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Soft lubrication, lift and optimality

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TIFF (Uncompressed) decompressor
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some history...

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Cedar sled from Lisht
Egypt 12th century BC

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are needed to see this picture.

Moving a statue in 12th century Egypt, note the man pouring lubricant

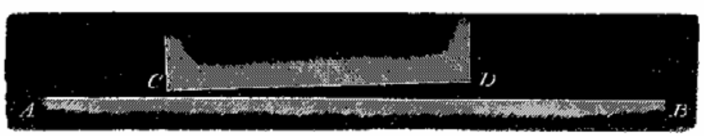
Lubrication Theory

- O. Reynolds - ‘On the theory of lubrication and its application to Mr. Beauchamp Tower’s experiments, including an experimental determination of the viscosity of olive oil.’ Phil. Trans. Roy. Soc. London 1886
- Reduction to lower dimensional systems
- Variety of rigid geometries considered: slider bearing, journal bearing etc.

Fig. 6.

Case 3. *Parallel Surfaces approaching with Tangential Motion.*—The lines

Fig. 4.



Case 1. *Parallel Surfaces in Relative Tangential Motion.*—In fig. 5 the su

Fig. 13.

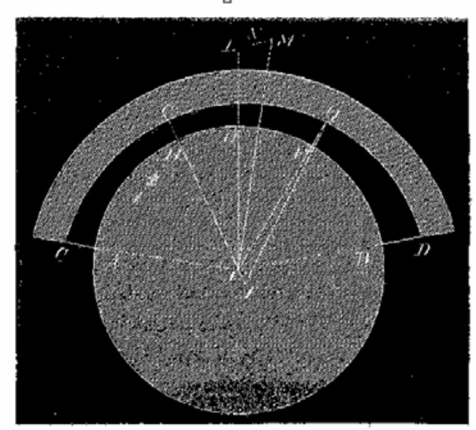
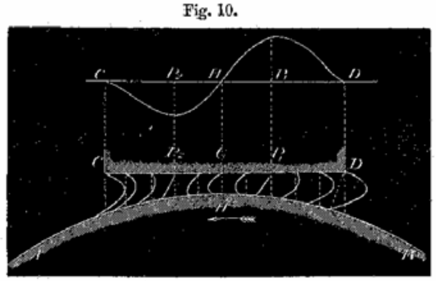


Fig. 10.



Applications

- Elastic deformation implies fluid pressures on the order of the elastic moduli
- High pressure industrial applications (Dowson & Higginson 59; O'Donoghue et al 67). Surface deformations arise alongside piezoviscous (high pressure produces a change in the fluid viscosity) and thermoviscous (thermal heating changes the fluid viscosity) effects. Young's modulus ~ Fluid pressure ~ GPa

- Biological and polymer applications : soft, complex architecture, fluid-infiltrated, electrokinetic..... modulus ~ fluid pressure ~ 1MPa

  e.g cartilage biomechanics (Frank & Grodzinsky 87), red blood cells in capillaries (Secomb et al 98), 10 MPa for rubber (Martin et al 02), polymer brushes (Klein et al. 91).

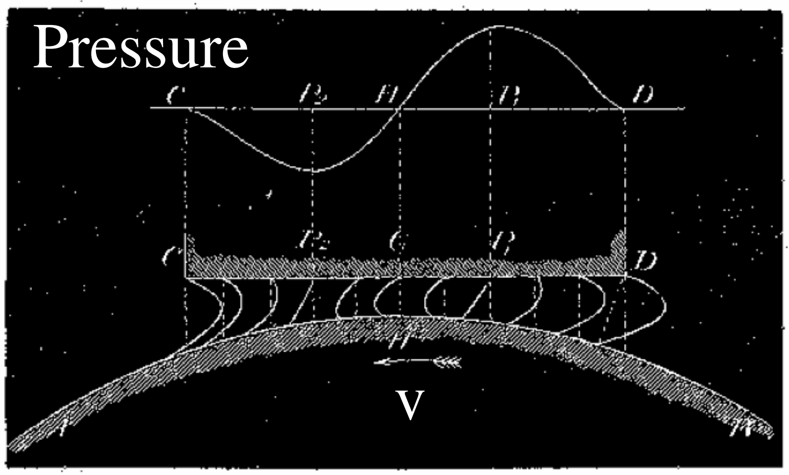
For a given application, what influence does the choice of material have on function?

Model problem

Consider a symmetric (2D) contact moving parallel to a flat surface

Strategy: specify the kinematics calculate the forces

Fig. 10. Reynolds 1886



gap thickness

$$h(x) = h_0\left(1 + \frac{x^2}{2h_0R}\right) + H(x)$$

$l \sim \sqrt{2h_0R}$ so we neglect inertial forces

$$\mathrm{Re}_g = \frac{\rho V^2/l}{\mu V/h_0^2} << 1 \text{ since } h_0 \ll R$$

Navier Stokes equations

$$\rho(\partial_t \mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v}) = \mu\nabla^2\mathbf{v} - \nabla p$$

density velocity viscosity pressure

Continuity

$$\nabla\cdot\mathbf{v} = 0$$

Dimensionless variables

$$x = \sqrt{2h_0R}X, \qquad z = h_0Z, \qquad p = \frac{\sqrt{2R}\mu V}{h_0^{3/2}}P,$$

$$h = h_0H, \qquad u = VU, \qquad w = \frac{V\sqrt{h_0}}{\sqrt{2R}}W,$$

Reynolds equation

$$0 = \partial_X(6H + H^3\partial_X P)$$

$$P(\infty) = P(-\infty) = 0$$

The Reynolds Equation

Reynolds equation

$$0 = \partial_X(6H + H^3 \partial_X P)$$

$$P(\infty) = P(-\infty) = 0$$

H = gap thickness

To close the problem we need to specify how the surface deforms due to the fluid pressure i.e. solve the coupled elastic and hydrodynamic problems

we can see reversibility in this equation: a symmetric contact moving tangentially to a surface generates no lift

