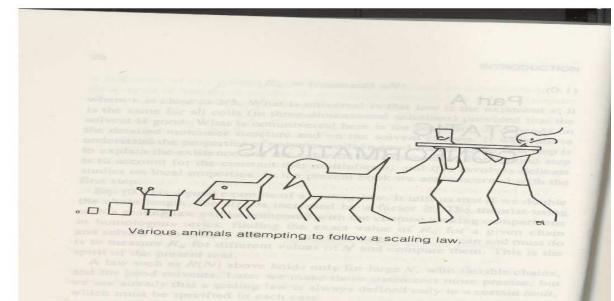
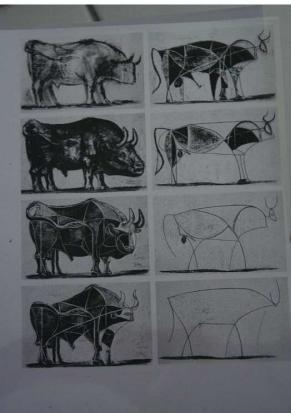
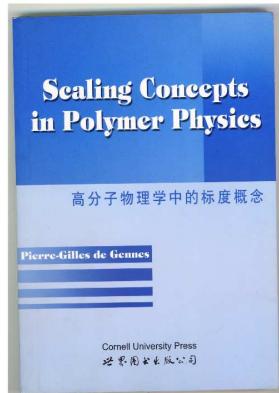


# *Polymers in confined geometries*

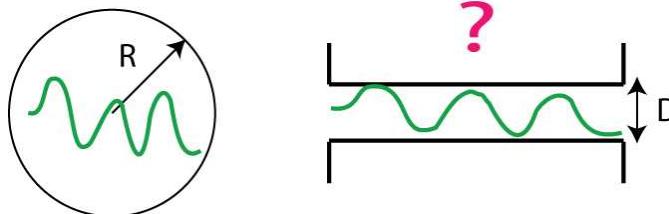
Françoise Brochard  
Université Paris 6, Inst. Curie



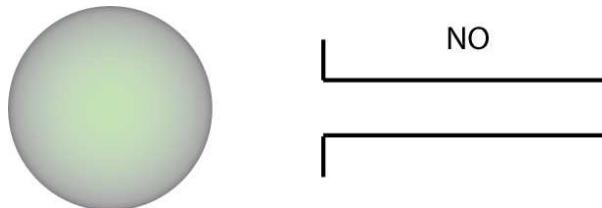
What Pierre-Gilles did all along his career, with the elegance of an artist was to draw white lines for science in large strokes, astonishingly simple, yet so pioneering.

# Three classes of polymers

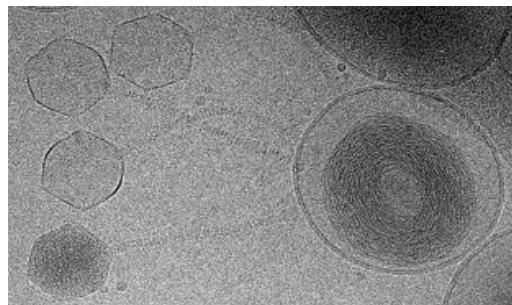
Flexible



Rigid: proteins, colloids



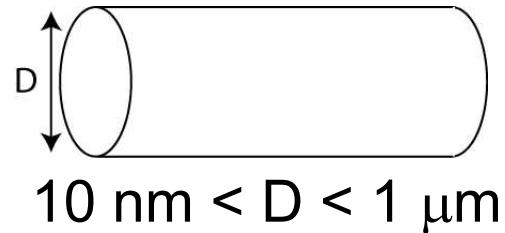
Semi-flexible: DNA



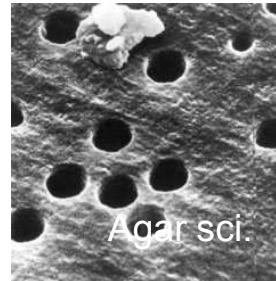
Transfer of T5 genome  
into proteoliposomes

O. Lambert et al, PNAS 2000, 97

$d=1$   
**Rigid tubes**



- Porous rocks
- Oil recovery
- Xanthans  
(Saffman Taylor)



