

# Quantum Computing at Microsoft

Dave Wecker



HOW'S YOUR  
QUANTUM COMPUTER  
PROTOTYPE COMING  
ALONG?

GREAT!



Dilbert.com DilbertCartoonist@gmail.com

THE PROJECT EXISTS  
IN A SIMULTANEOUS  
STATE OF BEING BOTH  
TOTALLY SUCCESSFUL  
AND NOT EVEN  
STARTED.




4-17-12 ©2012 Scott Adams, Inc./Dist. by Universal Uclick

CAN I  
OBSERVE  
IT?


THAT'S  
A TRICKY  
QUESTION.



Thurston  
... for circle  
Unique upto  $SL(2, \mathbb{C})$



Fixes up  
 $i$ th vertex



$l = r + t$



# STATION

Q

Handwritten mathematical notes on a chalkboard:

- Left side:  $\frac{P(\dots)}{1}$  and  $\sum abc$
- Middle:  $+ \delta \phi \dots P_2(\dots)$
- Right side: A diagram with nodes and arrows, labeled with  $E$ ,  $L_0$ ,  $L_1$ ,  $L_2$ ,  $j(z)$ , and  $\langle f(\tau) f(0) \rangle$ .



2006

“ MAJORANA PARTICLE  
GLIMPSED IN LAB. ”

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BBC NEWS

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○  
2012



## Bits

# Microsoft Spends Big to Build a Computer Straight Out of Science Fiction

By JOHN MARKOFF

SAN FRANCISCO — Microsoft is putting its considerable financial and engineering muscle into the experimental field of quantum computing as it works to build a machine that could tackle problems beyond the reach of today's digital computers.

There is a growing optimism in the tech world that quantum computers, superpowerful devices that were once the stuff of science fiction, are possible — and may even be practical. If these machines work, they will have an impact on work in areas such as drug design and artificial intelligence, as well as offer a better understanding of the foundations of modern physics.

Microsoft's decision to move from pure research to an expensive effort to build a working prototype underscores a global competition among technology companies, including Google and IBM, which are also making significant investments in search of breakthroughs.

In the exotic world of quantum physics, Microsoft has set itself apart from its competitors by choosing a different path. The company's approach is based on "braiding" particles known as qubits — which physicists describe as existing in just two dimensions — to form the building blocks of a supercomputer that will exploit the unusual properties of subatomic parti-

edge that barriers still remain to building useful quantum machines, both at the level of basic physics and in developing new kinds of software to exploit certain qualities of devices known as qubits that hold out the possibility of computing in ways not possible for today's digital systems.

Unlike conventional transistors, which can be only on or off at any one time, representing a dig-

## Planning a prototype of a superpowerful quantum computer.

ital 1 or 0, qubits can exist in superposition, or simultaneously in both states. If qubits are placed in an "entangled" state — physically separated but acting as though they are deeply intertwined — with many other qubits, they can represent a vast number of values simultaneously. A quantum computer would most likely consist of hundreds or thousands of qubits.

Microsoft began funding research in the field in 2005 when it quietly set up a laboratory known as Station Q under the leadership of the mathematician Michael Freedman.

Microsoft now believes that it is close enough to designing the basic qubit building block that the

Todd Holmdahl, a veteran engineering manager who will direct the Microsoft effort. Over the years, he has led various Microsoft projects, including its Xbox video game machine and the yet-to-be-released HoloLens augmented reality system.

"Once we get the first qubit figured out, we have a road map that allows us to go to thousands of qubits in a rather straightforward way," Mr. Holmdahl said.

There is still a debate among physicists and computer scientists over whether quantum computers that perform useful calculations will ever be created.

A variety of alternative research programs are trying to create qubits using different materials and designs. The Microsoft approach, known as topological quantum computing, is based on a field that took on new energy when this year's Nobel Prize in Physics was awarded to three scientists who had done fundamental work in forms of matter that may exist in just two dimensions.

Mr. Holmdahl's project will also include the physicists Leo Kouwenhoven of Delft University, Charles M. Marcus of the University of Copenhagen, David Reilly of the University of Sydney and Matthias Troyer of E.T.H. Zurich.

They will all become Microsoft employees as part of the Artificial Intelligence and Research Group that Microsoft recently created under the leadership of one of its top technical



JAN C. BATES FOR THE NEW YORK TIMES

Todd Holmdahl, who has led Microsoft projects like the Xbox gaming console, will direct the quantum computing efforts.

and superconductors," Dr. Marcus said. The researchers recently made a "remarkable breakthrough" in their ability to control the materials used to form qubits, he said. Most of the competing approaches involve cooling quantum computers to near absolute zero temperatures.

So far, there are relatively few proven algorithms that could be used to solve problems more quickly than today's digital computers. One early effort, known as Shor's algorithm, would be used to factor numbers, giving hope that quantum computers might be used in the future for breaking codes.

That would potentially have world-shaking consequences because modern electronic commerce is built on cryptographic systems that are largely unbreakable using conventional digital computers. Other proposed approaches might allow faster searching of databases or perform machine learning algorithms, which are being used to make advances in computer vision and speech recognition.

More immediately, however, these tools might advance the basic understanding of physics, a possibility the physicist Richard P. Feynman mentioned when he speculated about the idea of a quantum computer in 1992.

cists say the decision to try to build a topological quantum

confidence that the company



# STATION

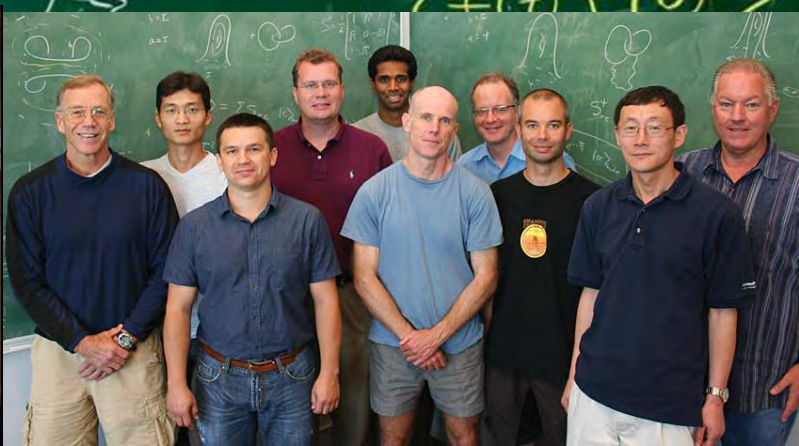
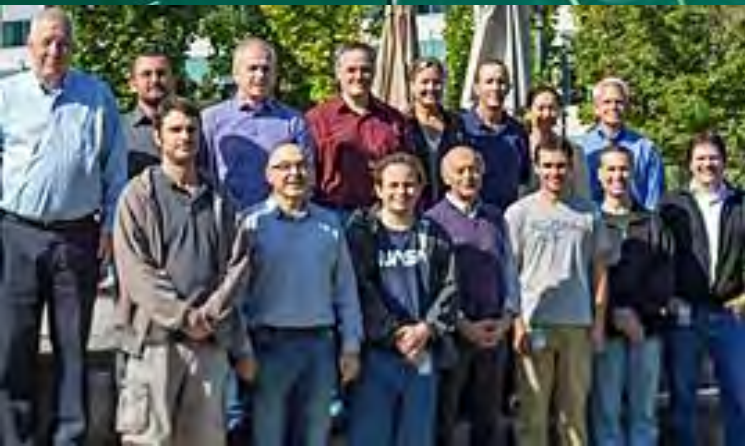
Q

[www.microsoft.stationq.com](http://www.microsoft.stationq.com)

REDMOND

SYDNEY, DELFT, COPENHAGEN

SANTA BARBARA



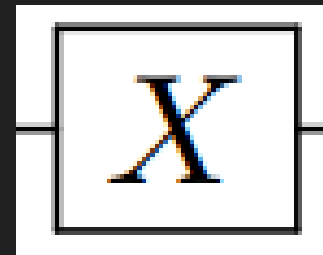
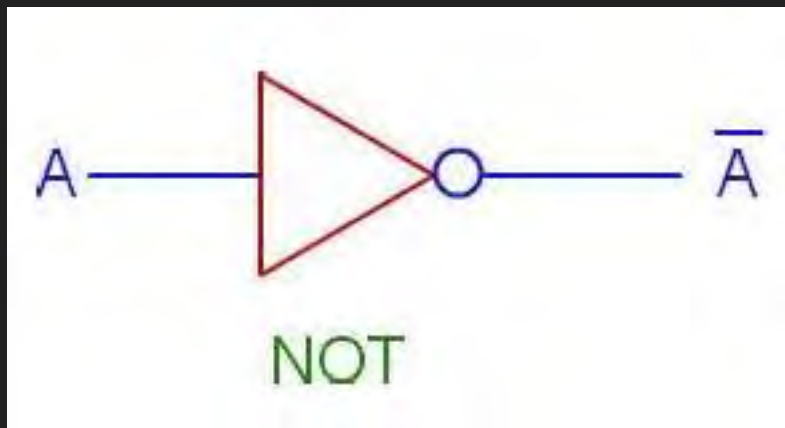
# Classical vs. Quantum Computing

Basic unit: **bit** = 0 or 1

Computing: **logical** operation

Basic unit: **qubit** = unit vector  $\alpha|0\rangle + \beta|1\rangle$

Computing: **unitary** operation



$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \beta \\ \alpha \end{bmatrix}$$

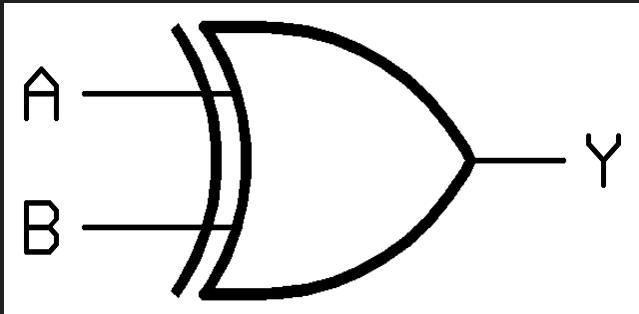


# Classical vs. Quantum Computing

Basic unit: **bit** = 0 or 1

Computing: **logical** operation

Description: **truth table**



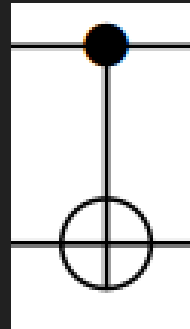
XOR gate

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

Basic unit: **qubit** = unit vector  $\alpha|0\rangle + \beta|1\rangle$

Computing: **unitary** operation

Description: **unitary matrix**



CNOT gate

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

# Classical vs. Quantum Computing

Basic unit: **bit** = 0 or 1

Computing: **logical** operation

Description: **truth table**

Direction: Most gates only run forward

Copying: Independent copies are easy

Noise: Manageable with minimal ECC

Storage: n bits hold **1** value from 0 to  $2^n - 1$

Input/Output: Linear

Computation:

An n-bit ALU performs 1 operation

Basic unit: **qubit** = unit vector  $\alpha|0\rangle + \beta|1\rangle$

Computing: **unitary** operation

Description: **unitary** matrix

Direction: Most gates are reversible (matrices)

Copying: Independent copies are **impossible**

Noise: Difficult to overcome. Sophisticated **QECC**

Storage: n qubits can hold  **$2^n$**  values

Input/Output: **sub**-Linear









Computation:

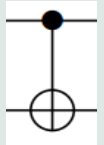
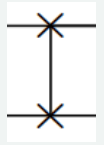

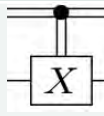
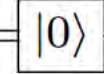
An n-qubit ALU performs  **$2^n$**  operations



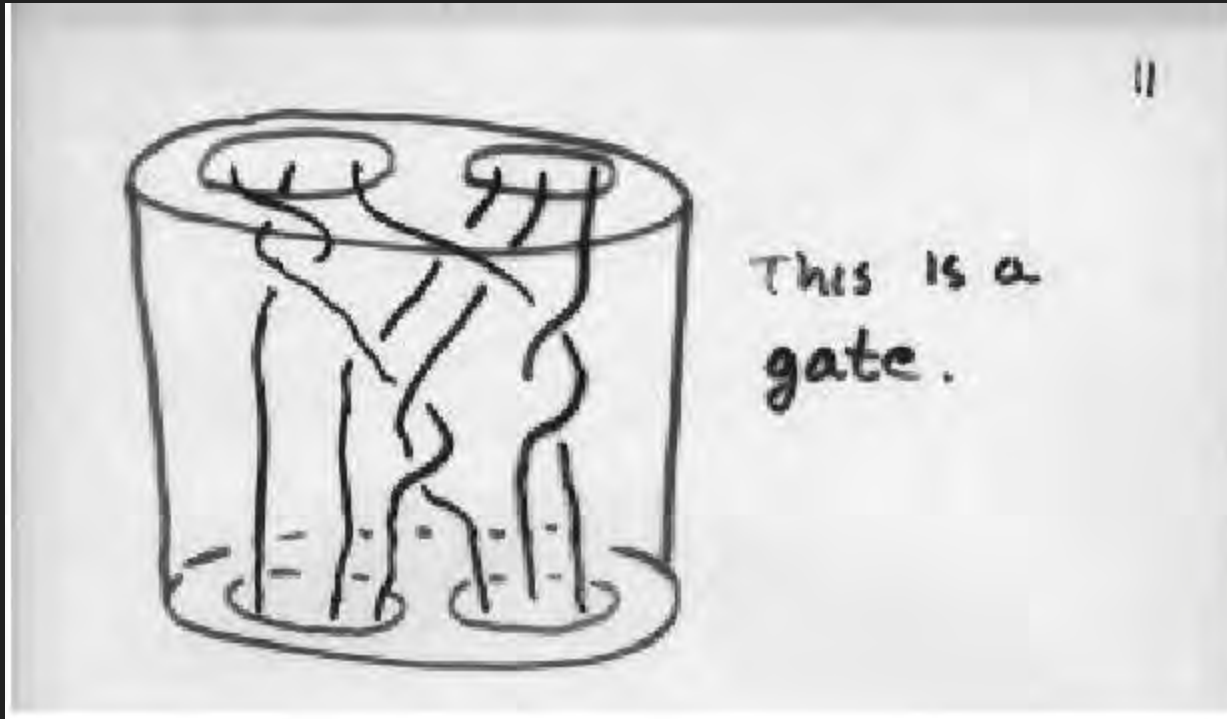
# Quantum Gates

**Evolution:**  $|\psi'\rangle = U|\psi\rangle$ , this may be realized by a Hamiltonian  $H = \frac{\ln(U)}{\Delta t}$

Type	Basis	U	Name	Sym
Pauli	$\{ 0\rangle,  1\rangle\}$	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$	X	
	$\{ 0\rangle,  1\rangle\}$	$\begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$	Y	
Z Rotation	$\{ 0\rangle,  1\rangle\}$	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$	Z	
$e^{i\pi/2}$	$\{ 0\rangle,  1\rangle\}$	$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$	S	
	$\{ 0\rangle,  1\rangle\}$	$\begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix}$	T	
	$\{ 0\rangle,  1\rangle\}$	$\begin{bmatrix} 0 & 1 \\ 1 & e^{i\pi/8} \end{bmatrix}$	R4	
Identity	$\{ 0\rangle,  1\rangle\}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	I	
Hadamard	$\{ 0\rangle,  1\rangle\}$	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$	H	

Type	Basis	U	Name	Sym
Controlled Not	$\{ 00\rangle,  01\rangle,  10\rangle,  11\rangle\}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$	CNOT (CX)	
	$\{ 00\rangle,  01\rangle,  10\rangle,  11\rangle\}$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	SWAP	
Measure	$\{ 0\rangle,  1\rangle\}$	Qubit to Bit	M	
Binary Control	$\{ 0\rangle,  1\rangle\}$	Conditional Application	BC	
Restore	$\{ 0\rangle,  1\rangle\}$	Bit to Qubit	Reset	

# Inspiration

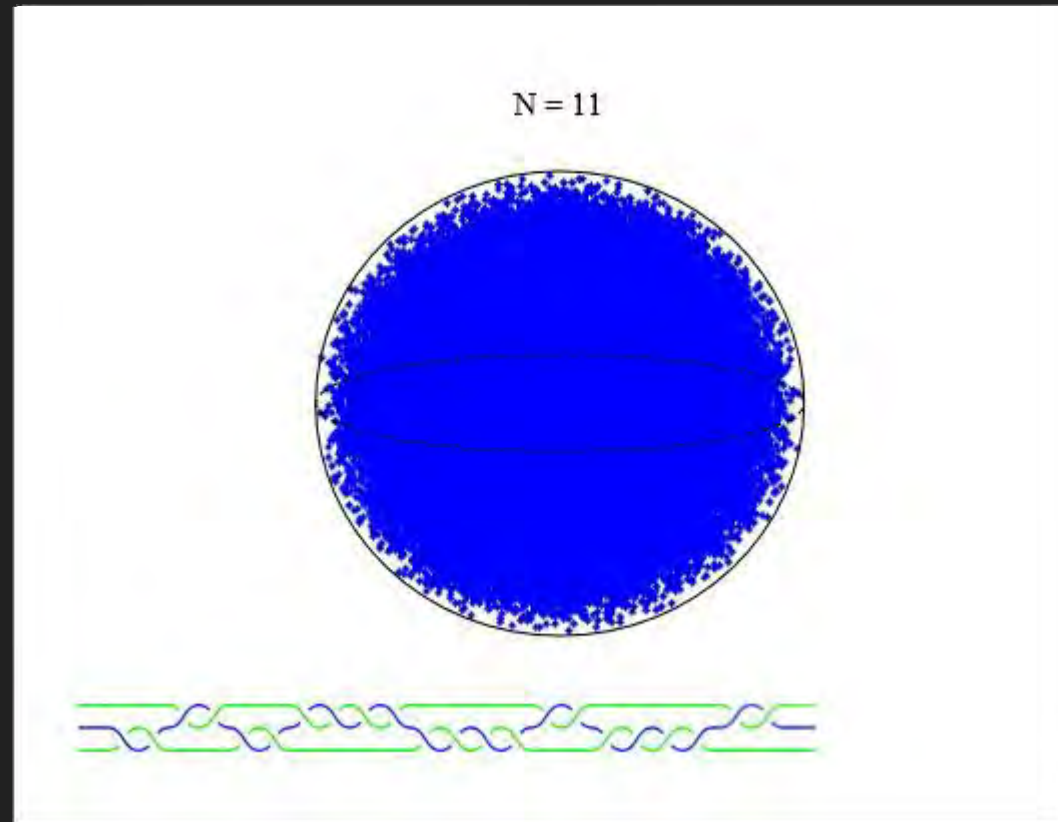


**From “A topological modular functor which is universal for quantum computation”**

Talk given by  
Michael Freedman at  
“[Mathematics of Quantum Computation](#)”,  
MSRI, Feb. 2000  
(available online).