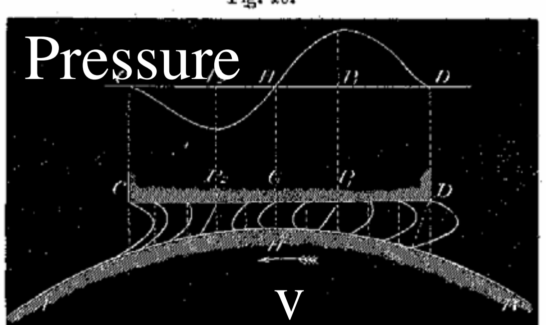
First consider the simplest relationship between pressure and surface deformation
(Lighthill 68; Johnson 85;
linear regime studied by Sekimoto and Leibler 93)

Elasticity can break the symmetry

$$H(X) = 1 + X^2 + \eta P(X)$$

H = dimensionless gap thickness

Fig. 10.



softness $\eta = \frac{\Delta H}{h_0} \sim \frac{p_0 H_l}{E\, h_0}$

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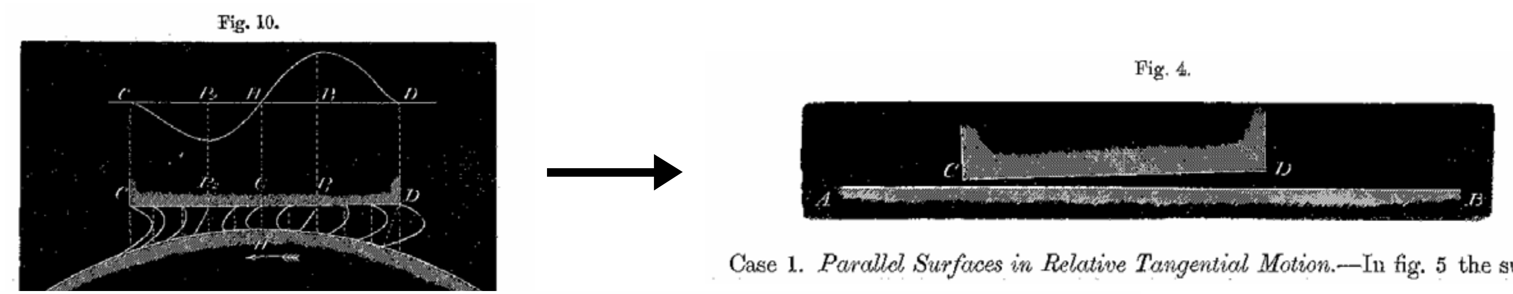
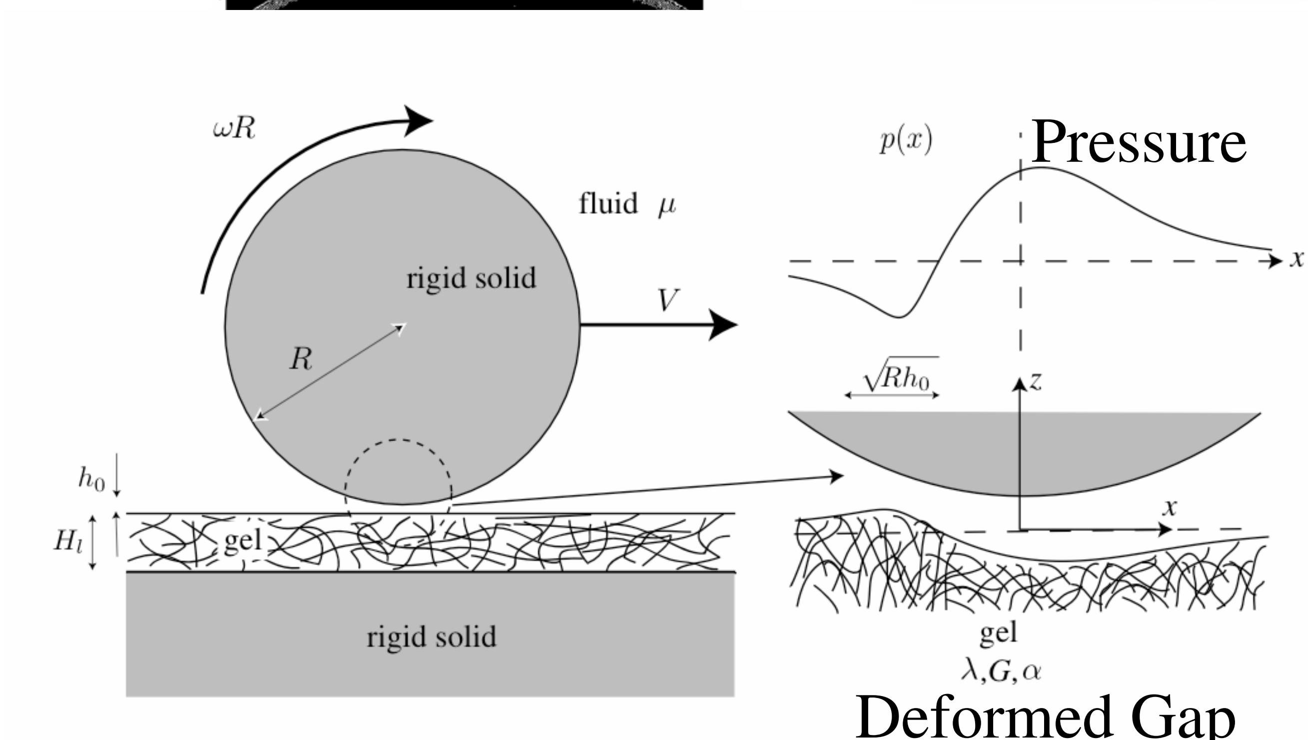
E = Young's modulus

