BLACK HOLE ENTROPY ENTANGLEMENT AND HOLOGRAPHIC SPACETIME

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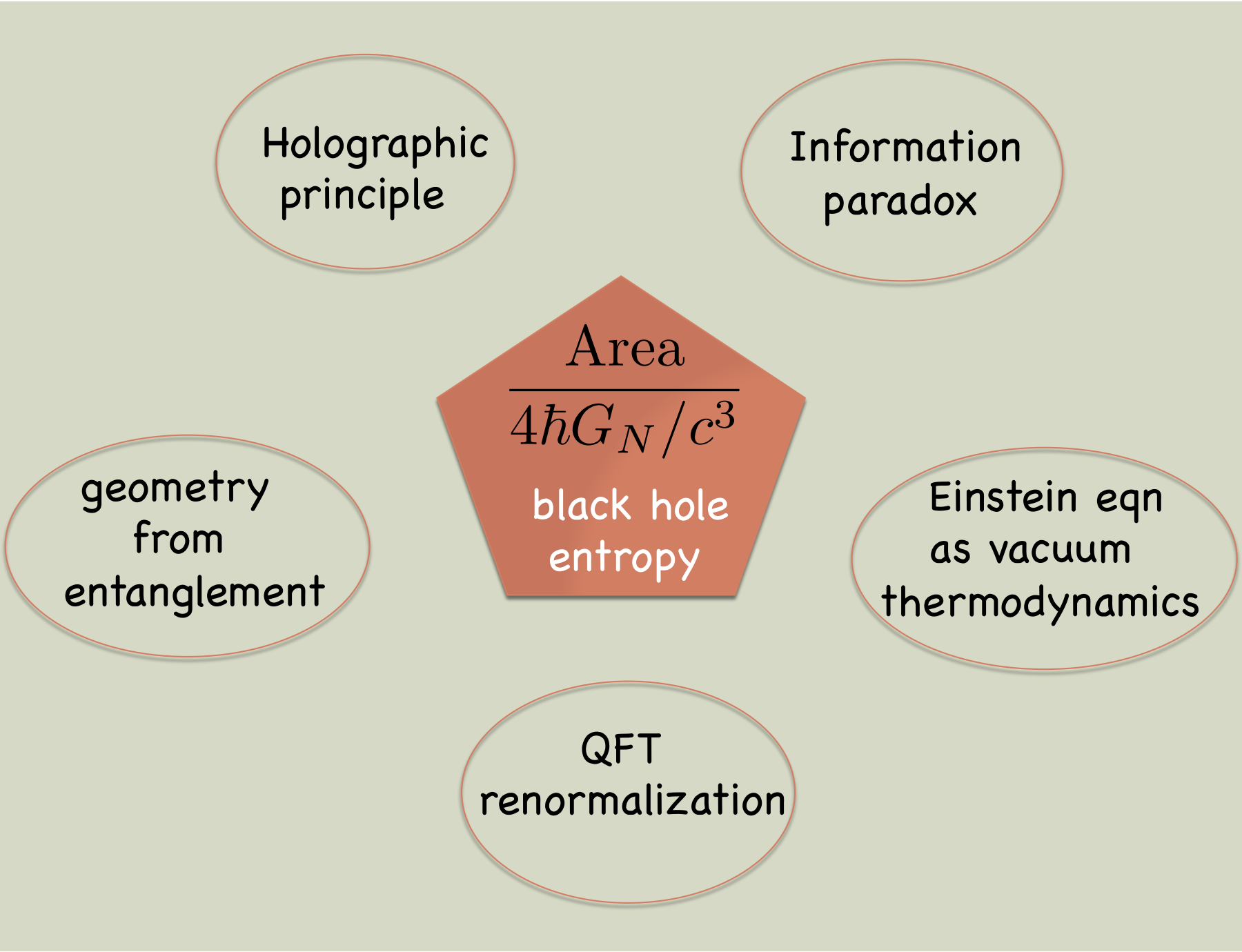
University of Maryland

Goddard Scientific Colloquium, Feb. 7, 2018

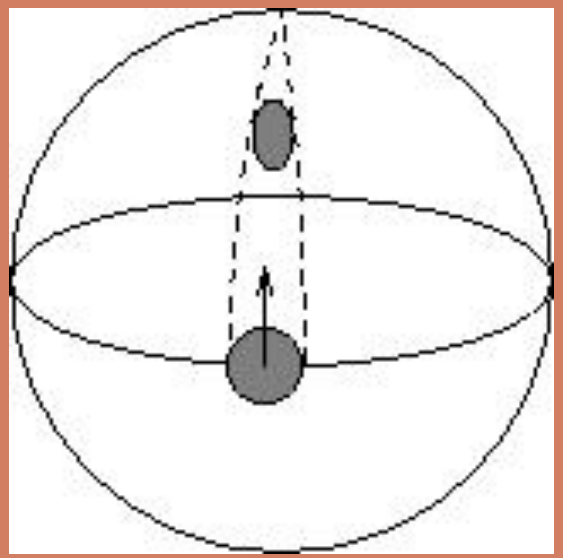

Holographic principle
Information paradox
$\frac{\text{Area}}{4\hbar G_N/c^3}$
black hole entropy
geometry from entanglement
Einstein eqn as vacuum thermodynamics
QFT renormalization

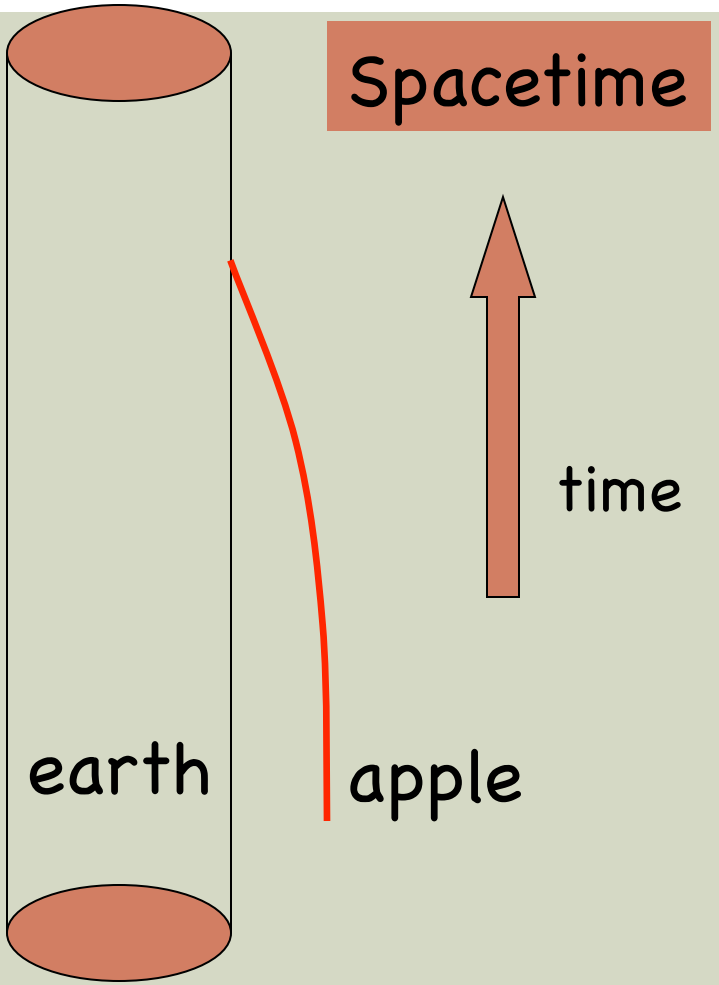
Albert Einstein, aged 33, 1912

GRAVITY IS CURVATURE OF SPACETIME

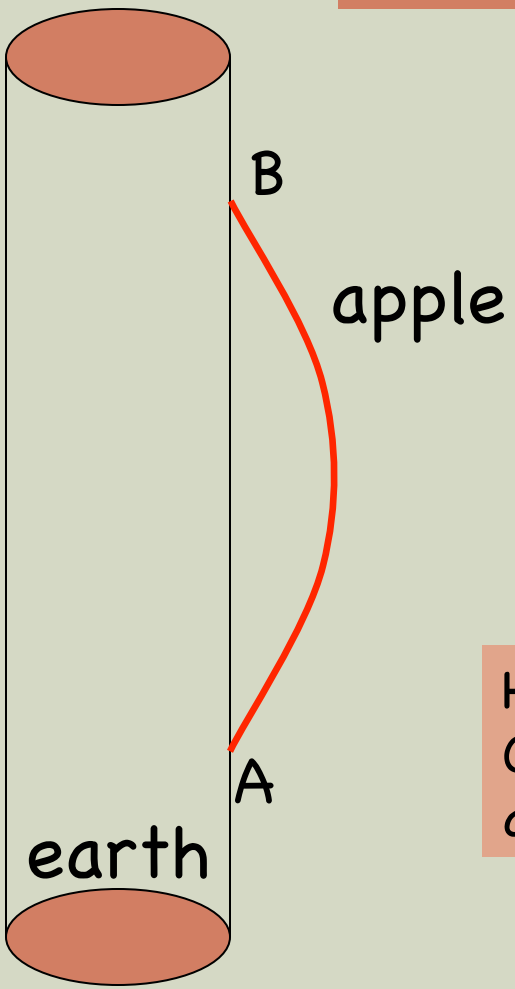
Spatial curvature analogy:

Initially parallel lines
don't stay parallel





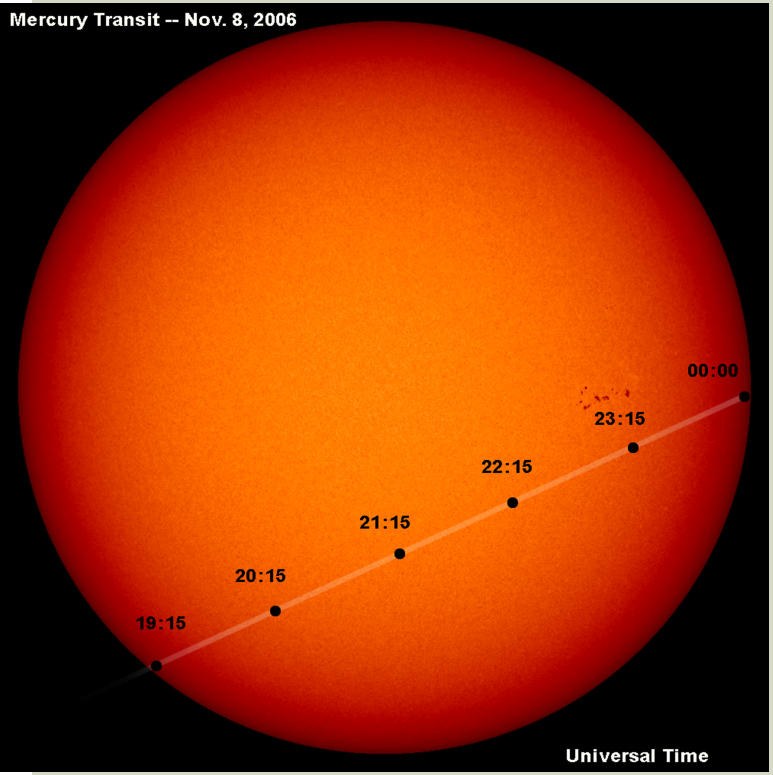
Time runs slower lower down!