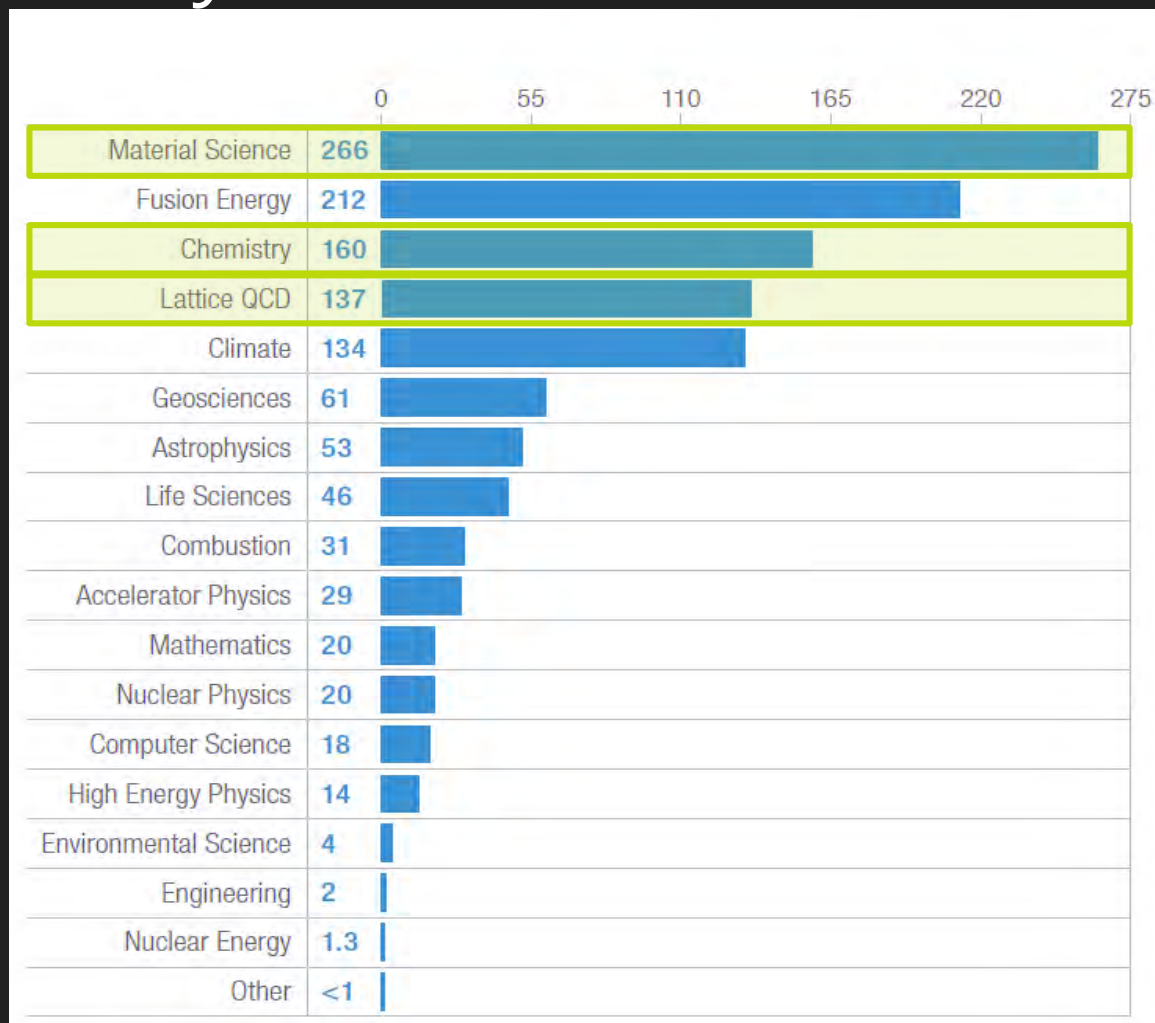
# Braiding



From: Nick Bonesteel talk at KITP UCSB

# Simulating physical systems

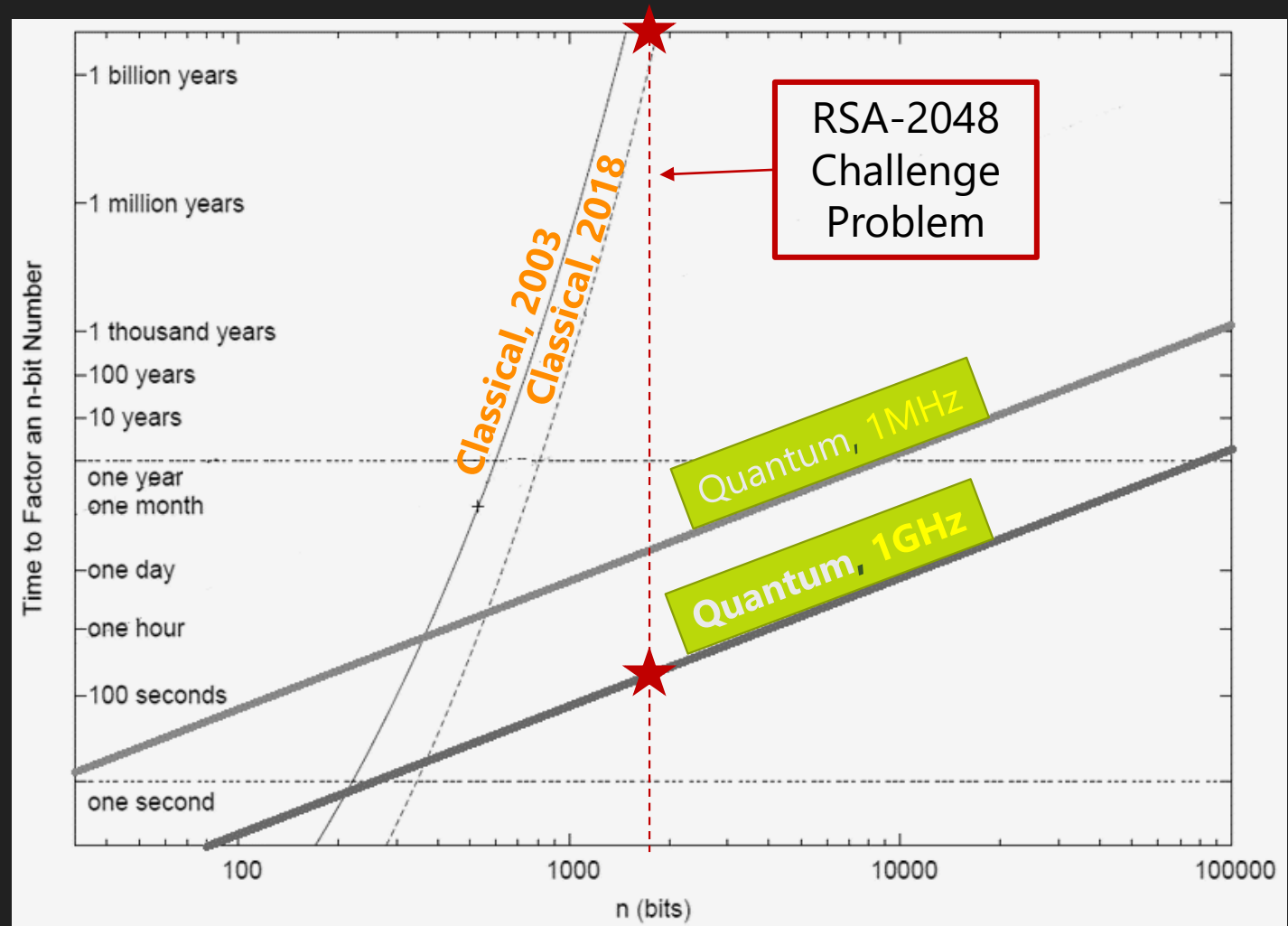
Hours (on Cray XE6)





# Breaking RSA

Time to Factor N-bit Number



Number of bits N

# Initial applications

Nitrogen  
fixation



100-200  
qubits

Carbon  
capture



100-200  
qubits

Materials  
science



100s-1000s  
qubits

Machine  
learning

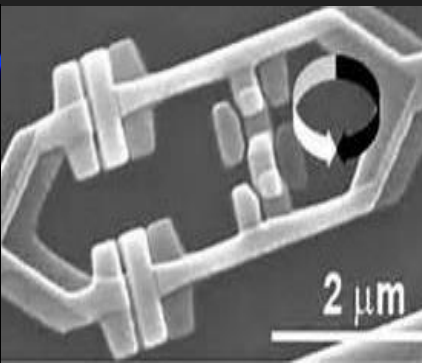


100s-1000s  
qubits

# Quantum hardware technologies



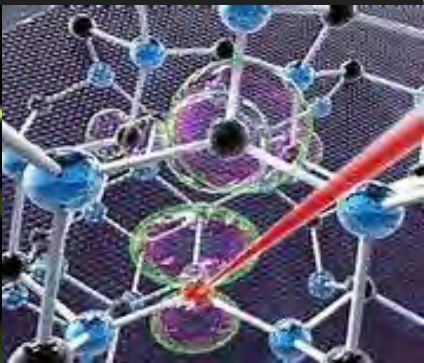
Ion traps



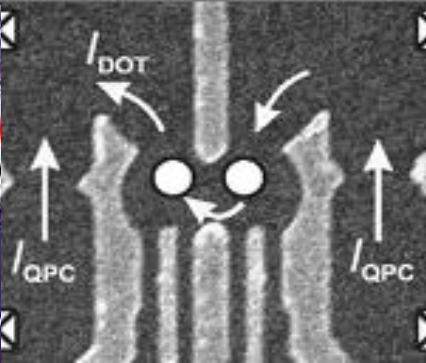
Superconductors



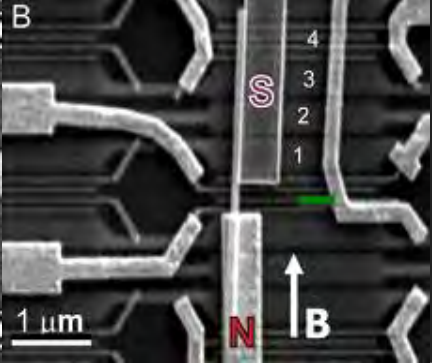
Linear optics



NV centers



Quantum dots



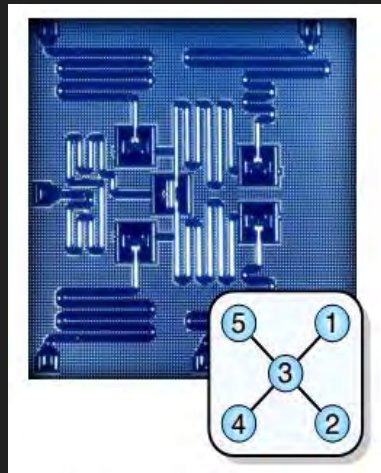
Topological



# Comparing quantum architectures

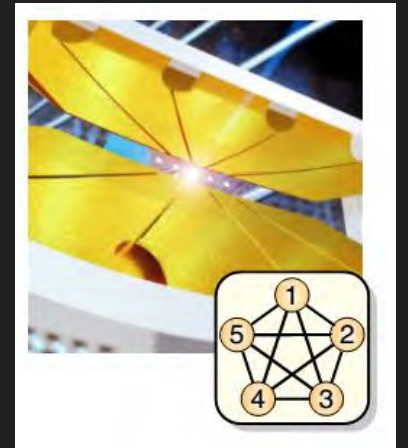
## IBM sc system

- five transmon qubits: JJ charge states, shunt capacitors, low sensitivity to charge noise
- Qubits are connected by microwave resonators
- Automatic calibration, twice a day
- Qubits drift between calibrations
- Addressing: qubit freqs around 5-5.4 GHz (all different)
- Fidelities:
  - Single qubit readout  $\sim 96\%$
  - Single qubit gate  $\sim 99.7\%$
  - Two-qubit gate  $\sim 96.5\%$
- Gate times:
  - Single qubit:  $\sim 130$  ns
  - Two qubit  $\sim 250$ - $450$  ns
- Decoherence times:
  - $T_1 \sim 60$   $\mu$ s
  - $T_2^*$  about equal to  $T_1$
- Native gate set:
  - CNOTs (ECR ZX-90), constrained to star shape
  - Single qubit: Pauli, H, S, T



## UMD ion trap system

- five  $^{171}\text{Yb}^+$  ions: hyperfine states, low sensitivity to B field, linear Paul trap, laser cooled to motional ground state
- Qubits are connected through pairs of Raman beams
- Manual calibration, 2 post-docs, 3 grad students
- Almost no drift between calibrations
- Addressing: qubit freqs = 12.642821 GHz (all the same)
- Fidelities:
  - Single qubit readout  $\sim 99.4\%$
  - Single qubit gate  $\sim 99.1\%$
  - Two-qubit gate  $\sim 97.0\%$
- Gate times:
  - Single qubit:  $\sim 20$   $\mu$ s
  - Two qubit  $\sim 250$   $\mu$ s
- Decoherence times:
  - $T_1 \sim$  several hours
  - $T_2^* \sim 0.5$  s
- Native gate set:
  - $\text{XX}(\chi)$  gate (Molmer-Sorensen) on any pair
  - Single qubit: any rotation





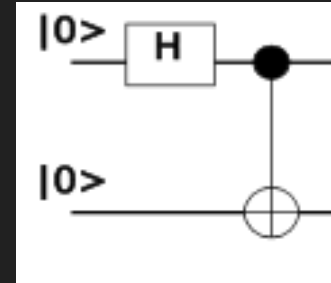
# Qubit Technologies

Realizations	Lifetimes	Gate Speed	ECC cost
Topological (Majorana)	1 minute	Nanoseconds	<b><math>10^1</math></b>
Flux Qubit	$/ 10^{10}$	same	$10^3 - 10^4$
Charge Qubit	$/ 10^{10}$	same	$10^3 - 10^4$
Transmon	$/ 10^7$	same	$10^3 - 10^4$
Ion Trap	$/ 10^2$	$10^3$ slower	$10^3 - 10^4$

- ECC is extremely painful (no "quantum refresh" like DRAM)
- Many can be fabricated with variations on standard semiconductor techniques

# Teleport: Quantum "Hello World"

- Alice entangles two qubits
- Bob takes one of them far away
- Alice is given a new qubit with a message
- Alice entangles it with her local part of the Bell pair
- Alice measure the local qubits, yielding 2 classical bits
- Alice transmits the two bits via classical channels
- Remotely, Bob applies gates as determined by the 2 bits
- Bob recovers the sent message



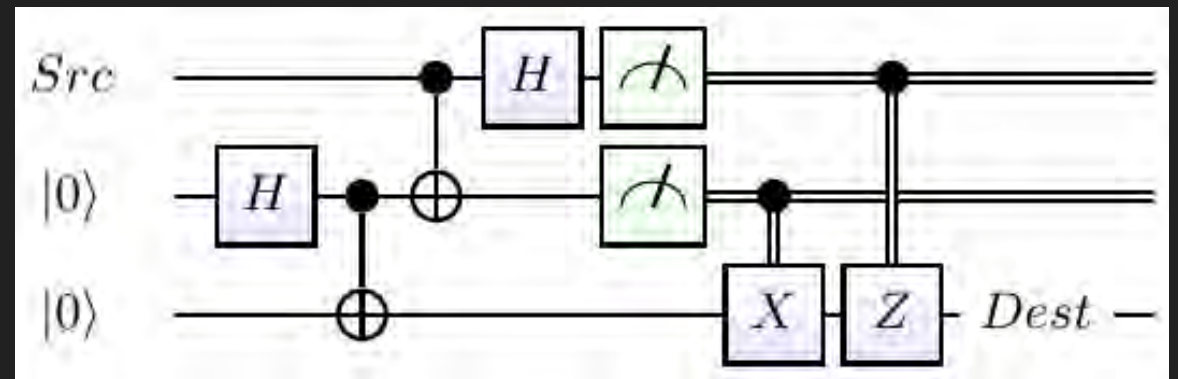
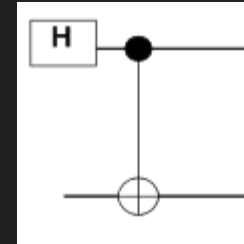
# Teleport: User Code

- Define a function to perform entanglement:

```
let EPR (qs:Qubits) = H qs; CNOT qs
```

- The rest of the algorithm:

```
let teleport (qs:Qubits) =  
  let qs' = qs.Tail  
  EPR qs'; CNOT qs; H qs  
  M qs'; BC X qs'  
  M qs; BC Z !!(qs,0,2)
```



# http://StationQ.github.io/Liquid/

LIQ*U*i|)

The Language Integrated Quantum Operations Simulator

It is very easy to get started with LIQ*U*i|) on your platform of choice

You just need

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    - Util.procStatsT Class

## Microsoft.Research.Liquid Namespaces

### Classes

Class	Description
App	Utilities for executing the Liquid application
Bit	Represents the measured value, in the computation
Circuit	The circuit representation of an operation
CMat	A dense matrix of complex numbers.
CSMat	A sparse matrix of complex numbers.
CVec	A block-sparse vector of complex numbers
Fermion	Hamiltonian simulation for fermionic system
Gate	A quantum gate.
GateOp	Gate operation type. This is used in Gate definitions
GrowPars	Parameters that control circuit growth. See the documentation for details.
Hamiltonian	Base class for Hamiltonian dynamics simulation
HamiltonianGates	A collection of gates that are useful for Hamiltonian simulation
Ket	Represents a state vector.
KrausOp	Entries for Kraus operators in Channel Gate
Noise	A complete noise model for a specific circuit
NoiseEvents	Noise statistics that are tracked for normal simulation
NoiseModel	A noise model for a particular type of gate
NoiseStat	Statistics tracked for each time that noise is applied
NoisyMats	Utility class for computing a Pauli rotation
Operations	The Operations module provides definition of existing gates.
QECC	Base class for quantum error correcting codes
Qubit	Represents a quantum bit. New Qubits are created by the simulator
RunMode	Trotterization types.
Spin	Hamiltonian for spin systems, such as the Ising model
SpinTerm	A single term in a Spin Hamiltonian.
Stabilizer	A stabilizer-based simulator based on CHP
Steane7	Implementation of a Steane 7-bit quantum error correcting code, $[[7,1,3]]$ , based on the QECC class.
Tests	A collection of sample Liquid simulations and tests, plus some utility routines to make it easier to write new samples.