**Nuclepore membrane filters**  
(pioneered by GE 1976)

Radiation-etching of polycarbonate films  
(Guillot et al, J. Appl. Phys. 52, 1981)



**Filters used in fabrication of cheese and low fat milk**

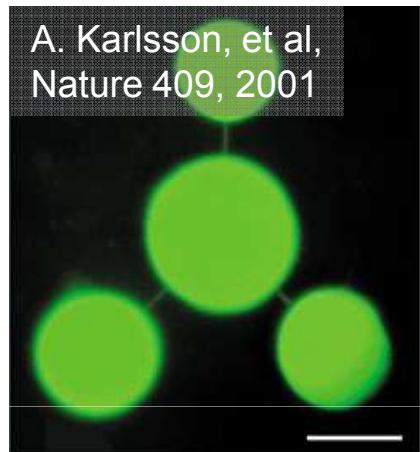


**Low fat milk producing cow?!**

$d=1$

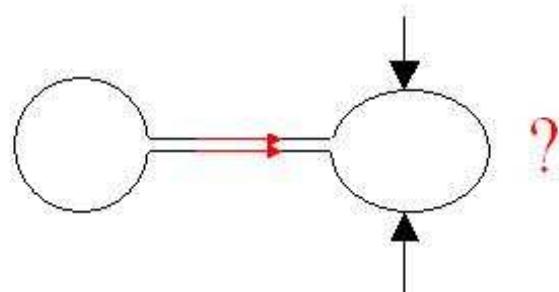
## Soft lipidic tubes

N. Borghi, K. Guevorkian, S. Kremer



Vesicle networks

Flows in soft lipidic tubes



QuickTime® and a Microsoft Video 1 decompressor are needed to see this picture.

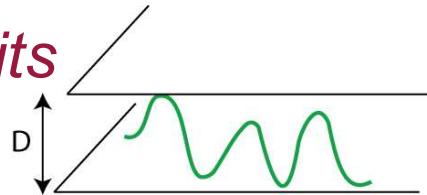
Hydrodynamic tether extrusion

P. G. Dommersnes, et al., EPL, 70 (2005)

QuickTime® and a decompressor are needed to see this picture.

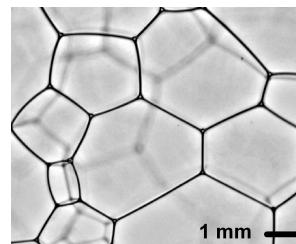
# $d=2$ confinement

*Rigid: slits*



*Soft: soap films*

D. Langevin, A.C.I.S, 89 (1989)



Polymers in foams

→ Stabilizing

→ Spreading (fluorinated foams)

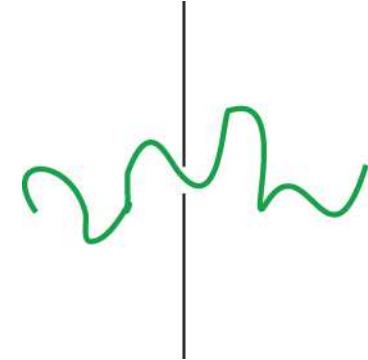
P. G. de Gennes, C. R. Acad. Sc. 289 (1979)

# $d=0$ , “nano holes”

Holes:

- Protein pores:  $\alpha$ -Hemolysin
- Nano fabricated pores

Translocation of polymer through  
a narrow hole under an electric  
field.

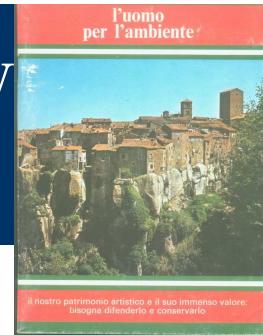


QuickTime™ and/or  
YUV420 codec decompressor  
are needed to see this picture.