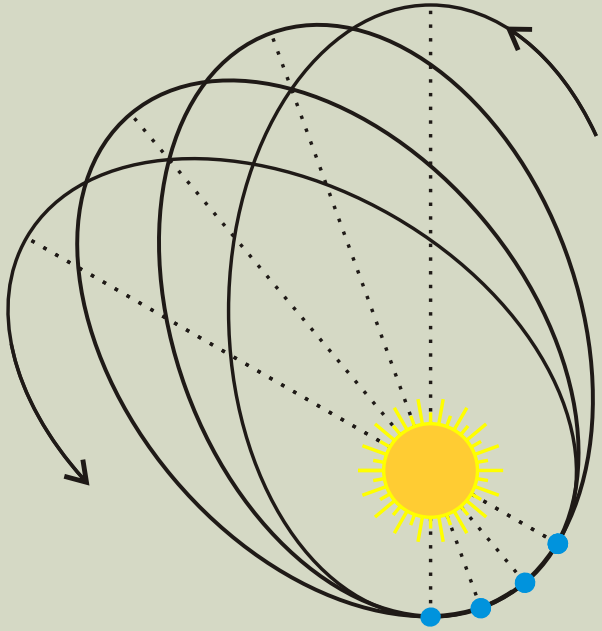
Apple free-fall
is the straightest path
in spacetime between A & B…
and the path of longest time.

How much slower?
One billionth of a second per year per foot at the earth's surface (g/c^2).

Spiral of Mercury's orbit: didn't fit Newton's theory, by 43"/century...



That's about 9 minutes advance time for the transit per century...

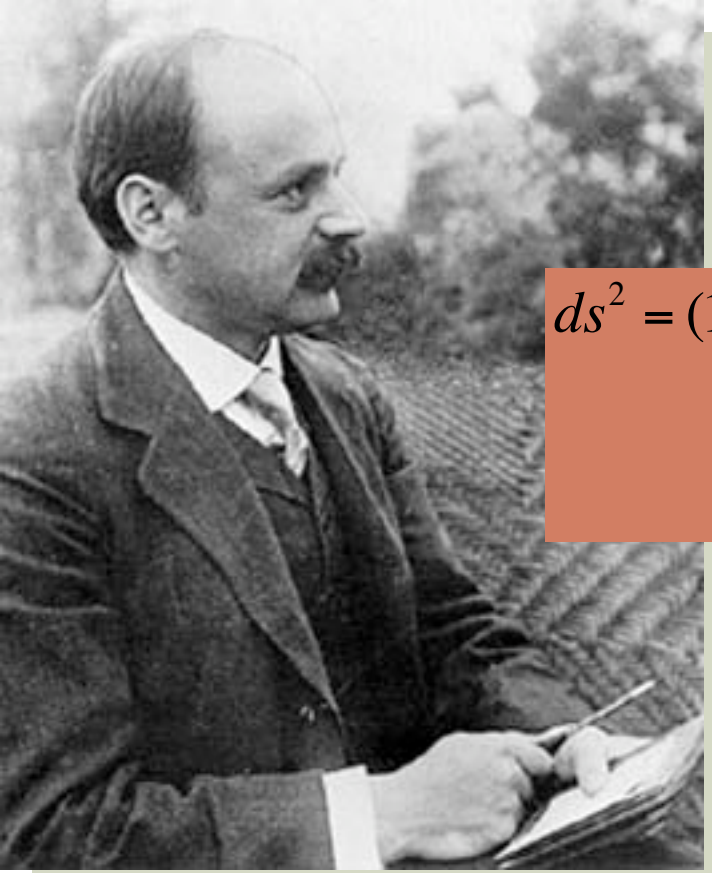


General relativity nailed it.

To calculate the rate of perihelion advance Einstein needed only the first approximation to the line element outside the sun:

$$ds^2 = (1 - r_S/r)dt^2 - (1 + r_S/r)dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$r_S = 2GM/c^2 = 3\text{km}$$



The Schwarzschild Singularity (1916)

$$ds^2 = (1 - r_S/r)dt^2 - (1 - r_S/r)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$r_S = 2GM/c^2 = 3\text{km} \quad \text{"Schwarzschild radius"}$$

Schwarzschild (1916): "in order that this discontinuity coincides with the origin" one should define the radial coordinate appropriately.

Droste (1916): "a moving particle can never pass that sphere because it would take an infinite amount of time"

The true, non-singular nature of the Schwarzschild "singularity" was not *widely* understood until 42 years later... but it was understood perfectly well by one man in 1932...

Georges Lemaitre

First person to understand the nature of the Schwarzschild singularity as an event horizon (1932).

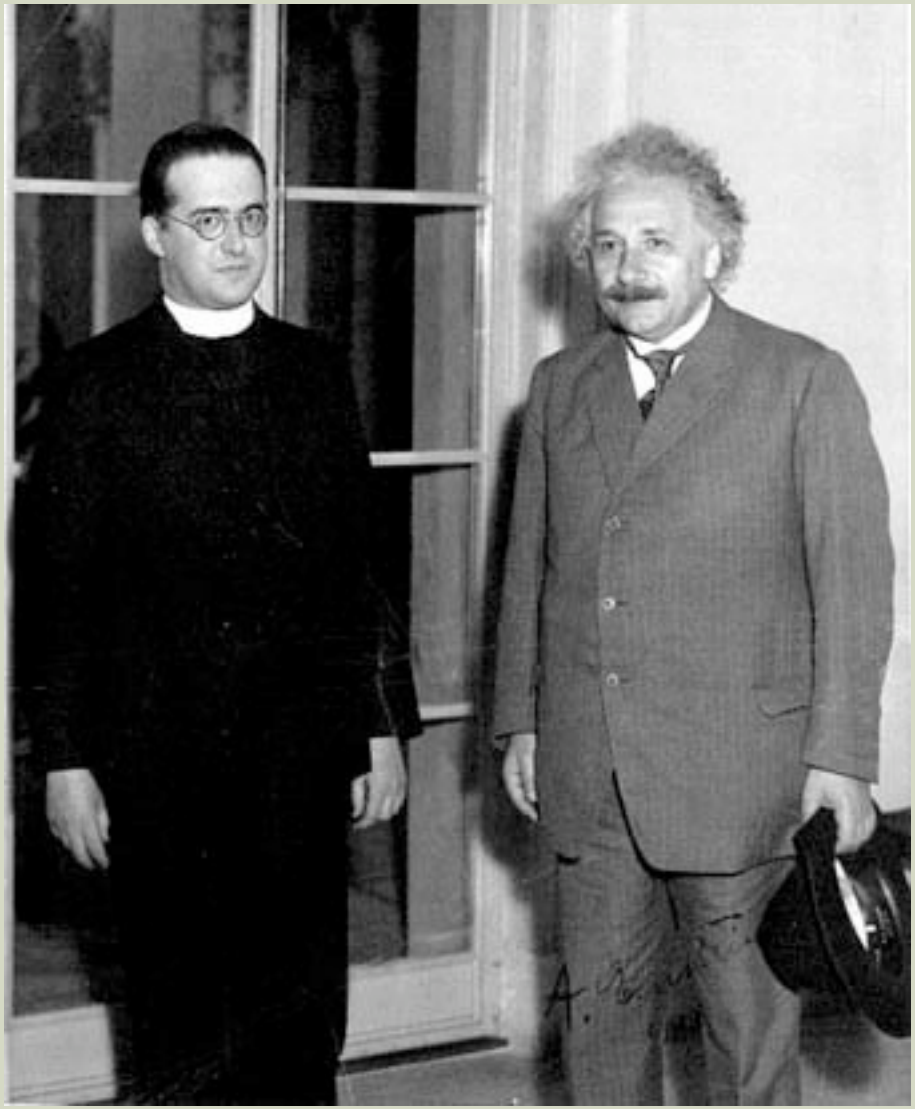
"The singularity of the Schwarzschild field is thus a fictitious singularity, analogous to that which appears at the horizon of the cylindrical original form of the de Sitter universe."

Belgian mathematician and physicist, Catholic priest,

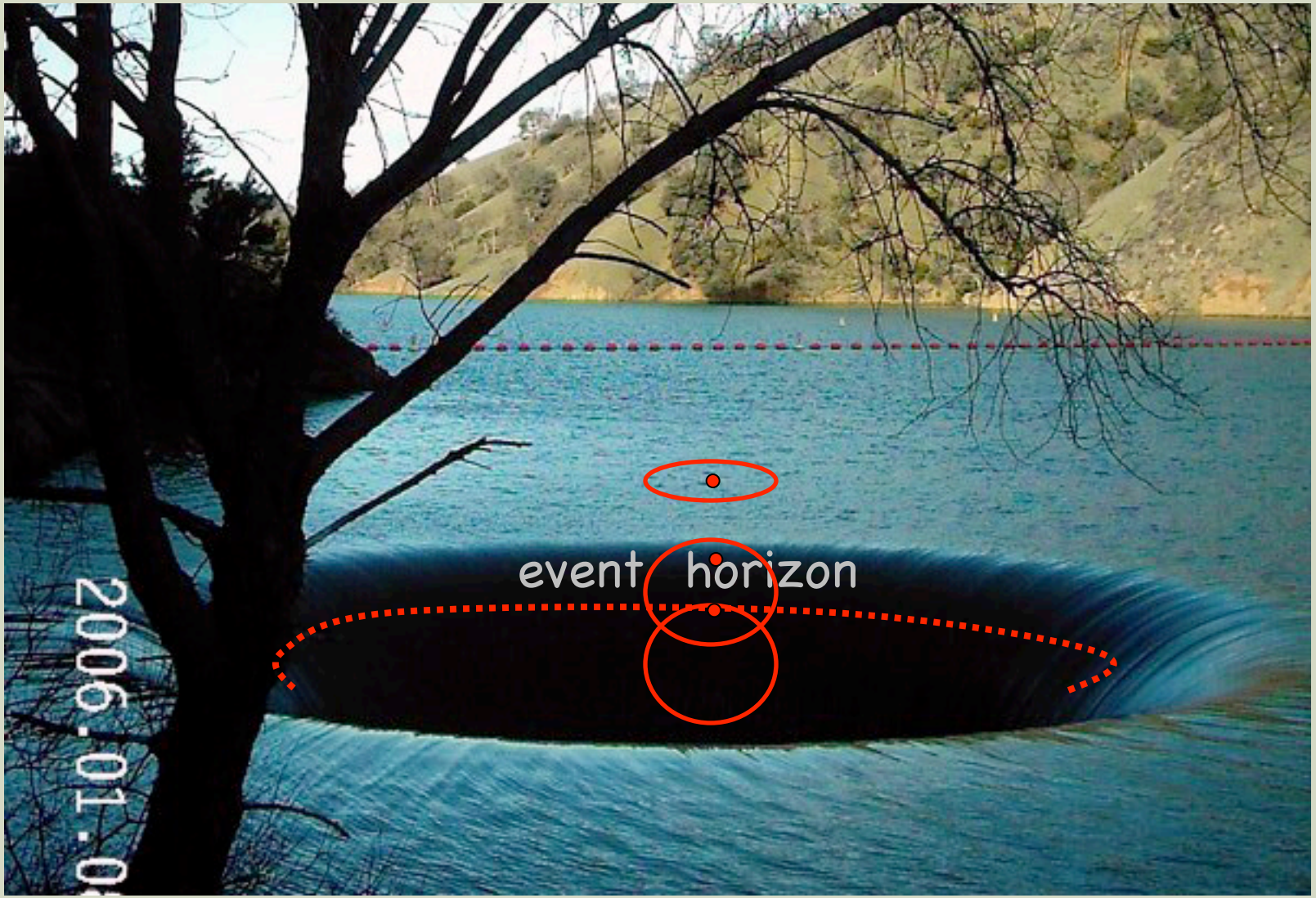
Educated at Cambridge and MIT

Pioneer of physical cosmology:

Expanding universe, cosmic fireball, origin of structure, beginning of time...