StationQ / Liquid

Watch 16 Star 74 Fork 13

Code Issues 1 Pull requests 0 Pulse Graphs

The Language-Integrated Quantum Operations (LIQ*U*i|>) Simulator <http://StationQ.github.io/Liquid>

30 commits 2 branches 0 releases 3 contributors

Branch: master	New pull request	Find file	Clone or download
alan-geller committed on GitHub Added code of conduct to README.md Latest commit d7fb788 23 days ago			
QuantumChallenge	Another typo		4 months ago
Samples	Added Kraus Ops and POVMs		4 months ago
UserCode	Added info on setting the tableau.		a month ago
bin	Added Stabilizer.Tableau property and a sample in UserCode/DaveWecker		2 months ago
docs	Fixed bosus hyperlink ref		a month ago
img	Copied over msr-quarc site		6 months ago
linux	Added Stabilizer.Tableau property and a sample in UserCode/DaveWecker		2 months ago
source	Copied over msr-quarc site		6 months ago
AzureGuide.md	Copied over msr-quarc site		6 months ago
GettingStarted.md	Copied over msr-quarc site		6 months ago
LICENSE.md	Copied over msr-quarc site		6 months ago
LiquidTikZ.tex	Copied over msr-quarc site		6 months ago
README.md	Added code of conduct to README.md		23 days ago

README.md

## The Language-Integrated Quantum Operations (LIQ*U*i|) simulator

### News

2016/07/02 Sample implementation of Spin.Test and Stabilizer.ShowState released

In UserCode\DaveWecker there are two new releases of sample implementations. The first shows how to implement the Spin.Test routine that's called by the `__Ferro()` example in LIQ*U*i|. There's also a new property on stabilizers called `Tableau` which will return the current (raw) stabilizers. See the `Tableau.fsx` sample in the same directory for further information. The `README.md` file is a good place to start.

2016/05/20 Channels/POVMs added

**Flexi**  
LIQ*U*i|) can run with built-in or as a runtime in a C# application

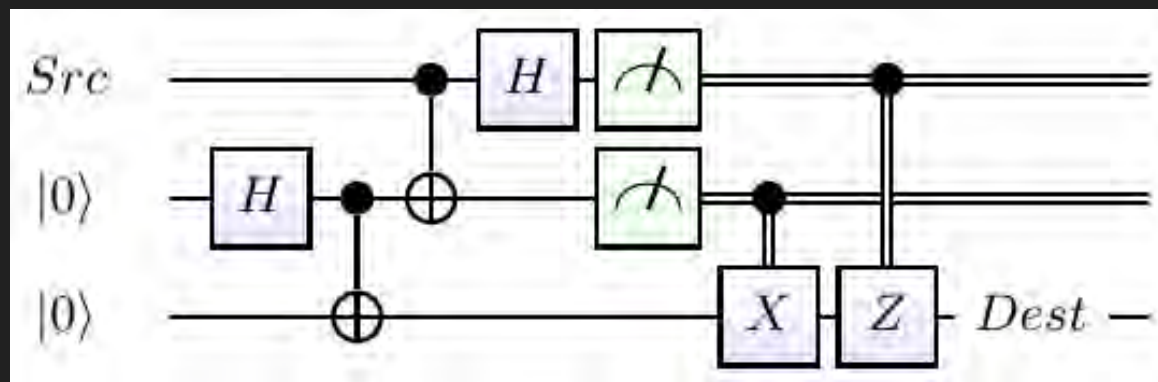
**Visuali**  
Circuits may be rendered in SVG graphics or LaTeX C formatting options

**Docum**  
LIQ*U*i|) comes with a lot including an extensive documentation (both com well as an extensiv

Refer to our getting



# Full Teleport Circuit in a Steane7 Code



3 qubits go to 27

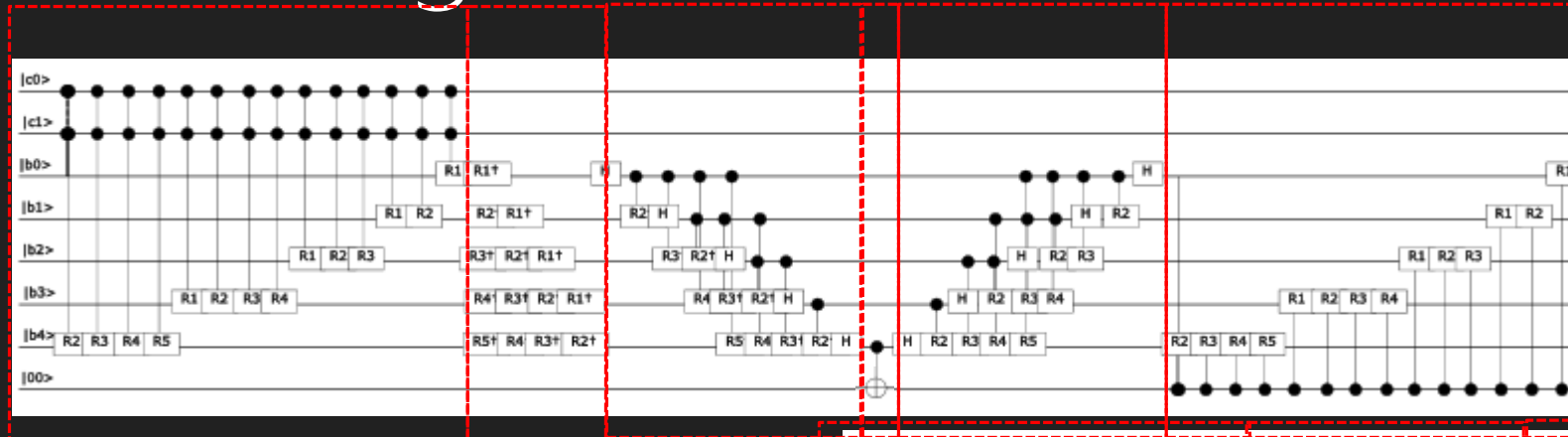


# Shor's algorithm: (4 bits $\cong$ 8200 gates)

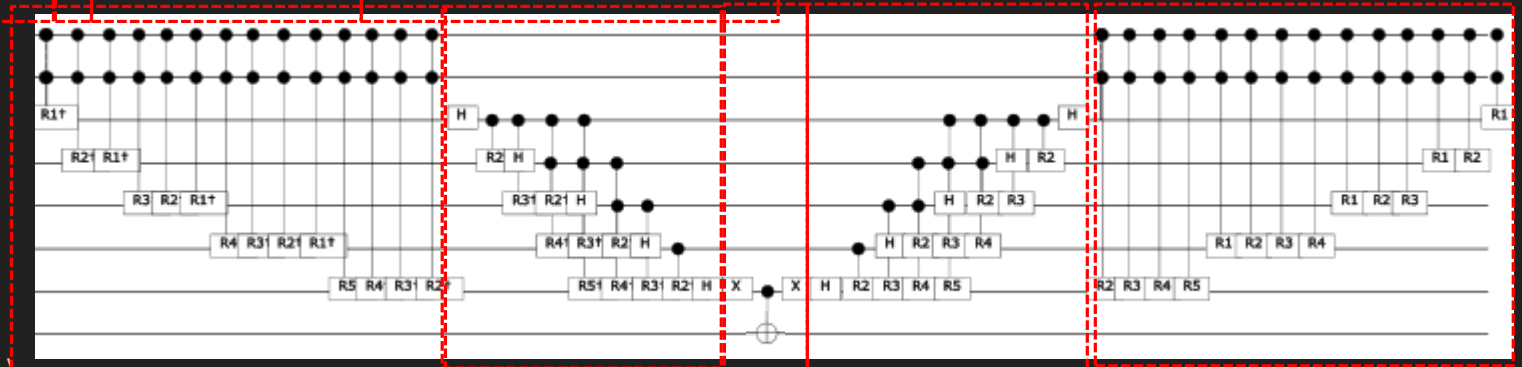


Circuit for Shor's algorithm using  $2n+3$  qubits – Stéphane Beauregard

# Shor's algorithm: Modular Adder



As defined in:  
**Circuit for Shor's algorithm using  $2n+3$  qubits**  
 – Stéphane Beauregard



let op (qs:Qubits) =

CCAdd a cbs

AddA' N bs

QFT' bs

CNOT [bMx;anc]

QFT bs

CAddA N (anc :: bs)

CCAdd' a cbs

// Add a to  $\Phi|b\rangle$

// Sub N from  $\Phi|a + b\rangle$

// Inverse QFT of  $\Phi|a + b - N\rangle$

// Save top bit in Ancilla

// QFT of  $a+b-N$

// Add back N if negative

// Subtract a from  $\Phi|a + b \bmod N\rangle$

QFT' bs

X [bMx]

CNOT [bMx;anc]

X [bMx]

QFT bs

CCAdd a cbs

// Inverse QFT

// Flip top bit

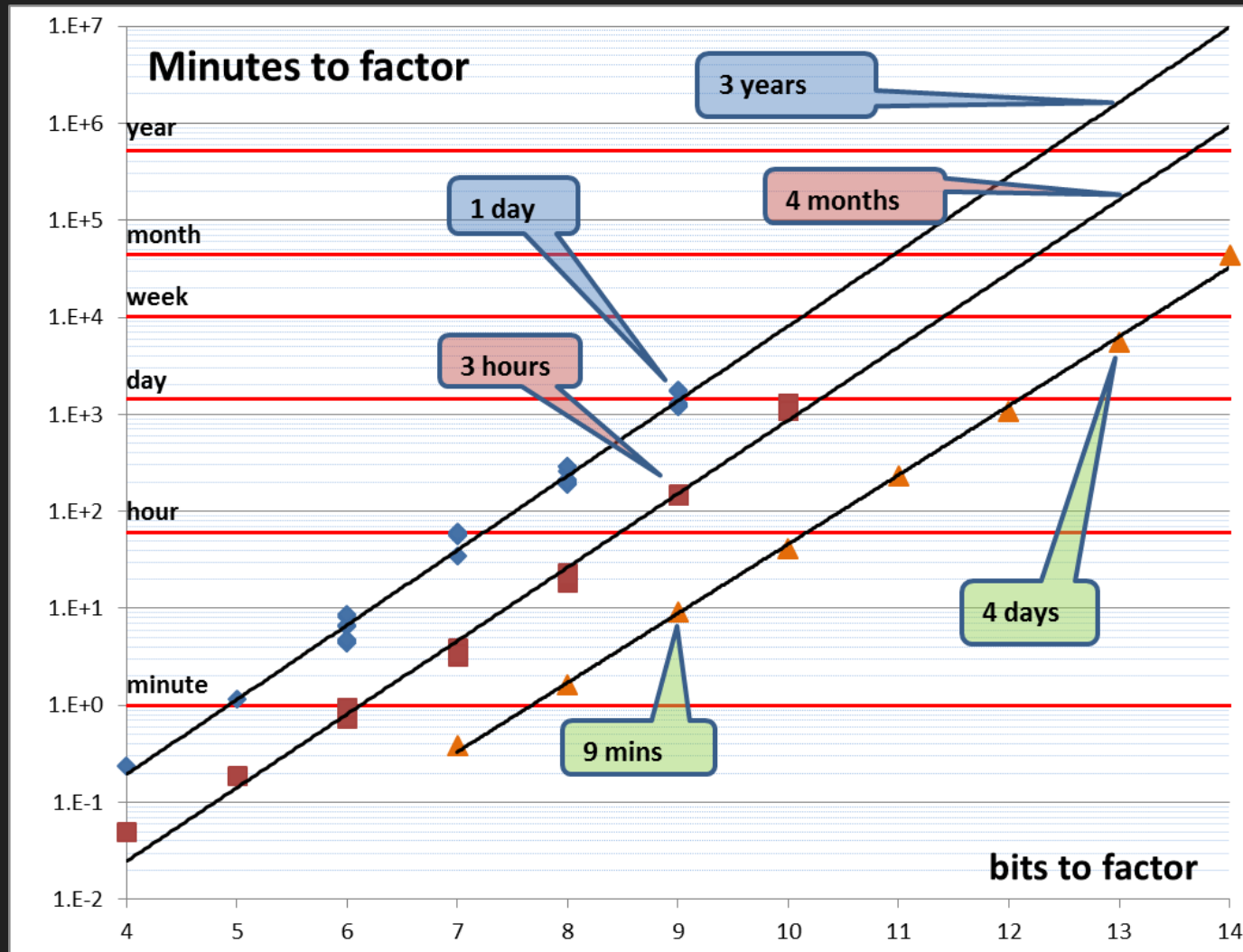
// Reset Ancilla to  $|0\rangle$

// Flip top bit back

// QFT back

// Finally get  $\Phi|a + b \bmod N\rangle$

# Shor's algorithm results





# Simulating quantum computers

- Need  $2^N$  complex numbers to store the wave function of  $N$  qubits
- $O(2^N)$  classical operations to perform a quantum gate on  $N$  qubits

Qubits	Memory	Time for one gate
10	16 kByte	microseconds on a watch
20	16 MByte	milliseconds on smartphone
30	16 GByte	seconds on laptop
40	16 TByte	seconds on supercomputer
50	16 PByte	seconds on top supercomputer
60	16 EByte	minutes on future supercomputer
70	16 ZByte	hours on potential supercomputer?
...	...	...
250	size of visible universe	age of the universe

# Quantum Chemistry

$$H = \sum_{pq} h_{pq} a_p^\dagger a_q + \frac{1}{2} \sum_{pqrs} h_{pqrs} a_p^\dagger a_q^\dagger a_r a_s$$

Can quantum chemistry be performed on a small quantum

computer: D. Imerman, Hastings, M. H.

As quantum computers will appear feasible applications of frequently met simulating qu of molecules computational perform quan the quantum molecule twice exactly. We first increase in the required incre executed is m quantum com problems, dra <http://arxiv.org/ab>

Ferredoxin ( $Fe_2S_2$ ) used in many metabolic reactions including energy transport in photosynthesis

- *Intractable on a classical computer*
- *Assumed quantum scaling: ~24 billion years ( $N^{11}$  scaling)*
- *First paper: ~850 thousand years to solve ( $N^9$  scaling)*
- *Second paper: ~30 years to solve ( $N^7$  scaling)*
- *Third paper: ~5 days to solve ( $N^{5.5}$  scaling)*
- *Fourth paper: ~1 hour to solve ( $N^3, Z^{2.5}$  scaling)*

# Molecules simulated in LIQUi|>



$H_2$



$HF$



$H_2O$



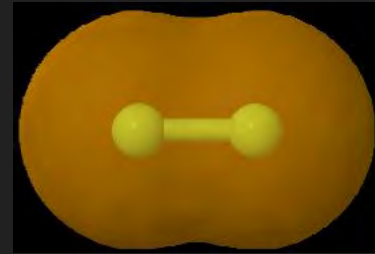
$NH_3$



$CH_4$



$HCl$



$F_2$

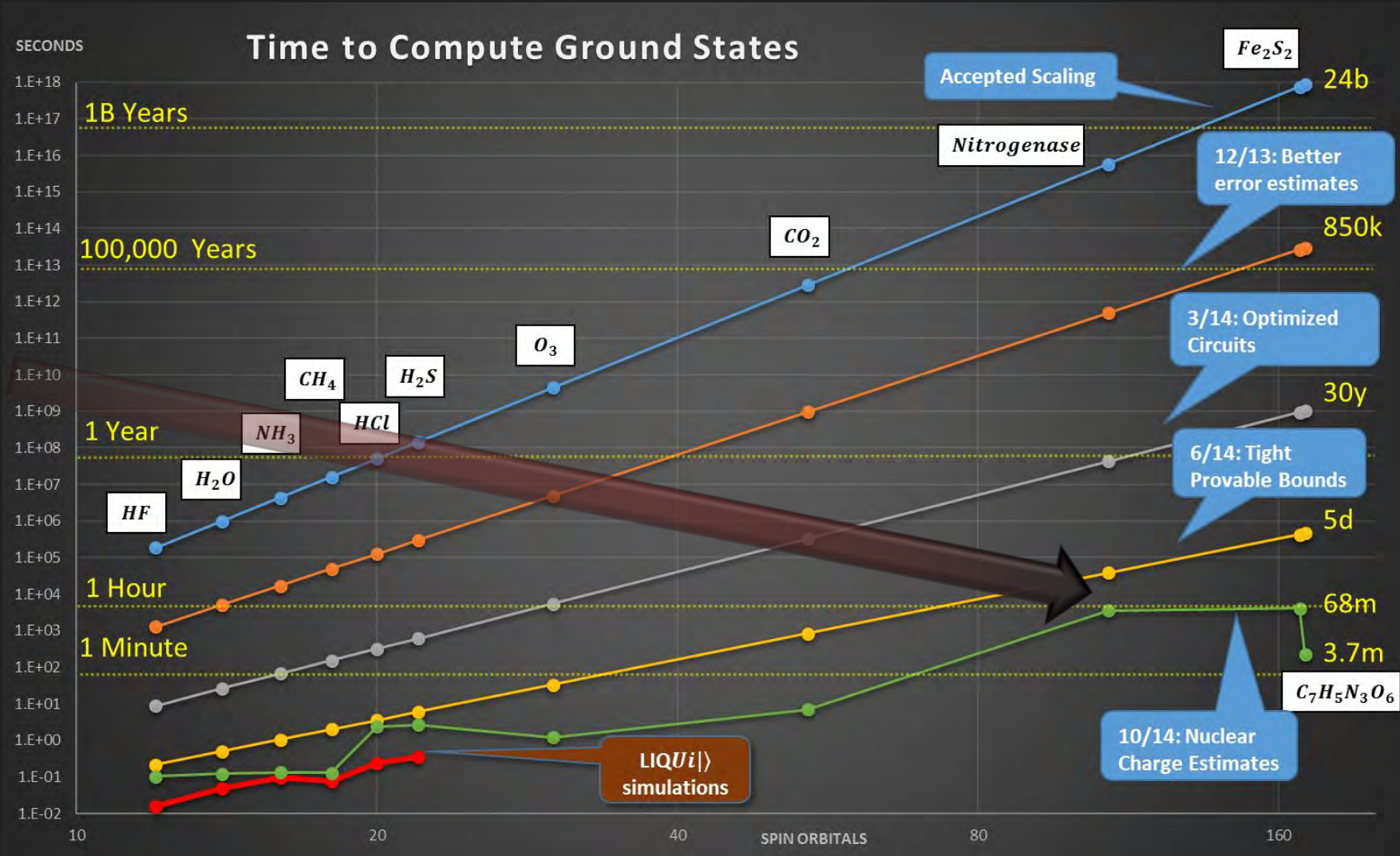
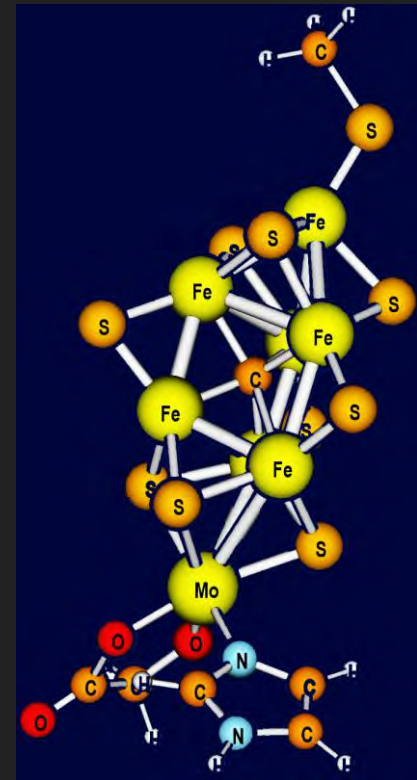