Lift

Symmetric contact transformed to resemble a slider bearing, which is well known to generate lift



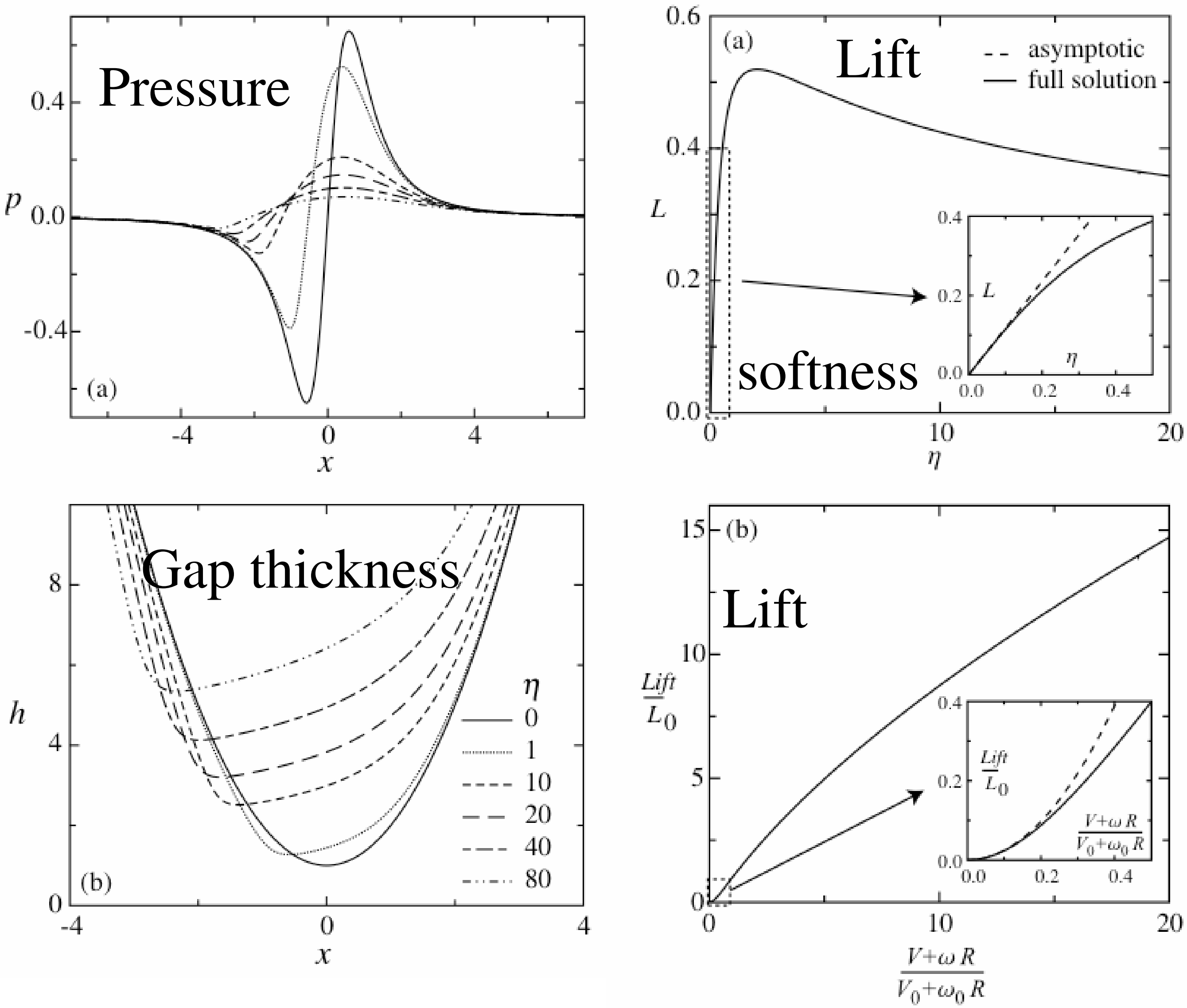


For tangential motion deformation leads to lift

for small deformations

$$F \sim \frac{\mu^2 V^2}{G} \frac{H_l R^{3/2}}{h_0^{7/2}}$$

nonlinear solution - Optimal softness



Increased deformation leads to greater asymmetry of the pressure distribution, but also decreased pressure since the mean gap thickness increases

