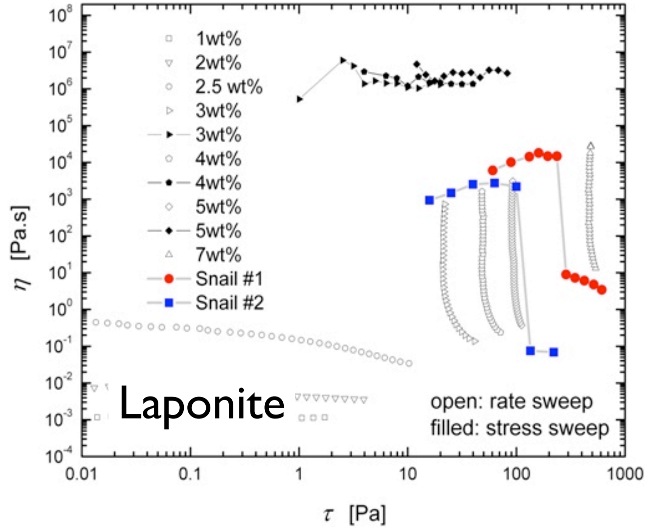
Nonlinear characteristics first measured by:

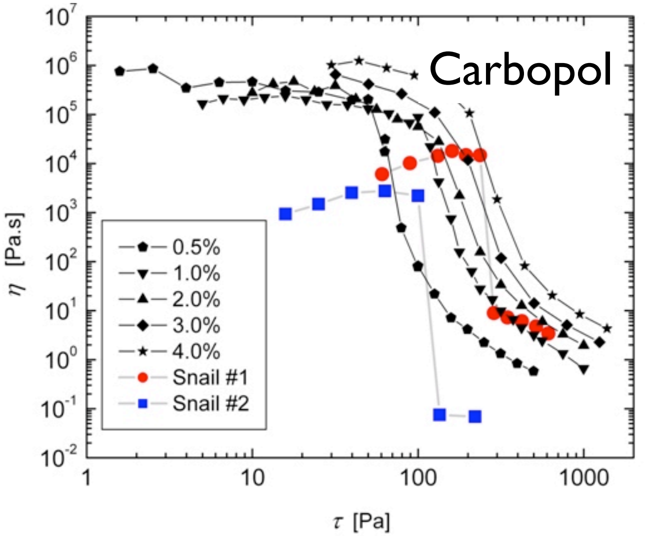
M. Denny, J. Exp. Biol. **91**, 195 (1981)

M. Denny, Nature **285**, 160 (1980)

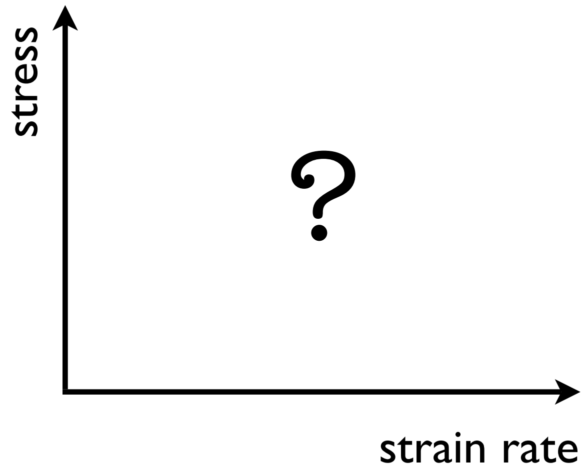
RoboSnail II







"Tune" Material Properties



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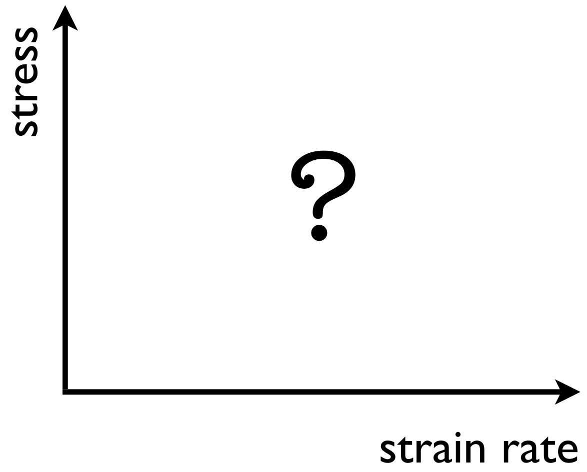