$H_2S$

Geometries and molecular models from <http://www.colby.edu/chemistry/webmo/>

# Simulation Evidence

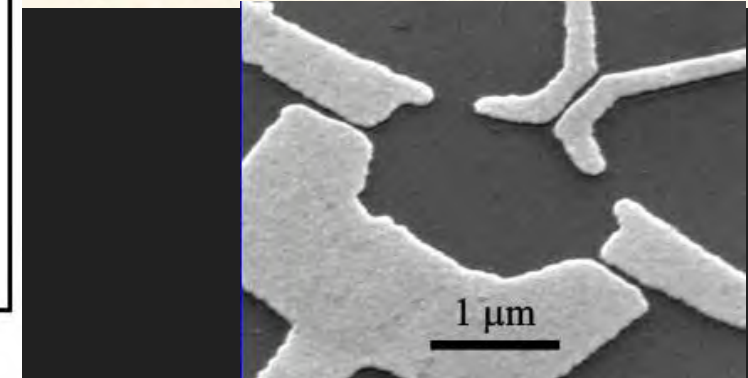
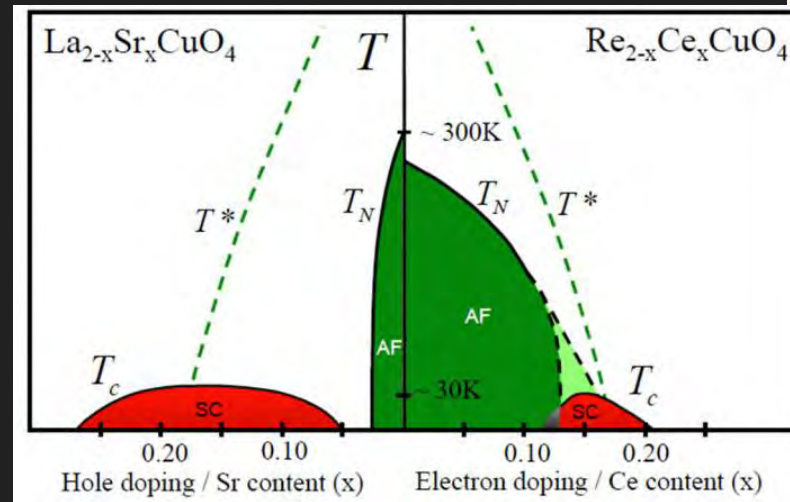
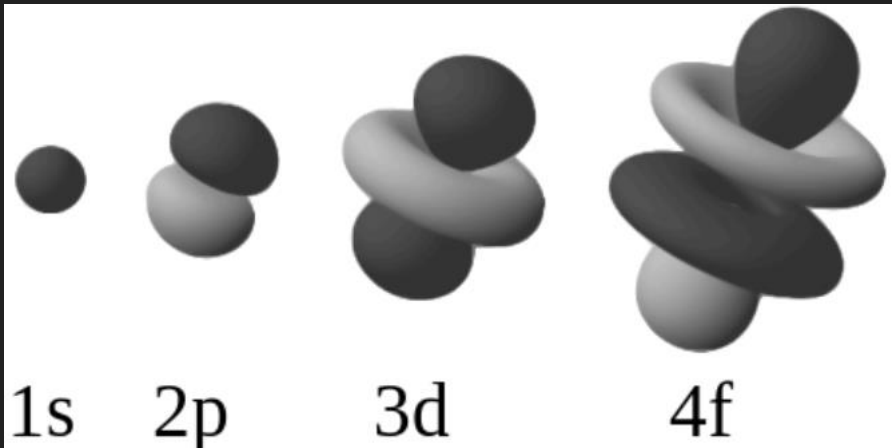
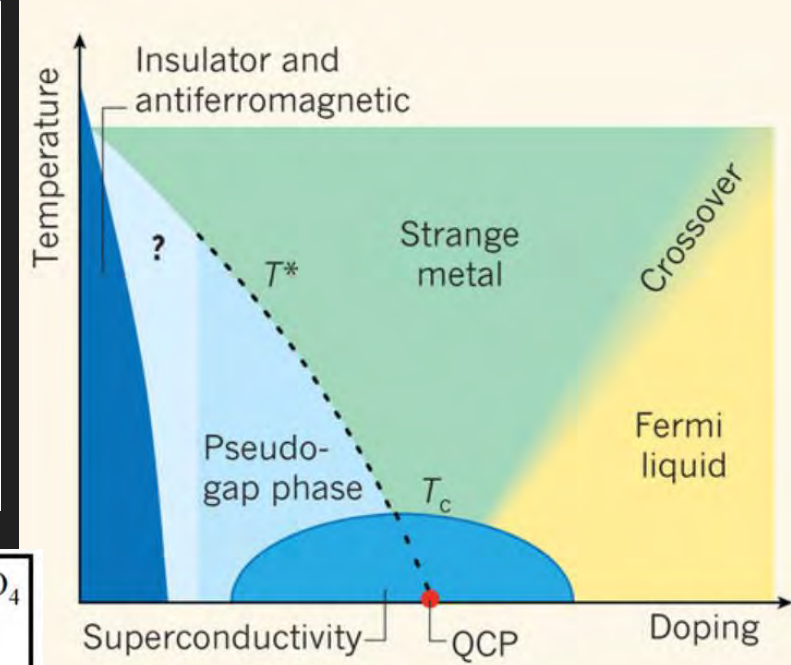
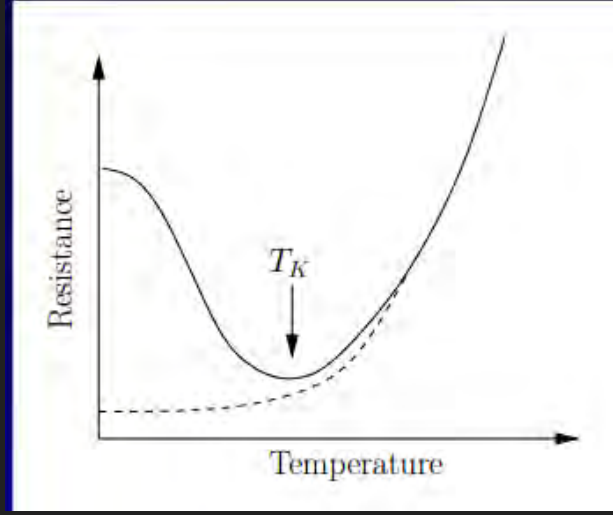
$$H = \sum_{pq} h_{pq} a_p^\dagger a_q + \frac{1}{2} \sum_{pqrs} h_{pqrs} a_p^\dagger a_q^\dagger a_r a_s$$





# Quantum Algorithms for Quantum Impurity Problems

- Mott Insulators
- Transition Metal Compounds
  - Cuprates (e.g., High  $T_c$  SC)
- Lanthanides and Actinides
- Kondo Physics (Low temperature Resistance) from Magnetic Impurities
  - Quantum Dots



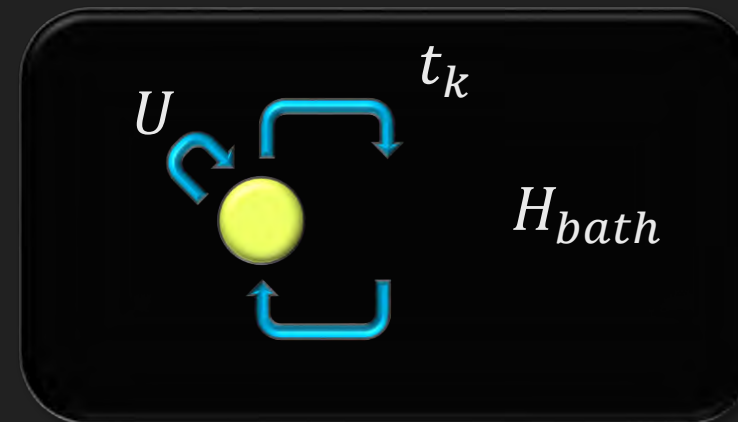
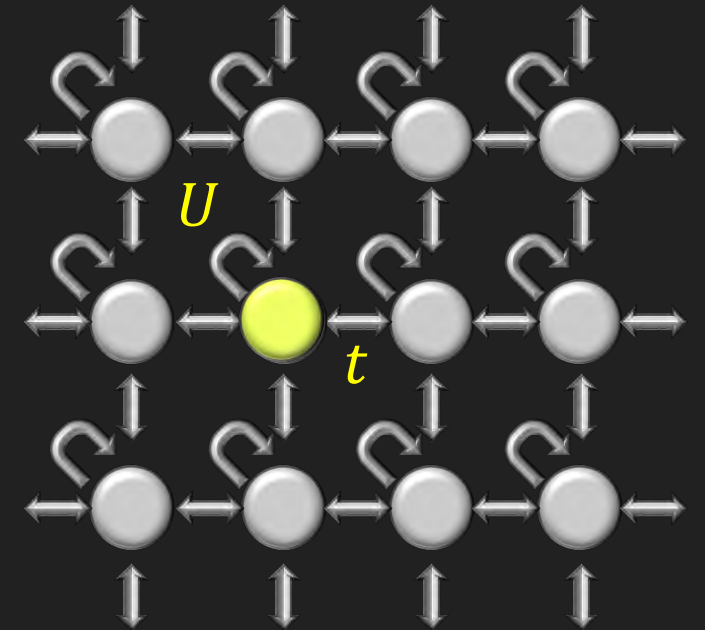


# Materials Modeling

$$H_{hub} = U \sum_i n_{i\uparrow} n_{i\downarrow} - t \sum_{\langle i,j \rangle, \sigma} c_{i\sigma}^\dagger c_{j\sigma}$$

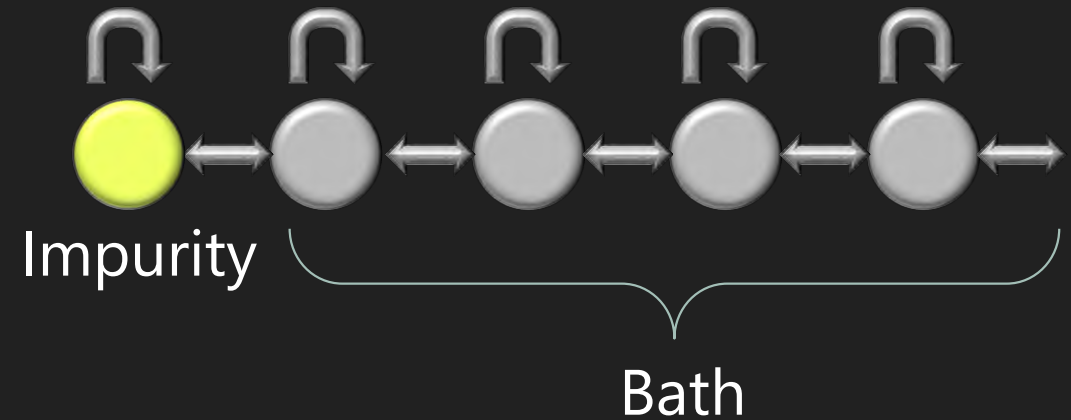
$$H_{imp} = U n_\uparrow n_\downarrow - \sum_{k, \sigma} (t_k c_\sigma^\dagger a_{k, \sigma}^{bath} + h.c.) + H_{bath}$$

- Solids have regular structure that can be modeled as lattices
- The Hubbard model only implements  $H_{pp}$  and  $H_{pqqp}$  terms
- This doesn't cover many of the materials we're interested in
- One can choose a single site in the lattice to model
- The effect of the rest of the lattice can be modeled in terms of its effect on this site



# Anderson Impurity Model

- Choose a single place in the lattice to model (the impurity). This may contain a collection of local sites
- The impurity is typically a full two-body model
- The effect of the rest of the lattice can be modeled in terms of its effect on the impurity (the bath) via a Dynamic Green's function  $G(\omega)$
- The bath may have many sites and interconnections

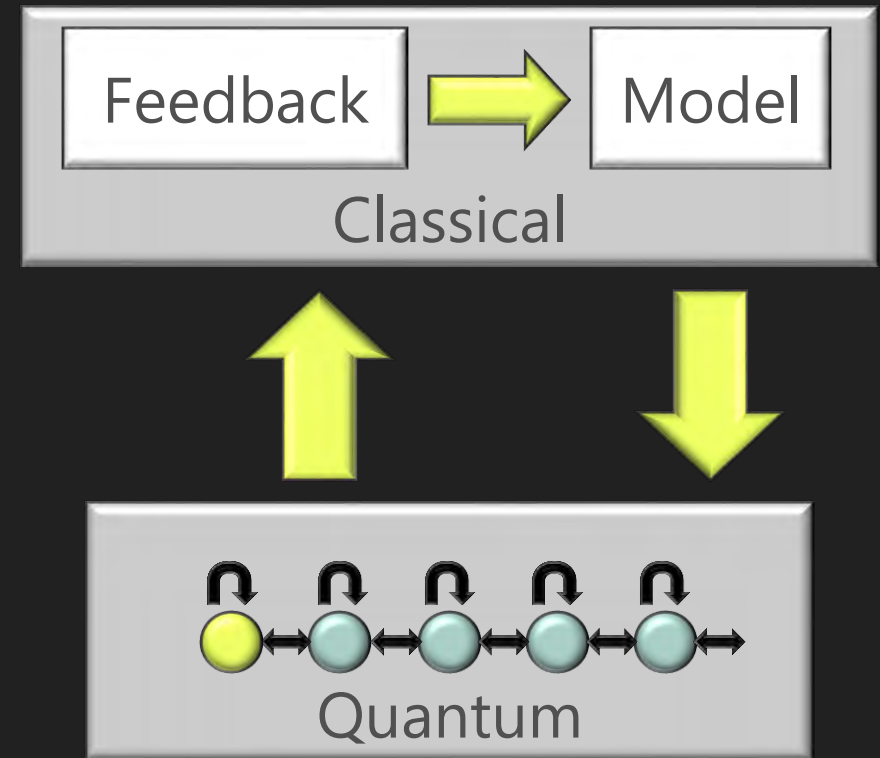


$$H = \sum_{ij} t_{ij} a_i^\dagger a_j + \frac{1}{2} \sum_{ijkl} w_{ijkl} a_i^\dagger a_j^\dagger a_k a_l + \sum_{ip} V_{ip} (a_i^\dagger a_p + a_p^\dagger a_i) + \sum_{pq} \epsilon_p a_p^\dagger a_q$$

# Dynamical Mean Field Theory (DMFT)

$$G_{\text{solver}}(\omega) \rightarrow \Sigma(\omega) \rightarrow G(k, \omega) \rightarrow G_n(\omega) \rightarrow \Delta_n(\omega)$$

- We can posit an initial model for a material
- Measure quantum simulations at many sites and frequencies deriving a dynamical Green's function
- Use feedback to update model
- Repeat until converged
- The resulting model is defined classically and may be used to efficiently investigate many questions about the material