[www.apmaths.uwo.ca/~mkarttu/](http://www.apmaths.uwo.ca/~mkarttu/)

# Flexible polymers in confined geometry

## Statics



**Flory:** Swollen chain  $R=N^\nu a$

$$\frac{F_{ch}}{kT} = \frac{R^2}{R_0^2} + NvC \quad \text{where } v = a^d \quad C = \frac{N}{R^d}$$

$$\frac{\partial F_{ch}}{\partial R} = 0 \quad R = N^{\frac{3}{d+2}}a$$

Ideal chain  $R_0=N^{1/2}a$

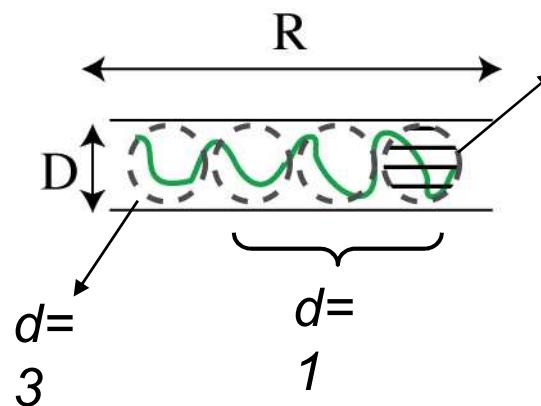
$$\frac{vN^2}{R_0^d} \square N^{2-\frac{d}{2}}, \quad d_c = 4$$

$$\begin{array}{c|c|c|c} d & 3 & 2 & 1 \\ \hline \nu & 3/5 & 3/4 & 1 \end{array}$$

$$d \square \quad \nu \square$$

*Excluded volume effects increases in confined geometries*

**PGG:** “Blobs”



$g$  monomers

$$R = \frac{N}{g} D = Na \left( \frac{a}{D} \right)^{2/3}$$

$$D = g^{3/5}a$$

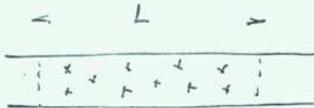
$$F_{conf} \square \frac{N}{g} kT \square R \frac{kT}{D}$$

$$f_c = \frac{kT}{D}$$

### III. MOBILITÉ DANS UN PORE

#### 1) APPROX. DEBYE

CHAINE = MILIEU  
POREUX HOMOGENE



ICI : PAS DE BACK FLOW

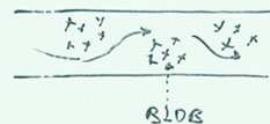
$$\frac{F}{V} = \mu = \frac{1}{N\eta_0} \quad \text{INCORRECT!}$$

#### 2. LOI D'ECHELLE

F. BROCHARD

FLUCTUATIONS

IMPORTANTES



$$F \approx \sum 6\eta D \nabla$$

NB BLOBS      STOKES

$$\mu \approx \frac{q_D}{N\eta D}$$

$$\mu = \frac{1}{N\eta_0 a} \left( \frac{D}{a} \right)^{2/3}$$

#### 3) COEFFICIENT DE DIFFUSION $D_c = \mu kT$

$$\text{PERMEABILITÉ DE MEMBRANE } K = \frac{J_{12}}{C_1 - C_2}$$

$$K = n_p D^2 f \frac{D_c}{E_a} f = \frac{C_{pore}}{C} = e^{-N/g} \quad \begin{matrix} \text{DEP. EN } p \text{ DOMINANTE} \\ \text{QUAL. OBSERVÉE} \end{matrix}$$

FRACTION TROUS    COEFF. PARTAGE    ÉPAISSEUR MEMBRANE

# Forced penetration

## *Hydrodynamic analysis*

S. Daoudi and F. Brochard, Macromolecules, (11) 1978

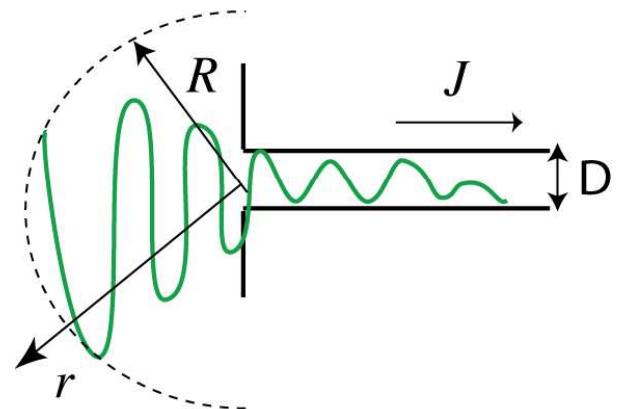
$$\frac{J}{r^3} \quad \dot{\gamma} > \frac{1}{\tau_z}$$