$$\eta = \frac{\Delta H}{h_0} \sim \frac{p_0 H_l}{E\, h_0}$$

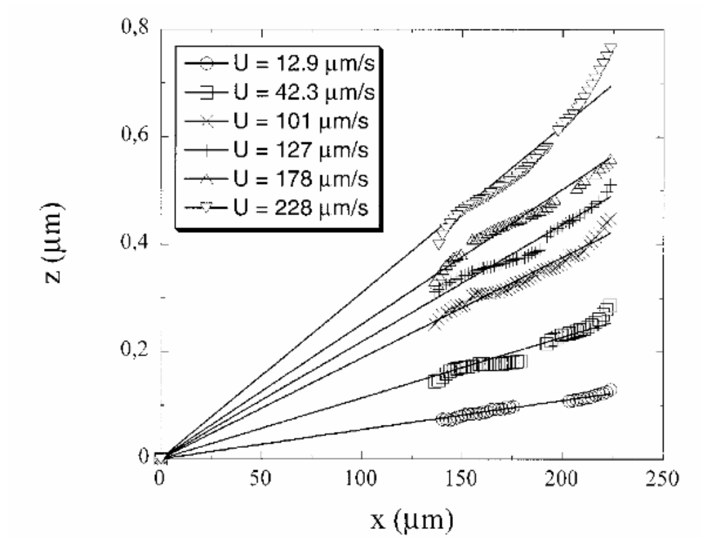
Relative motion between the two surfaces

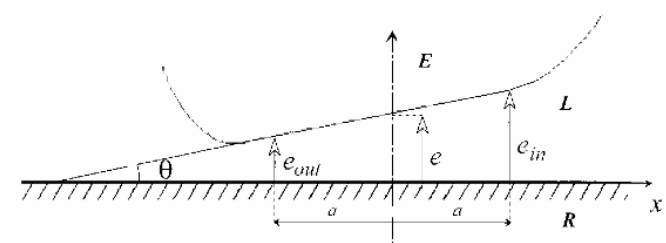
A sliding rubber sphere begins to resemble a slider bearing

PHYSICAL REVIEW E, VOLUME 65, 031605 2002

Wetting transitions at soft, sliding interfaces

A. Martin, J. Clain, A. Buguin, and F. Brochard-Wyart



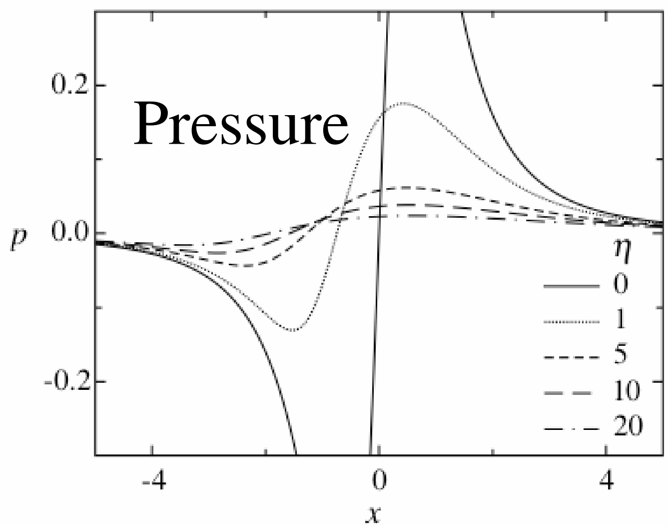


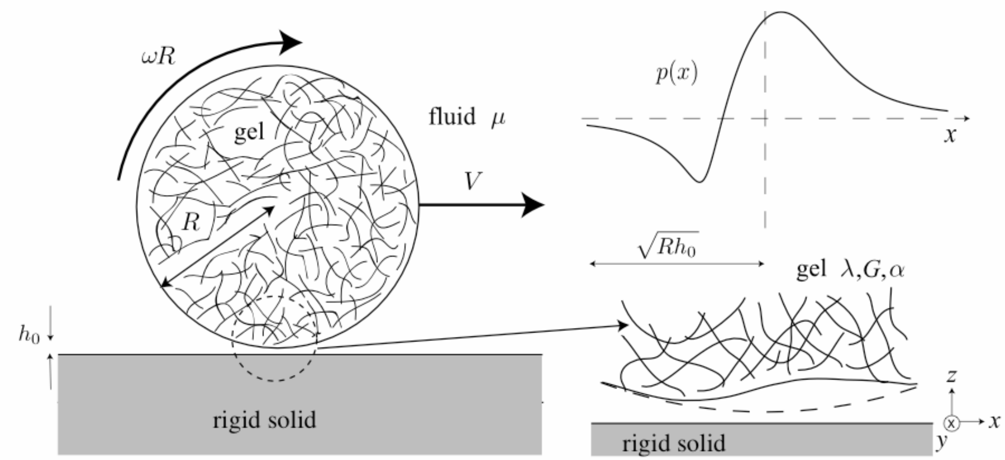
Deformable drops and bubbles moving in shear flows or in wall-bounded flows experience lateral migration (Karnis & Mason 1967; Magnaudet et al 2003).