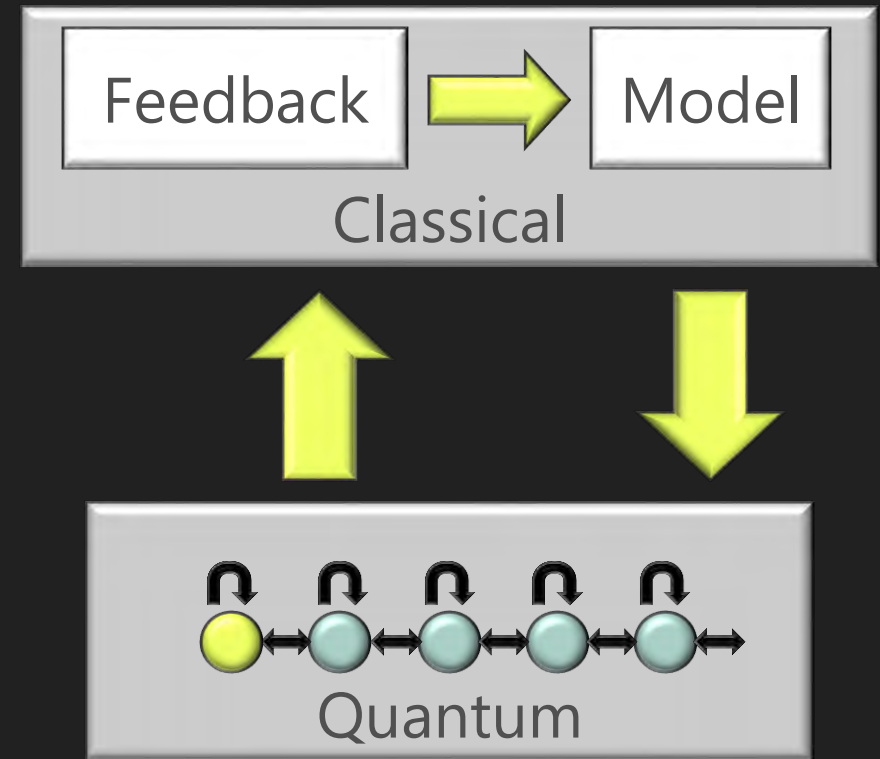
<http://arxiv.org/abs/1012.3609>

$$G_{\text{solver}}(\omega) = \langle c_i^\dagger(\omega) c_j(-\omega) \rangle$$

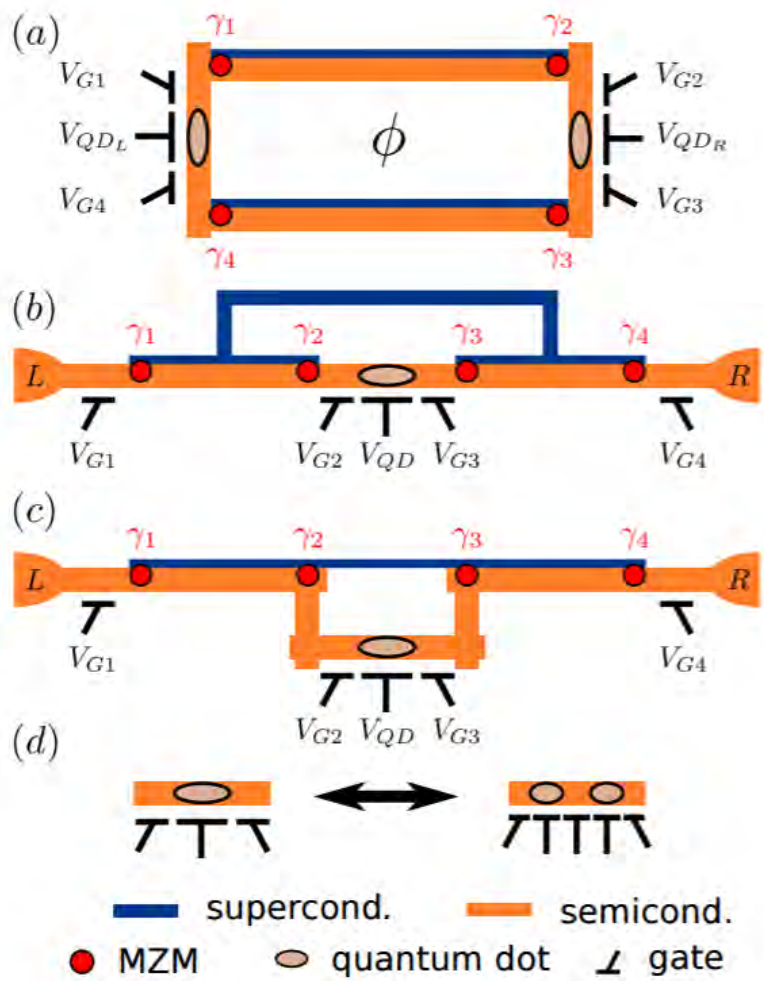
# Variational Eigensolver

$$E_{gs} = \sum_k \left( H_{FF} + \sum_i \sum_j \theta_{ij} H_j + M(H_k) \right)$$

- **Good:** Only need to stay coherent for prep, evolution and measurement
- **Bad:** Parameters discovered by sampling (quadratically worse than PE)
- Useful for small machines with physical qubits that have relatively short coherence times.
- Only needs few thousand gate executions before losing coherence

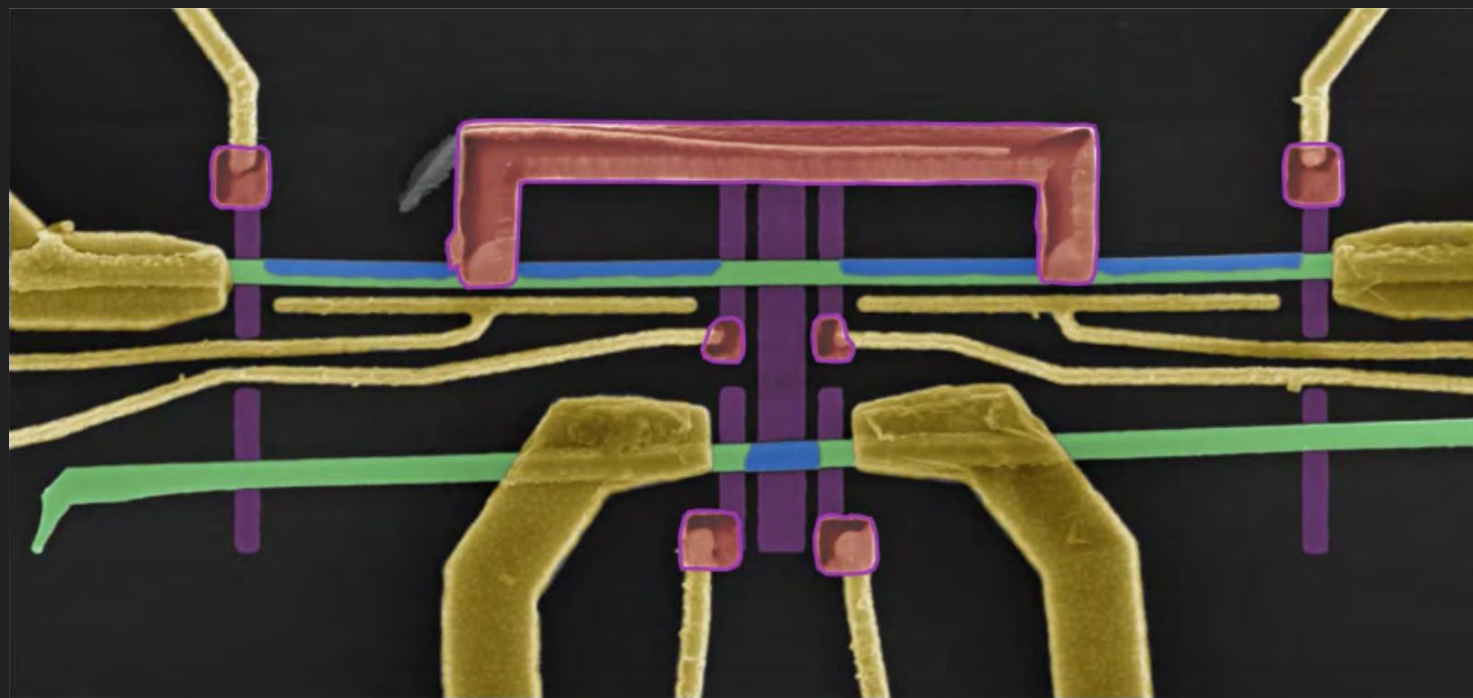


<http://arxiv.org/abs/1012.3609>



## Scalable Designs for Quasiparticle-Poisoning-Protected Topological Quantum Computation with Majorana Zero Modes

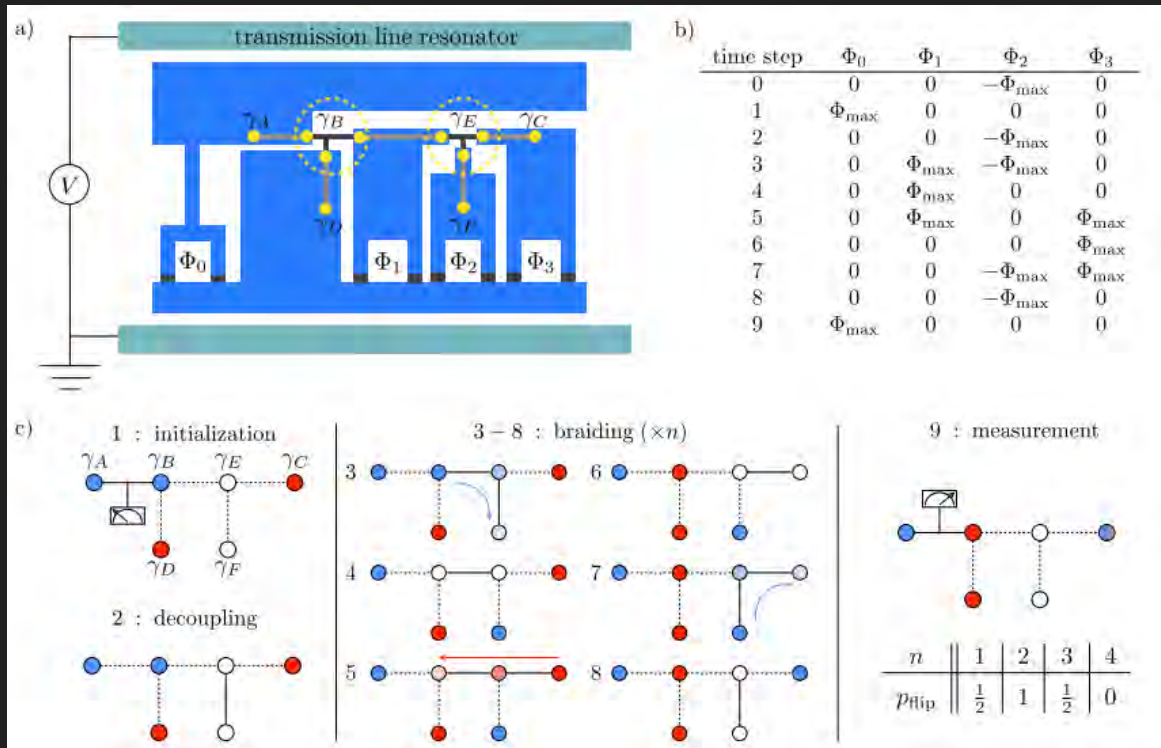
Torsten Karzig,<sup>1</sup> Christina Knapp,<sup>2</sup> Roman M. Lutchyn,<sup>1</sup> Parsa Bonderson,<sup>1</sup> Matthew Hastings,<sup>1</sup> Chetan Nayak,<sup>1,2</sup> Jason Alicea,<sup>3,4</sup> Karsten Flensberg,<sup>5</sup> Stephan Plugge,<sup>5,6</sup> Yuval Oreg,<sup>7</sup> Charles Marcus,<sup>5</sup> and Michael H. Freedman<sup>1,8</sup>



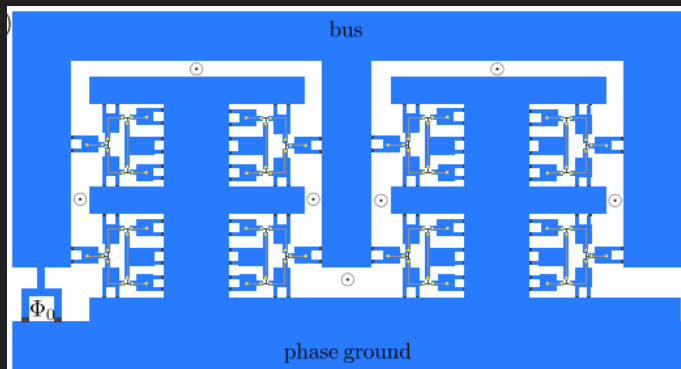
■ Au    ■ InAs    ■ Epitaxial Al    ■ Al bottom gates    ■ NbTiN w/ Al sticking layer



# Previously Proposed Architecture



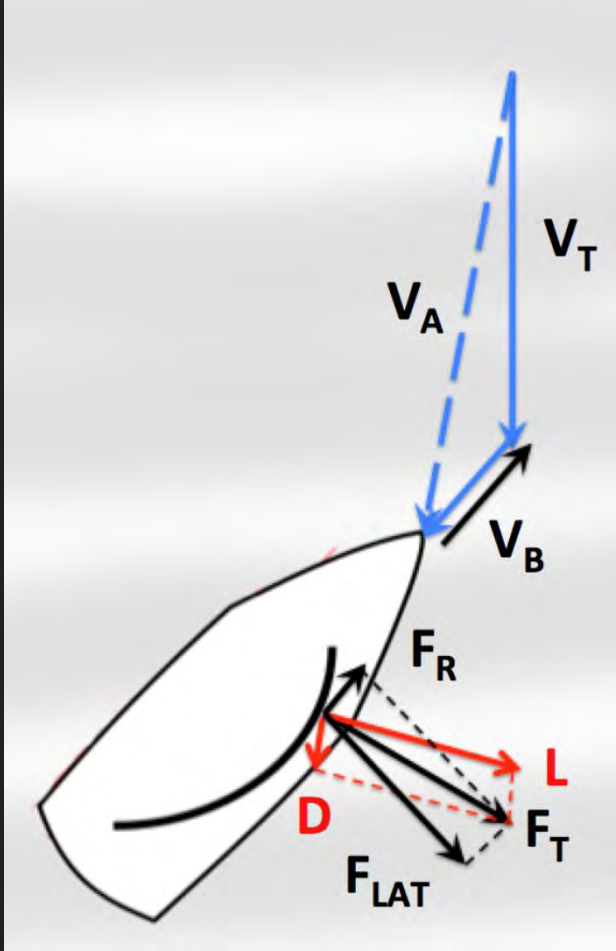
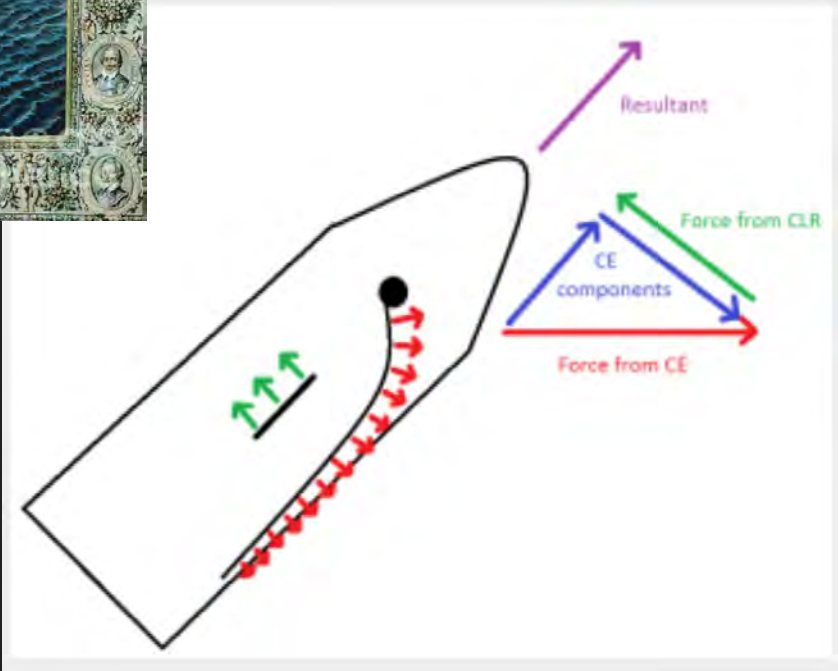
- Majoranas are the yellow dots
- Gray lines are nanowires
- Black lines are junctions
- Small black rectangles are Josephson Junctions
- Strong magnetic field must be **parallel** to all the nanowires and not affect the JJs... how?
- How do you keep coherence when changing directions in the junctions?