$$r^* = R \left( \frac{J}{kT / \eta} \right)^{1/3}$$

$$\frac{kT}{\eta R^3}$$

Affine deformation

$$\frac{R_\perp}{R} = \frac{r}{r^*}$$



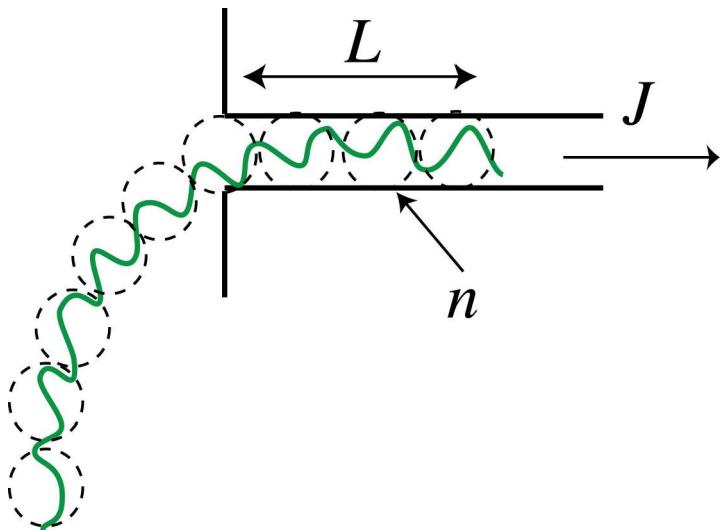
$$R_\perp = r = D \rightarrow r^* = R \rightarrow$$

$$J_C = \frac{kT}{\eta}$$

# Forced penetration

*Role of fluctuations = suction model*

P. G. de Gennes, 1984



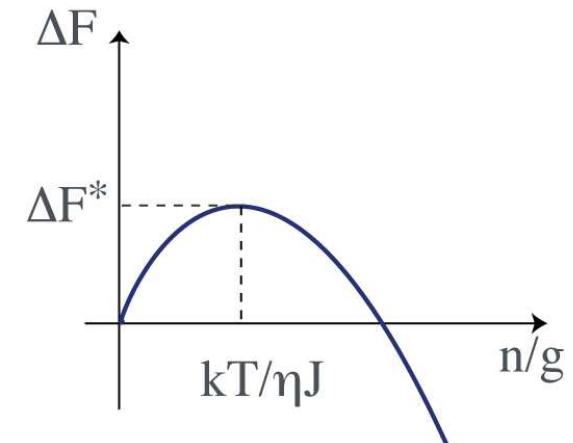
Force on a blob:  $\eta VD = \eta J/D$

Aspiration energy:  $\Delta F_A = -\frac{n}{g} \eta \frac{J}{D} L$

$$\Delta F = \Delta F_c + \Delta F_a = kT \left[ \frac{n}{g} - \eta \frac{J}{kT} \left( \frac{n}{g} \right)^2 \right]$$