A black hole analogy



Oppenheimer and Snyder (1939) analyzed *collapsing* matter and showed how an event horizon forms in a finite time.

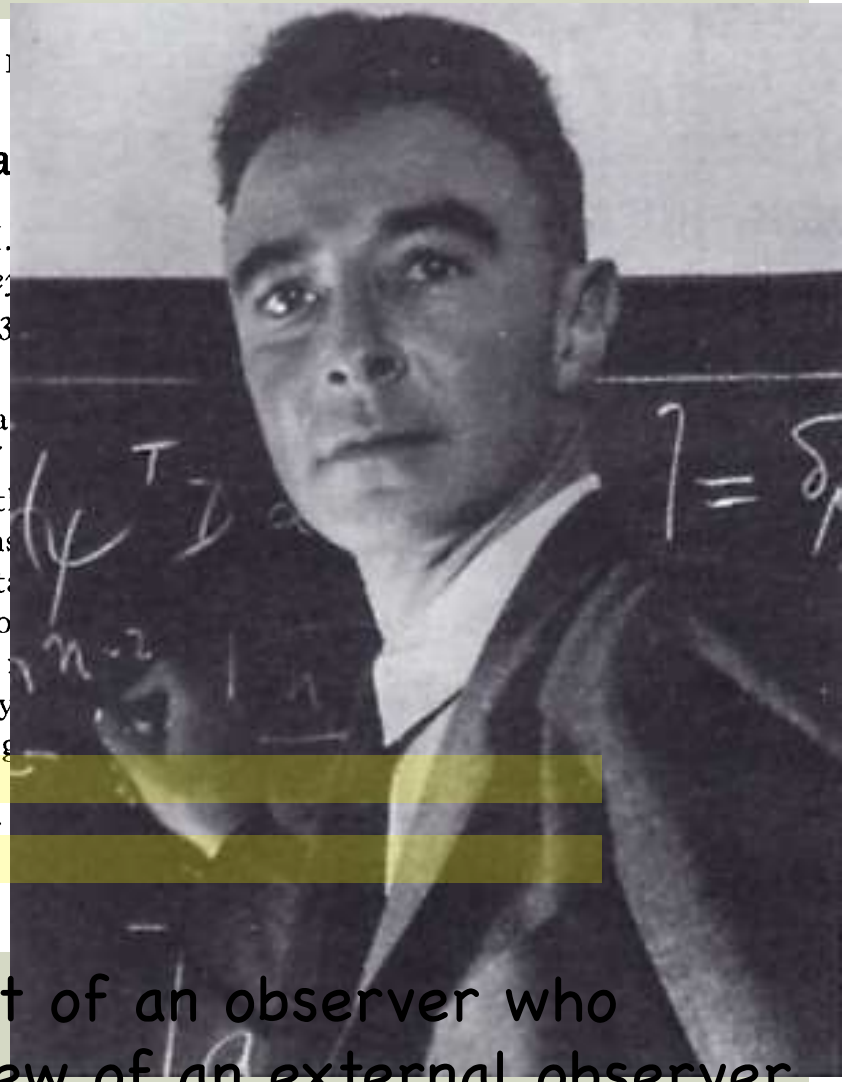
SEPTEMBER 1, 1939 PHYSICAL REVI

On Continued Gravitationa

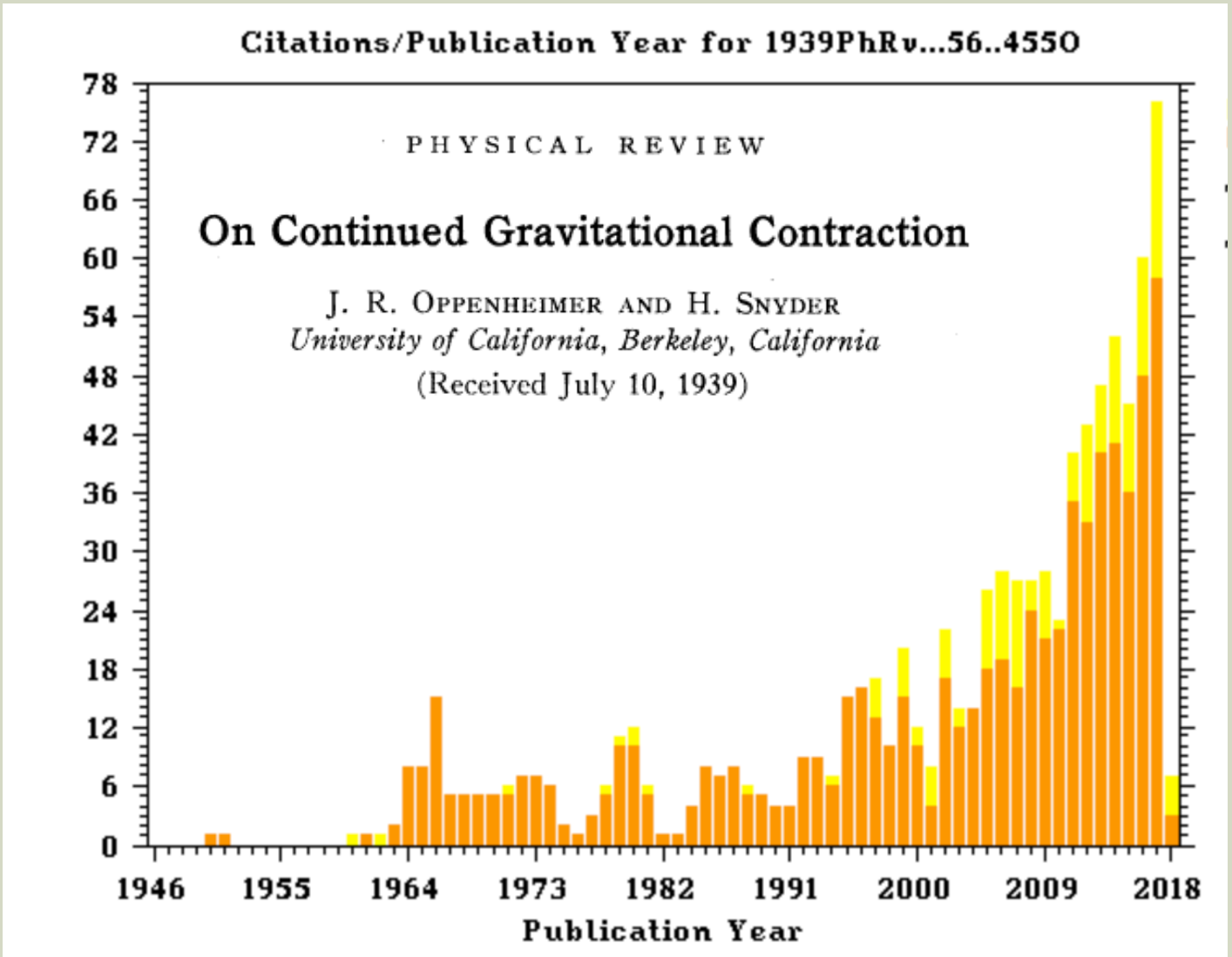
J. R. OPPENHEIMER AND H.
University of California, Berkele

(Received July 10, 193

When all thermonuclear sources of energy are exha
collapse. Unless fission due to rotation, the radiation of
radiation, reduce the star's mass to the order of that of t
indefinitely. In the present paper we study the solutions
which describe this process. In I, general and qualit
behavior of the metrical tensor as the contraction pro
proaches asymptotically its gravitational radius; light
gressively reddened, and can escape over a progressively
analytic solution of the field equations confirming these
case that the pressure within the star can be neglected.
server comoving with the stellar matter is finite, and for
masses, of the order of a day; an external observer sees
its gravitational radius.



First paper to discuss the viewpoint of an observer who falls in, and contrasted with the view of an external observer.


Citations/Publication Year for 1939PhRv...56..455O
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PHYSICAL REVIEW
On Continued Gravitational Contraction
J. R. Oppenheimer and H. Snyder
University of California, Berkeley, California
(Received July 10, 1939)
1946
1955
1964
1973
1982
1991
2000
2009
2018
Publication Year

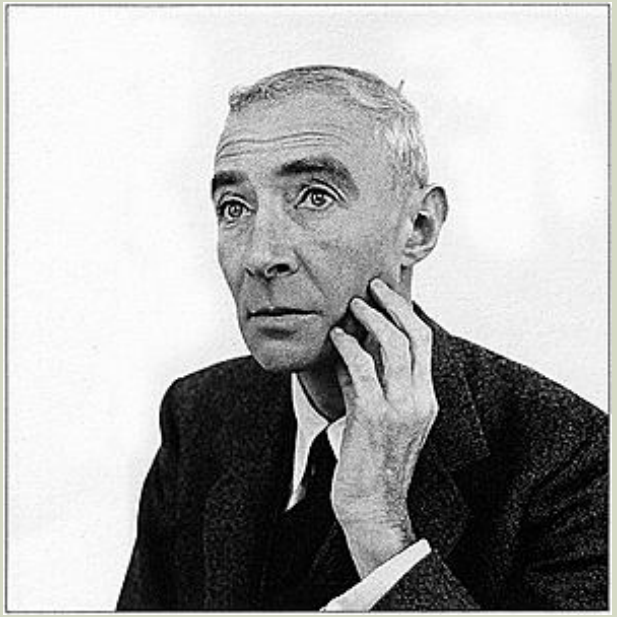
Twenty years later, it still wasn't accepted.

At 1958 Solvay Congress:

Wheeler

Wheeler:
The collapse theory "does not give an acceptable answer" to the fate of matter in such a star. (He thought matter would − or at least might − convert to radiation, which would escape and drain the energy.)

Oppenheimer:
"Would not the simplest assumption about the fate of a star more than the critical mass be this, that it undergoes continued gravitational contraction and cuts itself off from the rest of the universe?"