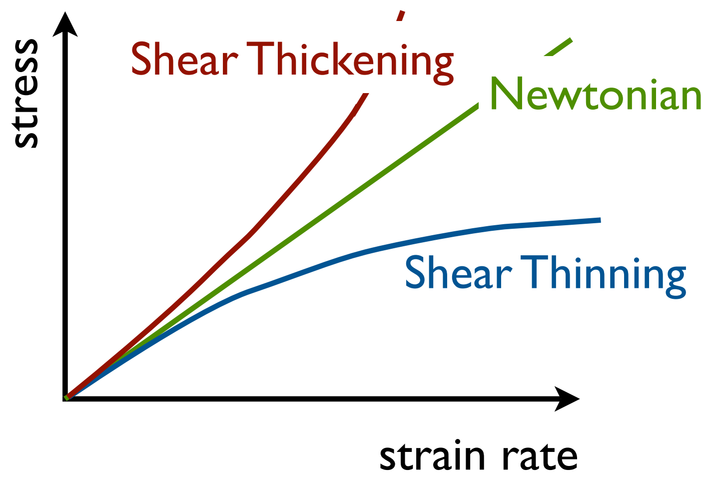

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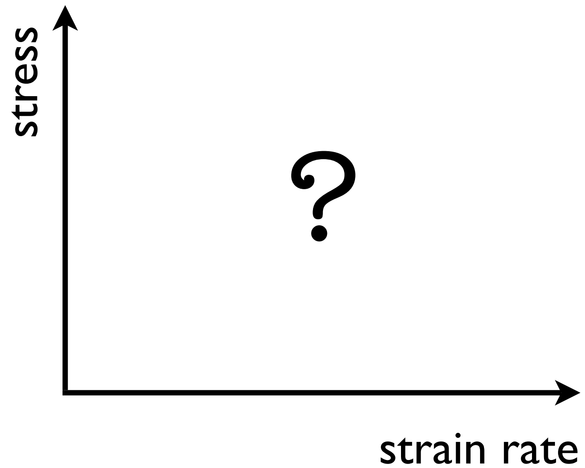

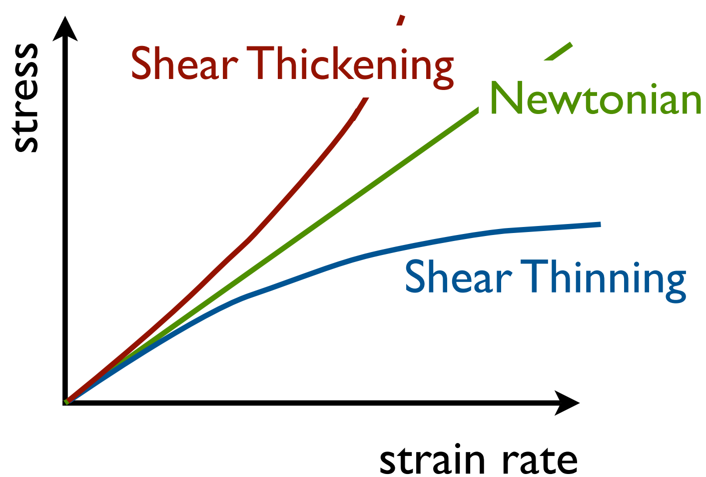

- Perturb rheology of Newtonian fluid (snail will crawl on any non-Newtonian fluid)
  - shear thickening
  - shear thinning
- Which material properties are “favorable”?

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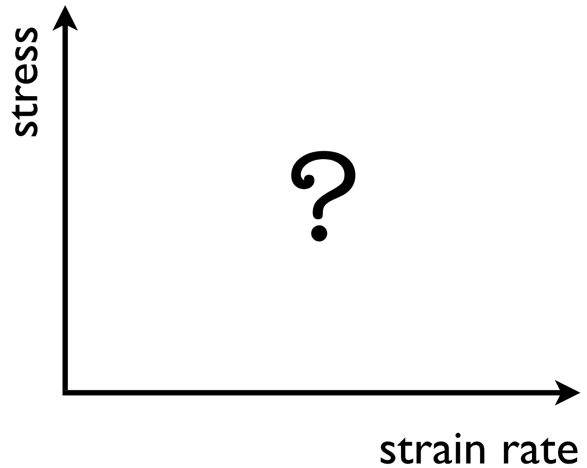