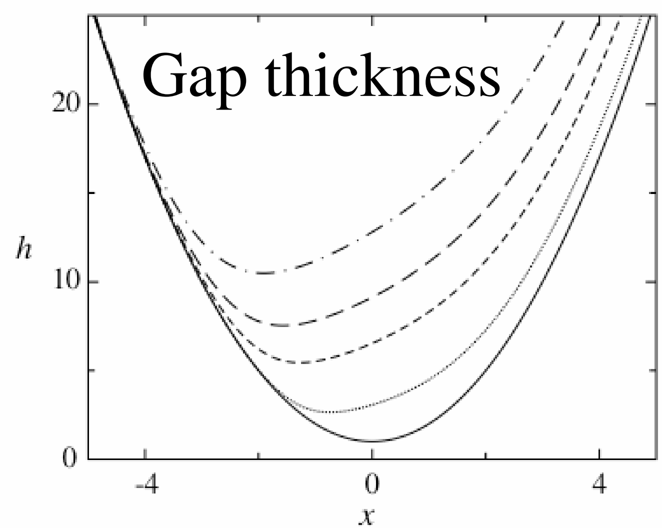
Soft slider

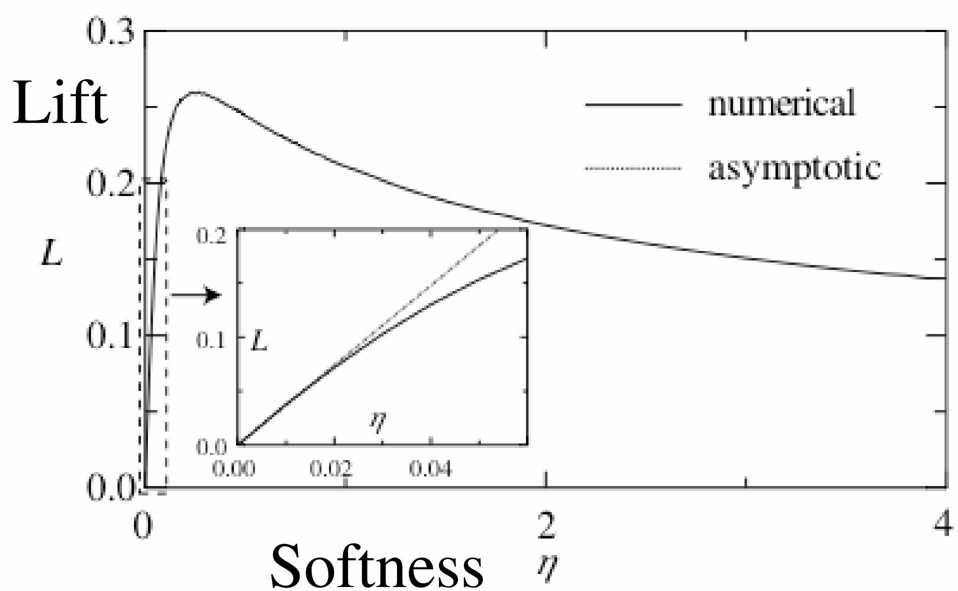
Different geometries and materials produce similar results

$$h = 1 + x^2 + \eta \int_{-\infty}^{\infty} dx' p(x') \log[\frac{Y}{(x-x')^2}]$$