Oppenheimer,1958
(by Richard Avedon)

Psychological resistance:

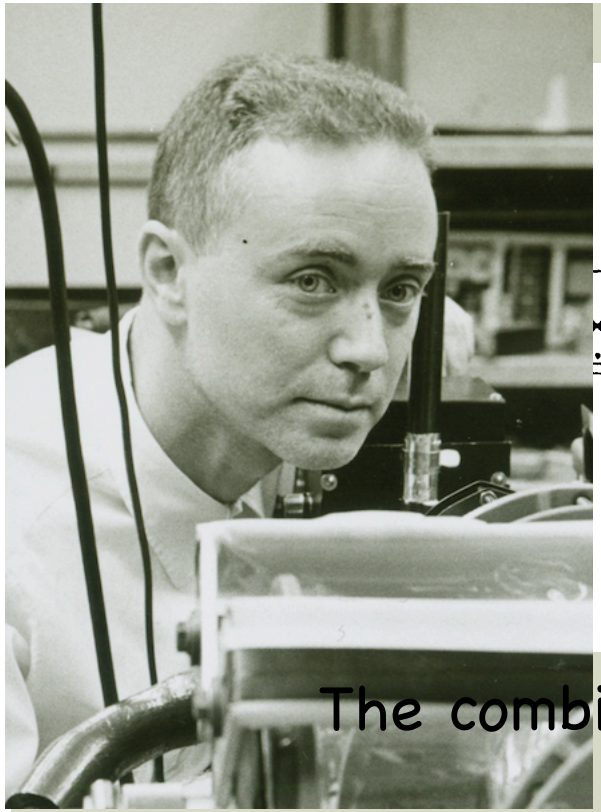
- It was difficult to conceive of collapse to nothing but empty curved space, and the disappearance of matter into the unknown ...
- Spacetime inside the horizon is *not static*: there is no time-translation symmetry.
- Also, missing was a *picture* that could unify the outside and inside views...

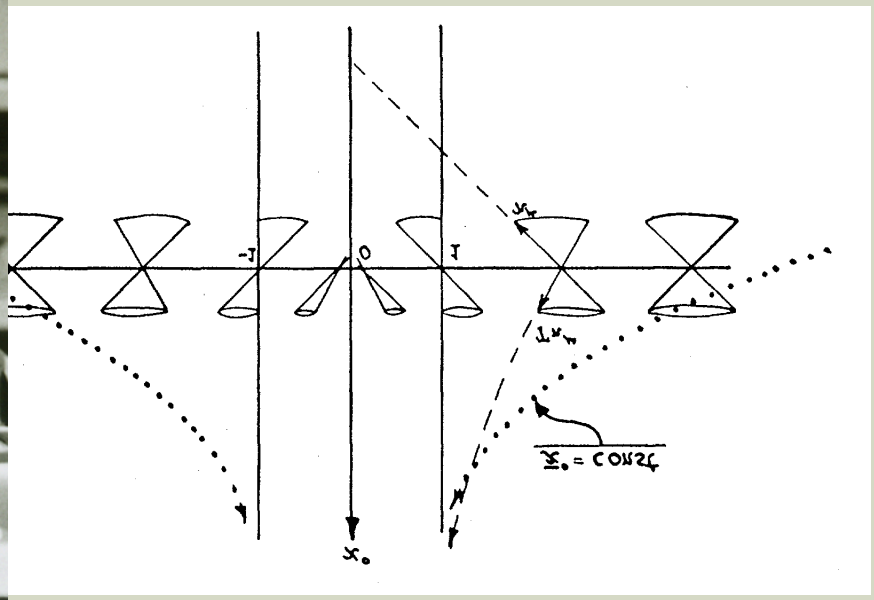
PHYSICAL REVIEW VOLUME 110, NUMBER 4 MAY 15, 1958

Past-Future Asymmetry of the Gravitational Field of a Point Particle

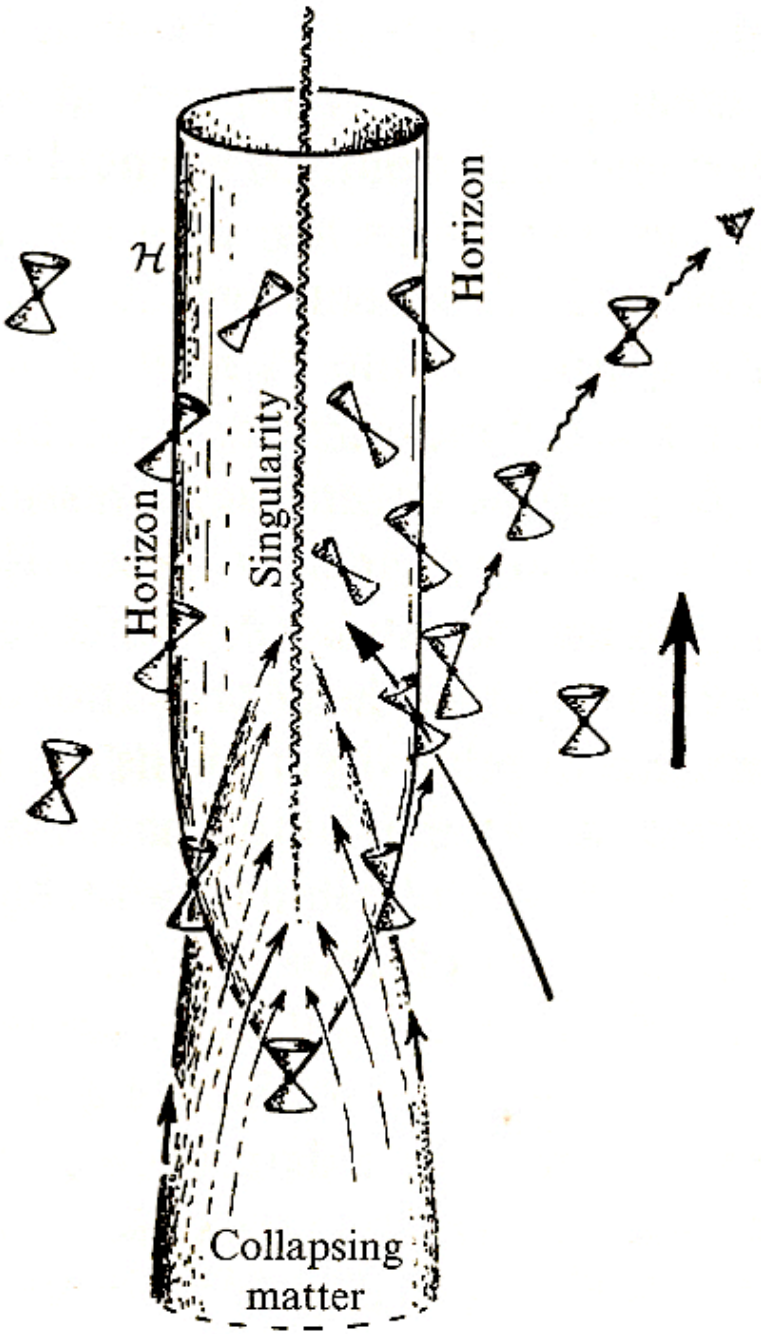
DAVID FINKELSTEIN
Stevens Institute of Technology, Hoboken, New Jersey, and New York University, New York, New York
(Received January 9, 1958)

The analytic extension of the Schwarzschild exterior solution is given in a closed form valid throughout empty space-time and possessing no irregularities except that at the origin. The gravitational field of a spherical point particle is then seen not to be invariant under time reversal for any admissible choice of time coordinate. The Schwarzschild surface $r=2m$ is not a singularity but acts as a perfect unidirectional membrane: causal influences can cross it but only in one direction. The apparent violation of the principle of sufficient reason seems similar to that which is associated with instabilities in other nonlinear phenomena.



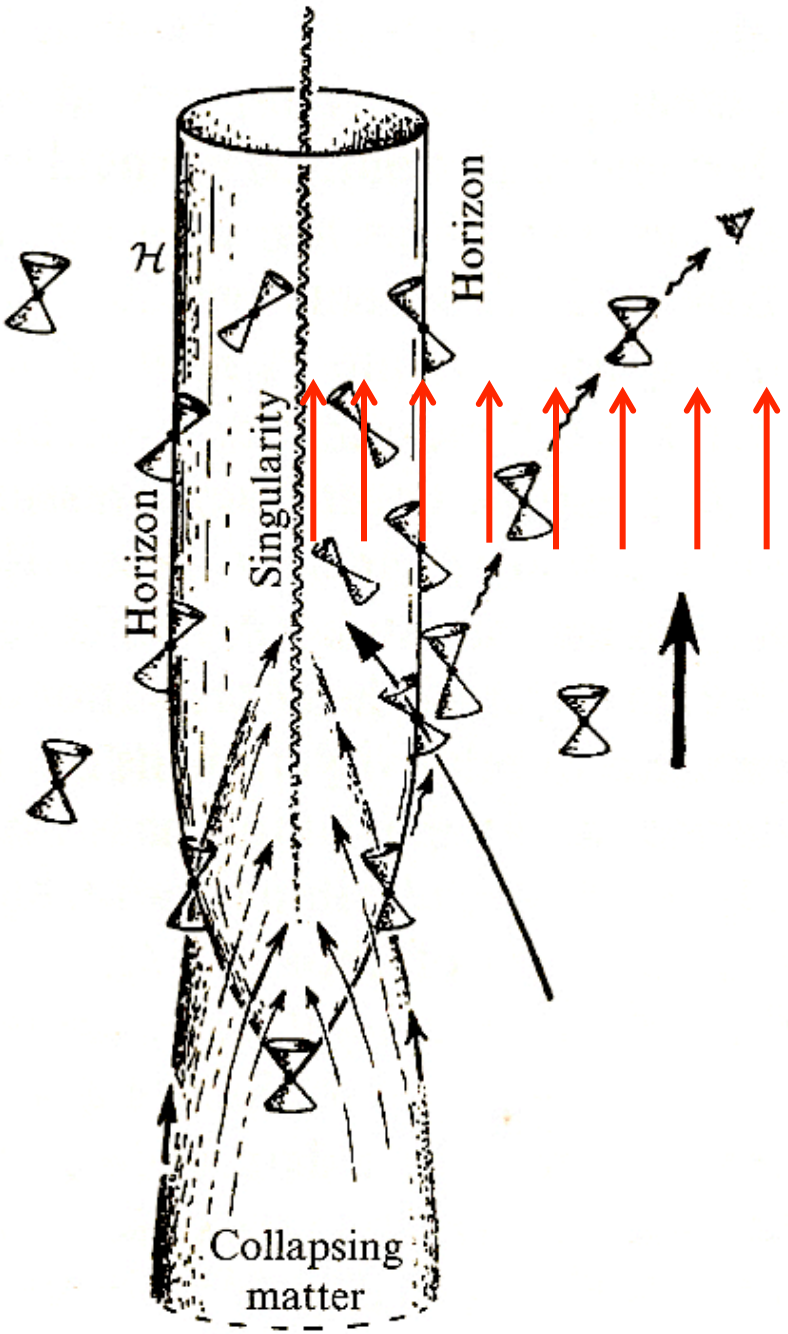


The combined outside-inside view visualized...



The light cones tip over and close off communication with the outside...

Picture by Roger Penrose
The Road to Reality (Knopf, 2005)

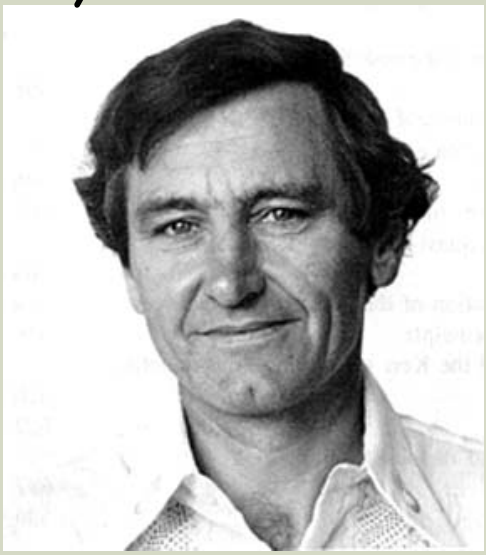


The "time" translation symmetry is *space* translation inside the horizon...