$$\rightarrow J_c = \frac{kT}{\eta}$$

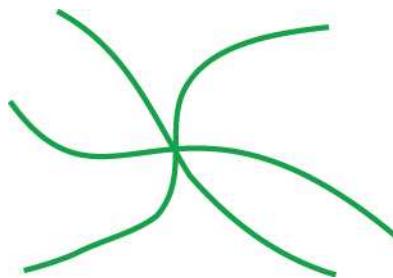
Penetration of first blob



$J_c$  does not depend on chain length,  $N$

# Role of topology

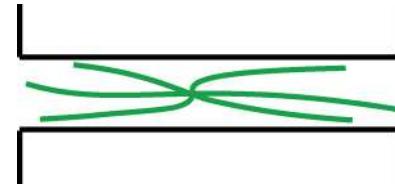
Stars



F. Brochard & P.G. de Gennes, CRAS Paris 323 (1996)

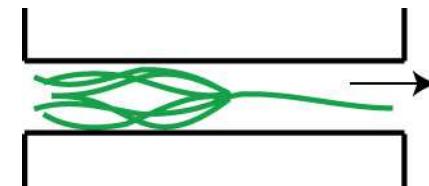
Symmetric

$$J_c^* = J_c f^2$$



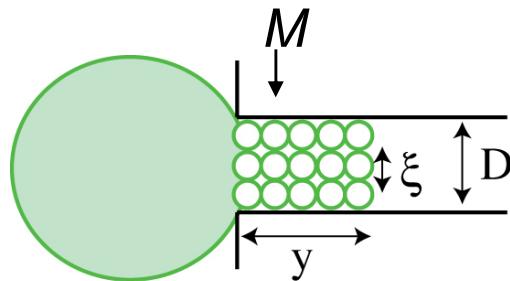
Asymmetric

$$J_{c1} = J_c^* \frac{D}{Naf^{1/2}}$$



# Branched Polymers

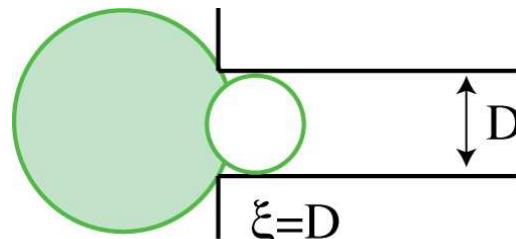
$$\left( \gamma = \frac{1}{4} \right)$$



C. Gay et al, Macromolecules, (29) 1996

$$\xi = \frac{D^{4/3}}{a^{1/3} M^{1/6}}$$

$$J = J_c \frac{D^4}{\xi^4} \quad f = \frac{D^2}{\xi^2}$$



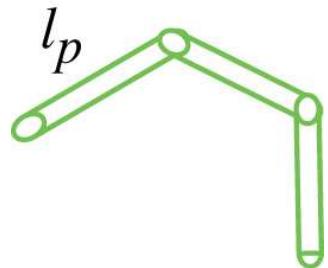
T. Sakaue, et al., EPL, (72) 2005

$$J_c = \frac{kT}{\eta} \quad !$$

$$f_V \quad \square \quad f_c \quad f_V \text{ increased faster than } f_c$$

Penetration of first blob ( $\xi=D$ ) limits the penetration

# DNA: semi-rigid polymers



$$v_B \square l_p^2 a$$

$$g = \frac{l_p}{a}$$

F. Brochard, et al, Langmuir (21) 2005

$$R_0 = N_D^{1/2} l_p = \sqrt{N a l_p}$$

$$\frac{F_{ch}}{kT} = \frac{R^2}{R_0^2} + \frac{F_{ve}}{kT}$$

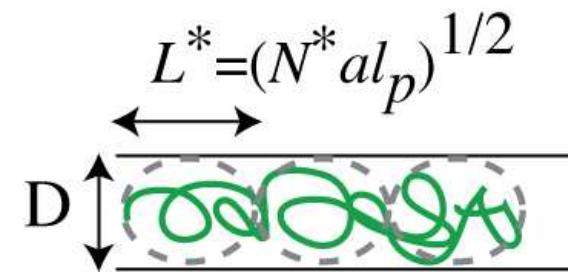
$d=3 \quad \frac{F_{ve}}{kT} < 1 \text{ if } N < N_{c3} = \left( \frac{l_p}{a} \right)^3 \square 10^6 !$

$d=1 \quad \frac{F_{ve}}{kT} = \left( \frac{N}{g} \right)^2 \frac{l_p^2 a}{R^2 D} < 1 \text{ if } N < N^* = \left( \frac{l_p}{a} \right)^{1/3} \left( \frac{D}{a} \right)^{4/3} \square 10^3$

Long DNA is ideal in 3d but swollen in nanotubes

$D > l_p$

$$L = \frac{N}{N^*} L^*$$



$D < l_p$

Reisner, et al., PRL 94 2005

$$F_{conf} = \frac{N}{N^*} \Delta F^*$$

$$\frac{\Delta F^*}{kT} = \frac{N^* al_p}{D^2}$$

$$f_c = \frac{F_{conf}}{L} = \frac{\Delta F^*}{L} \approx \left( \frac{N^* al_p}{D^2} \right)^{1/2} kT$$