After Hyart et al 2013

<http://arxiv.org/abs/1303.4379>

# Sailing into the Wind



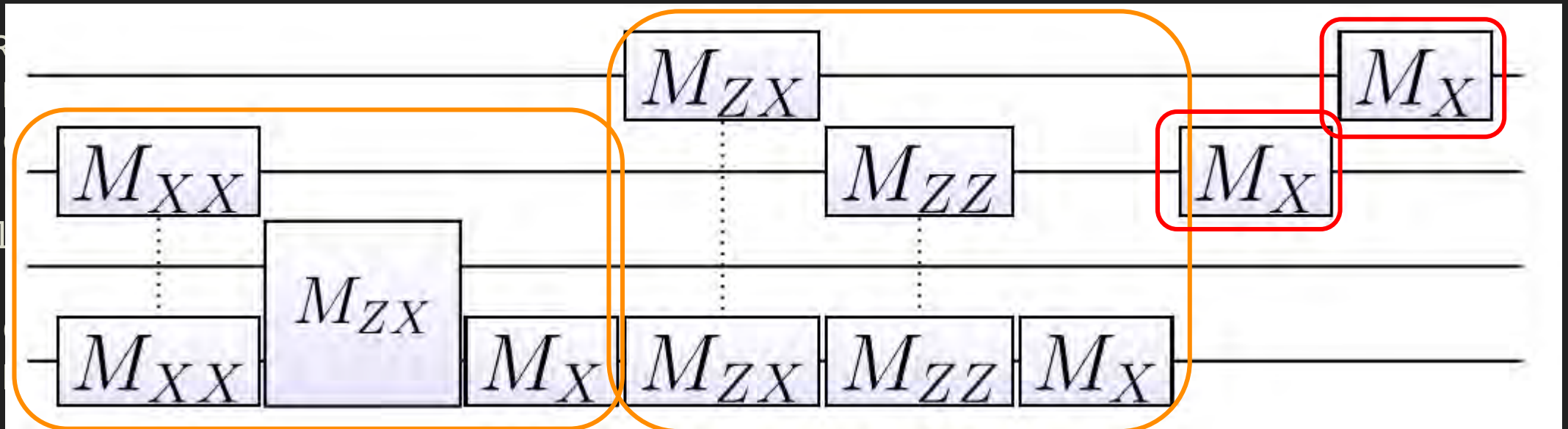
[https://en.wikipedia.org/wiki/Forces\\_on\\_sails](https://en.wikipedia.org/wiki/Forces_on_sails)

<http://hayward.peirce.me/the-physics-of-sailing-ce-and-clr/>

# Teleport: $SoLi| \rangle$ User Code

let EPR

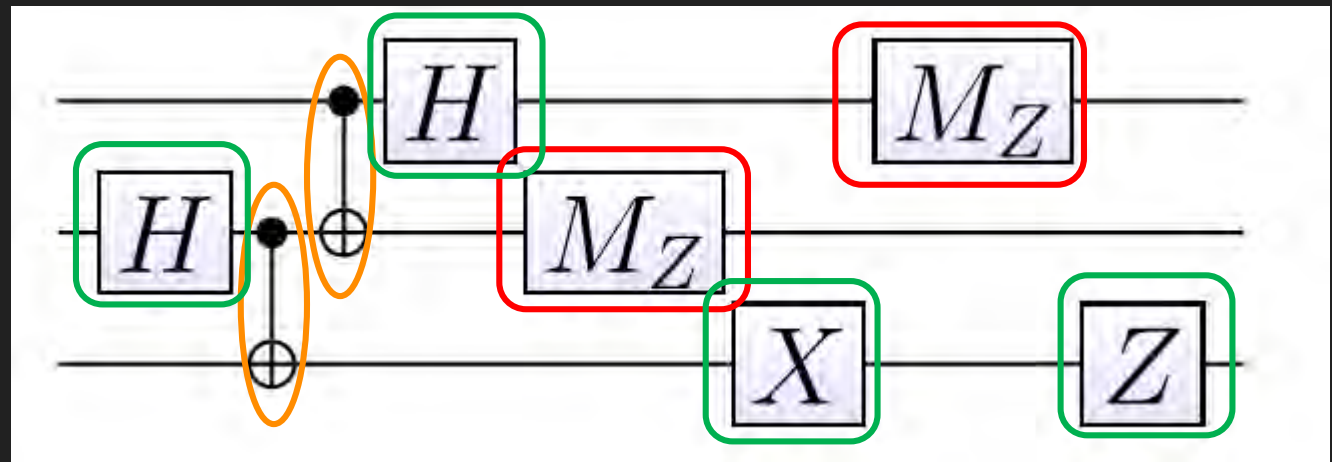
let Tel



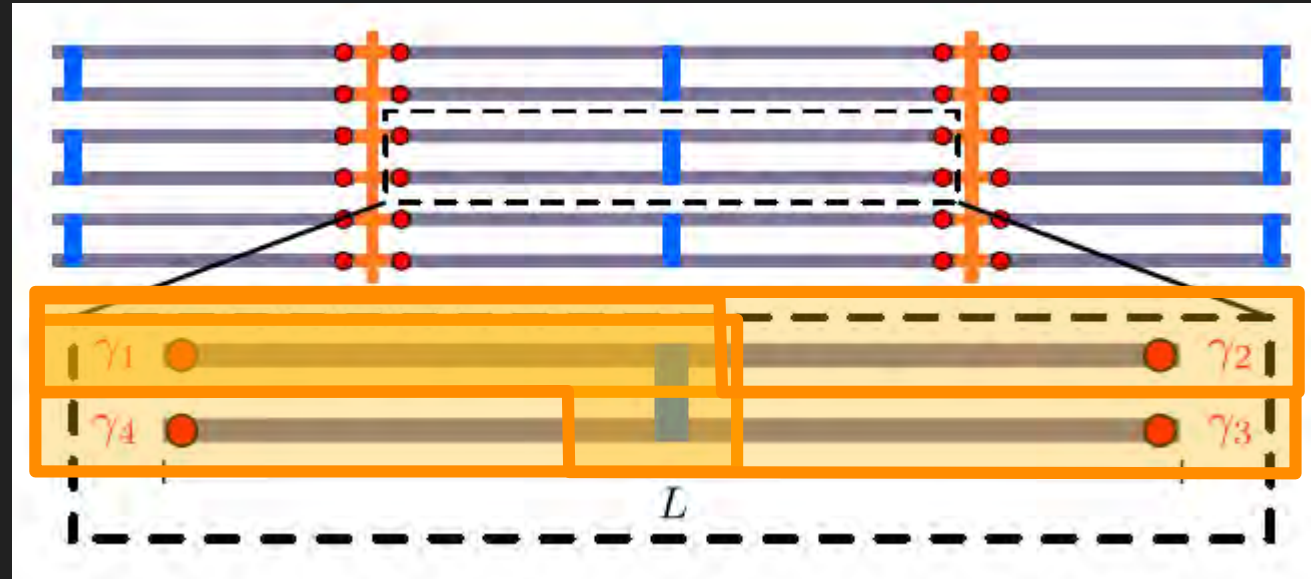
```

if JM "Z" [msg] = -1 then Z there

```



# Box Qubits: 1 Qubit Measurements



Measure two Horizontally =  $M_z$

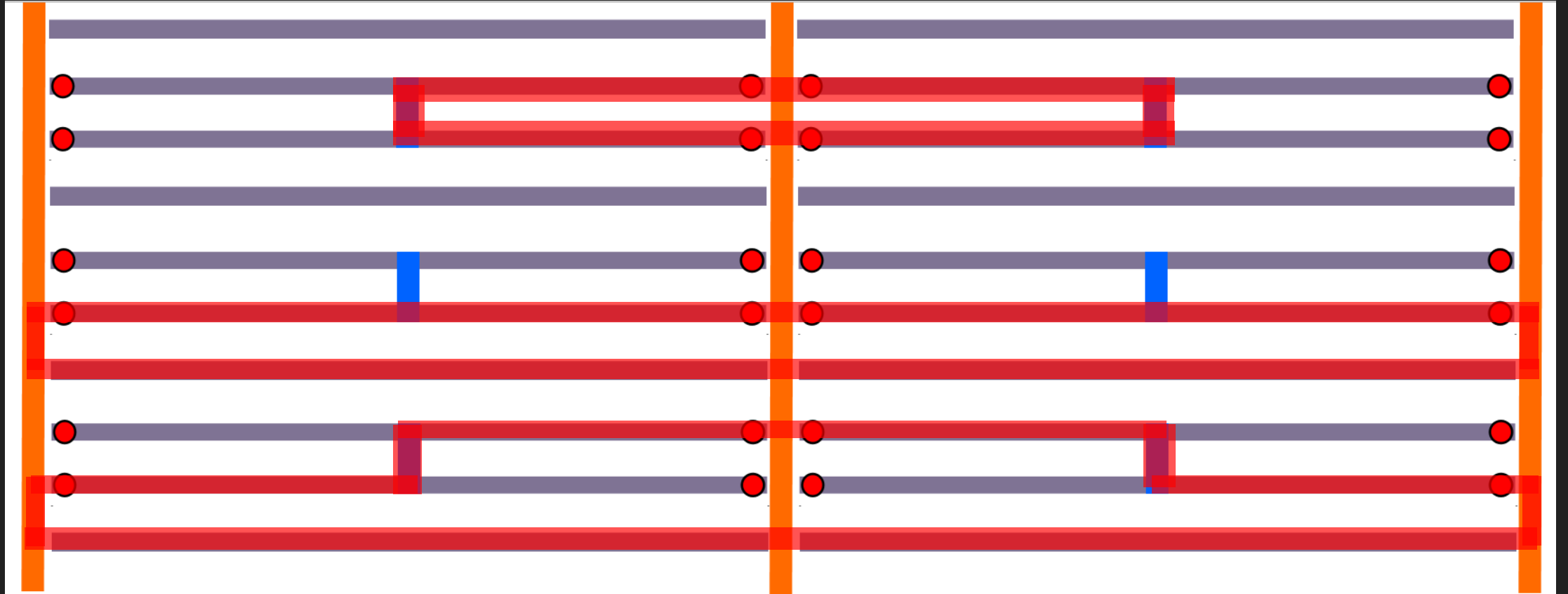
Measure two Vertically =  $M_x$

Measure two Diagonally =  $M_y$

**Scalable Designs for Quasiparticle-Poisoning-Protected Topological Quantum Computation with Majorana Zero Modes**

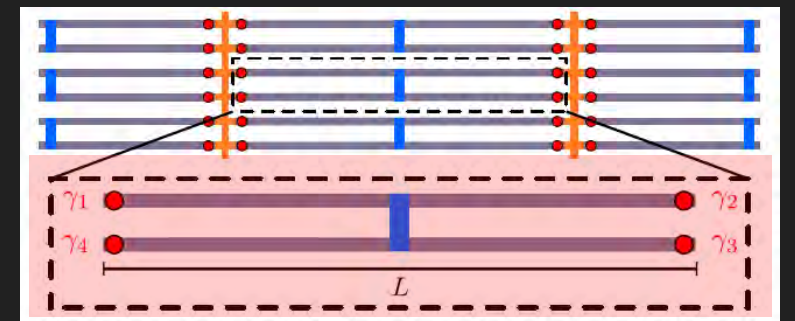
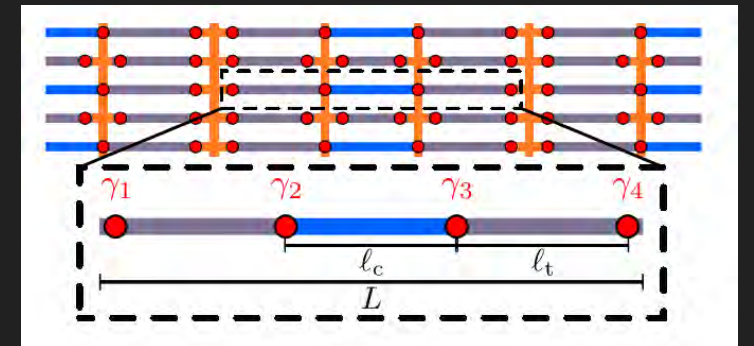
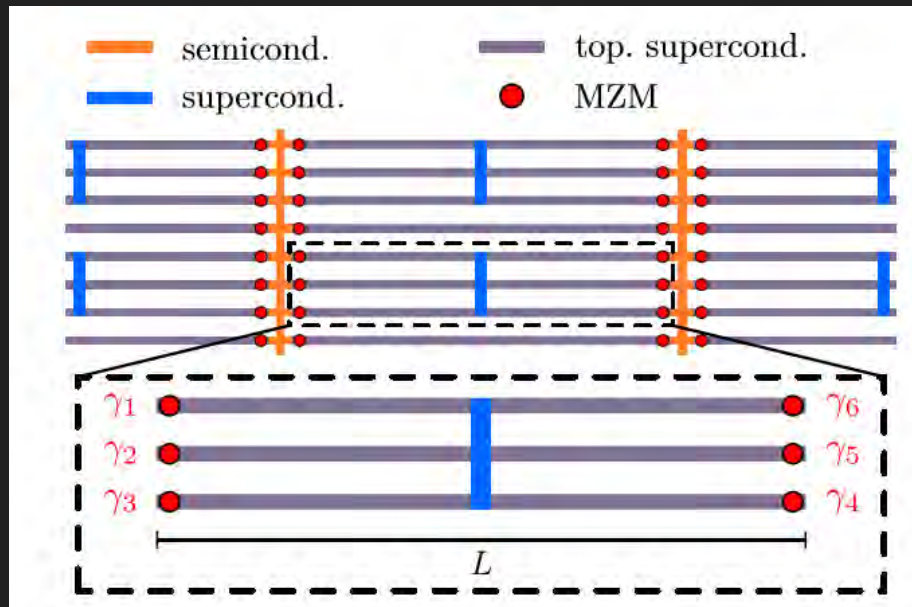
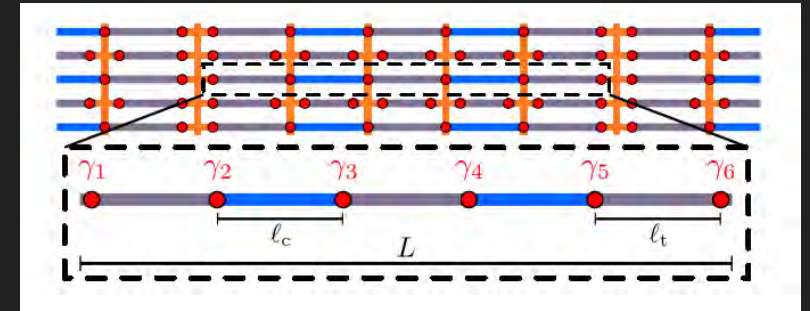
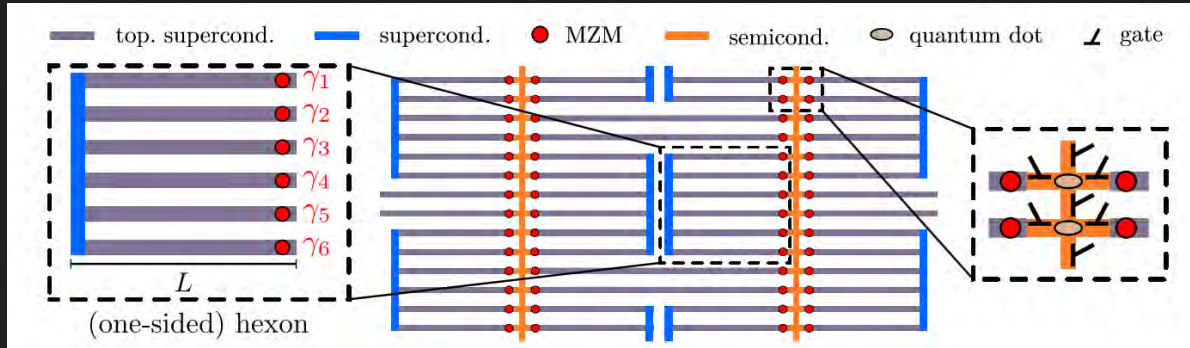
Torsten Karzig,<sup>1</sup> Christina Knapp,<sup>2</sup> Roman M. Lutchyn,<sup>1</sup> Parsa Bonderson,<sup>1</sup> Matthew Hastings,<sup>1</sup> Chetan Nayak,<sup>1,2</sup> Jason Alicea,<sup>3,4</sup> Karsten Flensberg,<sup>5</sup> Stephan Plugge,<sup>5,6</sup> Yuval Oreg,<sup>7</sup> Charles Marcus,<sup>5</sup> and Michael H. Freedman<sup>1,8</sup>

# Box Qubits: Horizontal 2 Qubit Measurements





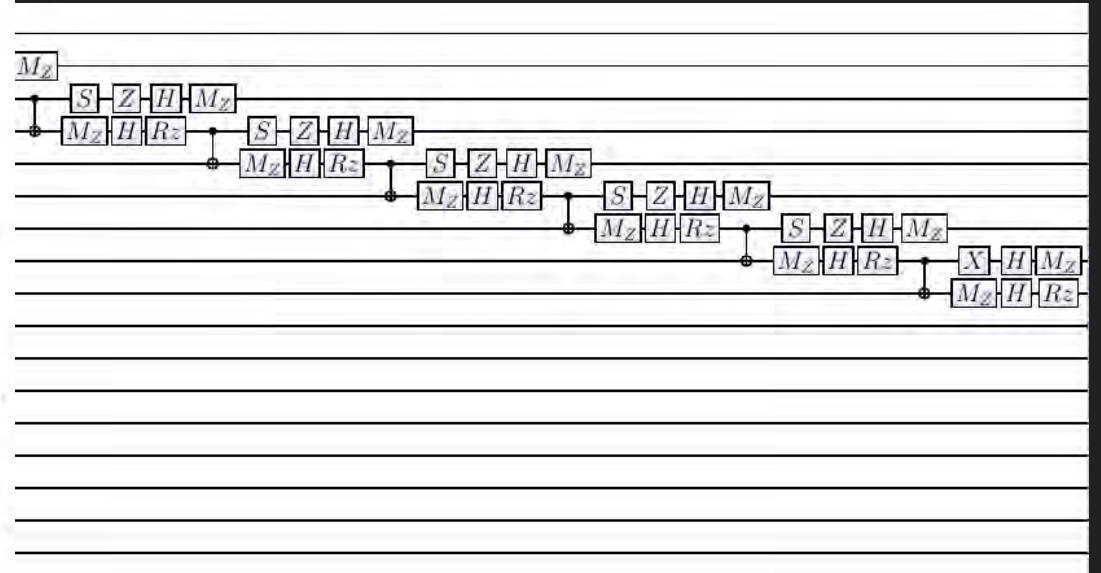
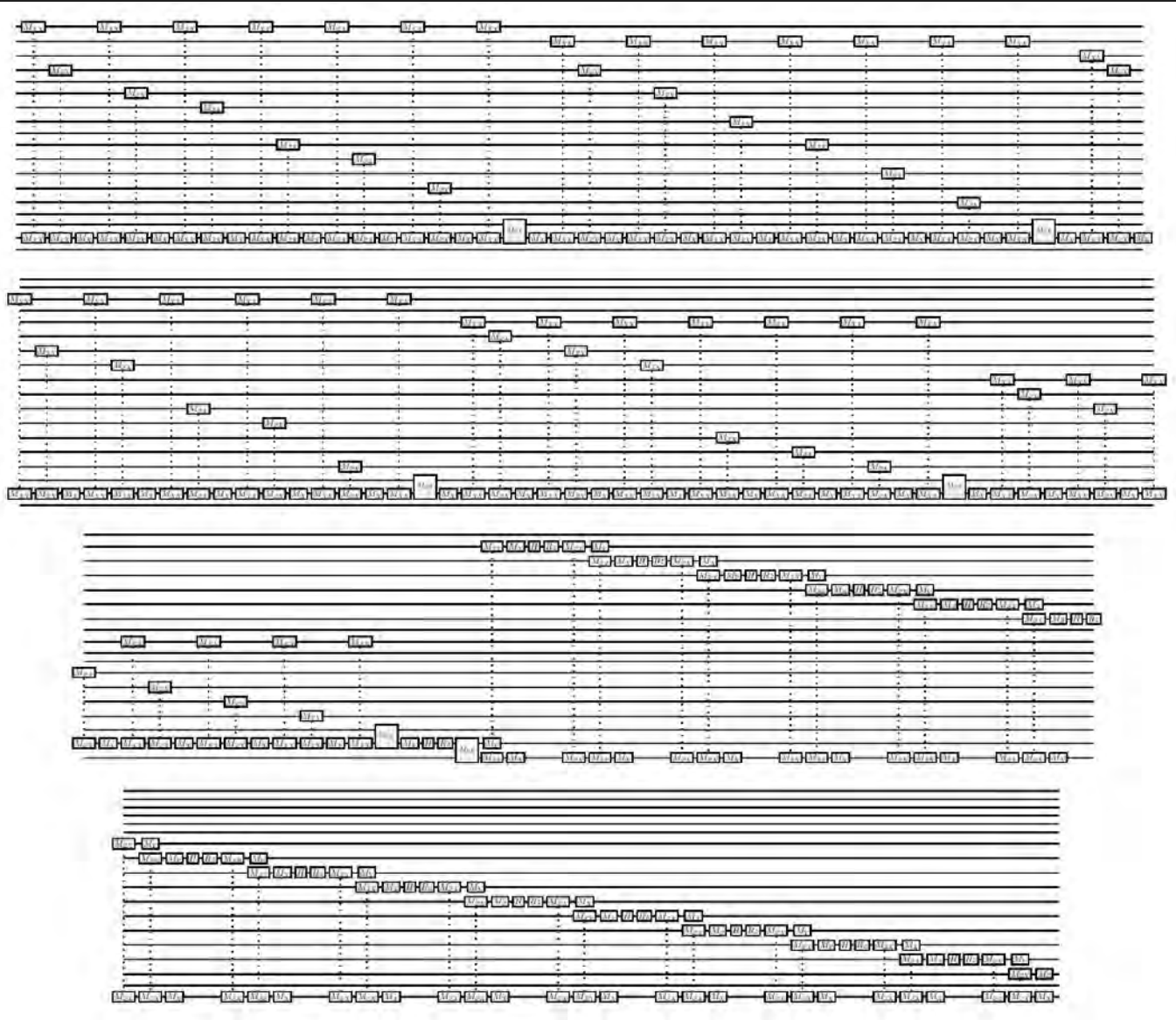
# Some of the choices: Hexons and Tetrons