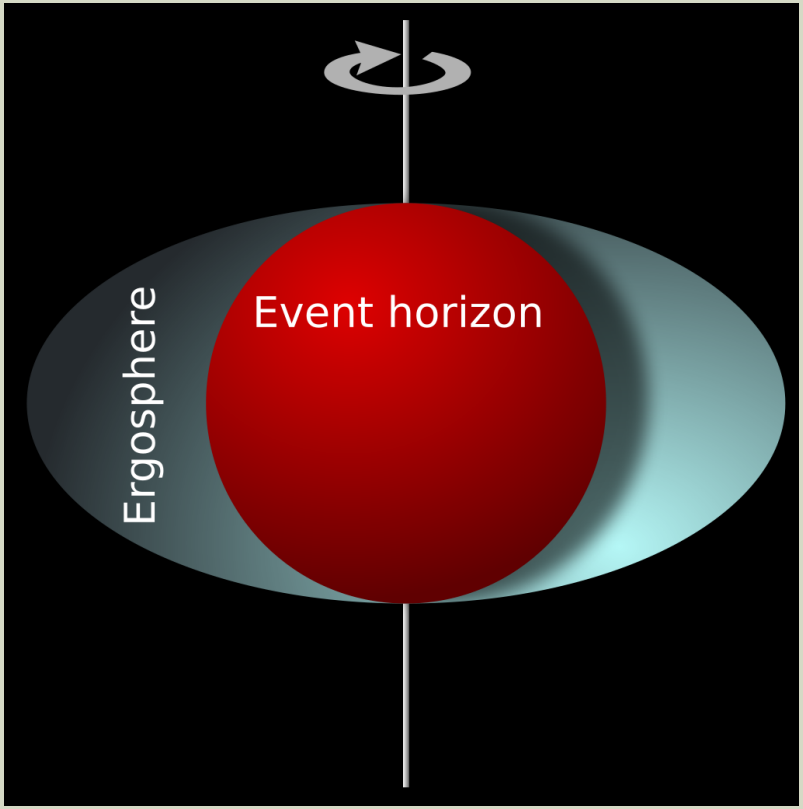
...so the conserved quantity is *momentum* inside the horizon, and can thus be negative there.

Picture by Roger Penrose
The Road to Reality (Knopf, 2005)

Roy Kerr in 1975



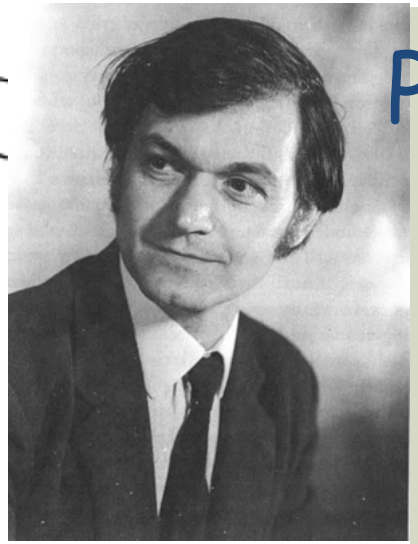
In 1963, Kerr found the exact, spinning black hole solution to Einstein's equation.



The time-translation symmetry becomes *spacelike* in the ergosphere.

This revealed that the rotational energy of a black hole could be tapped!

Penrose process



Extraction of rotational energy from a black hole.

The *mass* can decrease, but not the *horizon area.*

The process is reversible if and only if the area is *unchanged.*

area ⟺ *entropy*

?

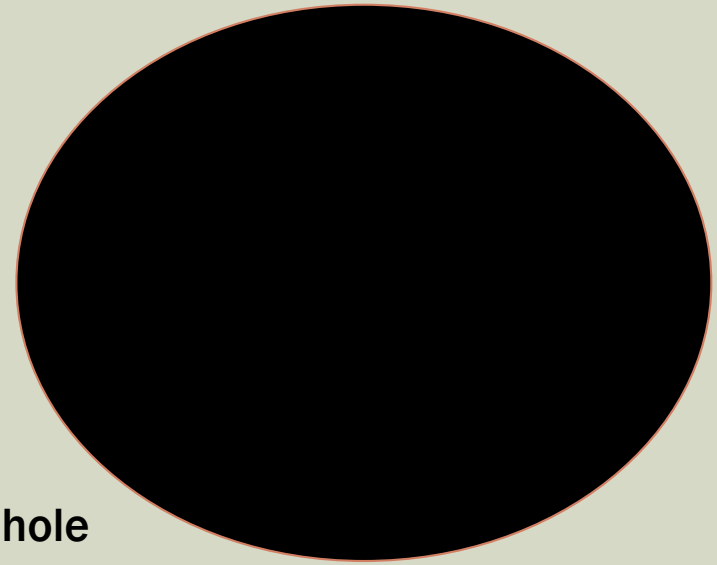
"Gravitational collapse: the role of general relativity"

Roger Penrose (1969)

Figure 5. Rotating "black hole" (Kerr-Newman solution with $m^2 > a^2 + e^2$). The inhabitants of the structures S and S^* are extracting rotational energy from the "black hole".

Wheeler to Bekenstein (1971):

"If I drop a teacup into a black hole,
I conceal from all the world the
increase of entropy."

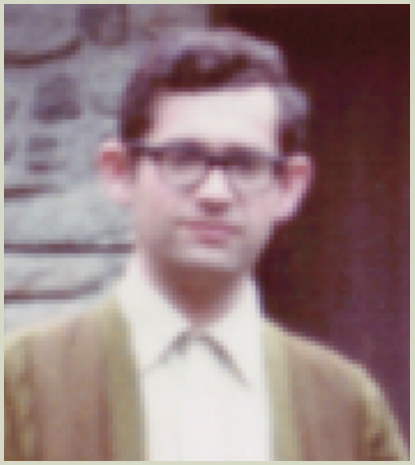


Black hole

$$\Delta S_{\text{outside}} < 0$$

Black hole entropy

Bekenstein, 1972



$$S_{BH} = \alpha(\text{Horizon Area}) \qquad \alpha \sim L^{-2}_{\text{Planck}}$$

$$L^2_{\text{Planck}} = \hbar G/c^3 = (10^{-33}\,\text{cm})^2$$

Why horizon area?

1. never decreases (classically)
2. extensive, local
3. GR & Heisenberg uncertainty relation implies minimum area increase is L^2_{Planck} independent of black hole mass, spin, charge

Generalized second law:

$$\Delta(S_{\text{outside}} + S_{\text{BH}}) \geq 0$$

?

Bekenstein's entropy → BH has a thermodynamic temperature:

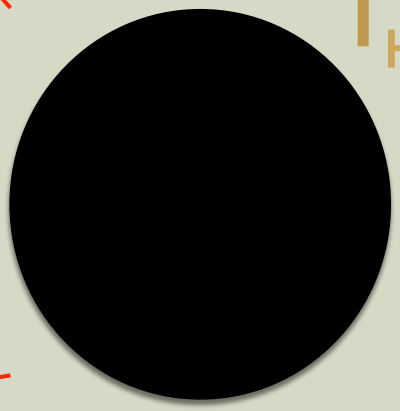
$$T_{BH} = (dS_{BH}/dE_{BH})^{-1} \sim \hbar \times (\text{surface gravity})$$

Validity of the GSL requires that this be a REAL temperature!

This real temperature became evident when Hawking considered a black hole immersed in the fluctuating vacuum of quantum fields.

Hawking was not seeking this --- it took him by surprise. He was checking the prediction, of Zeldovich and others, that a *rotating* black hole would spontaneously radiate.

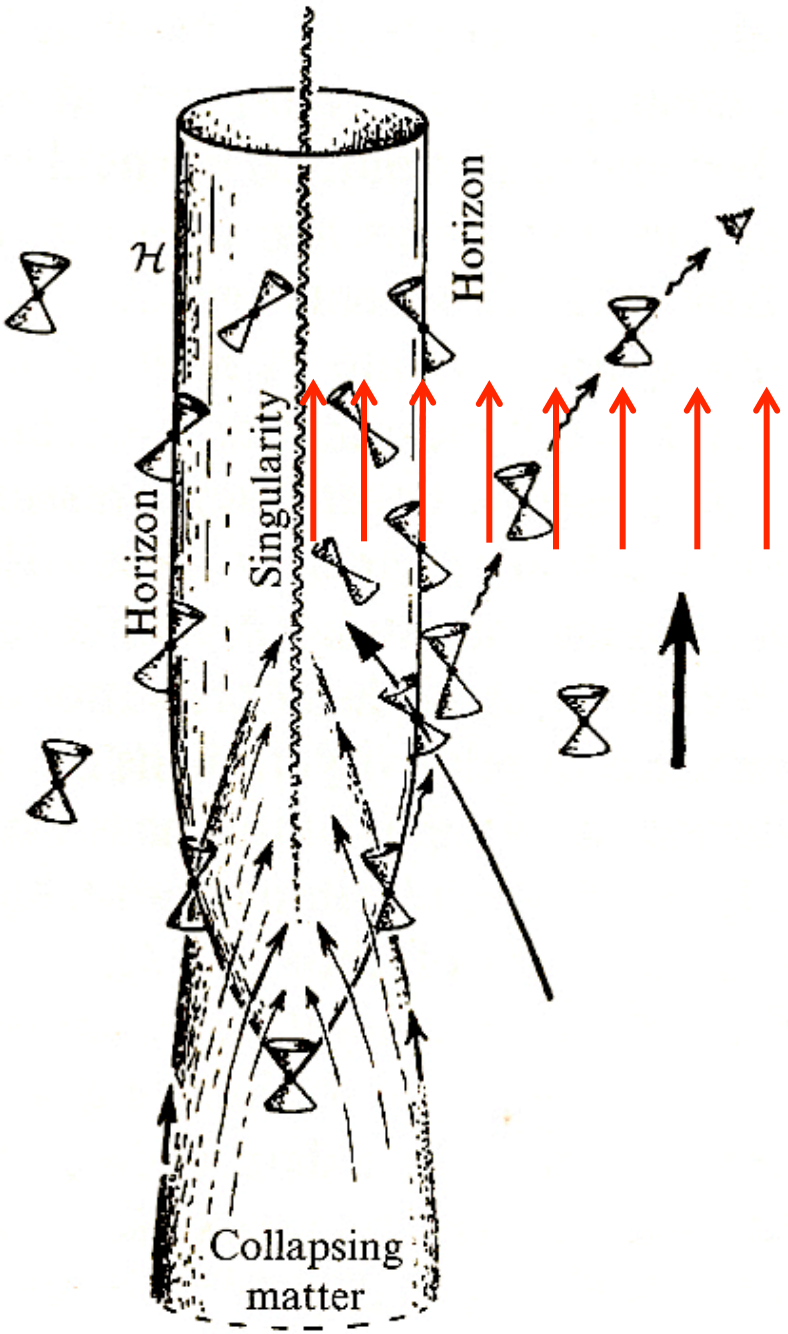
Hawking Temperature Saves GSL!