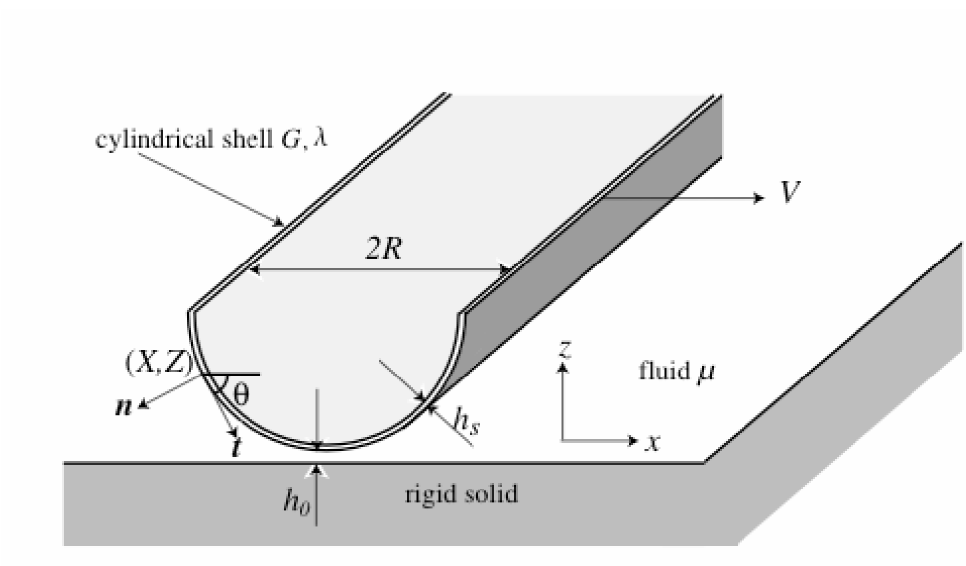


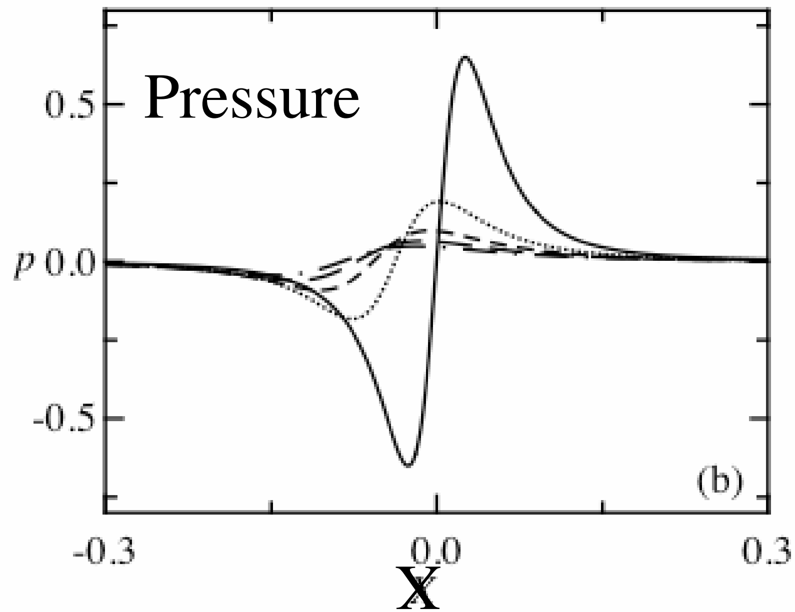


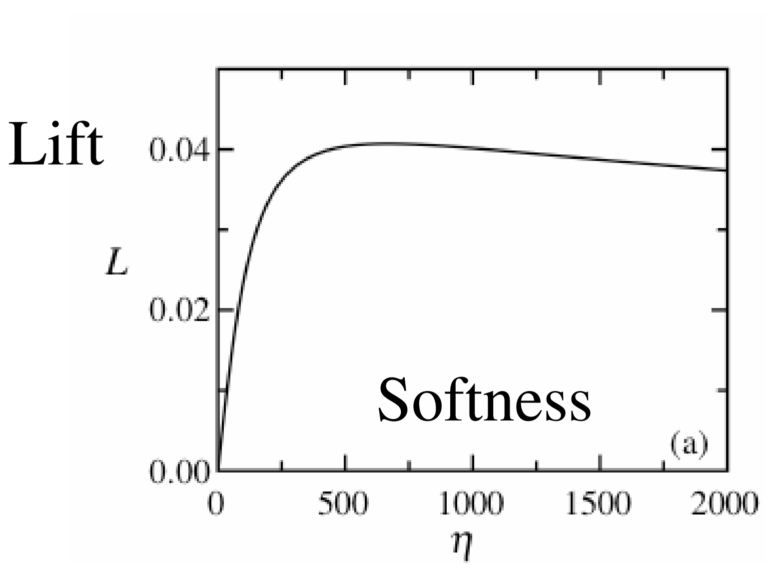




Cylindrical shell







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