## Forced penetration

- Electric field  $f_E = qEN = f_c \rightarrow E^* = E_1^* \frac{N^*}{N}$   
passage  $qE > \left( \frac{al_p}{N^*} \right)^{1/2} \frac{kT}{D^2}$  Barrier 1st blob

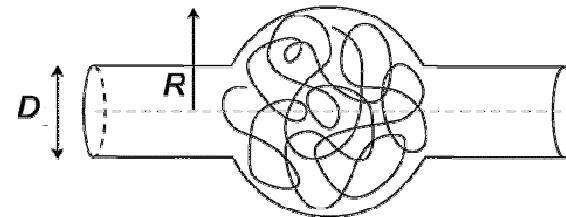
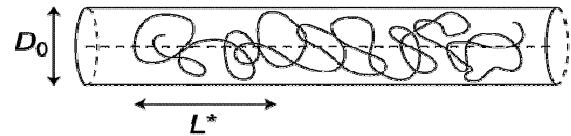
- Flow: ideal blobs

$$\text{Sponge } \xi = D^2/R^*$$

$$f_v = \frac{N}{N^*} \eta V R^* \left( \frac{R^*}{D} \right)^2 = \frac{N}{N^*} f_v^*$$

$$\text{passage } f_v^* = f_c \Rightarrow J_c^{SR} = \frac{kT}{\eta} \left( \frac{D}{R^*} \right)^2 = J_c \left( \frac{D}{l_p} \right)^{2/3} \left( \frac{a}{l_p} \right)^{2/3}$$

# Soft tubes Snake vs Globule

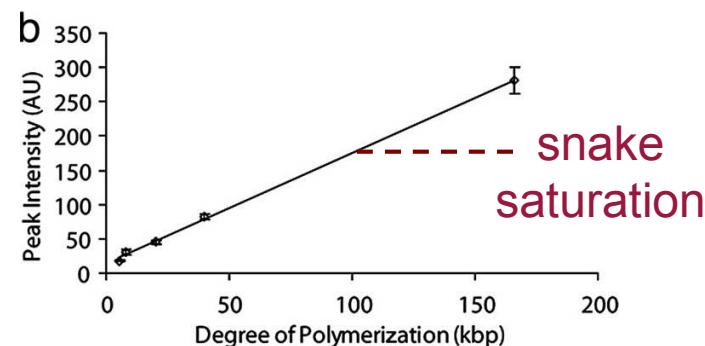


$$D > l_p$$

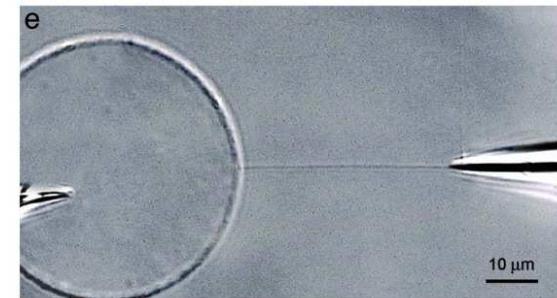
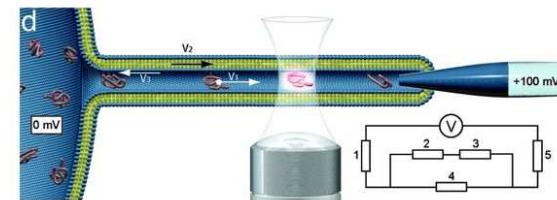
$$F_{\text{snake}} = \frac{Nal_p}{D^2} kT$$

$$F_{\text{Globule}} = \sigma R^2 + \frac{N^2 a^3 kT}{R^3}$$

$$F_{\text{snake}} = F_{\text{Globule}} \quad N_c = \frac{aD^4}{l_p^5} \left( \frac{\kappa}{kT} \right)^3$$