T_H

thermal radiation

Apparent GSL violation happens in *fluctuation-dominated regime*...

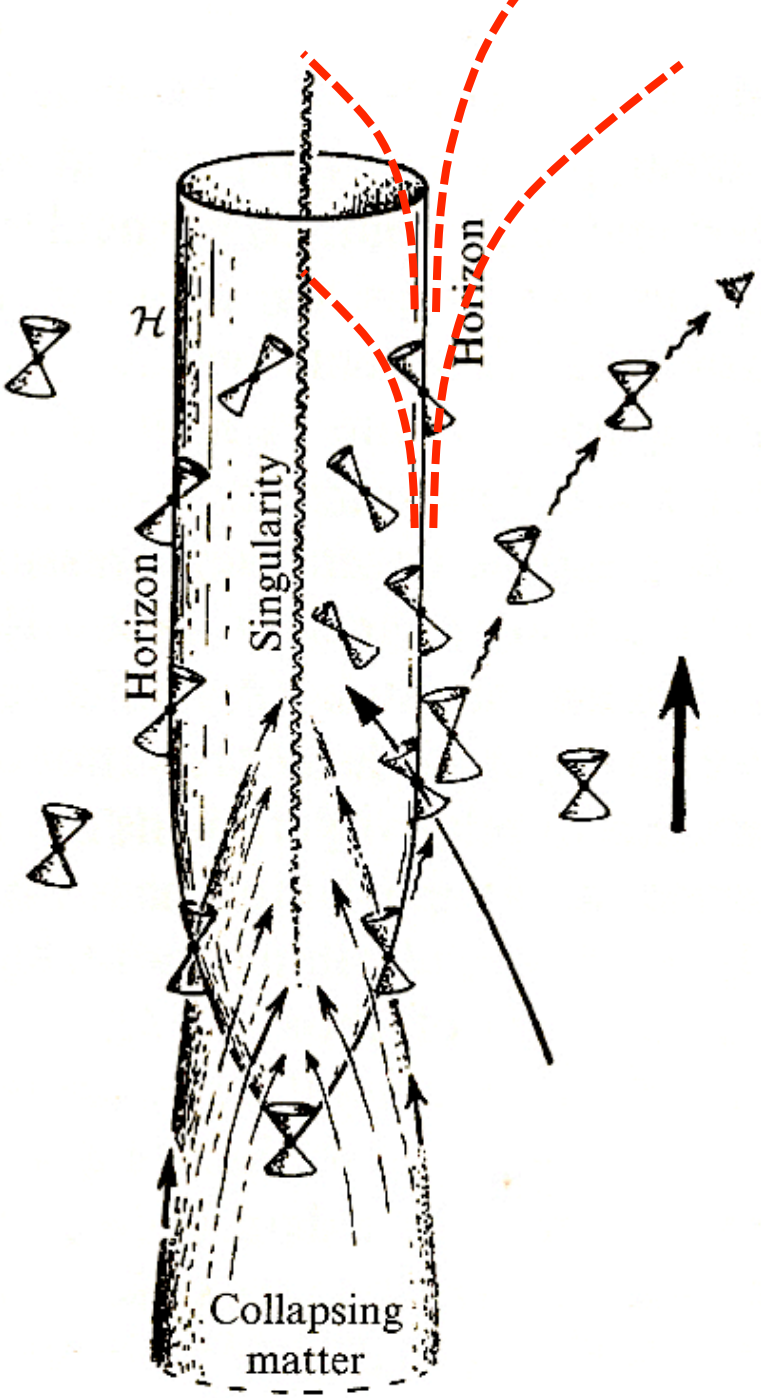


The "time" translation symmetry is *space* translation inside the horizon...

...so the conserved quantity is *momentum* inside, and can thus be negative.

This implies that the quantum field vacuum is unstable, which leads to ...

Picture by Roger Penrose
The Road to Reality (Knopf, 2005)



Hawking radiation:

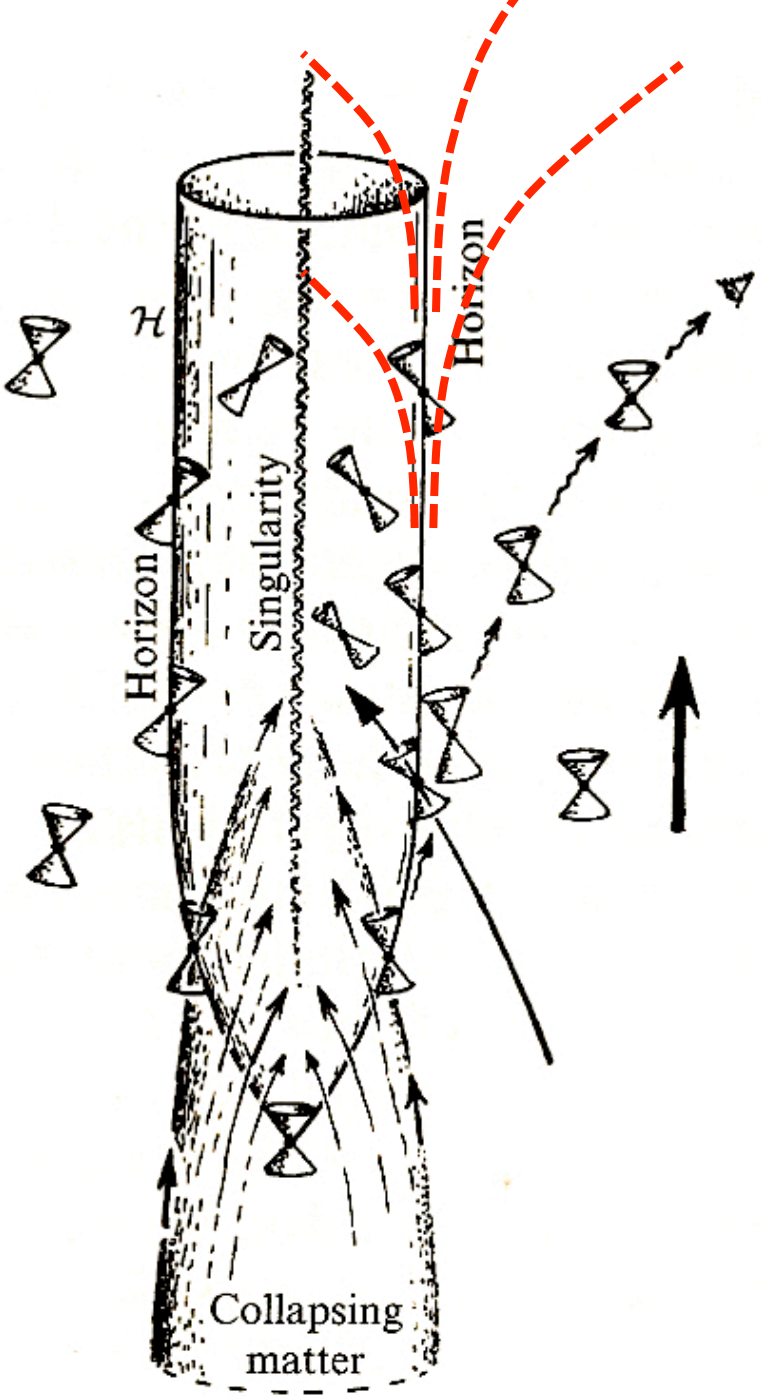
(1974)

Quantum field theory implies black holes have a temperature

$$T_{\rm H} = \hbar\kappa/2\pi$$

= 62 nK M_{sun}/M

surface gravity



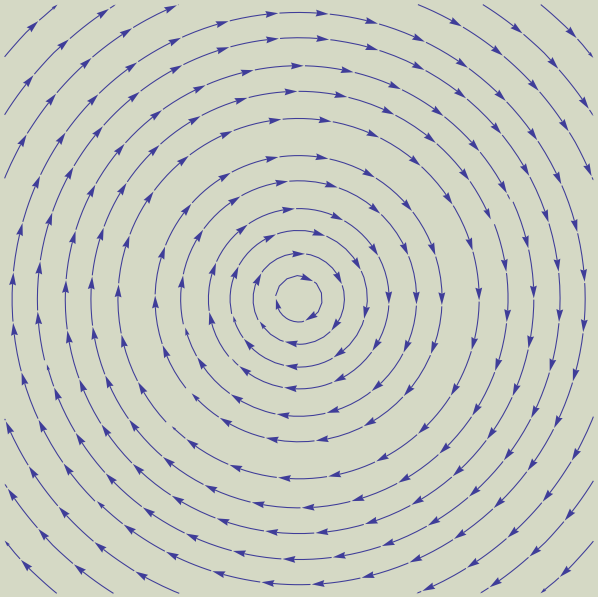
Hawking radiation:

(1974)

Tidal peeling of vacuum fluctuations corresponds to pair creation, with partners occupying negative energy states.

The pairs are present everywhere, even in the flat spacetime vacuum...

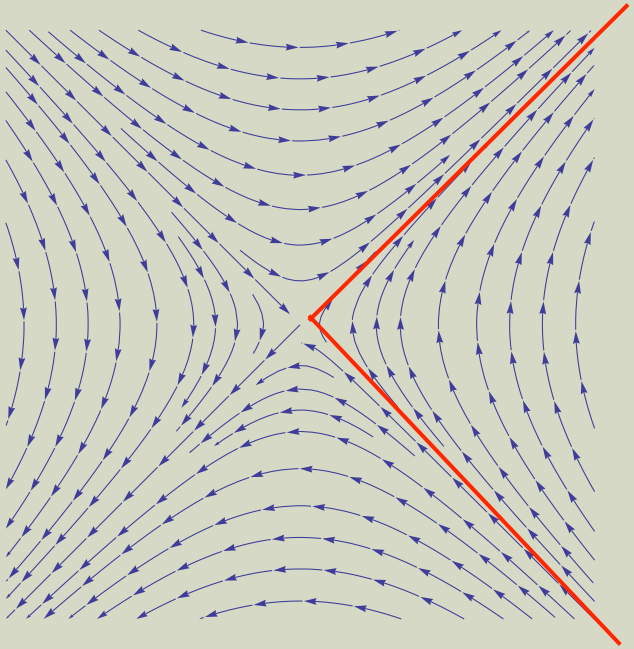
Euclidean space



Rotation symmetry

$$ds^2 = dx^2 + dy^2 = dr^2 + r^2 d\theta^2$$

Minkowski space

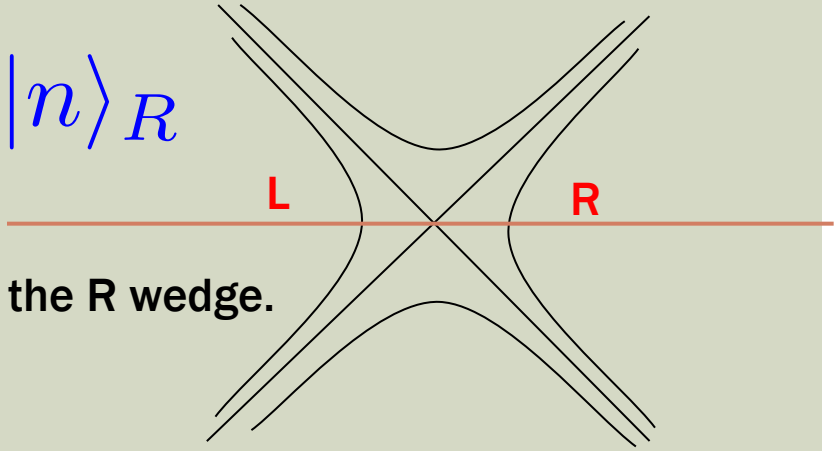


Lorentz boost symmetry

$$ds^2 = dx^2 - dt^2 = dl^2 - l^2 d\eta^2$$

THE ENTANGLED VACUUM

$$|0\rangle = \sum_n e^{-\frac{\pi}{\hbar} E_n^{\text{boost}}} \, |\bar{n}\rangle_L \, |n\rangle_R$$



The vacuum is a thermal state when restricted to the R wedge.