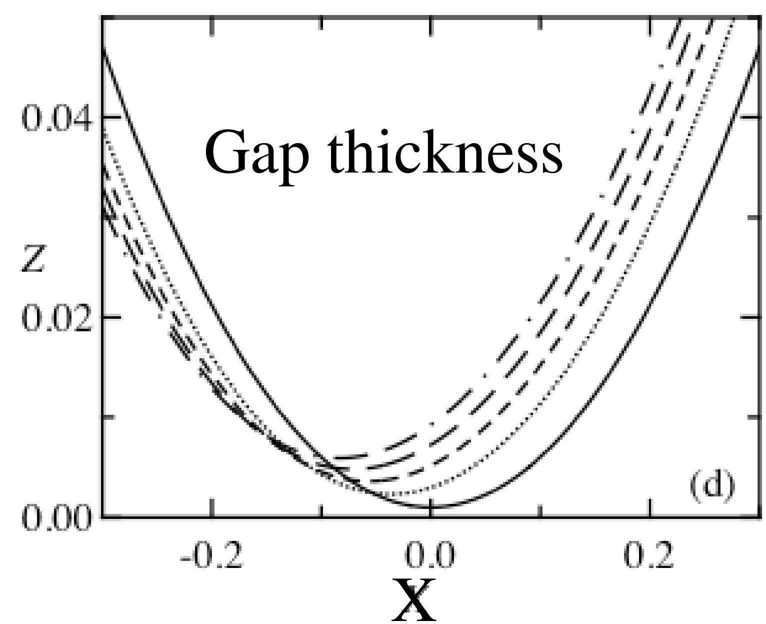
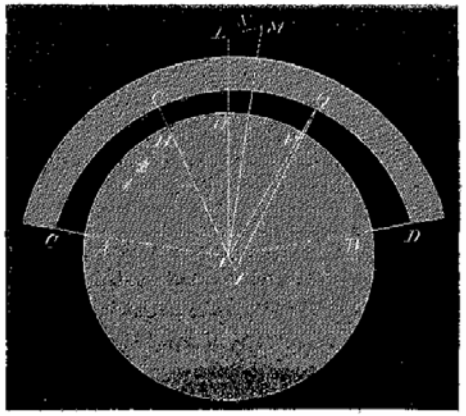
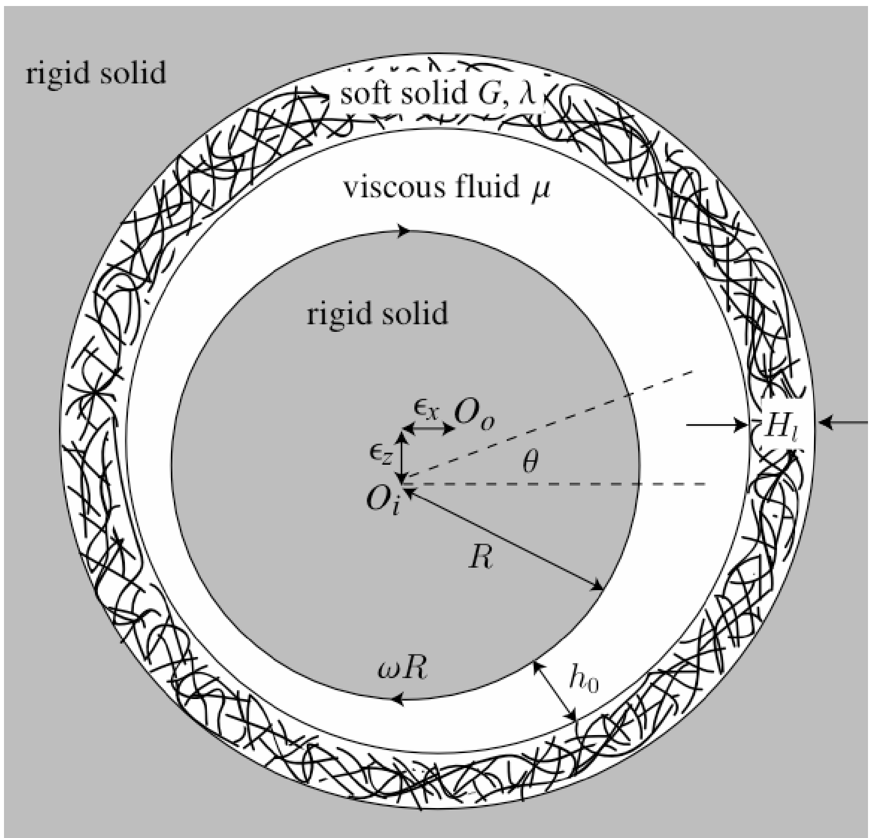
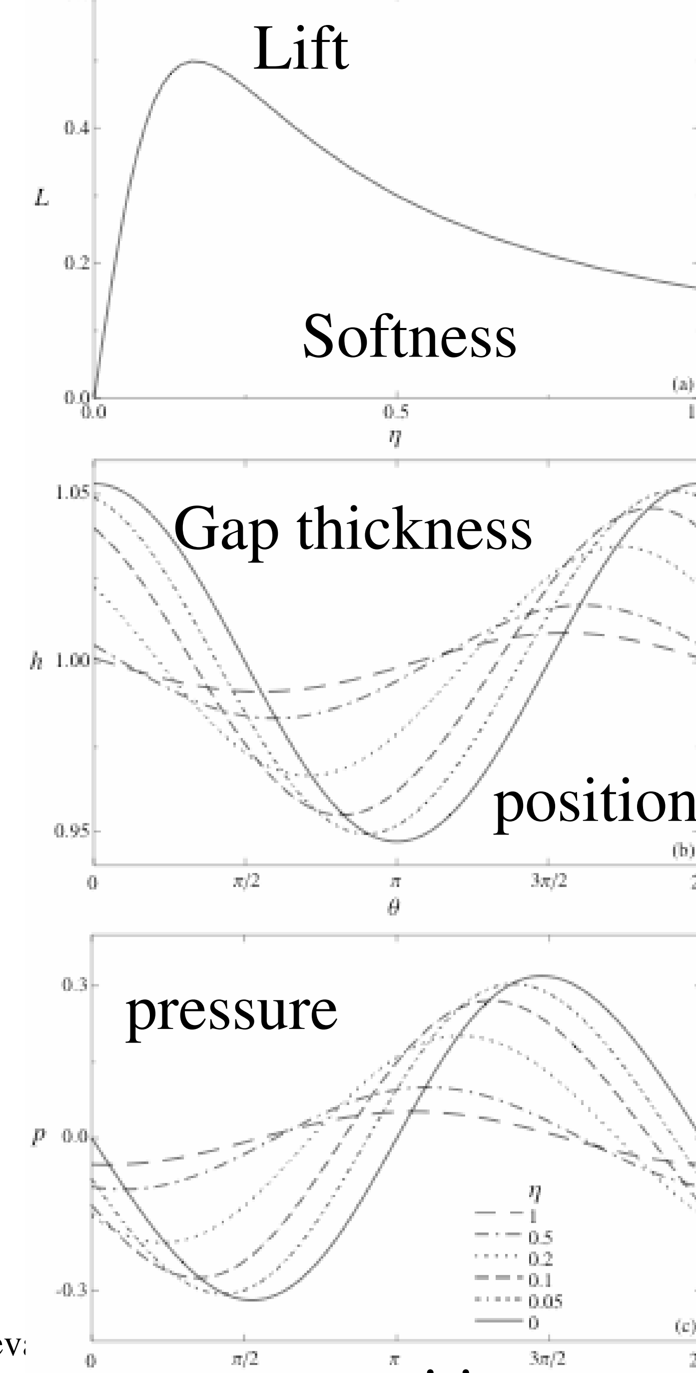


Fig. 13.