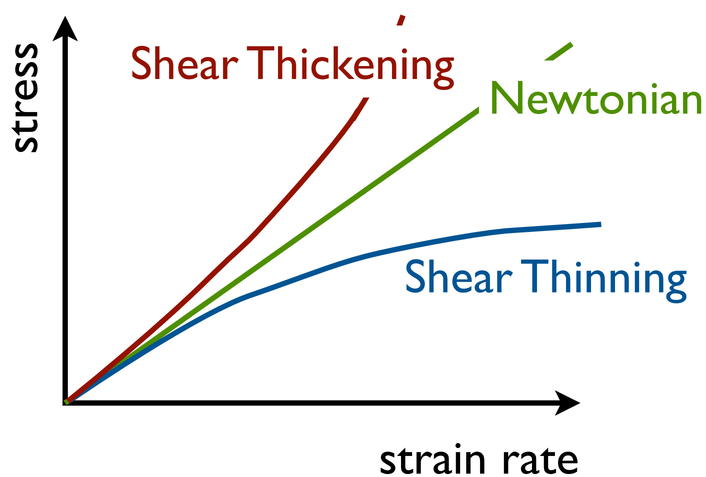

- Perturb rheology of Newtonian fluid (snail will crawl on any non-Newtonian fluid)

$$\dot{\gamma} = \frac{\sigma}{\mu}\left(1 - \epsilon\frac{|\sigma|}{\sigma_*}\right)$$

  - $\epsilon > 0$ shear thickening
  - $\epsilon < 0$ shear thinning

- Which material properties are "favorable"?

Rheology Cost Function

- Mechanical work done in crawling (= rate of viscous dissipation)

$$\mathcal{E} = \int_0^\lambda \int_0^h \sigma\dot{\gamma}\,\mathrm{d}y\,\mathrm{d}x. \quad \Rightarrow \quad \mathcal{E} = \frac{h\lambda}{4\mu}\langle\overline{\sigma}^2\rangle - \epsilon\frac{17}{96}\frac{h\lambda}{\mu\sigma_*}\langle\overline{\sigma}^2\,|\overline{\sigma}|\rangle + \ldots$$

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- Chemical cost associated with mucus production (~ flux in frame moving with snail)

$$Q = \int_0^h u_s(x) \mathrm{d}y \quad \Rightarrow \quad Q_s = \epsilon \frac{79}{432}\frac{h^2}{\mu\sigma_*}\langle \overline{\sigma} \, |\overline{\sigma}| \rangle \left(1 + \epsilon \frac{185}{8532\sigma_*} \frac{\langle \overline{\sigma}^3 \rangle - \langle |\overline{\sigma}| \rangle \langle \overline{\sigma} \, |\overline{\sigma}| \rangle}{\langle \overline{\sigma} \, |\overline{\sigma}| \rangle} + \dots \right)$$

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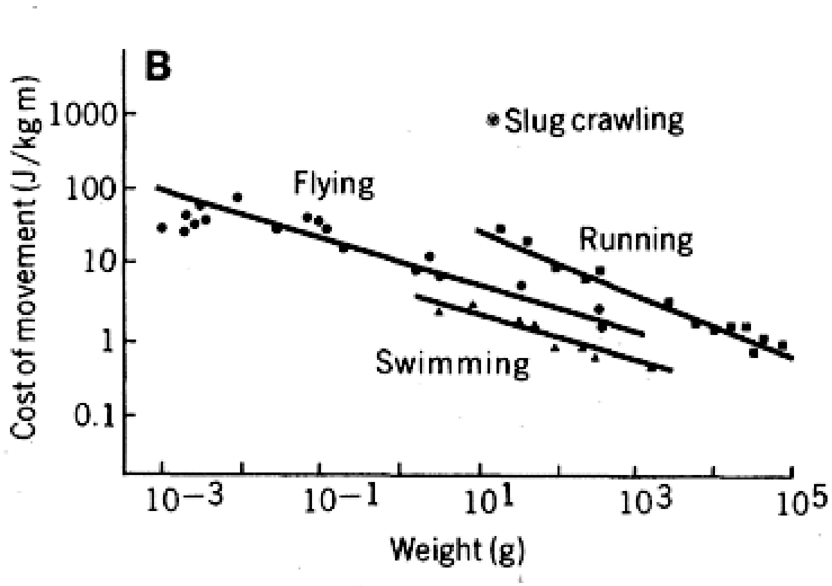
Cost of Locomotion



Mark Denny, *Science*, **208**, No. 4449 (1980)

"The high cost is primarily due to the cost of mucus production, which alone is greater than the total cost of movement for a mammal or reptile of similar weight, ..."