L & R sides are *entangled.*

$$|0\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle)$$

(Compare with a spin singlet)

VACUUM ENTANGLEMENT ENTROPY

(Sorkin '83, Bombelli, Koul, Lee, Sorkin '86)

$$S = -Tr(\rho_R \ln \rho_R)$$

$$\approx \int dA\, dl\, T^3_{\rm local} \propto \int_{l>\epsilon} dA\, dl\, l^{-3} \propto A/\epsilon^2$$

It seems to follow that:

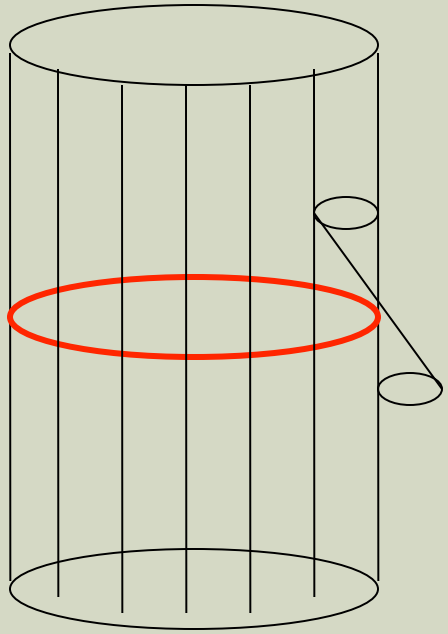
Black hole entropy includes – and may be 100% – vacuum entanglement

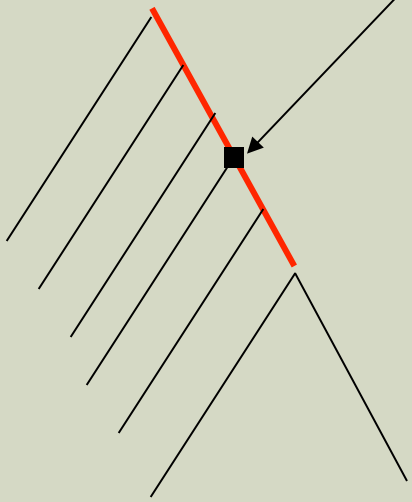
Could agree with BH entropy if $\epsilon \sim l_{\rm Planck} = \hbar G/c^3$

It seems natural that **gravity would cut off the entanglement of pairs separated by less than the Planck length, since they would be engulfed in a "virtual black hole".**

LOCAL CAUSAL HORIZONS

Stationary black hole horizon





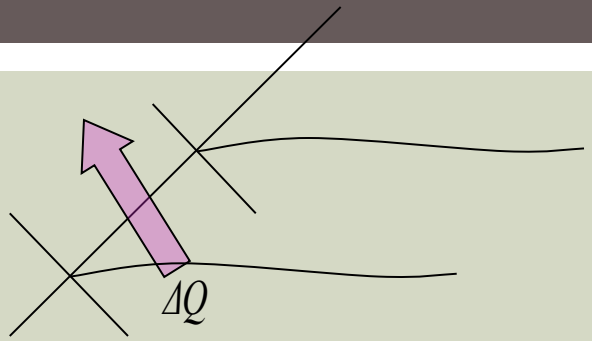
Arbitrary “equilibrium” point p

<u>Equivalence principle</u>: An approximate local Minkowski space with Lorentz boost symmetry exists around p

Local horizon

Boundary of the past of the red line (2-surface)

LOCAL HORIZON THERMODYNAMICS



Postulate for all such horizons

1. The horizon system is a 'heat bath', with universal entropy area density. $S = \alpha A$

2. Boost energy flux across the horizon is 'thermalized' at the Unruh temperature. $dS = \delta Q / T$

3. Energy conservation (energy-momentum tensor divergence-free

Implies focusing of light rays by spacetime curvature: the causal structure must satisfy Einstein field equation, with Newton's constant $G = \frac{1}{4\hbar\alpha}$

ENTANGLEMENT & GEOMETRY

- Vacuum entanglement proportional area
- More entanglement implies smaller Newton constant implies more rigid geometry
- Is entanglement the origin of geometry?

EINSTEIN'S EQUATION

Classical vacuum: any "small ball" of given volume has the same area as in flat spacetime.

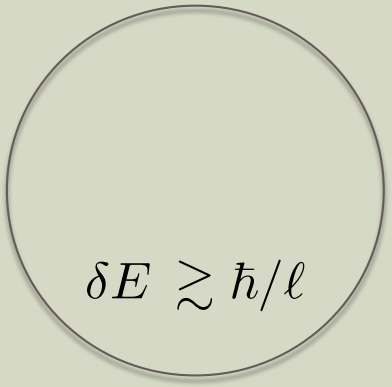
Semiclassical with matter:

area deficit of ball ~ G_N (energy inside)(radius of ball)

Is vacuum entanglement maximal?

Assume $S = S_{\rm UV} + S_{\rm IR}$, with $S_{\rm UV} = S_{\rm BH} = A/4\hbar G$

By changing the state of matter and geometry, can S be increased while holding the volume fixed?

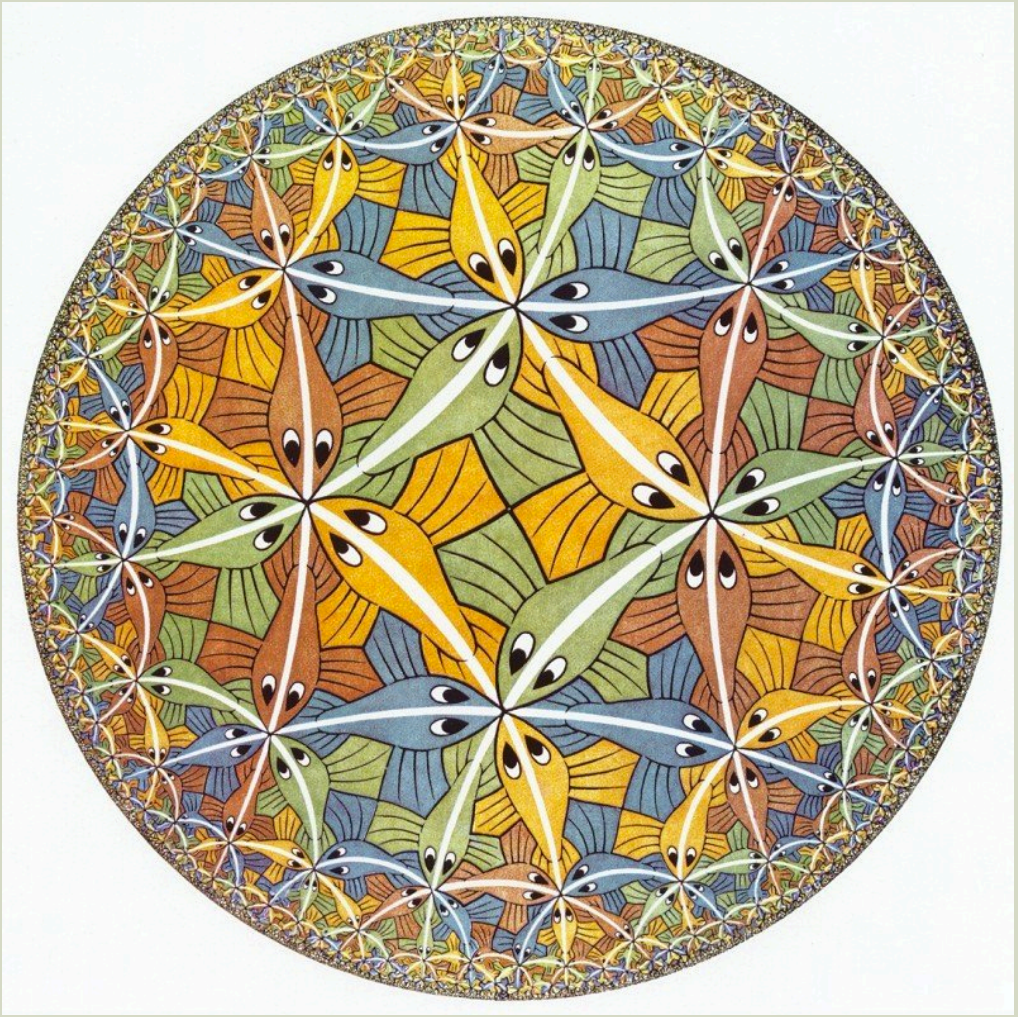


Entangled qbits

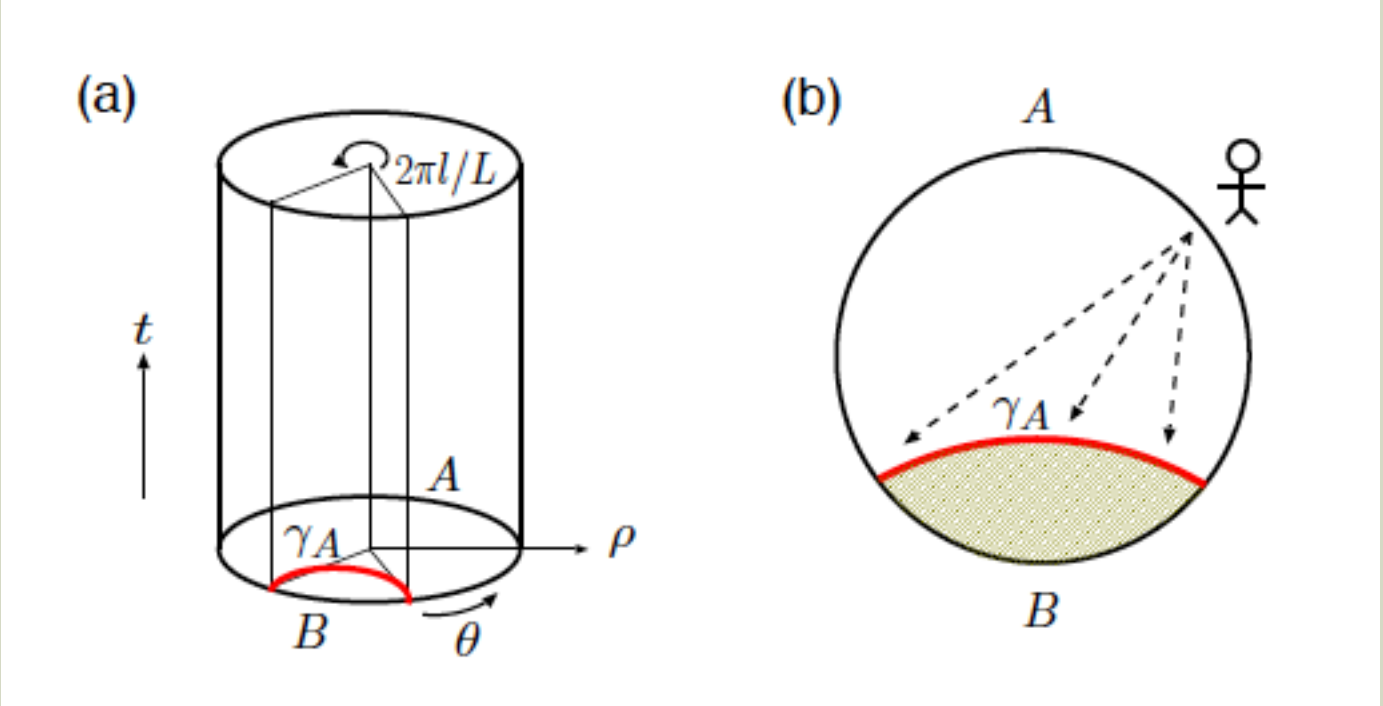
which by the Einstein equation implies $\delta A|_V \lesssim -\hbar G$