Journal Bearing

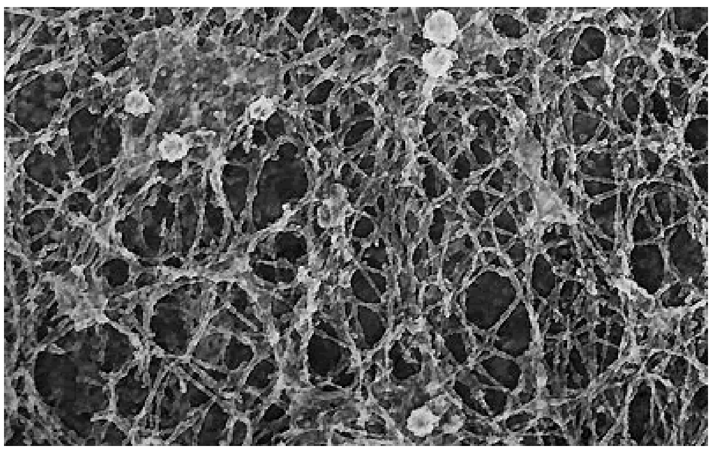




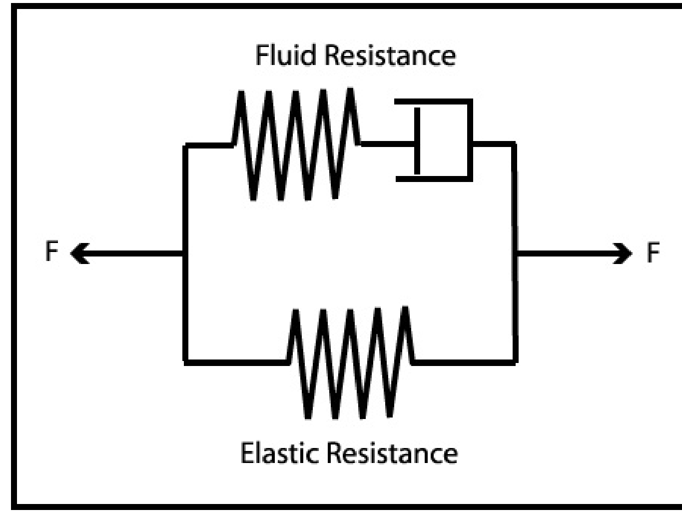
Independent of a selection of material constitutive laws

Soft materials are often squishy
what effect could this have?

cytoskeleton



Poroelasticity - schematic



$_{p} \sim \quad L^2 / k\, E$

L ~ smallest dimension

k ~ permeability

Stiffness is a function of the time scale

Effect remains the same:
there is an optimal softness

Cartilage

For typical operating conditions
for cartilage

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

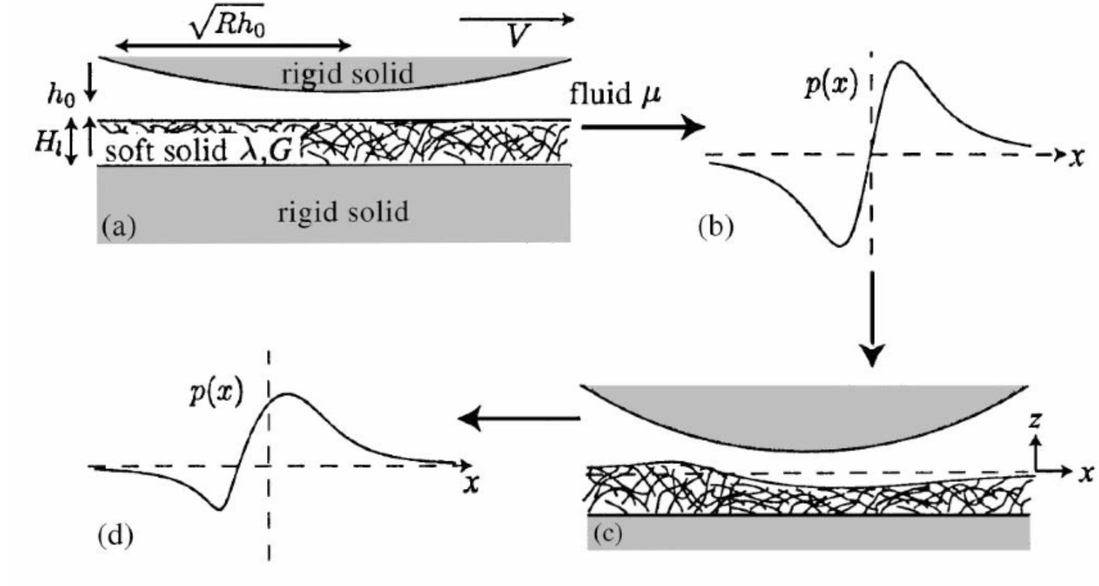
$$\eta \sim \frac{\mu V}{G} \frac{H_l R^{1/2}}{h_0^{5/2}} \sim 1$$

$H_l \sim 10^{-3}$ m
$G \sim 1$ MPa
$V \sim 0.01$ m/s
$R \sim 0.01$ m
$h_0 \sim 10^{-6}$ m

suggests a possible role for softness

effects of the detailed geometry, loading conditions and electrokinetics?

Conclusions



1. There is an optimal softness that produces a maximal lift between the surfaces
2. Experiments ? Applications ?
    1. - Elastomeric/poroelastic bearings
    2. - Cartilage ?

Jan M. Skotheim, L. Mahadevan,
November 2006