F. Brochard, et al., LANGMUIR 21 (2005)  
M. Tokarz, et al, PNAS 102 (2006)



# Holes = passage of polymers

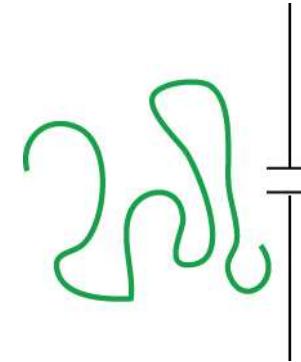
Neutral polymers: (POE)

Force: chemical potential  $\Delta\mu$

A. Oukhaled, et al, PRL, 98, 2007

P. Merzylak, et al., Biophys. J., 77, 1999

L. Movleanu, et al., Biophys. J., 84, 2003



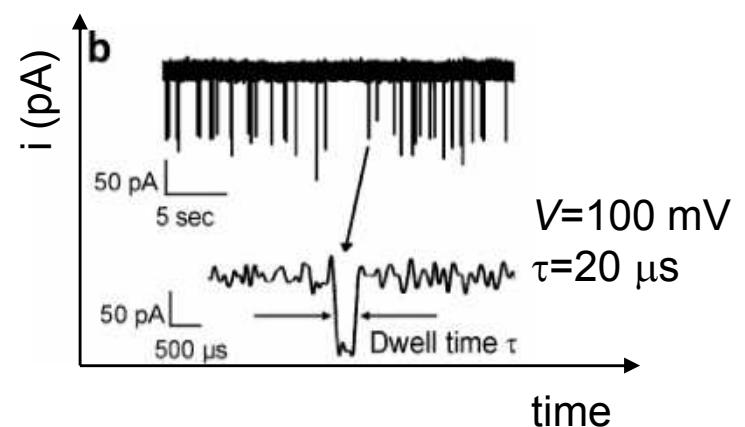
Charged polymers: (DNA - Polyelectrolytes)

Force: Electric field

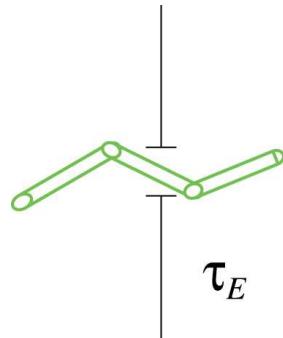
Observation: Blockade of the pore

M. Bates et al, Biophys. J. 84, 2003

J. Storm et al, Nanoletters, 5, 2005



# Problems of DNA entering a cell



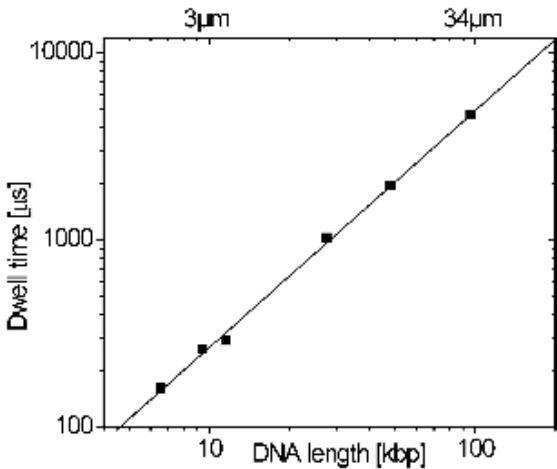
$$\eta l_p V = \frac{\Delta \mu}{a}$$

$$\tau_E = \frac{r_p}{V} \square 10^{-4} \text{ s}$$

$$\tau_p \square N$$

$$\tau_p \square N^{1.2}$$

$$\tau_{transfection} = \frac{\tau_E}{n} , n = \frac{C}{N} r_p^3 \square \text{ hours!}$$

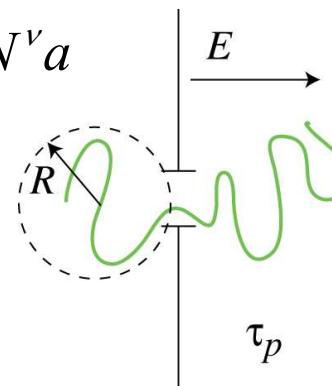


Fast DNA translocation

$$\tau_p \sim R^2 \sim N^{1.27}$$

J. Storm et al, Nanoletters, 5, 2005

QuickTime® and a decompressor are needed to see this picture.