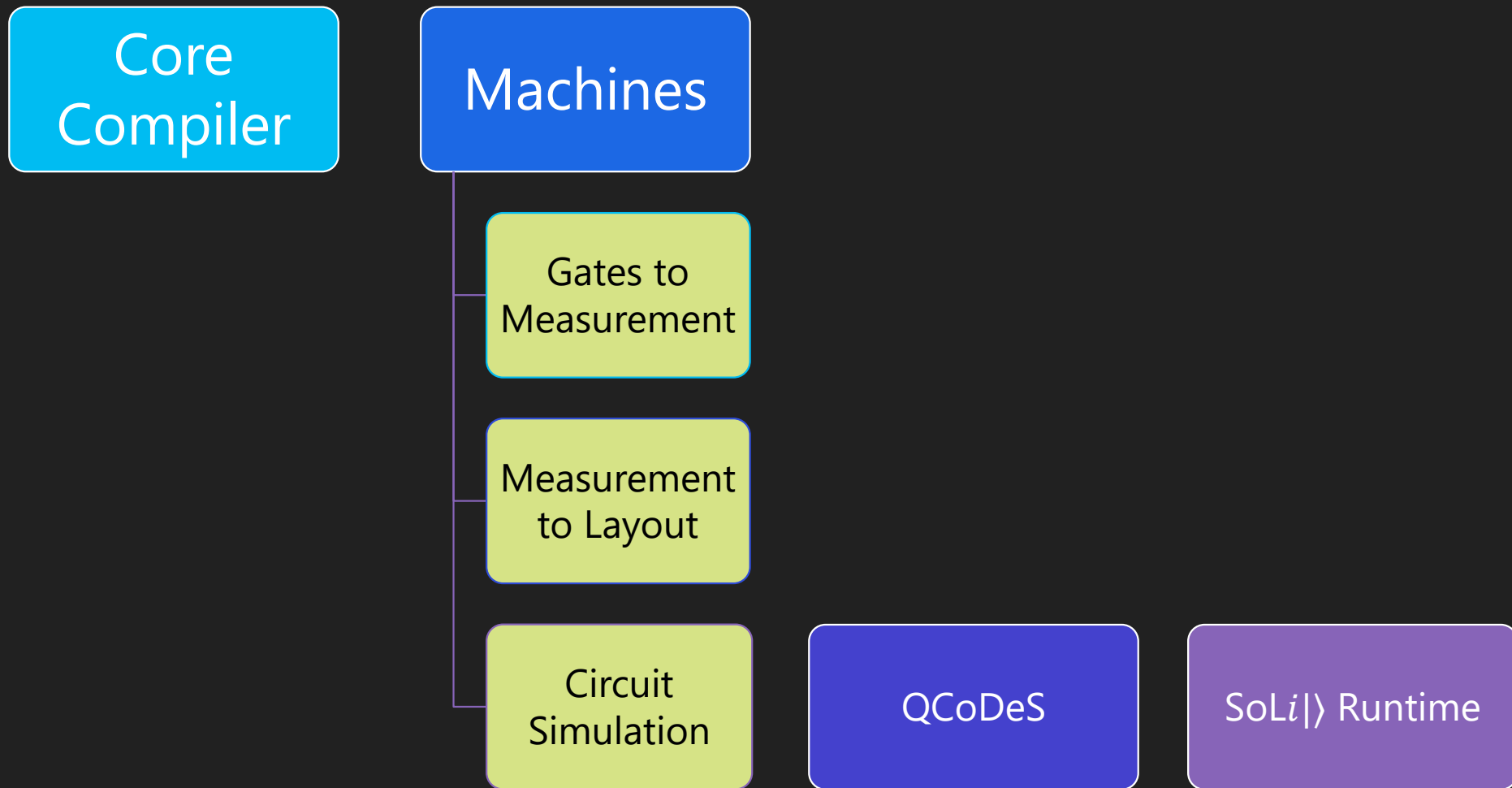
# What about the single qubit Clifford operators?

- Create a frame (where do the axes point?):  $\{x,y,z\}$
- Each slot  $(x,y,z)$  can contain any of:  $+X, +Y, +Z, -X, -Y, -Z$
- Operators are now defined as:
  - X: Flip signs of  $y$  and  $z$
  - Y: Flip signs of  $x$  and  $z$
  - Z: Flip signs of  $x$  and  $y$
  - H: Flip sign of  $y$  and interchange  $x$  and  $z$
  - S: Interchange  $x$  with  $-y$  and  $y$  with  $x$
- Measure in a basis  $(X,Y,Z)$  by looking at the entry for that basis  $(x,y,z)$ :
  - E.g., with  $\{Z,-Y,X\}$  an  $M_x$  is  $M_z$ ,  $M_y$  is  $-M_y$  and  $M_z$  is  $M_x$
  - Two qubit joint measurements are done according to each qubits' frame

# What about T gates?



# Compiling with SoLi|⟩

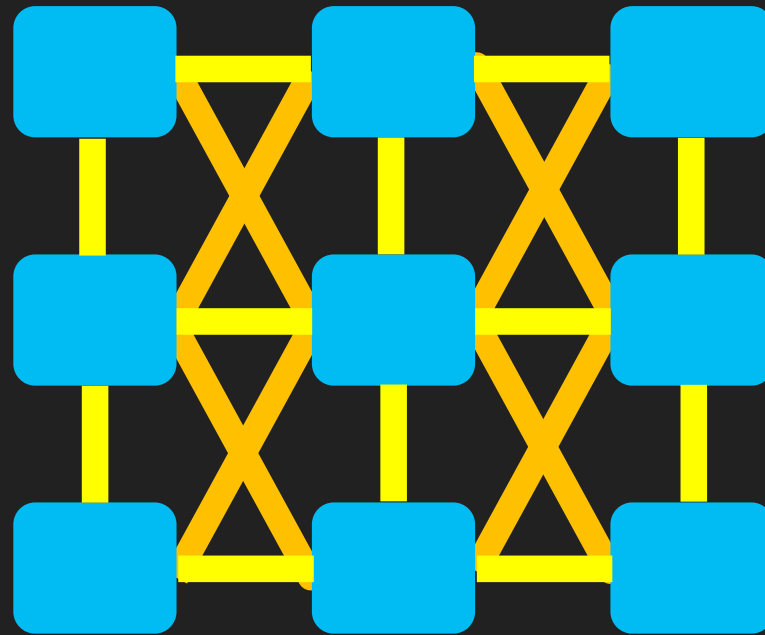




- <http://qcodes.github.io>
- Python package for running experiments
  - Runs either command-line or within a Jupyter notebook
  - Targets quantum computing experiments but is general purpose
- Plug-in model for device drivers
  - Note that these are high-level interaction models, not kernel drivers
- Integrated support for real-time plotting and monitoring
- Integrated support for structured data storage



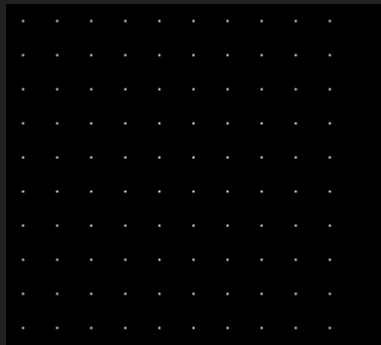
# Layout options



# Layout of T factory (DistillT)

DistillT: 1Q=65 2Q=100 LogQ=81 Frames=277

Connect	Dim	Data Rows	Phys Qubits	Data Teleport	Block Tele	Par Tele Depth
Rect	10x10	All	42	15/9	20/9	$4*(9+9)=72$
Rect	5x9	All	39	18/13	25/13	$4*(13+13)=104$
Rect	3x18	Half	39	40/31	40/31	$4*(31+31)=248$
Diag	3x9	Half	26	15/9	15/9	$4*(9+9)=72$
Diag	2x18	Half	34	39/24	36/24	$4*(24+24)=192$



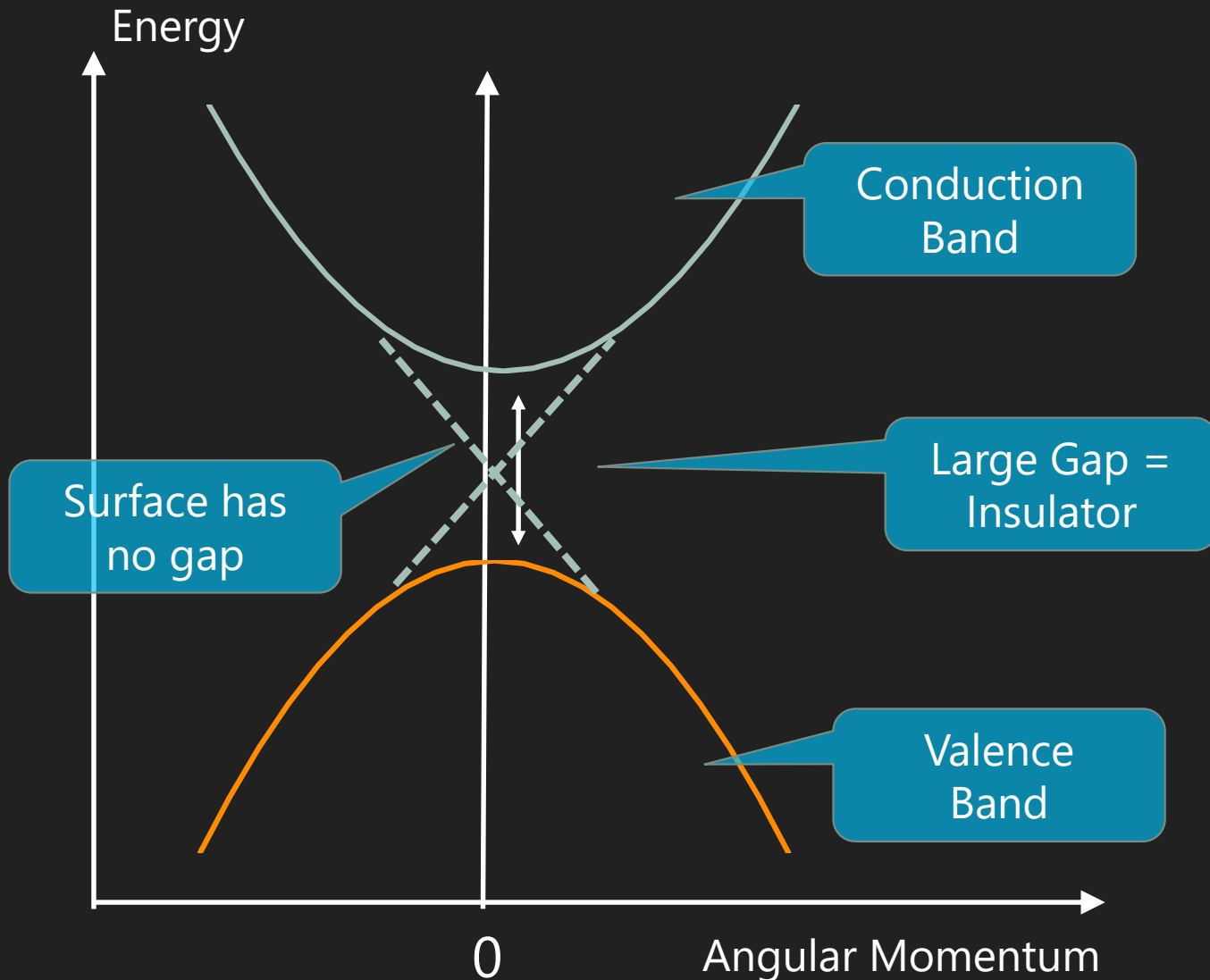


Thank You!



Photos courtesy of: Professor Charlie Marcus

# Topological Insulators

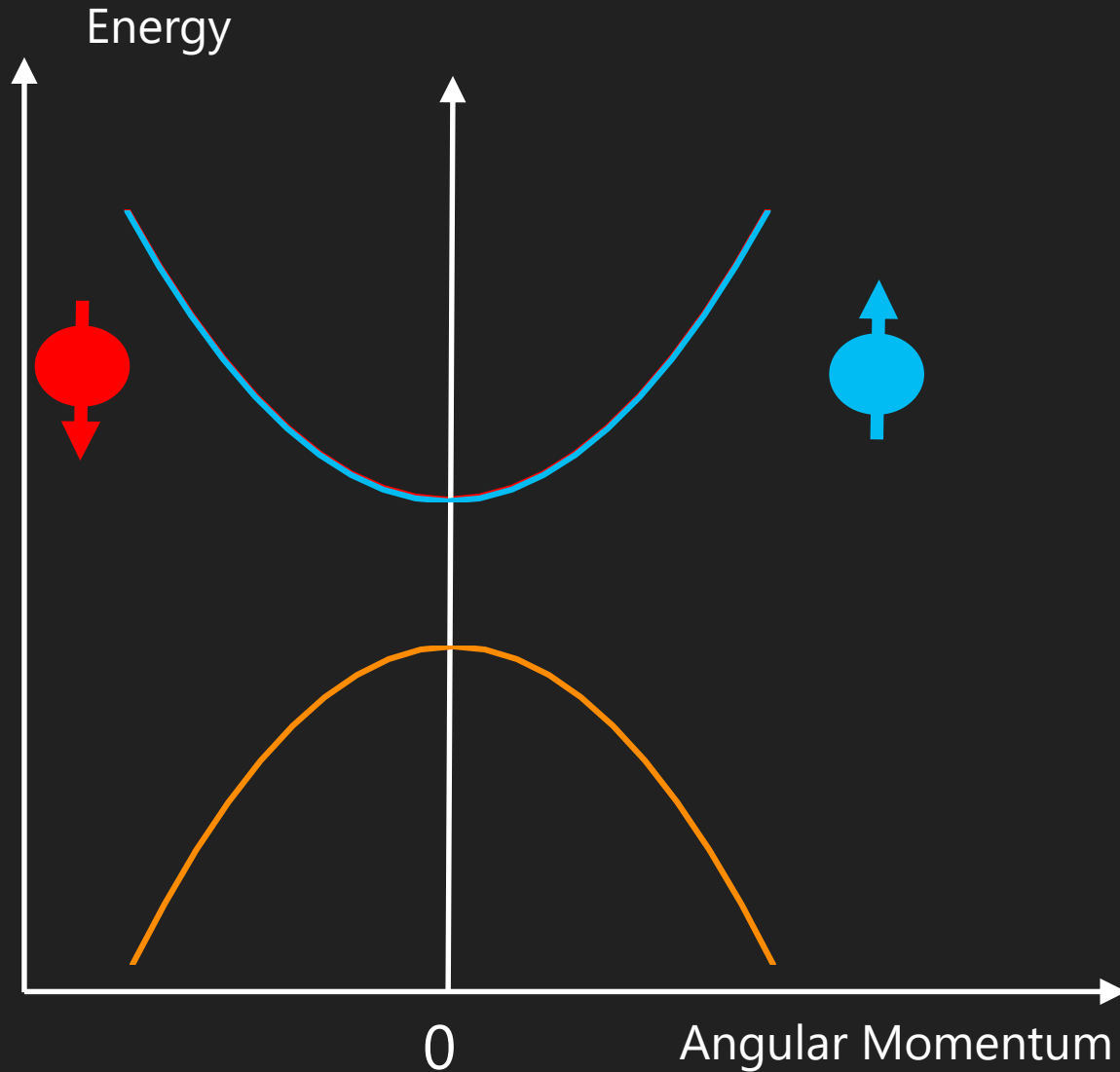


- Normal insulator (or un-doped semiconductor) has a large gap between the valence band and the conduction band
- A topological insulator has no gap at the boundary, acts as a conductor while the bulk is an insulator
- The surface of a 3D TI can be viewed as a 2D Electron Gas (2DEG)