hence $\delta S_{\rm tot} \lesssim 0$

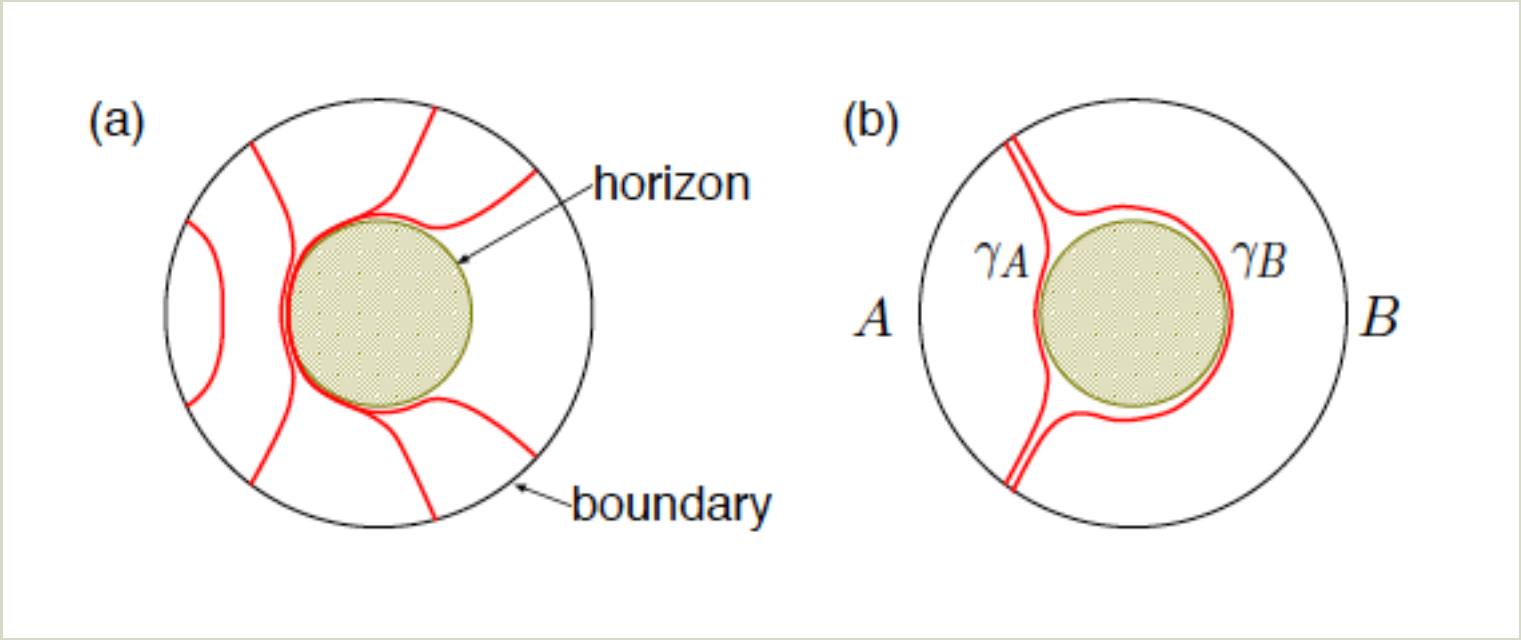
HYPERBOLIC SPACE



HOLOGRAPHY:
BOUNDARY ENTANGLEMENT = BULK AREA



THERMAL STATE = BLACK HOLE



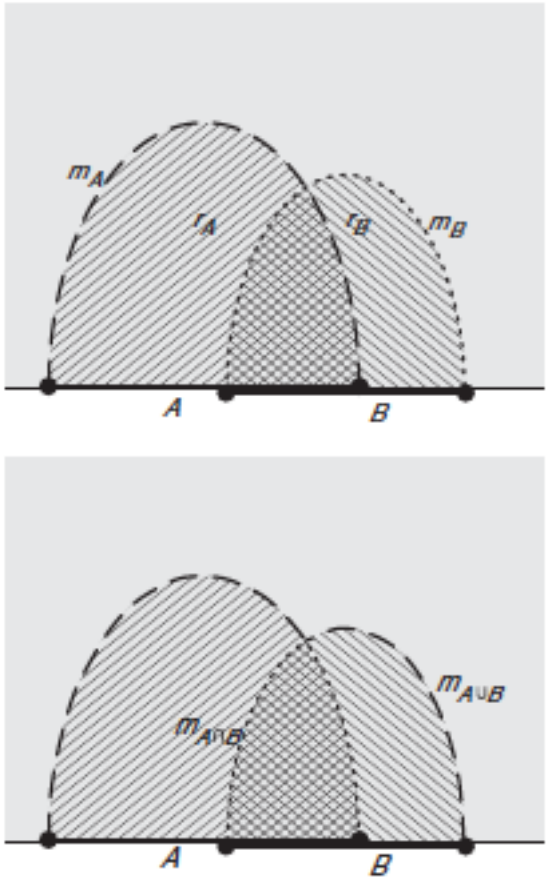
Strong sub-additivity of entanglement entropy:

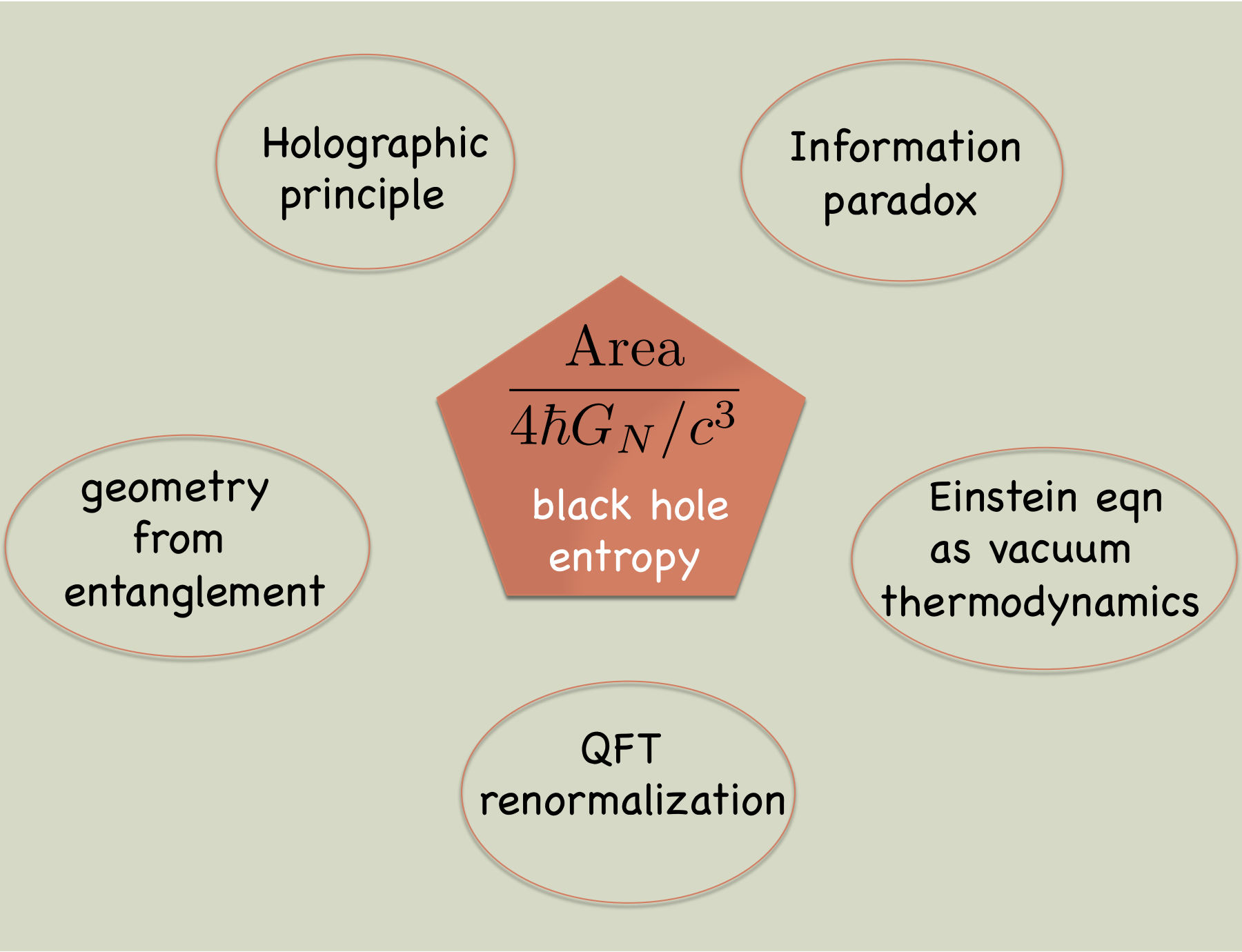
We begin by rewriting the inequalities (2) in the forms

$$S(A) + S(B) \geq S(A \cup B) + S(A \cap B),$$
$$S(A) + S(B) \geq S(A \setminus B) + S(B \setminus A), \tag{5}$$

where $A \setminus B = A \cap B^c$. We will prove the first inequality; the proof of the second one is very similar and is left as an exercise to the reader.

Let m_A, m_B be the minimal hypersurfaces in the bulk ending on ∂A, ∂B respectively, and r_A, r_B be the corresponding regions of the bulk (so that $\partial r_A = A \cup m_A$, $\partial r_B = B \cup m_B$). (See top of Fig. 3.) We now define the regions $r_{A \cup B} = r_A \cup r_B$, $r_{A \cap B} = r_A \cap r_B$. We can decompose the surfaces of these regions as usual into a part on the bound-




Holographic principle
Information paradox
$\frac{\text{Area}}{4\hbar G_N/c^3}$
black hole entropy
geometry from entanglement
Einstein eqn as vacuum thermodynamics
QFT renormalization