$$\eta R \dot{R} = f(\square eV / a)$$

$$\tau_p \square \frac{\eta}{f} R^2$$

[www.ks.uiuc.edu](http://www.ks.uiuc.edu)

PGG Physica A 274 (1999)  
PGG PNAS 96 (1999)

# Conclusions

**Tubes and slits** (citations: 3000, 2003<1200<2008)

- Solid:
  - polymer characterization and separation, oil separation
- Soft:
  - gene therapy, soap films stabilization
  - lipidic tubes: transport, nanoreactors

**Holes** (citations: 587, 2003<275<2008)

- Detection of polymer length
- Polymer length separation using minute amount of sample

**Goal:**

- Sequencing of DNA, RNA
- Transfection



# Acknowledgments

## Former students



- Biology  
J.-P. Thiery



M. Bornens



Y.-S. Chu



M. Théry



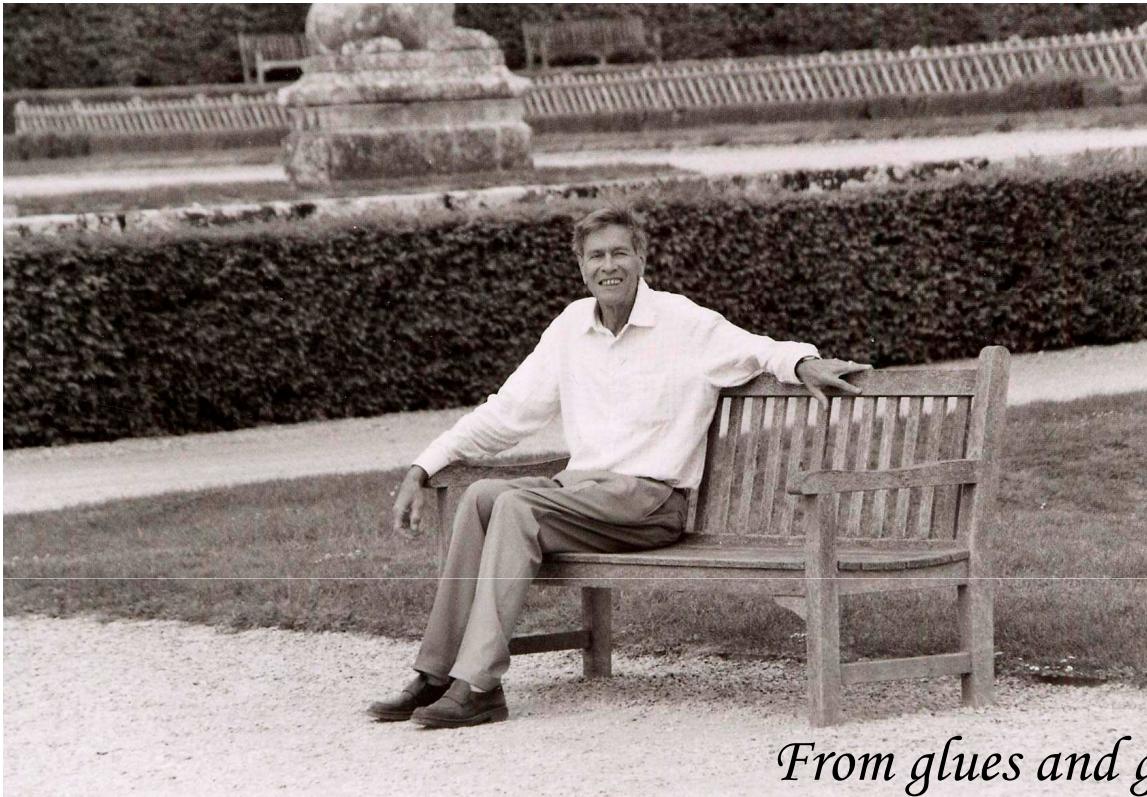
Axel Buguin



Pierre Nassoy



Thanks Pierre Gilles for sharing with us your insatiable love for science



*From glues and grains to rheology  
Using notions of symmetry and analogy  
He illuminates the messy  
With math not so dressy  
But with insights deep from figurology*

Mahadevan, Boston 2006