*After Hasan, Kane 2010*

<http://arxiv.org/abs/1002.3895>

# Spin-Orbit Coupling



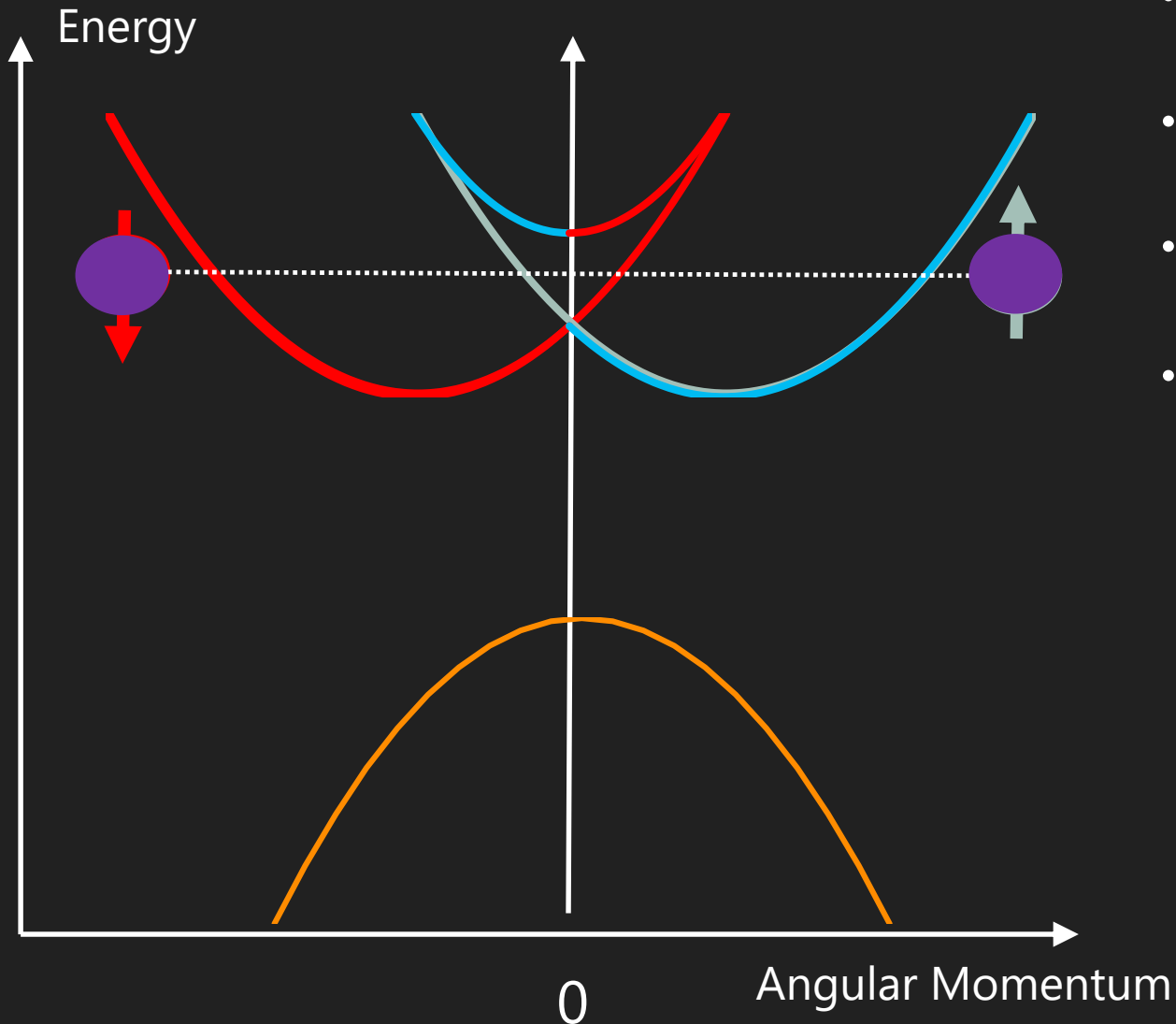
- If you make the TI out of large atoms, then the electrons are moving at relativistic velocity and there will have strong spin-orbit interaction (Rashba coupling).
- Electrons moving one way will be spin-up and the other way will be spin-down

$$H_{TI} = \int dx \psi^\dagger \left[ -\frac{\partial_x^2}{2m} - \mu - i\hbar v \partial_x \sigma^y \right] \psi$$

*After Jason Alicia, Winter 2010 Q Meeting*



# Making a P-Wave Superconductor



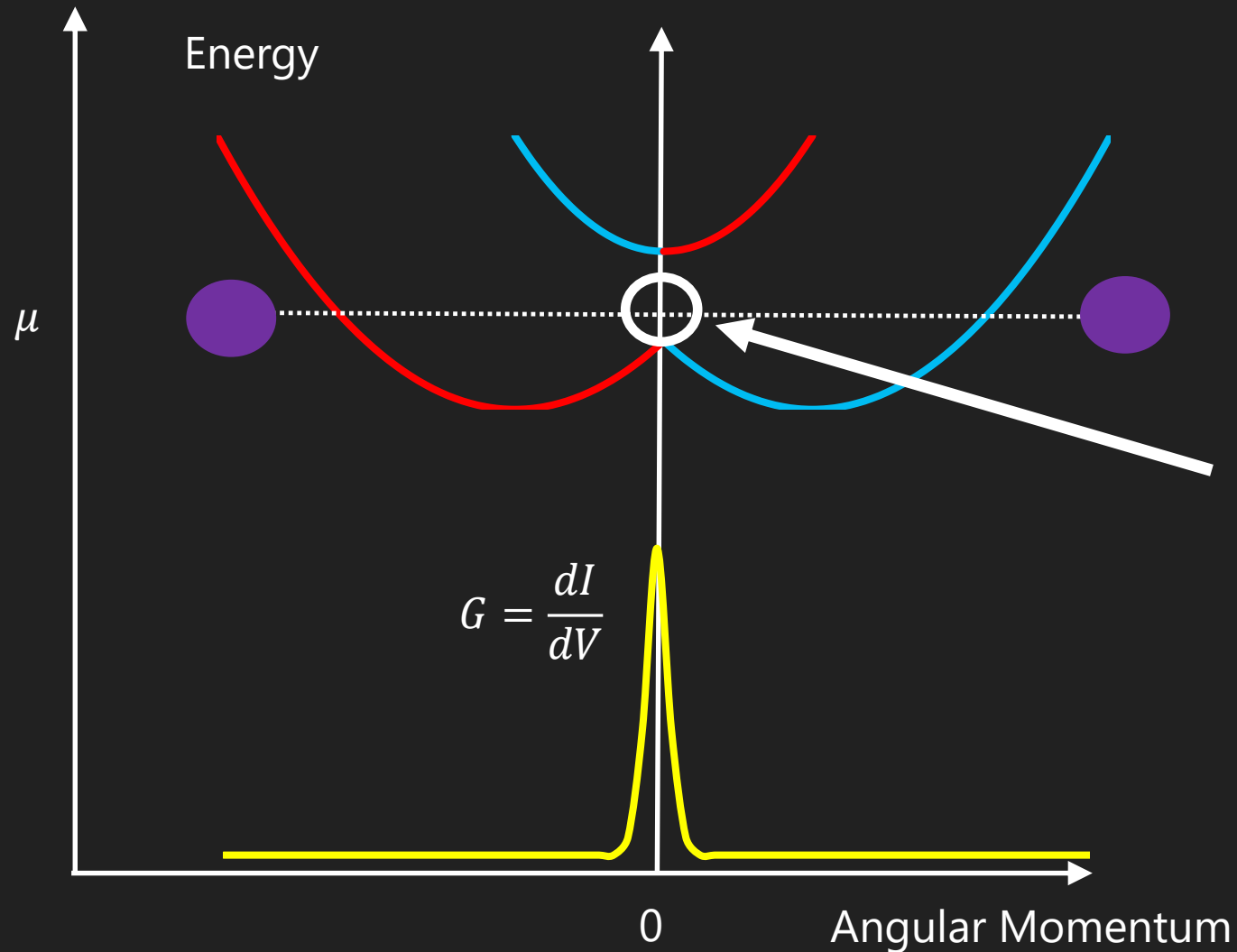
- Now apply a strong magnetic field ( $B$ ) perpendicular to the plane opening a gap (Zeeman field)
- Bring an S-Wave superconductor into close proximity to induce pairing of the electrons.
- We've made a chiral P-Wave superconductor ( $p_x + ip_y$ )
- If we have superconductivity in the bulk, it still has a gap that does not allow quasi-particle excitations (topological superconductor)

$$H_{TIB} = \int dx \psi^\dagger \left[ -\frac{\partial_x^2}{2m} - \mu - i\hbar v \partial_x \sigma^y - \frac{g\mu_B B}{2} \sigma^z \right] \psi$$

$$H_{p\text{-wave}} = H_{TIB} + (\Delta \psi_\uparrow \psi_\downarrow + h.c.)$$

*After Jason Alicia, Winter 2010 Q Meeting*

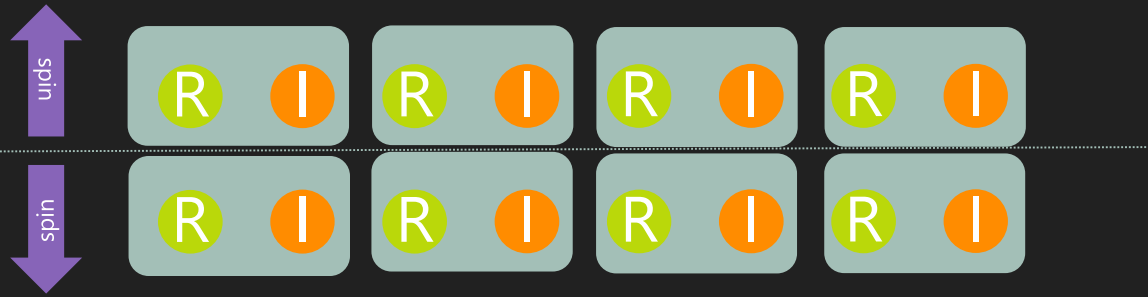
# Majorana Quasiparticle



- Set the chemical potential ( $\mu$ ) in the gap
- Measure differential conductance ( $dI/dV$ )
- Zero mode peak has:
  - no spin
  - no mass
  - no charge
- Signature of a Majorana quasiparticle
- Only appears at defects or edges

*After Leo Kouwenhoven, Summer 2012 Q Meeting*

Insulator:



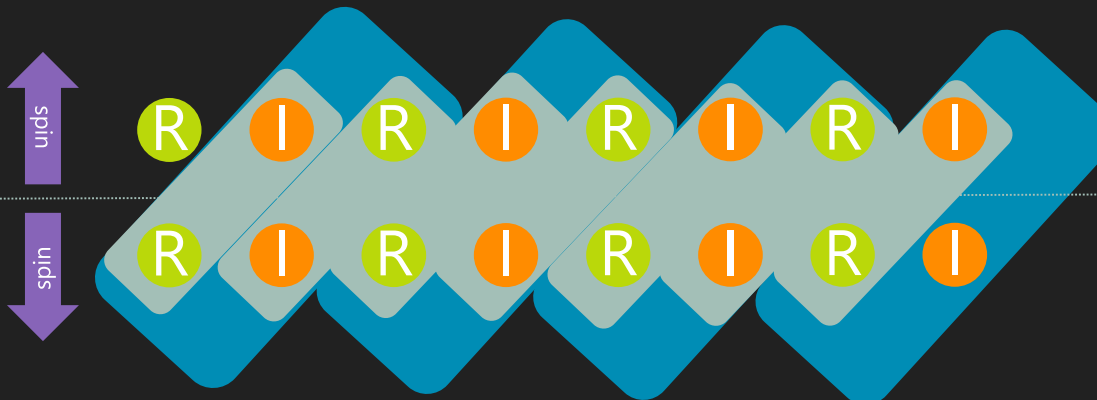
Electron

Pair

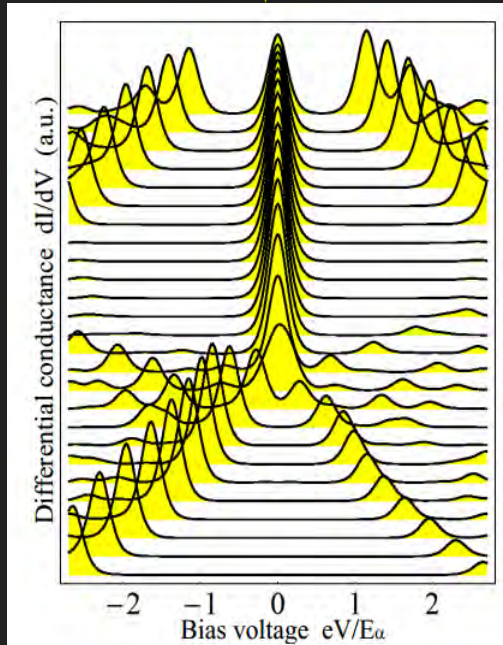
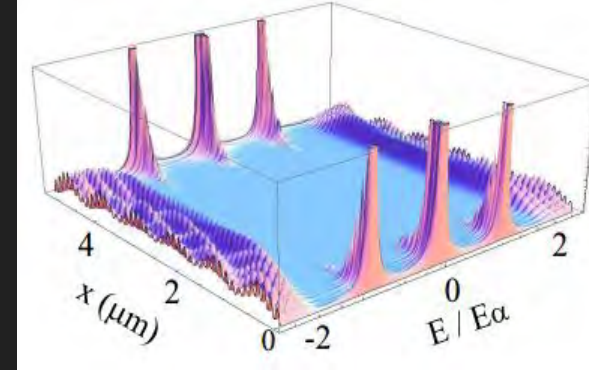
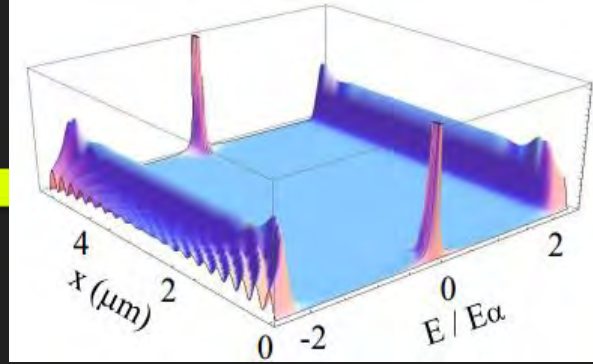
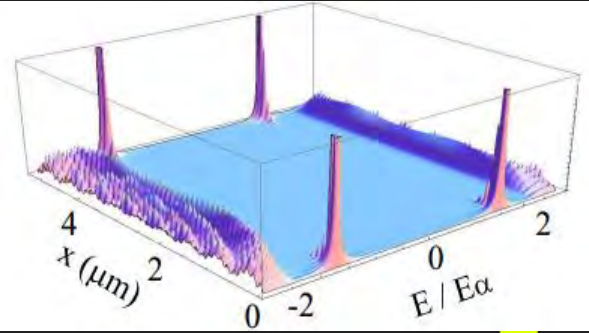
Normal S-Wave  
Superconductor:



Topological  
Superconductor:



# Signature of a MF in a magnetic field



- In a wire, we get the zero modes at the ends of the wire.
- If we vary the magnetic field, we can see the zero mode only appears when the field is strong enough to open a gap.
- If the field is stronger than the spin-orbit coupling, we get back into a non-topological phase

After Stanescu, Lutchyn, Das Sarma 2011  
<http://arxiv.org/abs/1106.3078>

# Periodic Table of Topological Insulators

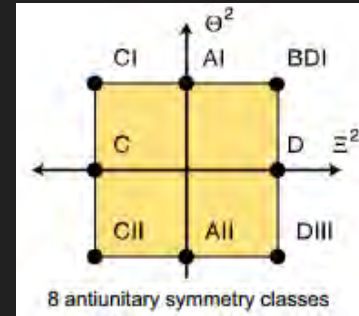
Symmetry				$d$							
AZ	$\Theta$	$\Xi$	$\Pi$	1	2	3	4	5	6	7	8
A	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$
AIII	0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0	$\mathbb{Z}$	0
AI	1	0	0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
D	0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
DIII	-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$	0
AII	-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0	$\mathbb{Z}$
CII	-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0	0
C	0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
CI	1	-1	1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0

Symmetries:  $\pm 1$  = Square of symmetry

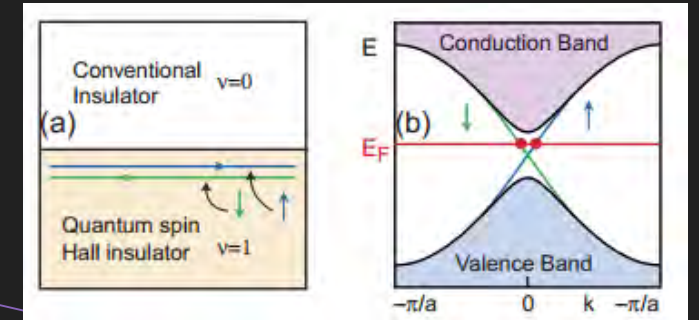
$\Theta$  = Time reversal:  $\Theta H(\mathbf{k})\Theta^{-1} = +H(-\mathbf{k})$

$\Xi$  = Particle-Hole:  $\Xi H(\mathbf{k})\Xi^{-1} = -H(-\mathbf{k})$

$\Pi$  = Chiral  $\propto \Theta\Xi$ :  $\Pi H(\mathbf{k})\Pi^{-1} = -H(\mathbf{k})$



Altland-Zirnbauer Random Matrix Classes  
<http://arxiv.org/abs/cond-mat/9602137>



Integer Quantum Hall Effect (GaAs)

Carbon nanotubes

TI (*BiSb*)

Superfluid  $^3\text{He} - B$

Majorana chain (*InSb*)

$p_x + ip_y$  (*SrRuO<sub>4</sub>*)

TI (*HgTe*)

Bott Periodicity:  $d \rightarrow d + 8$

After Hasan, Kane 2010 <http://arxiv.org/abs/1002.3895>  
 and Kitaev 2009 <http://arxiv.org/abs/0901.2686>  
 and Freedman et. al. 2010 <http://arxiv.org/abs/1005.0583>