→ shear thinning

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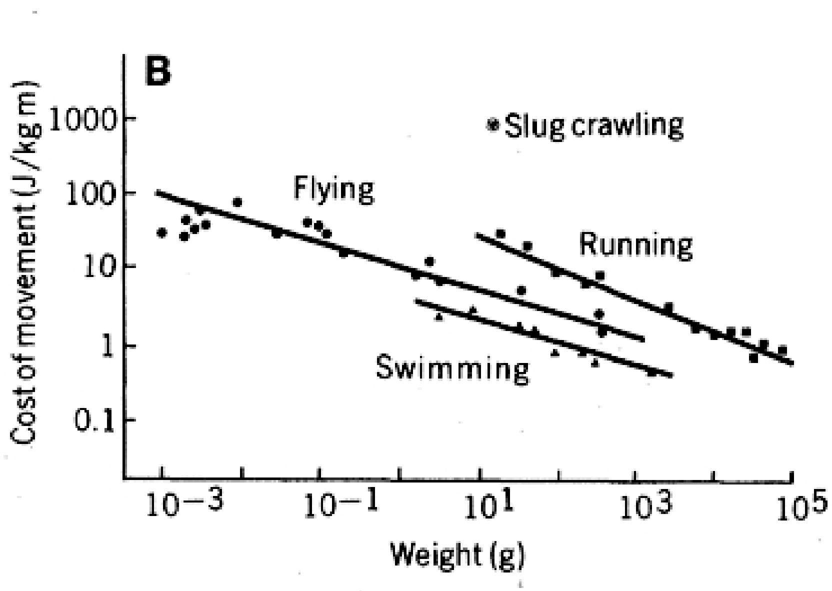
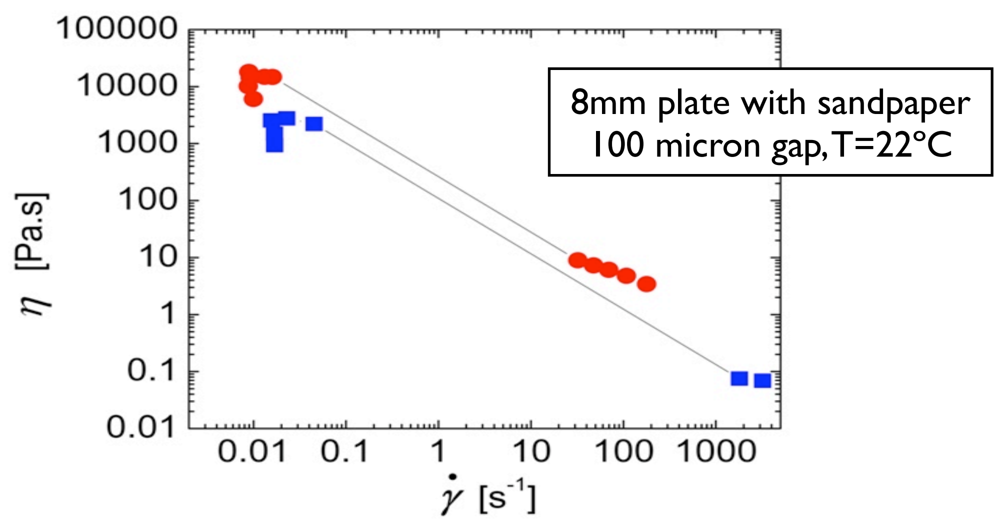


Mark Denny, *Science*, **208**, No. 4449 (1980)

Pedal mucus from common garden snail, *Helix aspera* is strongly shear-thinning.

"The high cost is primarily due to the cost of mucus production, which alone is greater than the total cost of movement for a mammal or reptile of similar weight, ..."

→ shear thinning



Final Comments

- 3-link (and n-link) swimmer (low Reynolds number)
  - ☑ Optimizing kinematics
  - ☑ Trade-off between efficiency and robustness in biological systems?
- Snails
  - ☑ Rely on the nonlinear response of pedal mucus to crawl
  - ☑ We can "tune" viscous material properties to find which weakly nonlinear response is energetically favorable → shear thinning
  - ☑ Mechanical wall-climber

Acknowledgments

Collaborators:

- Daniel Tam (GS, MIT) - Optimizing 3-link swimmer
- Brian Chan (GS, MIT) - Robosnails + mechanical swimmer
- Eric Lauga (MIT, Dept of Math) - Optimizing crawling

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- NSF

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Tuesday 10:45-11:10
Slip, Swim, Mix, Pack: Fluid Mechanics at the Micron Scale

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